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Mathematical Model for the Optimization of Municipal Solid Waste Management

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Solid waste management (SWM) is central to any nation experiencing rapid expansion via urbanization, migration, and population growth. Waste management is crucial in every country since it can directly affect people's health and the environment. For example, cholera outbreaks in some developing countries like Nigeria are common in congested areas, especially during the particular season. Therefore, efficient and effective Municipal Solid Waste Management (MSWM) is necessary. This study developed a mixed-integer optimization model for MSW of Kano State Nigeria. The model optimizes the total cost of SWM, which includes the cost of transporting different types of waste between other locations plus the fixed cost of establishing and maintaining/operating some facilities. The analysis further reveals that the government should establish 20 standard collection centers having a capacity of 60 tons. With one combusting and hazardous centers each having capacity of 391 and 81 tons, respectively, and two recycling, composting, and disposal centers, each having a total of 240, 200, and 113 tons, respectively. Incorporating the recovery process in the SWM policy reduces the number of disposal centers, and more than 80% of the daily generated wastes are recoverable. Hence, the government would save considerable resources (costs) and generate revenues from the approach once implemented.

Keywords: municipality, solid waste management, mathematical programming, cost optimization, Kano, Nigeria

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INTRODUCTION

Municipal solid waste, in particular, is usually made up of complex biodegradable and non-biodegradable substances (Barma et al., 2014). The compositional volumes and weights vary from one location to another, depending on population size and culture/lifestyle. Municipal solid waste contains not only "valuable" and often reusable materials such as metals, glass, paper, plastics, and food remain, but also an ever-increasing amount of hazardous waste.

Some wastes such as lead, manganese from batteries, cadmium, arsenic from florescent tubes, pesticides, bleaches are hazardous. Discarded electronic sets such as computers, toys, handsets, and television waste a range of toxic chemicals that occur in solvents, paints, disinfectants, and wood preservatives.

The SWM is one of the challenging issues in urban cities due to various interrelated factors such as operational costs and environmental concerns. As one of the most significant constraints of municipal SWM, the cost can be effectively economized by efficient planning approaches (Asefi et al., 2015; Barma and Modibbo, 2022). The SWM continues to be a big challenge in urban areas worldwide, especially in villages, where solid waste increases at an alarming rate, particularly in underdeveloped countries (Sabeen et al., 2016). United Nations Environment Programme (2015) opined that the first Global Waste Management Outlook, published by the United Nations Environment Programme (UNEP) and the International Solid Waste Association (ISWA) in 2015, highlighted the need for greater detail on the generation and management of waste at the regional level. The first Africa Waste Management Outlook published by the UNEP in June 2018 responded to this global call. The Africa Waste Management Outlook sets out the current state of SWM in Africa, including waste governance, the associated environmental, social and economic impacts of waste, and the opportunities waste provides through appropriate solutions and financing mechanisms. The problem of MSW in developing countries is a significant concern to the government; this problem becomes much more problematic in Nigeria, where production is continuously increasing because of the increase in population pressure and some socio-economic factors. Among other third-world countries, Nigeria is witnessing an unprecedented growth of cities in recent times. It was also observed that the country's high population figure has a series of implications for every aspect of people's socio-economic and cultural lifestyle. The rapid industrialization and population explosion in Nigeria have led to the migration of people from rural areas to the cities, which generates a lot of MSW daily. Indeed, MSW is expected to increase significantly shortly (Omole and Alakinde, 2013).

Barma et al. (2014) asserted that SWM involves activities associated with the generation, storage and collection, transfer and transport, treatment, and disposal of solid wastes. The management of MSW requires proper infrastructure, maintenance, and upgrade for all activities. These have become increasingly expensive and complex due to continuous and unplanned growth in urban centers. The difficulties in providing the desired level of public service in the urban centers are often attributed to the poor financial status of managing municipal corporations. Generally, SWM includes the processes associated with collecting, transportation, treatment, recycling, and disposal safely, hygienic, and cost-effective. The successful SWM requires the appropriate site selection of the waste management system's facilities, such as recycling and disposal facilities and transportation of wastes among the facilities. The extensiveness and complexity of the factors affecting SWM (e.g., limited resources such as land, investment costs, and operational costs) make it challenging to implement (Asefi et al., 2015) correctly. World Bank (2002, 2003) opined that municipal SWM is an essential part of the urban infrastructure that ensures the protection of the environment and human health. Proper management of solid waste is critical to the health and wellbeing of urban residents (Butu and Mshelia, 2014).

Studies pointed out by different researchers that some of the problems facing SWM in the Kano metropolitan are poor urban planning, finance, working materials, and availability of field workers (Muktar, 2008; Nabegu, 2012, 2013; Butu and Mshelia, 2014). In the light of the above, this study intends to carry out a critical assessment of SWM and develop a multi-objective mathematical model that will minimize the cost of SWM in the state. The study area comprises seven local governments (Kano Municipal, Tarauni, Fagge, Kumbotso, Dala, Gwale, and Nasarawa local governments).

Waste management is crucial for every country since it directly affects the health of its people and their environment. For example, in some developing countries like Nigeria, cholera outbreaks are common in congested areas, showery. Therefore, an efficient and effective MSW is necessary. Olapiriyakul et al. (2019) asserted that inefficient or poorly designed waste management systems affect society and the economy. For example, excessively long waste transportation routes can negatively impact a large share of the population and increase transportation costs. Also, the successful establishment of sustainable MSWM is dependent on the network design and transportation planning; thus, minimizing transportation costs is unavoidably one of the most critical issues in designing MSWM policy for any metropolitan.

Solid Waste Management in Kano

Like other states, Kano faces the issues of MSW that are uncollected in tons. The problem has become a severe challenge to the government and communities. Uncollected wastes block drainages and serve as harboring centers for mosquitos. Also, other infectious diseases arose, affecting citizens' health. As a result of improper planning, most residents have no access to waste collection centers (Nabegu, 2008). **Figures 1**, **2** show the pictures of some dumpsites/collection centers within the study area. Socio-economically, Kano is a commercial center and attracts an inflow of people. According to Butu and Mshelia (2014), garbage in the city is due to poor structures and rural migrants causing more hazardous substances in the areas. Despite all efforts made by the government for several years, the problem of SWM in Kano state is still compounding.

Waste Recovery Processes

Recovery processes have the advantages of generating income by selling the recovered materials and reducing the volume of waste that will reach the final disposal site or the combustors; it saves virgin material (in the case of paper, it saves trees). It reduces the production costs of some materials (e.g., aluminum) and the environmental damage that accompanies their production.

Recycling/Reusing Process in Kano

Recycling and reusing processes are taking place in Kano. The presence of a plastic recycling plant at/near the Maimalari dumping site indicates that plastic recycling is taking place. According to Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029 (2018), the management of the plastic recycling plant estimates that they sell between 200 and 300 Tones/month of recycled plastics. Plate II below shows how plastic wastes are sorted into a different color.

Apart from plastic waste, some materials found in the solid waste stream are also being recycled in Kano. **Table 1** below

FIGURE 1 | Solid waste at uncontrolled open dumped site. Source: Hotoro, Tarauni Local Government Area, Kano State, Nigeria, 2020.

shows waste composition measured from 162 Households in September 2017.

Composting

This is the controlled biological decomposition of organic solid waste material under aerobic conditions to produce compost to fertilize agricultural land. The composting process seemed to be not taking place in Kano. Even though there is one official composting center at Dorayi, supported by UNDP and theoretically designed to produce about 10 60 kg bags of compost per day, the plant/center is presently inactive (Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029, 2018). But looking at the fraction/percentage of organic waste in the waste composition daily generated, given in the above table, organic waste is the second to the most considerable fraction, which is the added advantage to the composting plan if the SWM policy is well-planned and effectively monitored/managed.

Energy Recovery

The waste to energy process is a type of resource recovery. The combustible portion of the solid waste is used as a fuel to produce some form of reusable energy such as steam and electricity. This process is not taking place in Kano presently, but it is recent (i.e., in the first quarter of this year, 2021); the Kano state government publically showed the intention to start the process of waste-to-energy to produce/generating electricity from waste. This is why the state government presently invites a private company to take over the activities of REMASAB, which took about 18 years (2003–2021) to control the actions of SWM in Kano. If the dream of the Kano state government of incorporating waste into the energy process in the state SWM system becomes realistic, the heap of garbage seen everywhere and along the roadside of the metropolitan will be reduced drastically or vanish with time. It can be observed from **Table 3** that the most considerable fraction is polythene. It is presently not recycled and combustible waste with a high heating value. It can be used for combustion to produce thermal energy and electricity, sold to people and companies and generate more income for the government.

LITERATURE REVIEW

The rapid population growth coupled with urbanization poses challenges to several societies in developing nations and, by extension, the environment. One such challenge is SWM. Pollution generates infectious diseases as a result of improper waste disposal. The emission of greenhouse gases (GHGs) affects the surroundings and exposes individuals and communities to a higher health risk (Lyeme et al., 2017). According to Kalu et al. (2017), SWM in Nigeria can be categorized into three agencies administratively-local, state and federal (national) bodies. Kalu

FIGURE 2 | Sorting plastic according to color. Source: Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029 (2018).

TABLE 1 | Waste composition measured from 162 Households in September 2017.

Waste type	Proportion	Comments
Polythene	25.1%	This seems very high—possibly a function of the high use of plastic bags, but needs checking. Currently mainly not recycled
Organic	21.9%	Lower than might have been expected, although visual inspections also reveal little evidence of high volumes of organic waste in the waste stream. Currently mainly not recycled
Plastic	11.2%	Useful to disaggregate plastic to PET, PVC etc. most being recycles
Paper and cardboard	9.6%	Most being recycled
Fines	9.1%	
Textiles	7.1%	Not being recycled
Metal	5.2%	Being recycled
Others	4.1%	
Glass	3.7%	Being recycled
Mixed electronics	3.0%	Being recycled

Source: Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029 (2018).

et al. (2017) further stated that the SWM is the function of the municipality primarily. However, the efficiency of the sector is challenging significantly. Especially, some prominent cities within Nigeria, such as "Aba, Enugu, Owerri, Port-Harcourt, Kaduna, Lagos, and Ibadan," where piles of MSW are often observed.

Many researchers worked on minimization of the cost of SWM. Many techniques and models were developed, but only a few emphasized recycling and reuse processes. Asefi et al. (2015) asserted that several authors studied SWM modeling in the past several decades; the early models on SWM dealt with specific aspects of the problem such as vehicle routing or transfer station siting while having practical shortcomings such as neglect of recycling centers. A few researchers factored out the over-simplicity of the model with a single waste type and recycling centers. Chinchodkar and Jadhav (2017) developed a transportation model for the SWM on Dumping Ground in Mumbai, India. They only considered the distance and cost of transporting solid waste from generation nodes to final disposal sites. They aimed to reduce the existing costs of managing the solid waste of damping ground at Mumbai. They did not give much consideration to the recovery of the solid waste. Sabeen et al. (2016) developed a model to minimize the cost of municipal solid waste in Pasir Gudang Johor, Malaysia. They used the technique of reuse and recycling. They created a flow chart to show how the proposed recycling process occurs. They compared the current expenditures with the expenditures of the proposed model when reuse/recycling takes place. The comparison result indicated a reasonable reduction in the cost of municipal SWM of the city when the reuse/recycling process is applied. Asefi et al. (2015) developed a mathematical model to minimize transportation and facilities establishment costs. The defined problem consists of concurrent site selection of the locations of the system's facilities (e.g., transfer stations, treatment-, recycling-, and disposal centers) from the candidate locations and the determination of routes and amounts of shipments among the selected sites to minimize the total cost of transportation and facility establishment. Barma et al. (2014) developed a multi-objective Programming model for reducing the cost of evacuation of the volume of waste at various collection centers in municipalities. The model was tested using data collected from the federal capital territory Abuja Nigeria. The model did not consider costs involving recycling/reuse centers. Such as opening, operating and maintenance costs and the costs of transporting the recycled/reused materials from recycling/reuse centers to either market or other factories are also the constituents of the total cost of managing solid waste.

The practical application of SWM mathematical models as tools for decision making by municipal solid waste planners in developing countries is still a big challenge (Barma and Modibbo, 2022). A considerable amount of research has been done in the last two decades on various aspects of SWM, and several economically based optimization models for waste streams allocation and collection vehicle routes, have been developed (Nganda, 2007). The solid waste models that have been developed in the last two decades have varied in goals and methodologies. These goals are reliable waste generation prediction, facility site selection, facility capacity expansion, facility operation, vehicle routing, system scheduling, waste flow, and overall system operation. Some techniques used include linear programming, integer programming, mixed integer programming, non-linear programming, dynamic programming, goal programming, gray programming, fuzzy programming, quadratic programming, stochastic programming, two-stage programming, interval-parameter programming, and geographic information systems (Hasit and Warner, 1981; Ghose et al., 2006; Barma et al., 2014; Barma and Modibbo, 2022).

A general framework for the selection of suitable waste to energy technologies in a systematic fashion has been studied and proposed in respect of a sustainable MSW management system recently (Farooq et al., 2021). Similarly, an application-based study has been conducted to recycle waste synthetics rubbers and plastic to improve consumption (Ki et al., 2021). In the study, waste plastic films have been applied to construct roads and infrastructure. Ahmadini et al. (2021) incorporated green investment in inventory and supply chain management to address the environmental issues and ensure sustainability devoid of waste from greenhouse gases. Appolloni et al. (2021) propose a hybrid approach based on an integrated multi-criteria decision analysis-Analytic Hierarchy Process to assess and identify risk in e-waste management via a new index. The study suggested new risk awareness indicators.

Most relevant models in SWM have multiple objectives and therefore require the use of Multi-objective Optimization Models. In this regard, this work focuses on identifying the main features of multi-objective optimization models implemented in SWM problems worldwide. To learn the best practices and identify possible gaps concerning the Kano metropolitan situation, such as the optimization criteria that drive the problem solution (parameters). Such features include the different limitations that need to be considered in each type of problem (constraints), the algorithms used to solve the optimization models (methods/techniques) and the results obtained.

MATERIALS AND METHODS

The data for this study is collected from journals, scavengers, and vendors of solid waste to get the prices of recoverable and reusable wastes. A mixed-integer mathematical programming model was formulated to analyze the data using lingo software version 18.0.

Current SWM Policy Practice in Kano Metropolitan

According to Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029 (2018), Kano metropolitan generates about 1,200 tons of solid waste per day. Out of this figure (1,200 tones per day), only 460.1 tones per day can be collected by REMASAB, leaving more than 50% uncollected every day. This is why a heaping amount of waste is seen almost everywhere in metropolitan areas. Recovery processes in Kano (as stated above) mainly include; plastic waste recycling centers, metallic waste recycling centers, aluminum waste recycling centers, and decomposed substances as fertilizer. In most cases, generation/collection centers serve as processing centers where waste treatment/separation and indiscriminate open burning occur. Also, recyclable waste, whether hazardous or non-hazardous, are mostly locally separated by scavengers (bola boys) and then taken to vendors then to recycling/reuse centers. Thus, most of the waste residue produced after selecting the recyclable/reusable wastes is burnt, buried or transferred to final disposal sites (dumpsites) by trucks if the collection centers are accessible. The Schematic diagram of the solid waste flow of current SWM practice in Kano metropolitan is shown in **Figure 3**.

Proposed Mixed-Integer Mathematical Programming Model

The proposed model has been formulated as a mixed-integer mathematical model. It optimizes the objective of minimizing the total cost of SWM, which includes the cost of transporting different types of waste between other locations plus the fixed cost of establishing and maintaining/operating some facilities. The nodes of the transportation network consist of collection nodes, recycling nodes, composting nodes, hazardous nodes, combusting (incineration) nodes, final disposal nodes, and a combination of any of the above. The proposed mixed-integer mathematical model was formulated to determine the establishment of recycling, composting, combusting, and hazardous centers at a minimum cost. Due to the realization that measuring transportation costs per trip is more relevant to most of the cities of developing countries. As the current

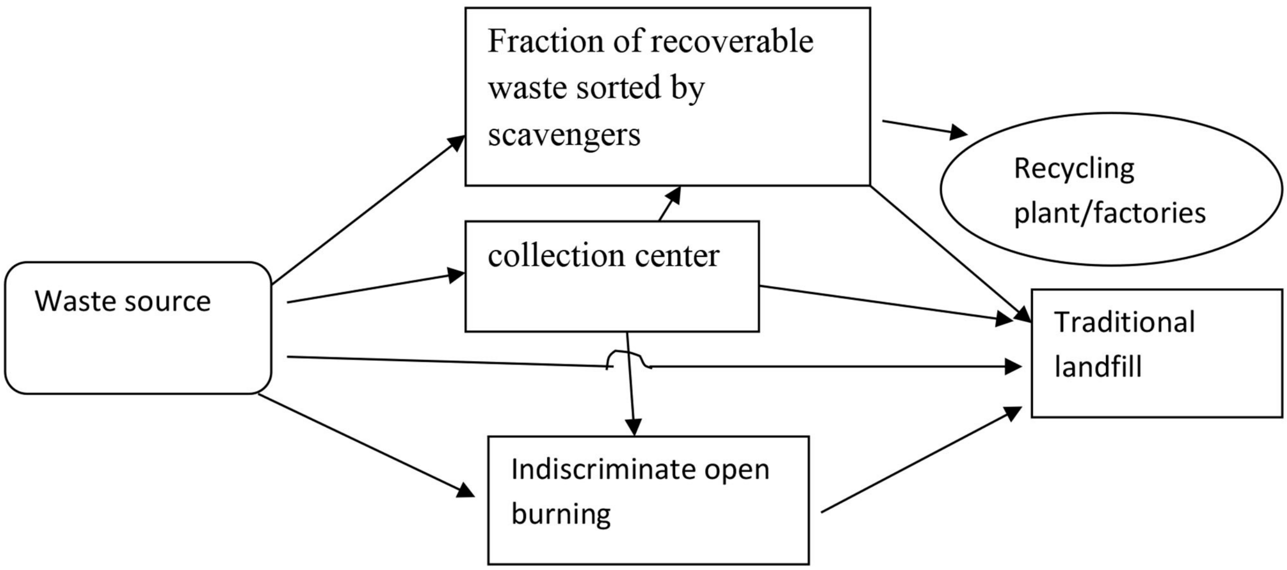

FIGURE 3 | The Schematic diagram of the solid waste flow of current SWM practice in Kano metropolitan. Source: Accord-Cadre Ville et Changement Climatique en Afrique Sub-Saharienne (CICLIA), Client Project Reference: AFD/DOE/EBC/CLD | ACH-2017-026 Internal project reference: A17FDD029 (2018).

situation of Kano metropolitan, where the technology to measure waste as it is carried away from the waste sources is not available, we may want to measure transportation costs in terms of costs per trip of a truck from waste collection center j to any of the centers or from one center to another. The planning horizon is a day, i.e., decisions are to be taken on a day to day basis.

Description of the Conceptual Framework of the Proposed Model for the MSWM System

The main focus of the model is to plan the MSW management by defining the refuse flows that have to be sent to recycling, composting, combusting, and hazardous waste centers or to the final disposal site. Several treatment plants and facilities between the collection center j (j = 1, 2, 3, …, J) and last disposal site d (d = 1, 2, 3, …, D) will be included within the desired MSW mathematical model: plants for recycling, composting, combusting, hazardous waste, production of refuse-derived fuel (RDF), and treatment of organic material for fertilizer and final disposal site; from waste sources (residences, markets, schools, restaurants, institutions, hotels etc.), all sort of wastes produced daily will be moved to collection center j at the expense of generators, and of course, some fractions of recyclable/reusable waste are bought/collected and directly taken to vendors/recycling/reuse center s by scavengers. Collection center s are the officially known/adapted points where wastes of a different kind from nearby places (waste sources) are dumped, after which they will be loaded/moved to other processing plants than to the final disposal site d. Recycling/reusing waste material center is the point where recycling recyclable waste materials such as glass, white paper, aluminum, newspapers, cardboard, and ferrous plastics are technically feasible. The advantages of recycling waste materials are reducing the amount of waste that reaches the final disposal site or the combustors; it saves virgin material. It reduces some of the materials' production costs (e.g., aluminum) and the environmental damage. Composting center is where composting takes place. Composting requires using the organic fraction of waste to prepare compost for the fertilization of agricultural land. The advantages of composting are: composting saves on the costs of landfilling or incineration and generates revenues from the sales of the composted fertilizer. The final disposal site is the final destination where the waste residue reaches either directly or after passing through different processes. It utilizes a land area to collect the waste with or without separation. Its advantage is that all waste (except hazardous materials) can be dumped without separation. Combusting center (incinerator) is a facility that absorbs the waste and transforms it into heat or energy. Thus, electricity can be generated, reducing the volume of waste that will be moved to a final disposal site. A hazardous center is a point where harmful wastes are dumped. Toxic wastes are produced from industries, manufacturers, hospitals, and other sectors. Poisonous wastes have detrimental health effects on humans and the environment. The primary sources of hazardous waste are the industrial, hospital, and manufacturing processes; other generators can be household or commercial.

Figure 4 above shows the flow of all sorts of wastes within the network between collection center j ($j = 1, 2, 3, \ldots, J$) and final disposal site d ($d = 1, 2, 3, \ldots, D$). At collection center j, the separation process is assumed to be started, in which all wastes are sorted into major different categories g ($g = 1, 2, 3, \ldots, 6$, $g = 1$ represents recyclable waste, $g = 2$ represents compostable waste, $g = 3$ represents

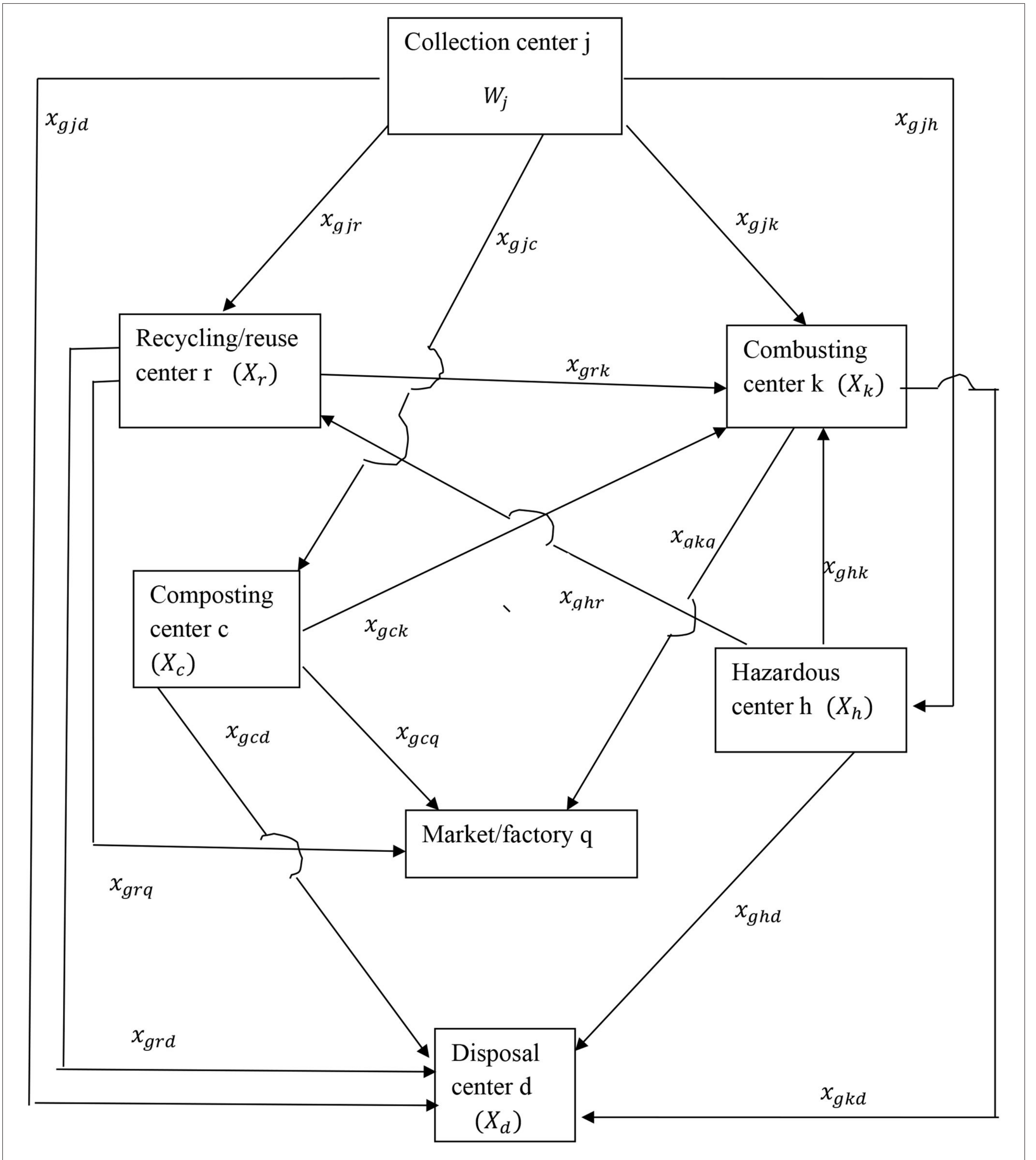

FIGURE 4 | The Schematic diagram of the proposed model for the MSWM system. Source: authors.

combustible waste residue, $g = 4$ represents hazardous waste, $g = 5$ represents incombustible waste residue, $g = 6$ means recovered material). x_{gjr} is a unit amount of recyclable waste that will be *transported from collection center j to* recycling center r (r = 1,2,3,…, R), x_{gjc} is a unit amount of compostable waste that will be *transported from collection center j* to composting center

c ($c = 1, 2, 3, \ldots, C$), x_{gjh} is a unit amount of hazardous waste that will be *transported from collection center j* to hazardous center h (h = 1, 2, 3, …, H), x_{gjk} is a unit amount of combustible waste that will be *transported from collection center j* to combusting center k (k = 1, 2, 3, …, K), and x_{gjd} is a unit amount of incombustible waste residue that will be *transported from collection center j* to final disposal center d (d = 1, 2, 3, …, D). At recycling center r, after the recycling process takes place, the expected fractions of outcomes are the recovered material and the waste residues (combustible and incombustible recycling waste residues). x_{grd} is a unit amount of incombustible recycling waste residue *that will be transported from recycling center r* to final disposal center s d. x_{grk} is a unit amount of combustible recycling waste residue *transported from recycling center r* to combusting center k and x_{grq} is a unit amount of recovered material that will be *transported from recycling center r* to either market/other factories q (q = 1, 2, 3, …, Q). At composting center c composting process takes place, the expected fractions of outcomes are: the recovered material and the waste residues (combustible and incombustible composting waste residues) in winch x_{gcq} is a unit amount of recovered material that will be *transported from composting center c* to market/other factories q, Also x_{gck} is a unit amount of combustible composting waste residue *transported from composting center c* to combusting center k and x_{gcd} is a unit amount of incombustible composting waste residue that will be *transported from composting center c* to final disposal center s d. At hazardous center h separation process continues, the expected fractions of outcomes are the recyclable/reusable hazardous waste and the waste residues (combustible and incombustible hazardous waste residues). x_{ghr} is a unit amount of recyclable/reusable hazardous waste that will be *transported from hazardous center h to* recycling/reuse center r (r = 1,2,3, …, R), x_{ghk} is a unit amount of combustible hazardous waste residue *that will be transported from hazardous center h* to combusting center k. x_{ghd} is a unit amount of incombustible hazardous waste residue *transported from hazardous center h* to final disposal centers d. There are two advantages in moving combustible residues from recycling/reuse center r, composting center c, and hazardous center h to combusting center k, instead of moving them directly to final disposal center d; Firstly, the volume of waste residues moving to final disposal center d will be drastically reduced along the way. Secondly, the combustible waste residues will serve as raw material/input for the combusting process to produce thermal energy/electricity when combusted at combusting center k. At the combusting center, the k combusting process takes place. The expected outcomes are the recovered material and incombustible combusting waste residues. x_{gkq} is a unit amount of recovered material that will be *transported from combusting center k* to market/other factories q and x_{gkd} is a unit amount of incombustible combusting waste residue *transported from combusting center k* to final disposal center d.

Assumptions of the Proposed Model

i. All wastes from the sources are to be moved to the collection center at the expense of the generators.
ii. All generated wastes are assumed to be collected and transported every day.
iii. Sorting and separation of significant types of waste are assumed to start from the collection center s.
iv. All categories of wastes are assumed to be correctly sorted at the collection center s and sent to their respective treatment center s.

Sets and Indices of the Model

$d = 1, 2, \ldots, D$: location of final disposal center (landfill).
$i = 1, 2, \ldots, I$: location of waste sources.
$j = 1, 2, \ldots, J$: location of collection points.
$k = 1, 2, \ldots, K$: location of combusting center (incinerators).
$r = 1, 2, \ldots, R$: location of recycling/reuse center s.
$c = 1, 2, \ldots, C$: location of composting center.
$h = 1, 2, \ldots, H$: location of hazardous center.
$q = 1, 2, \ldots, Q$: location of other factories/ markets.
$l = 1, 2, \ldots, L$: capacity of a center.
$g = 1, 2, \ldots, G$: waste type.

Variables of the Proposed Model

This section presents and defined the decision and binary variables used in the study for clarification and understanding.

Decision Variables

$x_{gjr}, x_{gjc}, x_{gjd}, x_{gjh},$ *and* x_{gjk} = unit amount of recyclable waste, compostable waste, incombustible waste residue, hazardous waste, and combustible waste residue in tones per day from collection center j to recycling/reuse center r, to composting center c, to final disposal center d, to hazardous center h, and to combusting center k, respectively.

x_{grq}, x_{grd}, and x_{grk}, = unit amount of recovered material, incombustible recycling waste residue and combustible recycling waste residue in tones per day from recycling/reuse center r to other factories/market q, to final disposal center d and combusting center k, respectively.

$x_{gcd},$ x_{gcq}, *and* x_{gck} = unit amount of incombustible composting waste residue, recovered material (composted fertilizer), and combustible waste residue in tons per day from composting center c to final disposal center d, market q, and combusting center k, respectively.

x_{ghr}, x_{ghd}, *and* x_{ghk} = unit amount of recyclable hazardous waste, incombustible hazardous waste residue and combustible hazardous waste residue in tons per day from hazardous center h to recycling/reusing center r, final disposal center d, and combusting center k, respectively.

x_{gkd} and x_{gkq} = unit amount of incombustible combusting waste residue (ashes) in tons per day from combusting center k to final disposal center d and unit amount of recovered material (electricity in kilowatts) from combusting center k to market/other factories q.

$X_j, X_r, X_c, X_h, X_k, X_d$ = total amount of waste in tons transported to collection j, recycling/reuse center r, composting center c, hazardous center h, combusting center k, and final disposal center d, respectively.

Binary Variables

ɱ = (0, 1), it is 1 if the unit amount of recyclable waste will be transported from collection center j to recycling/reusing center r, 0 otherwise.

ɩ = (0, 1), it is 1 if the unit amount of recyclable waste will be transported from collection center j to composting center c, 0 otherwise.

ʎ = (0, 1), it is 1 if the unit amount of hazardous waste will be transported from collection center j to hazardous center h, 0 otherwise.

ʖ = (0, 1), it is 1 if the unit amount of recyclable waste will be transported from collection center j to combusting center k, 0 otherwise.

ɹ = (0, 1), it is 1 if the unit amount of incombustible waste residue will be transported from collection center j to final disposal center d, 0 otherwise.

ɻ = (0, 1), it is 1 if the unit amount of recovered waste will be transported from recycling/reuse r to market/other factory q, 0 otherwise.

ɥ = (0, 1), it is 1 if the unit amount of combustible recycling waste residue will be transported from recycling/reuse center r to combusting center k, 0 otherwise.

ʉ = (0, 1), it is 1 if the unit amount of incombustible recycling waste residue will be transported from recycling/reuse center r to final disposal center d, 0 otherwise.

ƶ = (0, 1), it is 1 if the unit amount of recovered material (composted fertilizer) will be transported from composting center c to market/other factory q, 0 otherwise.

ʔ = (0, 1), it is 1 if the unit amount of combustible composting waste residue will be transported from composting center c to combusting center k, 0 otherwise.

ɟ = (0, 1), it is 1 if the unit amount of incombustible composting waste residue will be transported from composting center c to final disposal center d, 0 otherwise.

Ɏ = (0, 1), it is 1 if the unit amount of hazardous waste will be transported from hazardous center h to recycling/reusing center r, 0 otherwise.

ƙ = (0, 1), it is 1 if the unit amount of incombustible hazardous waste residue will be transported from hazardous center h to final disposal center d, 0 otherwise.

ɜ = (0, 1), it is 1 if the unit amount of combustible hazardous waste residue will be transported from hazardous center h to combusting k, 0 otherwise.

ȣ = (0, 1), it is 1 if the unit amount of combusting residual waste residue (ashes) will be transported from combusting center k to final disposal center d, 0 otherwise.

Ł = (0, 1), it is 1 if the unit amount of recovered material (thermal energy/electricity) will be transported/sold from combusting center k to market/other factories q, 0 otherwise.

Data/Parameters

$w_1 + w_2 + \ldots + w_n$ = sum of daily generated waste from different collection center s within the metropolitan.

W_j = all generated wastes per unit per day at collection j.

$C_{jr}, C_{jc}, C_{jk}, C_{jh}, C_{jd}$ = cost (in Naira) per day of transporting significant categories of waste from collection center j to recycling/reuse center r, composting center c, combusting k, hazardous center h, and final disposal center d, respectively.

C_{rq}, C_{rk}, C_{rd} = cost (in Naira) per day of transporting recycled material from recycling/reuse center r to market/other factories q, cost (in Naira) per day of transporting combustible recycling waste residue from recycling/reuse center r to combusting center k and cost (in Naira) per day of transporting incombustible recycling waste residue from recycling/reuse center r to final disposal center d, respectively.

C_{cq}, C_{ck}, C_{cd} = cost (in Naira) per day of transporting recovered material from composting center c to market/other factories q, combustible composting waste residue from composting center c to combusting center k and incombustible composting waste residue from composting center c to final disposal center d, respectively.

C_{hr}, C_{hk}, C_{hd} = cost (in Naira) per day of transporting waste from hazardous center h to recycling/reuse center r, combustible dangerous waste residue from hazardous center h to combusting center k and incombustible hazardous waste residue from hazardous center h to final disposal center d, respectively.

C_{kq}, C_{kd} = cost (in Naira) per day of transporting the unit amount of recovered material from combusting center k to market/other factories q and cost (in Naira) per day of transporting the unit amount of incombustible combusting waste residue (ash) from combusting center k to final disposal center d.

$L_j, L_r, L_c, L_h, L_k, L_d$, = maximum available size/capacity of collection j, recycling/reuse center r, composting center c, hazardous center h, combusting center k, and final disposal center d, respectively.

fC_r, fC_c, fC_k, fC_h = fixed cost (in Naira) of establishing and maintaining recycling/reuse center r, composting center c, combusting center k, and hazardous center h, respectively.

MC_j, MC_d = cost of managing collection center j and final disposal center d, respectively.

o_p = fraction (in kilogram) of recoverable plastic waste at collection center j .

Ω_a = fraction (in kilogram) of recoverable aluminum waste at collection center j.

δ_m = fraction (in kilogram) of recoverable metallic waste at collection center j.

O_o = fraction (in kilogram) of others recoverable waste at collection center j.

π_c = fraction (in kilogram) of compostable waste at collection center j.

π_k = fraction (in kilogram) of combusting waste at collection center j.

pr_h = percentage of recoverable hazardous waste at hazardous center h.

π_h = fraction (in kilogram) of hazardous waste at collection center j.

π_d = fraction (in kilogram) of waste residue at collection center j.

H_{jr} = waste handling cost to manage the unit amount of recyclable waste at collection center j.

H_{jc} = waste handling cost to manage the unit amount of compostable waste at collection center j.

H_{jh} = waste handling cost to manage the unit amount of hazardous waste at collection center j.

H_{jk} = waste handling cost to manage the unit amount of combustible waste residue at collection center j.

H_{jd} = waste handling cost to manage the unit amount of incombustible waste residue at collection center j.

H_{hr} = waste handling cost to manage the unit amount of recyclable hazardous waste hazardous center h.

H_{hk} = waste handling cost to manage the unit amount of combustible hazardous waste residue at hazardous center h to combusting center k.

H_{hd} = waste handling cost to manage the flow of incombustible hazardous waste residue from hazardous center h.

H_{cq} = waste handling cost to manage the unit amount of recovered material at composting center c.

H_{ck} = waste handling cost to manage the unit amount of combustible composting waste residue at composting center c.

H_{cd} = waste handling cost to manage the unit amount of incombustible composting waste residue at composting center c.

H_{rq} = waste handling cost to manage the unit amount of recovered waste at recycling/reusing center r.

H_{rk} = waste handling cost to manage the unit amount of combustible recycling/reuse waste residue at recycling/reusing center r.

H_{rd} = waste handling cost to manage the unit amount of incombustible recycling/reuse waste residue at recycling/reusing center r.

H_{kq} = waste handling cost to manage the unit amount of recovered material (thermal energy/electricity) at combusting center k.

H_{kq} = waste handling cost to manage the unit amount of incombustible combusting waste residue (ashes) at combusting center k.

Constraints of the Model

The model is under several reasonable constraints. In general, the constraints include; flow balance (mass balance) constraints, capacity constraints, facility establishment constraints, goal constraints, non-negative variable constraints, and binary variable constraints.

Flow Balance (Mass Balance) Constraints

The incoming amount of wastes at any facility in the SWM system must be equal to the outgoing amount of wastes at that facility after processing.

The sum of daily generated waste ($w_1 + w_2 + \ldots + w_{140}$) from 140 different collection centers within the metropolitan must be equal to the total daily generated waste (W_j)

$$w_1 + w_2 + \ldots + w_{140} = \sum_{j \in J} W_j = 1200 \tag{1}$$

The unit amount of recyclable waste (x_{gjr}) that will be moved from collection center j to recycling/reuse center r constitutes the fractions of recoverable plastic, recoverable aluminum, recoverable metal, and recoverable other wastes found in the total daily generated waste (W_j)

$$\sum_{j \in J} \sum_{r \in R} x_{gjr} = \sum_{j \in J} \acute{\omega}_p W_j + \sum_{j \in J} \Omega_a W_j + \delta_m W_j + \sum_{j \in J} \H{O}_o W_j, \tag{2}$$

$\acute{\omega}_p$ = fraction of recyclable plastic waste in the daily generated waste.

Ω_a = fraction of recyclable aluminum waste in the daily generated waste.

δ_m = fraction of recyclable metallic waste in the daily generated waste.

$\H{O}_o$ = fraction of recyclable others waste in the daily generatedw̃aste.

The unit amount of compostable waste (x_{gjc}) found in the total daily generated waste (W_j) that will be moved from collection center j to composting center c is given as:

$$\sum_{j \in J} \sum_{c \in C} x_{gjc} = \sum_{j \in J} \pi_c W_j \tag{3}$$

π_c = fraction of composting waste in the daily generated waste (W_j).

The unit amount of compostable waste (x_{gjk}) found in the total daily generated waste (W_j) that will be moved from collection center j to combusting center k is given as:

$$\sum_{j \in J} \sum_{k \in K} x_{gjk} = \sum_{j \in J} \pi_k W_j \tag{4}$$

π_k = fraction of combusting waste residue in the daily generated waste (W_j).

The unit amount of hazardous waste (x_{gjh}) found in the total daily generated waste (W_j) that will be moved from collection center j to hazardous center h is given as:

$$\sum_{j \in J} \sum_{h \in H} x_{gjh} = \sum_{j \in J} \pi_h W_j \tag{5}$$

π_h = fraction of hazardous waste in the daily generated waste (W_j).

The unit amount of incombustible waste residue (x_{gjd}) found in the total daily generated waste (W_j), that will be moved from collection center j to final disposal center d is given as:

$$\sum_{j \in J} \sum_{d \in D} x_{gjd} = \sum_{j \in J} \pi_d W_j \tag{6}$$

π_d = fraction of incombustible waste residue in the daily generated waste (W_j).

The sum of recyclable waste (x_{gjr}) moved from collection center j to recycling/reuse center r and recyclable hazardous waste (x_{ghr}) moved from hazardous center h to recycling/reuse center r must be equal to the total unit amount of recyclable waste (X_r) transported to recycling/reuse center r

$$\sum_{j\in J}\sum_{r\in R} x_{gjr} + \sum_{h\in H}\sum_{r\in R} x_{ghr} = X_r, \tag{7}$$

The sum of all fractions of combustible waste residues from collection center j (x_{gjk}), from composting center c (x_{gck}), from recycling/reuse center r (x_{grk}), and from hazardous center h (x_{ghk}), moved to combust center k must be equal to the total unit amount of waste (X_k) transported to combusting center k

$$\sum_{j\in J}\sum_{k\in K} x_{gjk} + \sum_{r\in C}\sum_{k\in K} x_{gck} + \sum_{r\in R}\sum_{k\in K} x_{grk} + \sum_{h\in H}\sum_{k\in K} x_{ghk} = \sum_{k\in K} X_k, \tag{8}$$

The sum of all fractions of incombustible waste residues from collection center j (x_{gjd}), from recycling/reuse center r (x_{grd}), from composting center c (x_{gcd}), from recycling/reuse center r (x_{grd}), and from hazardous center h (x_{ghd}) moved to final disposal center d must be equal to the total unit amount of waste (X_d) transported to final disposal center d

$$\sum_{j\in J}\sum_{k\in K} x_{gjd} + \sum_{r\in C}\sum_{k\in K} x_{grd} + \sum_{r\in R}\sum_{k\in K} x_{gcd} + \sum_{h\in H}\sum_{k\in K} x_{ghd} + \sum_{k\in K} x_{gkd} = \sum_{d\in D} X_d, \tag{9}$$

The sum of all fractions of compostable waste (x_{gjc}) moved from collection center j to composting center c must be equal to the total unit amount of waste (X_c) moved from collection center j to composting center c

$$\sum_{j\in J}\sum_{c\in C} x_{gjc} = \sum_{c\in C} X_c, \tag{10}$$

The sum of all fractions of hazardous waste (x_{gjh}) moved from collection center j to hazardous center h must be equal to the total unit amount of hazardous waste (X_h) transported to hazardous center h.

$$\sum_{j\in J}\sum_{h\in H} x_{gjh} = \sum_{h\in H} X_h, \tag{11}$$

Expected Outcomes From Processed Waste at Different Facilities

(a) At recycling center r, treatment of recyclable waste (X_r) transported to recycling/reuse center takes place. The expected outcomes are; recovered material and recycling waste residue (combustible and incombustible recycling waste residue).

(i) ***Recovered material at recycling center r***

The fraction of recovered material (x_{grq}) that will be moved to market/other factories q

$$\sum_{r\in R}\sum_{d\in Q} x_{grq} = \sum_{r\in R}\sum_{d\in Q} pr_q X_r, \tag{12}$$

pr_q = percentage of recovered material from recyclable waste that will be moved to market/other factories q.

X_r = the total amount in tones of recyclable waste transported to recycling center r.

(ii) ***Combustible recycling waste residue at recycling center r***

The fraction of combustible recycling waste residue (x_{grk}) that will be moved to combusting center k is given as:

$$\sum_{r\in R}\sum_{k\in K} x_{grk} = \sum_{r\in R}\sum_{k\in K} pr_k X_r, \tag{13}$$

pr_k = percentage of combustible waste residue from recyclable waste that will be moved to combusting center k.

X_r = the total amount in tones of recyclable waste transported to recycling center r.

(iii) ***Incombustible recycling waste residue at recycling center r***

The fraction of incombustible recycling waste residue (x_{grd}) that will be moved to the final disposal site d

$$\sum_{r\in R}\sum_{d\in D} x_{grd} = \sum_{r\in R}\sum_{d\in D} pr_d X_r, \tag{14}$$

pr_d = percentage of incombustible recycling waste residue from recyclable waste that will be moved to final disposal site d.

X_r = the total amount in tones of recyclable waste transported to recycling/reuse center r.

(b) At composting center c, treatment of compostable waste (X_c) transported to composting center c takes place. The expected outcomes are; recovered material and composting waste residue (combustible and incombustible composting waste residue).

(i) ***Combustible composting waste residue at composting center c***

The fraction of combustible composting waste residue (x_{gck}) that will be moved to combusting center k is given as:

$$\sum_{c\in C}\sum_{k\in K} x_{gck} = \sum_{c\in C}\sum_{k\in K} pc_k X_c, \tag{15}$$

pc_k = percentage of combustible composting waste residue from composting center c that will be moved to combusting center k.

X_c = total amount in tones of compostable waste transported to composting center c.

(ii) ***Incombustible composting waste residue at composting center c***

The fraction of incombustible composting waste residue (x_{gcd}) that will be moved to final disposal site d is given as:

$$\sum_{c\in C}\sum_{d\in D} x_{gcd} = \sum_{c\in C}\sum_{d\in D} pc_d X_c, \tag{16}$$

pc_d = percentage of incombustible composting waste residue from composting waste that will be moved to final disposal site d.

X_c = total amount in tones of compostable waste transported to composting center c.

(iii) ***Recovered material from composting at composting center c***

The fraction of recovered material (x_{gcq}) that will be moved to market/other factories q is given as:

$$\sum_{c \in C} \sum_{q \in Q} x_{gcq} = \sum_{c \in C} \sum_{q \in Q} pc_q X_c, \tag{17}$$

pc_q = percentage of composted material from composting center c that will be moved to market/other factories q.

X_c = total amount in tones of compostable waste transported to composting center c.

(c) At hazardous center h, treatment of waste (X_h) transported to hazardous center h takes place. The expected outcomes are; recyclable hazardous waste and hazardous waste residue (combustible and incombustible hazardous waste residue).

i. ***Combustible hazardous waste at hazardous center h***

The fraction of combustible hazardous waste residue (x_{ghk}) that will be moved to combusting center k is given as:

$$\sum_{h \in H} \sum_{k \in K} x_{ghk} = \sum_{h \in H} \sum_{k \in K} ph_k X_h, \tag{18}$$

ph_k = percentage of combustible hazardous waste residue from hazardous waste that will be moved to combusting center k.

X_h = the total amount in tones of hazardous waste transported to hazardous center h.

ii. ***Incombustible hazardous waste residue at hazardous center h***

The fraction of incombustible hazardous waste residue (x_{ghd}) that will be moved to final disposal site d is given as:

$$\sum_{h \in H} \sum_{d \in D} x_{ghd} = \sum_{h \in H} \sum_{d \in D} ph_d X_h, \tag{19}$$

ph_d = percentage of incombustible hazardous waste residue from hazardous waste that will be moved to final disposal site d.

X_h = the total amount in tones of hazardous waste transported to hazardous center h.

iii. ***Recyclable hazardous waste at hazardous center h***

The fraction of recyclable hazardous waste (x_{ghr}) that will be moved from hazardous center h to recycling center r

$$\sum_{h \in H} \sum_{r \in R} x_{ghr} = \sum_{h \in H} \sum_{d \in D} ph_r X_h, \tag{20}$$

ph_r = percentage of recyclable/reusable hazardous waste at hazardous center h that will be moved to recycling/reusing center r.

X_h = the total amount in tones of hazardous waste transported to hazardous center h.

(d) At combusting center k, treatment of combustible (X_k) transported to combusting center k takes place. The expected outcomes are; recovered material (thermal energy/electricity) and incombustible combusting waste residue (ashes).

i. ***Recovered material from combustion at combusting center k***

The fraction of recovered material (x_{gkq}) that will be moved from combusting center k to market/other factories q is given as:

$$\sum_{k \in K} \sum_{q \in Q} x_{gkq} = \sum_{k \in K} \sum_{q \in Q} pk_q X_k, \tag{21}$$

pk_q = percentage of recovered (thermal energy/electricity) generated at combusting center k.

X_k = the total amount in tones of combustible waste residue transported to combusting center k.

ii. ***Incombustible combusting waste residue (ashes) at combusting center k***

The fraction of incombustible combusting waste residue (ashes) after combustion that will be moved to final disposal site d is given as:

$$\sum_{k \in K} \sum_{d \in D} x_{gkd} = \sum_{k \in K} \sum_{d \in D} pk_d X_k, \tag{22}$$

pk_d = percentage of combusting waste residue (ashes) at combusting center k that will be moved to final disposal center d.

X_k = the total amount in tones of combustible waste residue transported to combusting center k.

The capacity of facilities is limited to some factors such as equipment and human resources. Hence, a group of constraints are needed to display the maximum capacity of collection center j recycling/reuse center r, composting center c, combusting center k, hazardous center h, and final disposal center s d for a unit time.

The total daily generated waste (W_j) must be less than or equal to the sum of waste from all the collection centers in the metropolitan

$$\sum_{j \in J} W_j \leq \sum_{n=1}^{140} w_n, \tag{23}$$

The sum of all recyclable waste from collection center j and hazardous center h transported to recycling/reuse center r must be greater than or equal to the sum of all waste transported from recycling/reuse center r to combusting center k, final disposal center d, and market q, respectively

$$\sum_{j \in J} \sum_{r \in R} x_{gjr} + \sum_{h \in H} \sum_{r \in R} x_{ghr} \geq \sum_{r \in R} \sum_{k \in K} x_{grk} + \sum_{r \in R} \sum_{d \in D} x_{grd} + \sum_{r \in R} \sum_{q \in Q} x_{grq}, \tag{24}$$

The sum of all combustible waste residue transported from various facilities to combusting center k must be greater than or equal to the amount of waste transported from combusting center k to final disposal center d, and market q, respectively

$$\sum_{j\in J}\sum_{k\in K}x_{gjk}+\sum_{r\in C}\sum_{k\in K}x_{gck}+\sum_{r\in R}\sum_{k\in K}x_{grk}+\sum_{h\in H}\sum_{k\in K}x_{ghk}\geq\sum_{k\in K}\sum_{d\in D}x_{gkd}+\sum_{k\in K}\sum_{q\in Q}x_{gkq}, \quad (25)$$

The sum of compostable waste transported from collection center j to composting center c must be greater than or equal to the amount of waste transported from composting center c to combusting center k, final disposal center d, and market q, respectively

$$\sum_{j\in J}\sum_{c\in C}x_{gjc}\geq\sum_{c\in C}\sum_{k\in K}x_{gck}+\sum_{c\in C}\sum_{d\in D}x_{g\mathrm{cd}}+\sum_{c\in C}\sum_{q\in Q}x_{gcq}, \quad (26)$$

The sum of hazardous waste transported from collection center j to hazardous center h must be greater than or equal to the sum of the amount of waste transported from hazardous center h to combusting center k, final disposal center d, and recycling/reuse center r, respectively

$$\sum_{j\in J}\sum_{h\in H}x_{gjh}\geq\sum_{h\in H}\sum_{k\in K}x_{ghk}+\sum_{h\in H}\sum_{d\in D}x_{ghd}+\sum_{h\in H}\sum_{r\in R}x_{ghr}, \quad (27)$$

The sum of all daily generated waste (W_j) must not exceed the capacity (L_j) of the collection center j

$$\sum_{j\in J}W_j\leq L_j, \quad (28)$$

L_j = capacity of collection center j.

The sum of all recyclable waste from collection center j and hazardous center h to recycling/reuse center r must not exceed the capacity (L_r) of the recycling/reuse center r

$$\sum_{j\in J}\sum_{r\in R}\text{ɱ}x_{gjr}+\sum_{h\in H}\sum_{r\in R}\text{Ɏ}x_{ghr}\leq L_r, \quad (29)$$

L_r = capacity of recycling/reuse center r.

The sum of compostable waste transported from collection center j to composting center c must not exceed the capacity (L_c) of composting center c

$$\sum_{j\in J}\sum_{c\in C}\text{Ì}\,x_{gjc}\leq L_c, \quad (30)$$

L_c = capacity of composting center c.

The sum of all combustible waste residues from various facilities to combusting center k must not exceed the capacity (L_K) of the combusting center k

$$\sum_{j\in J}\sum_{k\in K}\text{ʖ}\,x_{gjk}+\sum_{r\in R}\sum_{k\in K}\text{ɥ}x_{grk}+\sum_{c\in C}\sum_{k\in K}\text{ʔ}\,x_{gck}+\sum_{h\in H}\sum_{k\in K}\text{ʒ}x_{ghk}\leq L_k, \quad (31)$$

L_k = capacity of combusting center k.

The sum of all hazardous waste transported from collection center j to hazardous center h must not exceed the capacity (L_h) of hazardous center h

$$\sum_{j\in J}\sum_{h\in H}\text{ʎ}\,x_{gjh}\leq L_h \quad (32)$$

L_h = capacity of hazardous center h.

The sum of all incombustible waste residues from various facilities to the final disposal center d must not exceed the capacity (L_d) of the final disposal center d

$$\sum_{j\in J}\sum_{d\in D}\text{ɹ}x_{gjd}+\sum_{r\in R}\sum_{d\in D}\text{ʉ}x_{grd}+\sum_{r\in R}\sum_{q\in Q}\text{ɟ}x\mathrm{gcd}+\sum_{h\in H}\sum_{d\in D}\text{ȣ}x_{ghd}+\sum_{k\in K}\sum_{d\in D}\text{ƙ}x_{gkd}\leq L_d, \quad (33)$$

L_d = capacity of final disposal center d.

According to the financial limitations/restrictions, the budgeted amount for the SWM should be greater than or equal to the total expenditure of the SWM activities for a given period. There are three types of costs in the model;

i. The waste transportation cost of moving different categories of waste between the different locations/facilities in the SWM system.
ii. The cost of handling/managing different types of waste at various locations/facilities.
iii. Fixed cost of establishing and maintaining SWM facilities.

These costs constitute all costs that will be minimized by the objective function (F) of the model.

Transportation cost is the cost of moving unit amounts of waste from one facility to another. In contrast, waste handling cost is the cost of waste processing/treatment at a particular facility.

i. ***Cost of transporting and handling recyclable waste***

C_{jr} = transportation cost of moving unit amount of recyclable waste $\left(x_{gjr}\right)$ *m*oved from collection center j to recycling/reuse center r.

H_{jr} = waste handling cost to manage the unit amount of recyclable waste at collection center j.

ɱ = (0, 1), it is 1 if the unit amount of recyclable waste will be transported from collection center j to recycling/reuse center r, 0 otherwise.

$\sum_{j\in J}\sum_{r\in R}$ ɱ$(C_{jr} + H_{jr})x_{gjr}$ = transportation cost plus waste handling cost of the unit amount of recyclable waste (x_{gjr}) moved from collection center j to recycling/reuse center r.

ii. ***Cost of transporting and handling composting waste***

C_{jc} = transportation cost of the unit amount of composting waste (x_{gjc}) *moved* from collection center j to composting center c.

H_{jc} = waste handling cost to manage the unit amount of composting waste at collection center j.

ι = (0, 1), it is 1 if the unit amount of composting waste will be transported from collection center j to composting center c, 0 otherwise.

$\sum_{j\in J}\sum_{c\in C}$ ι $(C_{jc} + H_{jc})\, x_{gjc}$ = transportation cost plus waste handling cost of the unit amount of composting waste (x_{gjc}) moved from collection center j to composting center c.

iii. ***Cost of transporting and handling combusting waste***

C_{jk} = transportation cost of the unit amount of combustible waste *residue* (x_{gjk}) moved from collection center j to combusting center k.

H_{jk} = waste handling cost to manage the unit amount of combustible waste residue at collection center j.

ƽ = (0, 1), it is 1 if the unit amount of combustible waste residue will be transported from collection center j to combusting center k, 0 otherwise.

$\sum_{j\in J}\sum_{k\in K}$ ƽ $(C_{jk} + H_{jk})x_{gjk}$ = transportation cost plus waste handling cost of the unit amount of combustible waste residue (X_{gjk}) moved from collection center j to combusting center k.

iv. ***Cost of transporting and handling hazardous waste***

C_{jh} = transportation cost of the unit amount of hazardous waste (x_{gjh}) *moved* from collection center j to hazardous center h.

H_{jh} = waste handling cost to manage the unit amount of hazardous waste at collection center j.

ʎ = (0, 1), it is 1 if the unit amount of hazardous waste will be transported from collection center j to hazardous center h, 0 otherwise.

$\sum_{j\in J}\sum_{h\in H}$ ʎ$(C_{jh}+ H_{jh})x_{gjh}$ = transportation cost plus waste handling cost of the unit amount of hazardous waste (x_{gjh}) moved from collection center j to hazardous center h.

v. ***Cost of transporting and handling incombustible waste residue***

C_{jd} = transportation cost of the unit amount of incombustible waste residue (x_{gjd}) moved from collection center j to final disposal center d.

H_{jd} = waste handling cost to manage the unit amount of incombustible waste residue at collection center j.

ɹ = (0, 1), it is 1 if the unit amount of incombustible waste residue will be transported from collection center j to final disposal center d, 0 otherwise.

$\sum_{j\in J}\sum_{d\in D}$ ɹ $(C_{jd} + H_{jd})x_{gjd}$ = transportation cost plus waste handling cost of the unit amount of incombustible waste residue (x_{gjd}) *moved* from collection center j to final disposal center d.

$$B_1 = \sum_{j\in J}\sum_{r\in R} \text{ɱ}(C_{jr} + H_{jr})x_{gjr} + \sum_{j\in J}\sum_{c\in C} \text{ι}\left(C_{jc} + H_{jc}\right)x_{gjc} + \sum_{j\in J}\sum_{k\in K} \text{ƽ}(C_{jk} + H_{jk}x)_{gjk} + \sum_{j\in J}\sum_{h\in H} \text{ʎ}(C_{jh} + H_{jh})x_{gjh} + \sum_{j\in J}\sum_{d\in D} \text{ɹ}\left(C_{jd} + H_{jd}\right)x_{gjd}, \quad (34)$$

where,

B_1 = sum of transportation cost and waste handling cost of the unit amount of recyclable waste (x_{gjr}), compostable waste (x_{gjc}) combustible waste residue (x_{gjk}), hazardous waste (x_{ghr}), and incombustible waste residue (x_{gjd}) moved from collection center j to recycling/reuse center r, composting center c, combusting center k, hazardous center h, and final disposal side d, respectively.

Transportation and Waste Handling Cost of the Unit Amount of Different Categories of Waste Moved From Recycling/Reusing Center r to Their Respective Locations

These are the costs of processing and moving different outcomes from recycling/reuse centers to their respective locations.

i. ***Cost of transporting and handling recovered material from recycling***

C_{rq} = transportation cost of the unit amount of recovered material (x_{grq}) *moved* from recycling center r to market/other factory q.

H_{rq} = waste handling cost to manage the unit amount of recovered material at recycling/reuse r.

ʮ = (0, 1), it is 1 if the unit amount of recovered material will be transported from recycling/reuse r to market/other factory q, 0 otherwise.

$\sum_{r\in R}\sum_{q\in Q}$ ʮ $(C_{rq} + H_{rq})\, x_{grq}$ = transportation cost plus waste handling cost of the unit amount of recovered material (x_{grq}) moved from recycling center r to market q.

ii. ***Cost of transporting and handling incombustible recycling waste residue***

C_{rd} = transportation cost of the unit amount of incombustible recycling waste residue (x_{grd}) moved from recycling center r to final disposal center d.

H_{rd} = waste handling cost to manage the unit amount of incombustible recycling waste residue at recycling/reuse r.

ʉ = (0, 1), it is 1 if the unit amount of incombustible recycling waste residue will be transported from recycling/reusing center r to final disposal center d, 0 otherwise.

$\sum_{r\in R}\sum_{q\in Q}$ ʉ$(C_{rd} + H_{rd})\, x_{grd}$ = transportation cost plus waste handling cost of the unit amount of incombustible recycling waste residue (x_{grd}) *moved* from recycling center r to final disposal center d.

iii. ***Cost of transporting and handling combustible recycling waste residue***

C_{rk} = transportation cost of the unit amount of combustible recycling waste residue (x_{grk}) *moved* from recycling center r to combusting center k.

H_{rk} = waste handling cost to manage the unit amount of combustible recycling waste residue at recycling/reuse r.

$\text{ɥ} = (0, 1)$, it is 1 if the unit amount of combustible recycling waste residue will be transported from recycling/reusing center r to combusting center k, 0 otherwise.

$\sum_{c\in C}\sum_{k\in K} \text{ɥ}\left(C_{rk}^{t} + H_{rk}\right) x_{grk}$ = transportation cost plus waste handling cost of the unit amount of combustible recycling waste residue (x_{grk}) *moved* from recycling center r to combusting center k.

$$B_2 = \sum_{r\in R}\sum_{q\in Q} \text{ɿ}\left(C_{rq} + H_{rq}\right) x_{grq} + \sum_{r\in R}\sum_{q\in Q} \text{ʉ}(C_{rd} + H_{rd})x_{grd} + \sum_{r\in R}\sum_{k\in K} \text{ɥ}\left(\hat{C}_{rk}^{t} + H_{rk}\right) x_{grk} \quad (35)$$

where,

B_2 = sum of the cost of transporting and waste handling of the unit amount of recovered material x_{grq}, incombustible recycling waste residue x_{grd}, *combustible recycling waste residue* x_{grk} to either market/other factory q, final disposal side d, and combusting center k, respectively.

Transportation and Waste Handling Cost of the Unit Amount of Different Categories of Waste Moved From Composting Center c to Their Respective Locations

These are the costs of processing and moving different outcomes from composting center c to their respective locations.

i. ***Cost of transporting and handling recovered material gained from composting***

C_{cq} = transportation cost of the unit amount of recovered material (composted fertilizer) (x_{gcq}) *moved* from composting center c to market/other/factory q.

H_{cq} = waste handling cost to manage the unit amount of recovered material (composted fertilizer) at composting center c.

$\text{ʔ} = (0, 1)$, it is 1 if the unit amount of recovered material (composted fertilizer) will be transported from composting center c to market/other factory q, 0 otherwise.

$\sum_{c\in C}\sum_{q\in Q} \text{ʔ}\left(C_{cq} + H_{cq}\right) x_{gcq}$ = transportation cost plus waste handling cost of the unit amount of recovered material (composted fertilizer) (x_{gcq}) moved from composting center c to market/other factory q.

ii. ***Cost of transporting and handling of incombustible composting waste residue***

C_{cd} = transportation cost of the unit amount of incombustible composting waste residue (x_{gcd}) *moved* from composting center c to final disposal center d.

H_{cd} = waste handling cost to manage the unit amount of incombustible composting waste residue at composting center c.

$\text{ȷ} = (0, 1)$, it is 1 if the unit amount of incombustible composting waste residue will be transported from composting center c to final disposal center d, 0 otherwise.

$\sum_{r\in R}\sum_{q\in Q} j\text{ȷ}\left(C_{cd} + H_{cd}\right) x_{\mathrm{gcd}}$ = transportation cost plus waste handling cost of the unit amount of incombustible composting waste residue (x_{gcd}) moved from composting center c to final disposal center d.

iii. ***Cost of transporting and handling of combustible composting waste residue***

C_{ck} = transportation cost of the unit amount of combustible composting waste residue (x_{gck}) *moved* from composting center c to combusting center k.

H_{ck} = waste handling cost to manage the unit amount of combustible composting waste residue at composting center c.

$\text{ƶ} = (0, 1)$, it is 1 if the unit amount of combustible composting waste residue will be transported from composting center c to combusting center k, 0 otherwise.

$\sum_{c\in C}\sum_{k\in K} \text{ƶ}(C_{ck} + H_{ck})x_{gck}$ = transportation cost plus waste handling cost of the unit amount of combustible composting waste residue (X_{gck}) moved from composting center c to combusting center k.

$$B_3 = \sum_{c\in C}\sum_{q\in Q} \text{ƶ}(C_{cq} + H_{cq})x_{gcq} + \sum_{r\in R}\sum_{q\in Q} \text{ȷ}(C_{cd} + H_{cd})x_{\mathrm{gcd}} + \sum_{c\in C}\sum_{k\in K} \text{ʔ}(C_{ck} + H_{ck})x_{gck} \quad (36)$$

where,

B_3 = sum of the cost of transporting and handling the unit amount of recovered material (composted fertilizer) x_{gcq}, incombustible composting waste residue x_{gcd}, *combustible waste residue* x_{gck} To either market or factory q, final disposal center d, and combusting center k, respectively.

Transportation and Waste Handling Cost of the Unit Amount of Different Categories of Waste Moved From Hazardous Center h to Their Respective Locations

These are the costs of processing and moving different outcomes from hazardous centers h to their respective locations.

i. ***Cost of transporting and handling of recyclable hazardous waste***

C_{hr} = transportation cost of the flow of unit amount of recyclable hazardous waste (x_{ghr}) from hazardous center h to recycling/reusing center r.

H_{hr} = waste handling cost to manage the unit amount of recyclable hazardous waste at hazardous center h.

$\text{ɏ} = (0, 1)$, it is 1 if the unit amount of hazardous waste will be transported from hazardous center h to recycling/reusing center r, 0 otherwise.

$\sum_{h\in H}\sum_{r\in R}\text{ɏ}\,(C_{hr}+H_{hr})\,x_{ghr}$ = transportation cost plus waste handling cost of the unit amount of recyclable hazardous waste (x_{ghr}) moved from hazardous center h to recycling/reuse center r.

ii. ***Cost of transporting and handling of incombustible hazardous waste residue***

C_{hd} = transportation cost of the unit amount of incombustible hazardous waste residue (x_{ghd}) moved from hazardous center h to final disposal center d.

H_{hd} = waste handling cost to manage the unit amount of incombustible hazardous waste residue at hazardous center h.

ȣ= (0, 1), it is 1 if the unit amount of incombustible hazardous waste residue will be transported from hazardous center h to final disposal center d, 0 otherwise.

$\sum_{h\in H}\sum_{d\in D}\text{ȣ}\,(C_{hd}+H_{hd})\,x_{ghd}$ = transportation cost plus waste handling cost of the unit amount of incombustible hazardous waste residue (X_{ghd}) moved from hazardous center h to final disposal center d.

iii. ***Cost of transporting and handling of dangerous combustible waste residue***

C_{hk} = transportation cost of the unit amount of combustible hazardous waste residue (x_{ghk}) *moved* from hazardous center h to combusting center k.

H_{hk} = *waste h*andling cost to manage the unit amount of combustible hazardous waste residue at hazardous center h.

ɜ= (0, 1), it is 1 if the unit amount of combustible hazardous waste residue will be transported from hazardous center h to combusting k, 0 otherwise.

$\sum_{h\in H}\sum_{k\in K}\text{ɜ}(C_{hk}+H_{hk})x_{ghk}$ = transportation cost plus waste handling cost of the unit amount of combustible hazardous waste residue (x_{ghk}) *moved* from hazardous center h to combusting center k.

$$B_4=\sum_{h\in H}\sum_{r\in R}\text{ɏ}\,(C_{hr}+H_{hr})\,x_{ghr}+\sum_{h\in H}\sum_{d\in D}\text{ȣ}(C_{hd}+H_{hd})x_{ghd}+\sum_{h\in H}\sum_{k\in K}\text{ɜ}(C_{hk}+H_{hk})x_{ghk}, \qquad (37)$$

where,

B_4 = sum of the cost of transporting and waste handling of the unit amount of recyclable hazardous waste x_{ghr}, incombustible hazardous waste residue x_{ghd}, *combustible hazardous waste residue* x_{ghk} to recycling/reuse center r, final disposal side d, and combusting center k, respectively.

Transportation and Waste Handling Cost of the Unit Amount of Ashes Moved From Combusting Center k to Final Disposal Site d, and Unit Amount of Thermal Energy/Electricity (in Kilowatts) Moved From Combusting Center k to Market q

These are the costs of processing and moving different outcomes from combusting center k to their respective locations.

i. ***Cost of transporting and handling of incombustible combusting waste residue***

C_{kd} = transportation cost of the unit amount of incombustible combusting waste residue (ashes) (x_{gkd}), moved from combusting center k to final disposal center d.

H_{kd} = waste handling cost to manage the unit amount of incombustible combusting waste residue at combusting center k.

ƙ= (0, 1), it is 1 if the unit amount of incombustible waste residue (ashes) will be transported from combusting center k to final disposal center d, 0 otherwise.

$\sum_{k\in K}\sum_{d\in D}\text{ƙ}(C_{kd}+H_{kd})x_{gkd}$ = transportation cost plus waste handling cost of the unit amount of incombustible waste residue (ashes) (x_{gkd}), *moved* from combusting center k to final disposal center d.

ii. ***Cost of transporting and handling of recovered material at combusting center k***

C_{kq} = transportation cost of the unit amount of recovered material (thermal energy/electricity) (x_{gkq}), moved from combusting center k to market q.

H_{kq} = handling cost to manage the unit amount of recovered material (thermal energy/electricity) (x_{gkq}) at combusting center k.

Ł= (0, 1), it is 1 if the unit amount of recovered material (thermal energy/electricity) will be transported/sold from combusting center k to market q, 0 otherwise.

$\sum_{k\in K}\sum_{q\in Q}\text{Ł}\left(C_{hq}+H_{kq}\right)\,x_{gkd}$ = transportation cost plus waste handling cost of the unit amount of recovered material (thermal energy/electricity) (x_{gkq}) moved from combusting center k to market/other factories q.

$$B_5=\sum_{k\in K}\sum_{d\in D}\text{ƙ}(C_{kd}+H_{kd})x_{gkd}+\sum_{k\in K}\sum_{q\in Q}\text{Ł}\left(C_{hq}+H_{kq}\right)\,x_{gkd}, \qquad (38)$$

where,

B_5 = sum of the cost of transporting and handling of the unit amount of recovered material (thermal energy/electricity) x_{gkq}, incombustible combusting waste residue x_{gkd}, to market/other factory q, and final disposal center d, respectively.

Fixed Cost of Establishing and Maintaining Center s/Facilities

These are the costs of establishing new facilities such as recycling/reuse center r, composting center c, combusting center k and hazardous center h, and maintaining collection center j, final disposal center d and the newly established center s.

fC_r = fixed cost of establishing and maintaining recycle/reuse center r.

ɜ_r = (0, 1), it is 1 if recycling center will be established at center r, 0 otherwise.

fC_c = fixed cost of establishing and maintaining composting center c.

ɜ_c = (0, 1), it is 1 if the composting center will be established at center c, 0 otherwise.

fC_k = fixed cost of establishing and maintaining combusting center k.

$ʒ_k = (0, 1)$, it is 1 if the combusting center will be established at center k, 0 otherwise.

fC_h = fixed cost of establishing and maintaining hazardous center h.

$ʒ_h = (0, 1)$, it is 1 if the hazardous center will be established at center h, 0 otherwise.

CM_j = fixed cost of maintaining collection center j.

CM_d = fixed cost of maintaining final disposal center d.

$$Y = \sum_{r \in R} ʒ_r fC_r + \sum_{c \in C} ʒ_c fC_c + \sum_{k \in K} ʒ_k fC_k + \sum_{h \in H} ʒ_h fC_h + \sum_{j \in J} CM_j + \sum_{d \in D} CM_d \tag{39}$$

where,

Y = sum of the fixed cost of establishing and maintaining recycle/reuse center r, composting center c, combusting center k and hazardous center h, cost of maintaining collection center j, and final disposal center d, respectively.

The total cost of the SWM system is given by

$$\sum_{i=1}^{5} B_i + Y \tag{40}$$

To establish a new center minimum amount of recyclable waste, compostable waste, combustible waste or hazardous waste is required.

To establish a new recycling/reuse center r, the total unit amount of recyclable waste (X_r) transported to recycling/reuse center r must be greater than or equal to the minimum capacity (CAP_r) of recycling/reuse center r.

$$X_r \geq CAP_r \tag{41}$$

To establish a new composting center c, the total unit amount of compostable waste (X_c) transported to composting center c must be greater than or equal to the minimum capacity (CAP_c) of composting center c.

$$X_c \geq CAP_c \tag{42}$$

To establish a new combusting center k, the total unit amount of compostable waste (X_k) transported to combusting center k must be greater than or equal to the minimum capacity (CAP_k) of combusting center k.

$$X_k \geq CAP_k \tag{43}$$

To establish a new hazardous center h, the total unit amount of hazardous waste (X_h) transported to hazardous center h must be greater than or equal to the minimum capacity (CAP_h) of hazardous center h.

$$X_h \geq CAP_h \tag{44}$$

The total number of each facility required to be established in the SWM system defends on the total daily generated waste (W_j) within the metropolitan. It also depends on the minimum and maximum capacities of the facilities specified by the MSWM authorities. This can be obtained as follows;

To establish new collection center j, the total unit amount of waste (X_j) transported to collection center j must be greater than or equal to the maximum capacity (L_j) of collection center j as indicated in constraint (28) above, then the number of collection center s (J) to be established can be obtained by dividing X_j *by* L_j as shown below;

$$\mathrm{J} = X_j/L_j, \mathrm{J} \geq 0, \text{integer} \tag{45}$$

To establish a new recycling/reuse center r, the total unit amount of waste (X_r) transported to recycling/reuse center r must be greater than or equal to the maximum capacity (L_r) of recycling center r as indicated in constraint (29) above, then the number of recycling/reuse center s (R) to be established can be obtained by dividing X_r *by* L_r as shown below;

$$\mathrm{R} = X_r/L_r, \mathrm{R} \geq 0, \text{integer} \tag{46}$$

To establish new composting center c, the total unit amount of waste (X_c) transported to composting center c must be greater than or equal to the maximum capacity (L_c) of composting center c as indicated in constraint (30) above, then the number of composting center s (C) to be established can be obtained by dividing X_c *by* L_c as shown below;

$$\mathrm{C} = X_c/L_c, \mathrm{C} \geq 0, \text{integer} \tag{47}$$

To establish new combusting center k, the total unit amount of waste (X_k) transported to combusting center k must be greater than or equal to the maximum capacity (L_k) of combusting center k as indicated in constraint (31) above, then the number of combusting center s (K) to be established can be obtained by dividing X_k *by* L_k as shown below;

$$\mathrm{K} = X_k/L_k, \mathrm{K} \geq 0, \text{integer} \tag{48}$$

To establish a new hazardous center h, the total unit amount of waste (X_h) transported to hazardous center h must be greater than or equal to the maximum capacity (L_h) of hazardous center h as indicated in constraint (32) above, then the number of hazardous center s (H) to be established can be obtained by dividing X_h *by* L_h as shown below;

$$\mathrm{H} = X_h/L_h, \mathrm{H} \geq 0, \text{integer} \tag{49}$$

To establish new final disposal center d, the total unit amount of waste (X_d) transported to final disposal center d must be greater than or equal to the maximum capacity (L_d) of final disposal center d as indicated in constraint (33) above, then the number of last disposal center s (D) to be established can be obtained by dividing X_d *by* L_d as shown below;

$$\mathrm{D} = X_d/L_d, \mathrm{D} \geq 0, \text{integer} \tag{50}$$

The non-negativity (variables) constraints are

$$x_{gjr}, x_{gjc}, x_{gjd}, x_{gjh}, x_{gjk}, x_{grq}, x_{grd}, x_{grk}, x_{gcd,}, \ x_{gcq}, \ x_{gck}, \\ x_{ghr}, x_{ghd}, \ x_{ghk}, \ x_{gkd}, x_{gkq}, \ W_j \geq 0 \quad (51)$$

The binary variables constraint are

ɱ, ɩ, ʎ, ʖ, ɹ, ʮ, ʯ, ʉ, ʑ, ʔ, ɟ, Ɏ, ƙ, ɜ, Ȣ, Ł,
ɜ$_r$ɜ$_c$, ɜ$_k$, ɜ$_h$, $= (1, 0)$ (52)

The goal is to minimize the total cost of the SWM system (F) subject to the constraints above. The complete mixed-integer mathematical model is stated below:

$$Min\ F = \sum_{i=1}^{5} B_i + Y$$

Subject to:

$$\sum_{j\in J} W_j \ < = \sum_{n\in N} w_n,$$

$$\sum_{j\in J}\sum_{r\in R} x_{gjr} + \sum_{h\in H}\sum_{r\in R} x_{ghr} \geq \sum_{r\in R}\sum_{k\in K} x_{grk} + \sum_{r\in R}\sum_{d\in D} x_{grd} + \sum_{r\in R}\sum_{q\in Q} x_{grq},$$

$$\sum_{j\in J}\sum_{k\in K} x_{gjk} + \sum_{r\in C}\sum_{k\in K} x_{gck} + \sum_{r\in R}\sum_{k\in K} x_{grk} + \sum_{h\in H}\sum_{k\in K} x_{ghk} \geq \sum_{k\in K}\sum_{d\in D} x_{gkd} + \sum_{k\in K}\sum_{q\in Q} x_{gkq},$$

$$\sum_{j\in J}\sum_{c\in C} x_{gjc} > = \sum_{c\in C}\sum_{k\in K} x_{gck} + \sum_{c\in C}\sum_{d\in D} x_{\mathrm{gcd}} + \sum_{c\in C}\sum_{q\in Q} x_{gcq},$$

$$\forall_{j\in J} \ , \ \forall_{c\in C}, \forall_{k\in K} \forall_{d\in D}, \forall_{q\in\ Q}$$

$$\sum_{j\in J}\sum_{h\in H} x_{gjh} \geq \sum_{h\in H}\sum_{k\in K} x_{ghk} + \sum_{h\in H}\sum_{d\in D} x_{ghd} + \sum_{h\in H}\sum_{r\in R} x_{ghr},$$

$$x_{gjr} = \acute{\omega}_p W_j + \Omega_a W_j + \delta_m W_j + O_o^{''} W_j, \forall_{j\in J}, \forall_{r\in\ R}$$

$$x_{gjc} = \pi_c W_j$$

$$x_{gjk} = \pi_k W_j$$

$$x_{gjh} = \pi_h W_j$$

$$x_{gjd} = \pi_d W_j$$

$$\sum_{j\in J}\sum_{r\in R} x_{gjr} + \sum_{h\in H}\sum_{r\in R} x_{ghr} = \sum_{r\in R} X_r,$$

$$\sum_{j\in J}\sum_{k\in K} x_{gjk} + \sum_{r\in C}\sum_{k\in K} x_{gck} + \sum_{r\in R}\sum_{k\in K} x_{grk} + \sum_{h\in H}\sum_{k\in K} x_{ghk} = \sum_{k\in K} X_k,$$

$$\sum_{j\in J}\sum_{d\in D} x_{gjd} + \sum_{r\in R}\sum_{d\in D} x_{grd} + \sum_{r\in R}\sum_{q\in Q} x_{\mathrm{gcd}} + \sum_{h\in H}\sum_{ds\in D} x_{ghd} + \sum_{k\in K}\sum_{d\in D} x_{gkd} = \sum_{d\in D} X_d,$$

$$\sum_{j\in J}\sum_{c\in C} x_{gjc} = \sum_{c\in C} X_c,$$

$$\sum_{j\in J}\sum_{h\in H} x_{gjh} = \sum_{h\in H} X_h,$$

$$\sum_{r\in R}\sum_{k\in K} x_{grk} = \sum_{r\in R}\sum_{k\in K} pr_k X_r,$$

$$\sum_{r\in R}\sum_{d\in D} x_{grd} = \sum_{r\in R}\sum_{d\in D} pr_d X_r,$$

$$\sum_{r\in R}\sum_{d\in Q} x_{grq} = \sum_{r\in R}\sum_{d\in Q} pr_q X_r,$$

$$\sum_{c\in C}\sum_{k\in K} x_{gck} = \sum_{c\in C}\sum_{k\in K} pc_k X_c,$$

$$\sum_{c\in C}\sum_{d\in D} x_{\mathrm{gcd}} = \sum_{c\in C}\sum_{d\in D} pc_d X_c,$$

$$\sum_{c\in C}\sum_{q\in Q} x_{gcq} = \sum_{c\in C}\sum_{q\in Q} pc_q X_c,$$

$$\sum_{h\in H}\sum_{k\in K} x_{ghk} = \sum_{h\in H}\sum_{k\in K} ph_k X_h,$$

$$\sum_{h\in H}\sum_{d\in D} x_{ghd} = \sum_{h\in H}\sum_{d\in D} ph_d X_h,$$

$$\sum_{h\in H}\sum_{r\in R} x_{ghr} = \sum_{h\in H}\sum_{d\in D} ph_r X_h,$$

$$\sum_{k\in K}\sum_{d\in D} x_{gkd} = \sum_{k\in K}\sum_{d\in D} pk_d X_k\ ,$$

$$\sum_{k\in K}\sum_{q\in Q} x_{gkq} = \sum_{k\in K}\sum_{q\in Q} pk_q X_k\ ,$$

$$\sum_{j\in J} X_j \geq \sum_{j\in J} L_j,$$

$$\sum_{r\in R} X_r \geq \sum_{r\in R} L_r,$$

$$\sum_{c\in C} X_C \geq \sum_{j\in J} L_C,$$

$$\sum_{h\in H} X_h \geq \sum_{j\in J} L_h,$$

$$\sum_{k\in K} X_k \geq \sum_{j\in J} L_k,$$

$$\sum_{d\in D} X_d \geq \sum_{j\in J} L_d,$$

TABLE 2 | Objective function value.

	Description	Values in Naira
Z*	Objective function	0.9104123E+08
TC	Transportation cost	1674291
Y	Total fixed costs	0.9000000E+08

*means Optimal value of Z.

TABLE 3 | Values of major categories waste with their percentages.

S/N	Waste type	Amount from the results	Capacity	No. of facilities to be established	Percentage of waste type
1	Daily generated waste	1,200 (maximum capacity in tons for the current scenario)	60.00000	20	100
2	Recyclable/reusable waste	421.2000	240.0000	2	35.1
3	Compostable waste	262.8000	200.0000	2	21.9
4	Combustible waste	386.4000	391.0000	1	32.2
5	Hazardous waste	80.40000	81.00000	1	6.7
6	Incombustible waste residue	49.20000	113.0000	2	4.1

$$X_r \geq CAP_r$$
$$X_c \geq CAP_c$$
$$X_k \geq CAP_k$$
$$X_h \geq CAP_h$$
$$x_{gjr}, x_{gjc}, x_{gjd}, x_{gjh}, x_{gjk}, x_{grq}, x_{grd}, x_{grk}, x_{gcd},$$
$$x_{gcq}, x_{gck},$$
$$x_{ghr}, x_{ghd}, x_{ghk}, x_{gkd}, x_{gkq}, W_j \geq 0$$

ɱ , ɩ, ʎ, ʖ, ɹ, ʮ, ʯ, ʉ, ʔ, ƶ, ɟ, Ɐ, ƙ, ɜ, Ȣ, Ł,

ʒ$_r$ʒ$_c$, ʒ$_k$, ʒ$_h$, $= (1, 0)$

RESULTS AND DISCUSSION

The mixed-integer non-linear model developed in this study has been solved using Lingo software version 18.0. According to the package, after 1,342 iterations, the optimal solutions are attained. The optimal integer values of the objective function and the values of major categories of waste with their percentages are shown in **Tables 2**, **3**, respectively.

Looking at the results in **Table 3**, the recoverable waste (Recyclable/reusable waste, Compostable waste, and Combustible waste) has more than 80% out of the daily generated waste. Thus, there is a need for the state government to intervene fully in the waste recovery processes to generate more revenue and reduce the volume of the waste that will reach the final disposal site.

Managerial Implications

As stated above, there are two types of waste in the Kano metropolis; the first group is the light and predominantly non-biodegradable waste, which comprises mainly tins, plastic materials and bottles (the recyclable and reusable waste comes from this group). Government is not directly benefiting from this group because the processes of recycling and reusing and recycling centers are owned chiefly by private individuals. These centers get their raw materials (recyclable wastes) from vendors/scavengers. The second group of solid waste is the heavier, predominantly biodegradable wastes that contain a high proportion of food scrap, ash, dirt, and vegetables. This group includes organic waste substances that can be used to prepare compost for the fertilization of agricultural land. This organic fraction of waste is composted naturally to fertilizer at either collection or disposal centers. The compost can easily freely be fetched by anybody who wishes to convey within the locality. Thus, the government is also not benefiting from this compost. Combustion is another recovery process where thermal energy and electricity can be obtainable. Which the government is not also benefiting from it. Should the government incorporate these recovery processes (i.e., recycling/reuse, composting for fertilizer, and combusting for energy), the heap of waste seen everywhere and along the roadside of the metropolitan will be drastically reduced. The government and people of Kano state would generate more revenues and job opportunities from the process.

CONCLUSION

Optimizing municipal solid waste is a challenging task for every concerned government. It involves many resources ranging from financial to human and machinery. Therefore, minimizing these costs is a welcome idea for policymakers and project executers. In this study, a conceptual model for MSW management is proposed. A mixed-integer mathematical model was formulated to determine the establishment of recycling, composting, combusting and hazardous centers at a minimum cost. The results show that; there is a need for the Kano state government to incorporate recovery processes in the municipal solid waste management policy because more than 80% of the daily generated waste can be recoverable, as evident from **Table 3**. The results further indicate that the state government should establish 20 standard collection centers, two recycling centers, two composting centers, one combusting center, one hazardous center, and two final disposal centers. It can be observed that only two disposal centers are needed in the process. The reason for that could be when recovery processes are incorporated in the state SWM policy, the waste volume that reaches the final disposal site will drastically be reduced. In such a way, only a tiny amount of incombustible waste residue will go to the definitive disposal sites.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

ETHICS STATEMENT

Written informed consent was not obtained from the minor(s)' legal guardian/next of kin for the publication of any potentially identifiable images or data included in this article.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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