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# Inherent individual differences in utility

***R. Duncan Luce****

*Institute for Mathematical Behavioral Sciences, University of California, Irvine, CA, USA*
**Correspondence: rdluce@uci.edu*

## 1 WHO KNOWS A UTILITY REPRESENTATION

Repeatedly it is alleged that this or that theory concerns a decision maker maximizing some utility function. But, as I have been at pains to discuss, the representation is not a creature of the decision maker but of the scientist studying the decision maker's behavior "The representation theorems go in only one direction: from behavioral and structural properties to numerical representations. The behavioral properties embody the scientific information that we, as scientists, have about the decision maker. It is the scientist, not the decision maker, who formulates both the properties (axioms) and the representation. The decision maker exhibits the behavior which, presumably, arises from some fairly complex neuronal processes…" (Luce, 2000, p. 25). This observation holds far more broadly than just in decision making.

## 2 UNIQUENESS OF THE REPRESENTATION

Uniqueness of representations is, of course, a very well trodden topic. The gist is that in addition to proving a representation theorem, one must also formulate a uniqueness theorem – how do the several representations relate? In most of the popular measurement theories, the uniqueness falls into one of Stevens' (1946, 1951) well known classification: nominal, ordinal, interval, log-interval, ratio, and absolute scales (see Krantz et al., 1971; Luce et al., 1990).

As I recently came to realize, when we are dealing with a preference weak order $\succsim$ and a binary operation $\oplus$, such as the concatenation of two sums of money, for which 0 money is an identity of $\oplus$ and $\oplus$ is commutative, associative, and monotonic in $\succsim$, things are not really quite as simple as has been long believed.

Recall that Hölder (1901) proved in this context the existence of an additive representation

$$\begin{aligned}\psi(x\oplus y)&=\psi(x\oplus 0)+\psi(0\oplus y)\\&=\psi(x)+\psi(y),\end{aligned}\tag{1}$$

which is unique up to a ratio scale.

A century later it was recognized that, in the context of utility theory, we also have the interplay between value and risk, which means that multiplication as well as addition is in play. See, e.g., (3) below. So the mapping should be into $\langle\mathbb{R},\geq,+,\times\rangle$, in which case the full set of polynomial (p-additive) representations are (see Luce, 2000, p.151, § 4.4.6)

$$\begin{aligned}&\psi(x\oplus y)\\&=\psi(x)+\psi(y)+\delta\psi(x)\psi(y)\quad \delta=-1,0,1.\end{aligned}\tag{2}$$

Of course, $\delta = 0$ is Hölder's original solution. The other two are quite different in that they are absolute scales (see Luce, 2010a). Here is the reason: when $\delta\neq 0$, (2) is equivalent to

$$1+\delta\psi(x\oplus y)=\big(1+\delta\psi(x)\big)\big(1+\delta\psi(y)\big).$$

But this expression is not invariant under any transformation of $\psi$ other than the identity. The fact they are absolute scales has several important implications.

## 3 SOME IMPLICATIONS OF THE p-ADDITIVE REPRESENTATION

### 3.1 THREE TYPES OF PEOPLE

There are three inherently different types of people corresponding to their values of $\delta$. The following simple behavioral criterion to decide a particular person's type was reported by Luce (2010a).

Let $(x, p; y, 1-p)$ denote the gamble where $x$ occurs with probability $p$ and $y$ with probability $1-p$ and suppose, as is true in a great many theories of utility, that

$$\begin{aligned}U(x,p;y,1-p)&=U(x)W(p)\\&\quad+U(y)W(1-p),\end{aligned}\tag{3}$$

where $\psi$ has been replaced in this context by $U$ for utility, $W$ is a strictly increasing subjective probability function, and $W(0) = 0$. Further, assume that

$$W(p)+W(1-p)=1.\tag{4}$$

Find the probability $p_{1/2}$ with subjective probability 1/2, i.e., such that for the representation (3) and for all $x > y$

$$\begin{aligned}\left(x,p_{\frac12};y,1-p_{\frac12}\right)\sim\left(y,p_{\frac12};x,1-p_{\frac12}\right)&\Leftrightarrow W(p_{\frac12})\\&=\frac12.\end{aligned}\tag{5}$$

I assume that $W(p) = p$ is satisfied for $p = 0$, $p = p_{1/2}$, possibly for $p = 1$, but not elsewhere. Note that it follows that relative to the line $f(p) = p$, the weighting function $W$ must be either S-shaped or inverse S-shaped. See Section 3.4 of Luce (2000) for a summary of empirical estimates which seem to agree with this prediction.

Then the criterion for $\delta$ is that for all $x > x' > y > y'$

$$\begin{aligned}&\delta=\left\{\begin{array}{c}1\\0\\-1\end{array}\right\}\Leftrightarrow\\&\left(x\oplus x',p_{\frac12};y\oplus y',1-p_{\frac12}\right)\left\{\begin{array}{c}>\\\sim\\<\end{array}\right\}\\&\left(x\oplus y,p_{\frac12};x'\oplus y',1-p_{\frac12}\right).\end{aligned}\tag{6}$$

Observe that under the ordering constraint on the consequences, the gamble on the left has a greater variance than the one on the right. For that reason, the three $\delta$ types are called, respectively, risk-seeking ($\delta = 1$), risk-neutral ($\delta = 0$), and risk-averse ($\delta = -1$). It would not surprise me that in the academic population, which is the source of many experimental respondents, risk-averse types will predominate except, perhaps, in schools of business.

It has yet to be shown empirically that this classification holds up in the population. But should it, one simply should not average data over people without, at the least, dealing with the three classes separately. Indeed, really only the $\delta = 0$ types should ever be averaged. Individual differences are

important. This has significant implications – e.g., many behavioral economic experiments should be redone taking into account individual types (Luce, 2010b).

### 3.2 UTILITY FORMS OF RISK TYPES

Three forms of utility follow from (2) and (6): there exist positive constants $\alpha$, $\kappa$, and $\lambda$ such that

$$U(x)=\left\{\begin{array}{c} e^{\kappa x}-1 \\ \alpha x \\ 1-e^{-\lambda x}\end{array}\right\}\Leftrightarrow \delta=\left\{\begin{array}{c} 1 \\ 0 \\ -1\end{array}\right\} \quad (7)$$

(Luce, 2010a). Note that for $\delta = 1$, this function is bounded from below by $-1$ and unbounded from above, whereas for $\delta = -1$ it is unbounded from below and bounded by 1 from above.

### 3.3 INTERPERSONAL COMPARISONS OF UTILITY

Because the $\delta \neq 0$ scales are absolute, interpersonal comparisons become meaningful, although for a mix of $-1$ and 1 types, only in the common interval $(-1,1)$ (Luce, 2010a). No such comparisons can be made involving risk-neutral types of people. If correct, this makes sense of the fact that classical utility theorists have found interpersonal comparisons impossible (e.g., Robbins, 1938; Harsanyi, 1977; Elster and Roemer, 1991; Hammond, 1991; Binmore, 2009) although most of us intuit that we regularly make such comparisons. And just how many of us are risk-neutral?

### 3.4 PSYCHOPHYSICAL SCALING IN GENERAL

As I have thought more about subjective intensity scales, I have come to realize that many other attributes of subjective intensity closely resemble the utility theory of risk except that most are defined only on the non-negative real numbers (Luce, under review). Indeed, the only exceptions that I know of involve what can be called binary senses that involve the close interaction of the two ears and of the two eyes. The binary theory coupled with certain loudness and brightness data simply rule out the two $\delta \neq 0$ cases.

Thus, when we ask a respondent to match subjective intensities of one modality to those of another modality, then, because of the three values of $\delta$ arise for all but the binary attributes, we get a complex of predicted results (Luce, under review, Table 1). This fact suggests an extensive experimental program to be done.

## 4 CLOSING REMARKS

I have made a strong claim here, namely, that a slight mathematical oversight – mapping just into addition when, for other theoretical reasons, multiplication is also involved – has put us on a misguided course for over a century. And that course may, in a number of ways, have been scientifically misleading.

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