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FSE-RBFNN-based LPF-AILC of finite time complete tracking for a class of time-varying NPNL systems with initial state errors

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The paper proposes a low-pass filter adaptive iterative learning control (LPF-AILC) strategy for unmatched, uncertain, time-varying, non-parameterized nonlinear systems (NPNL systems). To address the difficulty of nonlinear parameterization terms in system models, a new function approximator (FSE-RBFNN), which combines the radial basis function neural network (RBFNN) and Fourier series expansion (FSE), is introduced to model each time-varying nonlinear parameterized function. The adaptive backstepping method is used to design control laws and parameter adaptive laws. In the process of controller design, we may encounter the problem of too many derivatives, which can cause parameter explosions after derivatives. Therefore, we introduce a first-order low-pass filter to solve this problem and simplify the structure of the controller. As the number of iterations increases, the maximum tracking error gradually decreases until it converges to the nearby region, approaching zero within the entire given interval $[0, T]$, according to the Lyapunov-like synthesis. To mitigate the impact of initial state errors, a dynamically changing boundary layer is introduced, along with a series to deal with the unknown error upper bounds. Finally, two simulation examples prove the correctness of the proposed control method.

KEYWORDS

adaptive iterative learning control, time-varying non-parameterized nonlinear systems, backstepping method, Fourier series expansion-radial basis function neural network, initial state errors, low-pass filter

1 Introduction

Adaptive iterative learning control (AILC) is a useful control strategy for solving repetitive tracking control task problems for uncertain nonlinear systems. It continuously adjusts its control algorithm through iterative learning to gradually approach the ideal trajectory of the unknown system. AILC has extensive application value and promising development prospects for practical applications. Repeat systems include uncertain robotic manipulators and uncertain hard disk drivers. The task requirements specify that it can quickly achieve exact tracking as the number of iterations increases [1–4].

A non-parameterized nonlinear (NPNL) system refers to a dynamic characteristic that exhibits a complex nonlinear relationship and unknown parameters, making it difficult to design effective control strategies. It is particularly challenging to achieve high-precision

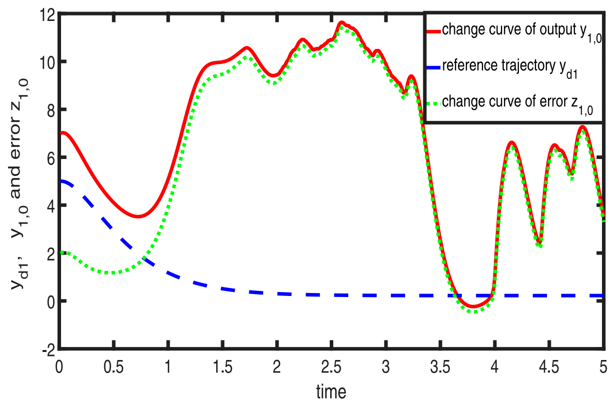

FIGURE 1
Variation in $y_0, y_{d1}, z_{1,0}$ over time without iteration.

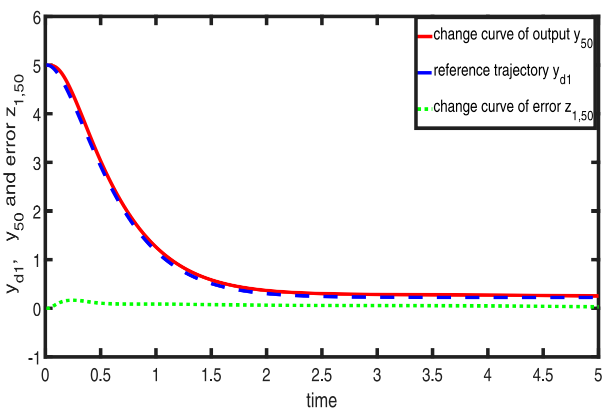

FIGURE 2
Variation in $y_{50}, y_{d1}, z_{1,50}$ over time during the 50th iteration.

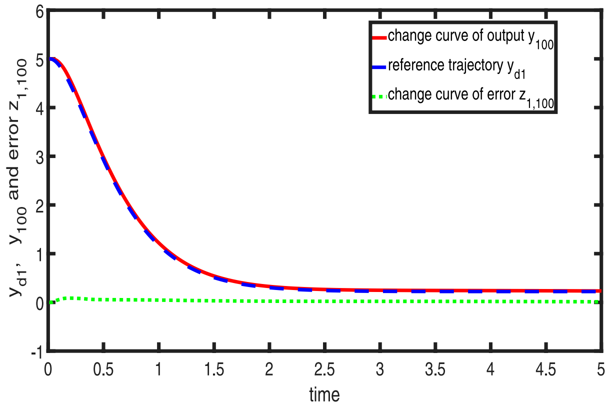

FIGURE 3
Variation in $y_{100}, y_{d1}, z_{1,100}$ over time during the 100th iteration.

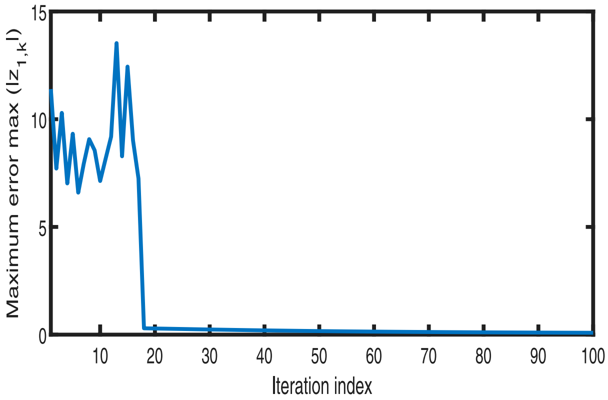

FIGURE 4
Variation in $max(|z_{1,k}|)$ according to the iteration index.

tracking and control within a limited time frame. Traditional control methods often require the establishment of a mathematical model for the system, but for the NPNL system, this step is usually very difficult or even impossible to complete. AILC technology has become an important method for solving these problems [5, 6].

There are many challenging problems in the research of AILC. This paper considers three difficult problems of AILC. The first problem is the processing problem of uncertain nonlinear parameterization terms with time-varying parameters. In the field of control, the control problem of nonlinear systems with uncertain time-varying parameters is very challenging. Adaptive control and robust control are common methods to deal with uncertain problems [7, 8]. Through learning, adaptive control can mitigate the impact of uncertainties. In order to handle uncertain nonlinear terms, adaptive control is often combined with some approximation methods, such as neural networks (NNs) and Fuzzy Logic Systems (FLSs). However, these adaptive controls only solve the uncertain linearly parameterized disturbances and ensure the stability of the system [7–20]. For the uncertain system, a fuzzy AILC was presented [21]. The composite energy function–adaptive iterative learning control (CEF–AILC) is an effective scheme for systems with time-varying disturbances [21–23]. Few AILC research results focus on uncertain, non-parameterized nonlinear systems [24–26]. Specifically, for systems with non-separable time-varying parameters, the tracking control problem on finite time intervals is still an open problem.

The second problem of AILC is ensuring complete tracking over a finite time interval when the initial state has errors. In these studies [27–31], the stability analysis section requires that initial state errors be strictly zero. Although the research on this problem is well done in traditional D-type or P-type ILC [32–41], it has not been well solved based on Lyapunov analysis for AILC. Specifically, in the presence of an initial state error, ensuring the system's completion of accurate tracking tasks within a specified time frame presents a complex challenge. [39] solved the tracking control problem of the unmatched uncertain NPNL systems. [41] solved the tracking problem of a class of high-order nonlinear systems with random initial state shifts, which relaxes the requirement of initial

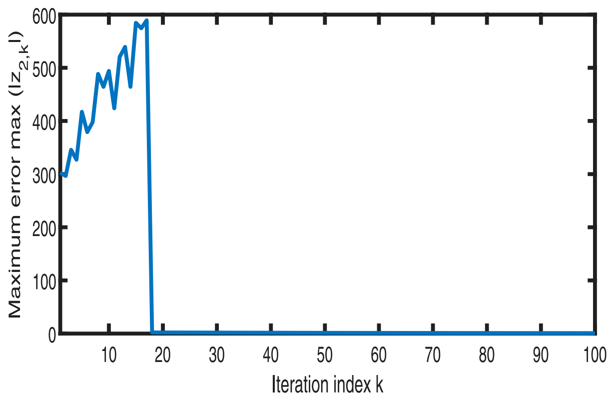

FIGURE 5
Variation in $max(|z_{2,k}|)$ according to the iteration index.

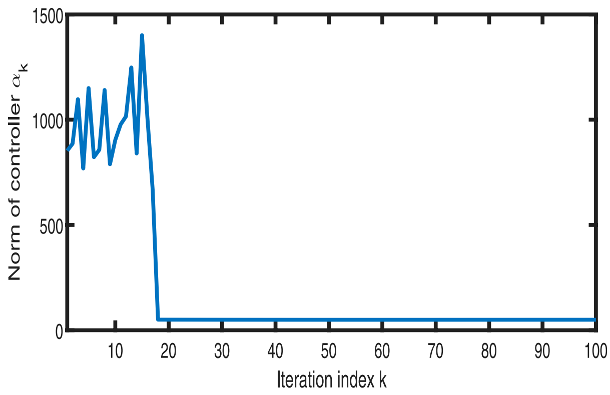

FIGURE 7
Variation in $\|\alpha_k\|$ according to the iteration index.

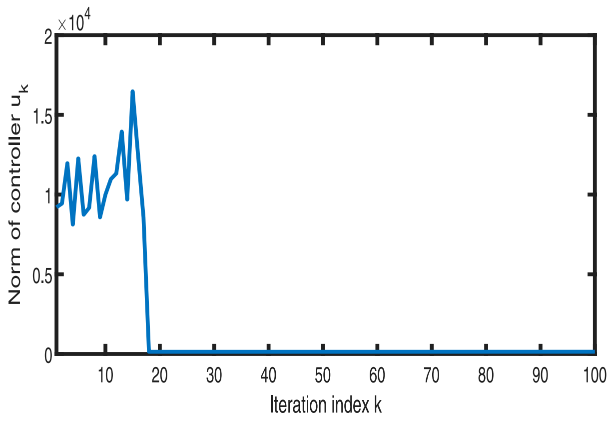

FIGURE 6
Variation in $\|u_k\|$ according to the iteration index.

positioning in ILC. So far, no relevant research results have been found for AILC applied to NPNL systems with uncertain time-varying parameters and initial state errors.

The last problem is parameter explosions after the derivative of the virtual controller. When designing a controller, we may encounter the problem of too many derivatives, which can cause parameter explosions after derivatives. Addressing this issue and streamlining the controller's structure to ensure the effective tracking of the non-parametric, nonlinear, time-varying system is a challenging and crucial problem. [42–44] employed a first-order low-pass filter to address the challenge of parameter explosions and achieve satisfactory performance. Therefore, we introduce a first-order low-pass filter to solve this problem and simplify the structure of the controller.

Motivated by the above discussion, we will use a low-pass filter AILC (LPF-AILC) method for uncertain time-varying NPNL systems. The AILC is given by the adaptive backstepping technique and Lyapunov-like theorem. In response to the difficult issues discussed above, the main contributions of this article are as follows:

1) An LPF-AILC strategy is proposed for a class of strongly time-varying, non-parameterized, nonlinear systems combined with a new approximation method.
2) The processing problem of uncertain time-varying nonlinear parameterization terms was solved. This is a very important and difficult problem. Specifically, in the field of AILC, no relevant research results have been found.
3) The difficulty problem of AILC is ensuring complete tracking on a given interval when the initial state has errors.
4) The problem of parameter explosions was solved by applying a derivative to the virtual controller and simplifying its structure.

In this paper, a combination of Fourier series expansion and radial basis function neural network (RBFNN) (FSE-RBFNNs) is used to model the uncertain, time-varying nonlinear dynamics by using their uniform approximation [24, 38]. An updating time-varying boundary layer is used to design the error function to deal with the initial state error. A common convergence series sequence is employed to mitigate the impact of approximation errors on the control performance of the system. A low-pass filter was introduced to solve the problem of parameter explosions resulting from the derivative of the virtual controller and simplify the structure of the controller. Theoretical analysis can demonstrate the bounded nature of all signals within the closed-loop system. The maximum value of errors will gradually converge to a narrow range close to zero as the boundary layer width satisfies the convergence condition with the number of iterations. Finally, two simulation examples are given to prove the effectiveness and correctness of the control method.

2 Problem description and mathematical foundations

2.1 Problem description

Uncertain time-varying NPNL systems are considered:

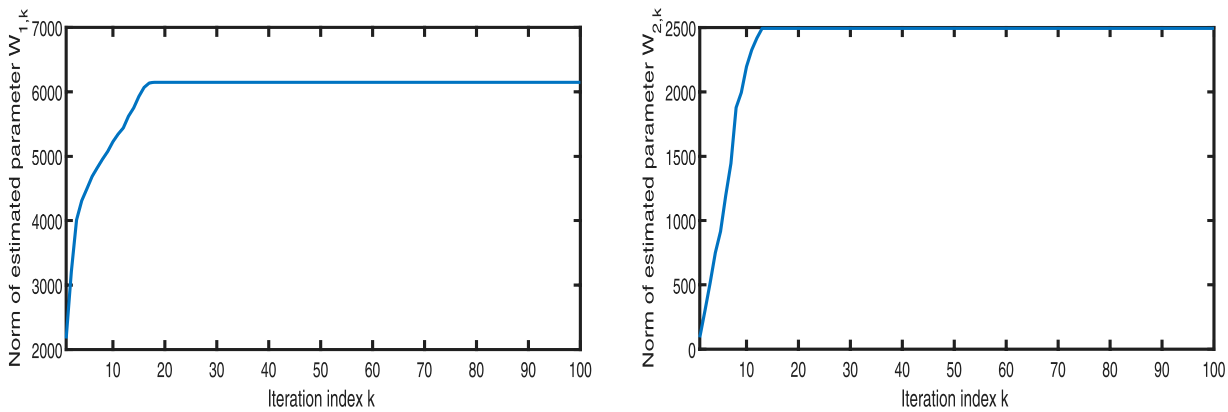

FIGURE 8
Variation in $\|\hat{W}_{1,k}\|$ and $\|\hat{W}_{2,k}\|$ according to the iteration index.

$$\begin{aligned}\dot{x}_{1,k} &= x_{2,k} + f_1\left(\bar{x}_{1,k}, \theta_1(t)\right) + g_1\left(\bar{x}_{1,k}\right)\\ \dot{x}_{i,k} &= x_{i+1,k} + f_i\left(\bar{x}_{i,k}, \theta_i(t)\right) + g_i\left(\bar{x}_{i,k}\right)\\ \dot{x}_{n,k} &= u_k + f_n\left(\bar{x}_{m,k}, \theta_n(t)\right) + g_n\left(\bar{x}_{n,k}\right)\\ y_k &= x_{1,k},\end{aligned} \tag{1}$$

where $\bar{x}_{i,k} = [x_{1,k}, \ldots, x_{i,k}]^T \in R^i$ and $x = \bar{x}_n$ represents measurable state vectors. $u_k \in R$ is the control input. $y_k \in R$ is the system output. $f_i(\bar{x}_{i,k}, \theta_i(t))$, $g_i(\bar{x}_{i,k})$, and $i = 1, 2, \ldots, n$ are uncertain time-varying functions, and $\theta_i(t)$ represents unknown time-varying parameters. k denotes the iteration time.

The design objective of this article is to find $u_k(t)$ for system (1) to ensure that $y_k(t)$ follows the ideal trajectory $y_{d1}(t)$ on $[0, T]$.

2.2 Mathematical foundations

The mathematical knowledge used in this article is provided with relevant references, and the specific definitions and principles will not be elaborated. Here, we only provide the conclusions that need to be used in this article.

In system (1), the processing of unknown time-varying, nonlinear, parameterized function terms $f(\chi_k, \theta(t))$ is a challenge. Since the function $\theta(t)$ is not known, $\theta(t)$ is expanded using Fourier series as $\theta(t) = M^T\Phi(t) + \delta_\theta(t), \|\delta_\theta(t)\| \le \bar{\delta}_\theta$; based on this, uncertain time-varying nonlinear functions $f(\chi_k, \theta(t))$ can be approximated as

$$f(\chi_k, \theta_k(t)) = W_k^T S(\chi_k, M_k^T\Phi(t) + \delta_{\theta,k}) + \delta_{f,k}. \tag{2}$$

A new FSE-RBFNN approximator is built:

$$G(\chi_k, t) = W_k^T S(\chi_k, M_k^T\Phi(t)), \tag{3}$$

representing $f(\chi_k, \theta_k(t))$ as

$$f(\chi_k, \theta_k(t)) = W_k^T S(\chi_k, M_k^T\Phi(t)) + \delta_k(\chi_k, t), \tag{4}$$

where

$$\delta_k(\chi_k, t) = \delta_{f,k} + W_k^T S(\chi_k, M_k^T\Phi(t) + \delta_{\theta,k}) - W_k^T S(\chi_k, M_k^T\Phi(t)). \tag{5}$$

Assumption 1: In the compact domain Ω_k, the weights W_k and M_k are constrained, and $\|W_k\| \le w_{m,k}$ and $\|M_k\| \le m_{a,k}$ with $w_{m,k}, m_{a,k}$ being unknown positive numbers.

Lemma 1[38]: For $(\chi_k, \theta_k(t)) \in \Omega_k$, $\delta_k(\chi_k, t)$ in (5) is bound, and

$$|\delta_k(\chi_k, t)| \le \overline{\delta_k}, \tag{6}$$

where $\overline{\delta_k}$ represents the supremum of $\delta_k(\chi_k, t)$.

Because W_k and M_k are unknown, we estimate them with $\hat{W}_k$ and $\hat{M}_k$, respectively. $\tilde{W}_k = \hat{W}_k - W_k$ and $\tilde{M}_k = \hat{M}_k - M_k$ are estimation errors.

Lemma 2[38]: In the surrogate model (4), the following conclusion holds:

$$\begin{aligned}&W_k^T S(\chi_k, M_k^T\Phi(t)) - \hat{W}_k^T S\left(\chi_k, \hat{M}_k^T\Phi(t)\right)\\ &= \tilde{W}_k^T\left(S\left(\chi_k, \hat{M}_k^T\Phi(t)\right) - \hat{S}_k'\hat{M}_k^T\Phi(t)\right) + \hat{W}_k^T\hat{S}_k'\tilde{M}_k^T\Phi(t) + d,\end{aligned} \tag{7}$$

where $\hat{S}_k' = [\hat{s}_{1,k}', \hat{s}_{2,k}', \ldots, \hat{s}_{p,k}'] \in \mathfrak{R}^{m\times p}$ with $\hat{s}_{i,k}' = (\partial s_i(\chi_k, \omega_k))/\partial\omega_k|_{\omega_k = \hat{M}_k\Phi(t)}$ and $i = 1, \ldots, p$, and the remainder d_k is bounded by

$$|d_k| \le \|M_k\|_F\|\Phi(t)\hat{W}_k^T\hat{S}_k'\|_F + \|W_k\|\|\hat{S}_k'\hat{M}_k^T\Phi(t)\| + |W_k|_1. \tag{8}$$

For the processing of the supremum of each error term, this article introduces the following typical series sequence:

Lemma 2[39] For a sequence $\Delta_k = \{\frac{1}{k^l}\}$, where $k = 1, 2, \cdots$ and $l \ge 2$, the following result exists:

$$lim_{k\to\infty}\Sigma_{i=1}^k \frac{1}{i^l} \le 2. \tag{9}$$

Assumption 2: The initial error value at the beginning of each iteration should meet $|z_{i,k}(0)| = \epsilon_{i,k}$ with $\epsilon_{i,k}$ being a convergence series sequence, where $i = 1, \ldots, n$.

Considering the initial errors, a new function $z_{\phi,k}$[34] is accepted:

$$\begin{aligned}z_{\phi,k} &= z_k - \phi_k(t) sat\left(\frac{z_k}{\phi_k(t)}\right)\\ \phi_k(t) &= \epsilon_k e^{-\eta t},\end{aligned} \tag{10}$$

where sat is the saturation function given as

$$sat\left(\frac{z_k}{\phi_k(t)}\right) = \begin{cases} 1 & \text{if} \quad z_k > \phi_k(t) \\ \frac{z_k}{\phi_k(t)} & \text{if} \quad z_k \le \phi_k(t) \\ -1 & \text{if} \quad z_k < -\phi_k(t) \end{cases},$$

with $\phi_k(t)$ being an updating time-varying boundary layer. When $lim_{k\to\infty} z_{\phi,k} = 0$ and considering assumption 2 again, we have $lim_{k\to\infty}|z_k| = 0$.

In order to prevent the problem of gradient explosion, we introduce the first-order low-pass filter β_k, which is given as follows:

$$\dot{\beta}_k = -\xi_k(\beta_k - \alpha_k), \tag{11}$$

where β_k results from filtering an instruction with α_k as its input, with α_k being the virtual controller, $\xi_k > 0$, and $\beta_k(0) = \alpha_k(0)$. Because part of α_k $\beta_k - \alpha_k$ cannot pass through the filter, an error compensation mechanism ζ_k is introduced to overcome the influence of the instruction filter. Therefore, a new function Z_k is introduced as follows:

$$Z_k = z_{\phi,k} - \zeta_k. \tag{12}$$

3 AILC design

Based on the above mathematical foundations, we present the specific controller design process.

3.1 Designing the AILC controller

Step 1: Denote $N_1 = \omega_{M1}^2$, which will be defined later. $z_{1,k} = x_{1,k} - y_{d1}$ and $z_{2,k} = x_{2,k} - \alpha_{1,k}$, where $\alpha_{1,k}$ is the virtual controller. Because the initial state values of the system have errors and gradient explosion, the new error functions $Z_{1,k}$ and $Z_{2,k}$ are given as

$$\begin{aligned} Z_{1,k} &= z_{1\phi,k} - \zeta_{1,k} \\ z_{1\phi,k} &= z_{1,k} - \phi_{1,k}(t) sat\left(\frac{z_{1,k}}{\phi_{1,k}(t)}\right) \\ z_{1,k} &= x_{1,k} - y_{d1} \\ \phi_{1,k}(t) &= \epsilon_{1,k} e^{-\eta_1 t}, \end{aligned} \tag{13}$$

$$\begin{aligned} Z_{2,k} &= z_{2\phi,k} - \zeta_{2,k} \\ z_{2\phi,k} &= z_{2,k} - \phi_{2,k}(t) sat\left(\frac{z_{2,k}}{\phi_{2,k}(t)}\right) \\ z_{2,k} &= x_{2,k} - \beta_{1,k} \\ \phi_{2,k}(t) &= \epsilon_{2,k} e^{-\eta_2 t}. \end{aligned} \tag{14}$$

We recall that

$$\dot{x}_{1,k} = x_{2,k} + f_1(\bar{x}_{1,k}, \theta_1(t)) + g_1(\bar{x}_{1,k}). \tag{15}$$

Given the derivative of $z_{1\phi,k}$,

$$\begin{aligned} \dot{z}_{1\phi,k} &= \begin{cases} \dot{z}_{1,k} - \dot{\phi}_{1,k} & \text{if} \quad z_{1,k} > \phi_{1,k}(t) \\ 0 & \text{if} \quad z_{1,k} \le \phi_{1,k}(t) \\ \dot{z}_{1,k} + \dot{\phi}_{1,k} & \text{if} \quad z_{1,k} < -\phi_{1,k}(t) \end{cases} \\ &= \dot{z}_{1,k} - sgn\left(z_{1\phi,k}(t)\right)\dot{\phi}_{1,k} \\ &= z_{2,k} + \beta_{1,k} + f_1(\bar{x}_{1,k}, \theta_1(t)) + g_1(\bar{x}_{1,k}) \\ &\quad - \dot{y}_{d1} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}. \end{aligned} \tag{16}$$

Therefore, the derivative of $Z_{1,k}$ with respect to time is as follows:

$$\begin{aligned} \dot{Z}_{1,k} &= z_{2,k} + \beta_{1,k} + f_1(\bar{x}_{1,k}, \theta_1(t)) + g_1(\bar{x}_{1,k}) \\ &\quad - \dot{y}_{d1} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k} - \dot{\zeta}_{1,k}. \end{aligned} \tag{17}$$

The error compensation mechanism is considered as follows:

$$\dot{\zeta}_{1,k} = \beta_{1,k} + \zeta_{2,k} - \eta_1 \zeta_{1,k} - \alpha_{1,k}. \tag{18}$$

Using Equation 18, we can find the time derivative of the error function as follows:

$$\begin{aligned} \dot{Z}_{1,k} &= z_{2,k} - \zeta_{2,k} + \eta_{1,k}\zeta_{1,k} + \alpha_{1,k} - \dot{y}_{d1} \\ &\quad + f_1(\bar{x}_{1,k}, \theta_1(t)) + g_1(\bar{x}_{1,k}) - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}. \end{aligned} \tag{19}$$

The unknown time-varying, nonlinear functions $f_1(\bar{x}_{1,k}, \theta_1(t))$ and $g_1(\bar{x}_{1,k})$ may be approximated by FSE-RBFNN and RBFNN, respectively.

$$\begin{aligned} f_1(\bar{x}_{1,k}, \theta_1(t)) &= W_{f1}^T S_{f1}(\bar{x}_{1,k}, M_1^T \phi_1(t)) + \delta_{f1} \\ g_1(\bar{x}_{1,k}) &= W_{g1}^T S_{g1}(\bar{x}_{1,k}) + \delta_{g1}, \end{aligned} \tag{20}$$

where δ_{f1} and δ_{g1} are the truncation errors after approximation and W_{f1} and W_{g1} are weight vectors.

Consider $\Delta_k = \frac{a}{k^l}$, $a > 0$, and $l \ge 2$. The virtual control law is designed as

$$\begin{aligned} \alpha_{1,k} &= -\hat{W}_{f1,k}^T S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T \Phi_1(t)\right) - \hat{W}_{g1,k}^T S_{g1}(\bar{x}_{1,k}) \\ &\quad - \hat{N}_{1,k}\frac{1}{\Delta_k} Z_{1,k} + \dot{y}_{d1} - \eta_1 z_{1,k}. \end{aligned} \tag{21}$$

By substituting Equations 20, 21 into Equation 19, we obtain

$$\begin{aligned} \dot{Z}_{1,k} &= z_{2,k} - \hat{N}_{1,k}\frac{1}{\Delta_k} Z_{1,k} - \zeta_{2,k} + \eta_{1,k}\zeta_{1,k} \\ &\quad + W_{f1}^T S_{f1}(\bar{x}_{1,k}, M_1^T \Phi_1(t)) + \delta_{f1} - \hat{W}_{f1,k}^T S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T \Phi_1(t)\right) \\ &\quad + W_{g1}^T S_{g1}(\bar{x}_{1,k}) + \delta_{g1} - \hat{W}_{g1,k}^T S_{g1}(\bar{x}_{1,k}) \\ &\quad - \eta_1 z_{1,k} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}(t) \\ &= Z_{2,k} - \hat{N}_{1,k}\frac{1}{\Delta_k} Z_{1,k} - \zeta_{2,k} + \eta_{1,k}\zeta_{1,k} \\ &\quad + W_{f1}^T S_{f1}(\bar{x}_{1,k}, M_1^T \Phi_1(t)) - \hat{W}_{f1,k}^T S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T \Phi_1(t)\right) \\ &\quad + W_{g1}^T S_{g1}(\bar{x}_{1,k}) - \hat{W}_{g1,k}^T S_{g1}(\bar{x}_{1,k}) + \delta_{f1} + \delta_{g1} \\ &\quad + \phi_{2,k}(t) sat\left(\frac{z_{2,k}}{\phi_{2,k}(t)}\right) + \zeta_{2,k} \\ &\quad - \eta_1 z_{1,k} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}(t) \\ &= Z_{2,k} - \hat{N}_{1,k}\frac{1}{\Delta_k} Z_{1,k} + \phi_{2,k}(t) sat\left(\frac{z_{2,k}}{\phi_{2,k}(t)}\right) \\ &\quad + W_{f1}^T S_{f1}(\bar{x}_{1,k}, M_1^T \Phi_1(t)) - \hat{W}_{f1,k}^T S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T \Phi_1(t)\right) \\ &\quad + W_{g1}^T S_{g1}(\bar{x}_{1,k}) - \hat{W}_{g1,k}^T S_{g1}(\bar{x}_{1,k}) + \delta_{f1} + \delta_{g1} \\ &\quad - \eta_1 z_{1,k} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}(t) + \eta_{1,k}\zeta_{1,k}, \end{aligned} \tag{22}$$

where $\hat{W}_{f1,k}$, $\hat{W}_{g1,k}$, $\hat{M}_{1,k}$, and $\hat{N}_{1,k}$ are estimations of W_{f1}, W_{g1}, M_1, and N_1, respectively. $\tilde{W}_{f1,k} = \hat{W}_{f1,k} - W_{f1}$, $\tilde{W}_{g1,k} = \hat{W}_{g1,k} - W_{g1}$, $\tilde{M}_{1,k} = \hat{M}_{1,k} - M_1$, and $\tilde{N}_{1,k} = \hat{N}_{1,k} - N_1$ are the estimation errors. It can be proved that the following result is correct.

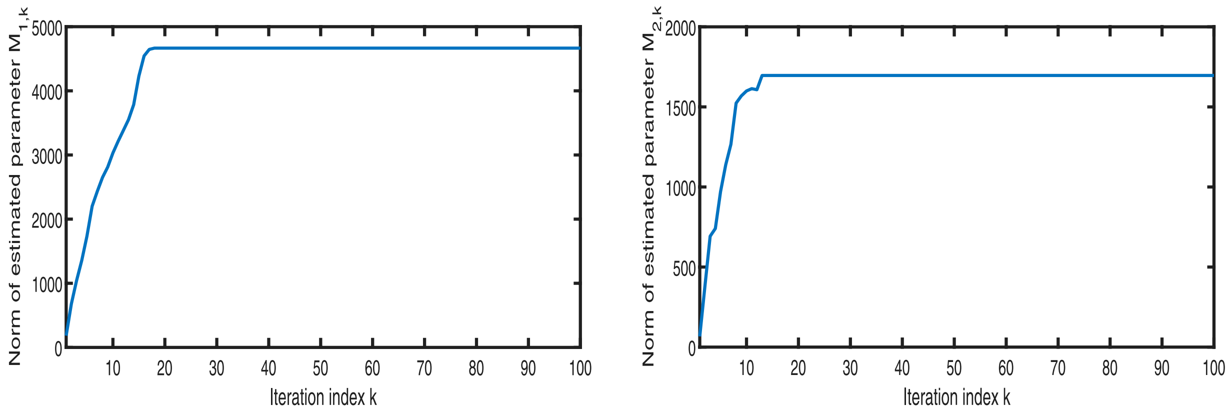

FIGURE 9
Variation in $\|\hat{M}_{1,k}\|$ and $\|\hat{M}_{2,k}\|$ according to the iteration index.

$$\begin{aligned} -\eta_1 z_{1,k} - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}(t) + \eta_1\zeta_{1,k} &= -\eta_1 z_{1\phi,k} - \eta_1\phi_{1,k}(t) sat\left(\frac{z_{1,k}}{\phi_{1,k}(t)}\right) \\ &\quad - sgn\left(z_{1\phi,k}\right)\dot{\phi}_{1,k}(t) + \eta_1\zeta_{1,k} \\ &= -\eta_1 z_{1\phi,k} + \eta_1\zeta_{1,k} \\ &\quad - sgn\left(z_{1\phi,k}\right)\left(\dot{\phi}_{1,k}(t) + \eta_1\phi_{1,k}(t)\right) \\ &= -\eta_1\left(z_{1\phi,k} - \zeta_{1,k}\right) \\ &= -\eta_1 Z_{1,k}. \end{aligned} \tag{23}$$

Using Equations 7, 23, Equation 22 can be rewritten as

$$\begin{aligned} \dot{Z}_{1,k} &= Z_{2,k} - \hat{N}_{1,k}\frac{1}{\Delta_k}Z_{1,k} - \eta_1 Z_{1,k} \\ &\quad + \tilde{W}_{f1}^T\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right) \\ &\quad + \hat{W}_{f1,k}^T\hat{S}'_{f1,k}\tilde{M}_{1,k}^T\Phi_1(t) - \tilde{W}_{g1}^T S_{g1}(\bar{x}_{1,k}) \\ &\quad + d_1 + \delta_{f1} + \delta_{g1} + \phi_{2,k}(t) sat\left(\frac{z_{2,k}}{\phi_{2,k}(t)}\right). \end{aligned} \tag{24}$$

Let $\omega_1 = d_1 + \delta_{f1} + \delta_{g1} + \phi_{2,k}(t) sat\left(\frac{z_{2,k}(t)}{\phi_{2,k}(t)}\right)$, where d_1 is the remaining term of the estimation error after FSE-RBFNN expansion, and d_i is also the same; then, Equation 24 becomes

$$\begin{aligned} \dot{Z}_{1,k} &= Z_{2,k} - \hat{N}_{1,k}\frac{1}{\Delta_k}Z_{1,k} - \eta_1 Z_{1,k} + \omega_1 \\ &\quad + \tilde{W}_{f1}^T\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right) \\ &\quad + \hat{W}_{f1,k}^T\hat{S}'_{f1,k}\tilde{M}_{1,k}^T\Phi_1(t) - \tilde{W}_{g1}^T S_{g1}(\bar{x}_{1,k}). \end{aligned} \tag{25}$$

Assumption 3 The remainder $\omega_i = d_i + \delta_{fi} + \delta_{gi} + \phi_{i+1,k}(t) sat\left(\frac{z_{i+1,k}(t)}{\phi_{i+1,k}(t)}\right)$ $(i = 1, 2, \ldots, n-1)$ is bounded with $|\omega_i| \le \omega_{Mi}$ and $\omega_{Mi} > 0$.

Remark 1: This assumption is easily satisfied because 1) d_i, δ_{fi}, and δ_{gi} are bounded and 2) when η_i is large enough, $\phi_{i,k}(t) sat\left(\frac{z_{i,k}(t)}{\phi_{i,k}(t)}\right)$ is sufficiently small.

The Lyapunov-like function is chosen as follows:

$$\begin{aligned} V_{1,k} &= \frac{1}{2}Z_{1,k}^2 + \frac{1}{2}\tilde{W}_{f1,k}^T\Gamma_{f11}^{-1}\tilde{W}_{f1,k} + \frac{1}{2}\tilde{W}_{g1,k}^T\Gamma_{g11}^{-1}\tilde{W}_{g1,k} \\ &\quad + \frac{1}{2}\tilde{M}_{1,k}^T\Gamma_{m11}^{-1}\tilde{M}_{1,k} + \frac{1}{2}\Gamma_{n11}^{-1}\tilde{N}_{1,k}^2, \end{aligned} \tag{26}$$

where Γ_{f11}, Γ_{g11}, Γ_{m11}, and Γ_{n11} are adjustable matrices, each being positive, definite, and symmetric. Consider the derivative of $V_{1,k}$ by system (25), we obtain

$$\begin{aligned} \dot{V}_{1,k} &= Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 \\ &\quad + \tilde{W}_{f1,k}^T\Gamma_{f11}^{-1}\left(\Gamma_{f11}\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right)Z_{1,k}\right. \\ &\quad \left. + \dot{\hat{W}}_{f1,k}\right) - \tilde{W}_{g1,k}^T\Gamma_{g11}^{-1}\left(\Gamma_{g11}S_{g1}(\bar{x}_{1,k})Z_{1,k} - \dot{\hat{W}}_{g1,k}\right) \\ &\quad + \tilde{M}_{1,k}^T\Gamma_{m11}^{-1}\left(\Gamma_{m11}\Phi_1(t)\hat{W}_{f1,k}^T\hat{S}'_{f1,k}Z_{1,k} + \dot{\hat{M}}_{1,k}\right) \\ &\quad - \hat{N}_{1,k}\frac{1}{\Delta_k}Z_{1,k}^2 + \omega_{1,k}Z_{1,k} + \Gamma_{n11}^{-1}\tilde{N}_{1,k}\dot{\hat{N}}_{1,k} \\ &\le Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 \\ &\quad + \tilde{W}_{f1,k}^T\Gamma_{f11}^{-1}\left(\Gamma_{f11}\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right)Z_{1,k}\right. \\ &\quad \left. + \dot{\hat{W}}_{f1,k}\right) - \tilde{W}_{g1,k}^T\Gamma_{g11}^{-1}\left(\Gamma_{g11}S_{g1}(\bar{x}_{1,k})Z_{1,k} - \dot{\hat{W}}_{g1,k}\right) \\ &\quad + \tilde{M}_{1,k}^T\Gamma_{m11}^{-1}\left(\Gamma_{m11}\Phi_1(t)\hat{W}_{f1,k}^T\hat{S}'_{f1,k}Z_{1,k} + \dot{\hat{M}}_{1,k}\right) \\ &\quad - \hat{N}_{1,k}\frac{1}{\Delta_k}Z_{1,k}^2 + \frac{1}{\Delta_k}\omega_{M1}^2 Z_{1,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{n11}^{-1}\tilde{N}_{1,k}\dot{\hat{N}}_{1,k} \\ &= Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 \\ &\quad + \tilde{W}_{f1,k}^T\Gamma_{f11}^{-1}\left(\Gamma_{f11}\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right)Z_{1,k}\right. \\ &\quad \left. + \dot{\hat{W}}_{f1,k}\right) - \tilde{W}_{g1,k}^T\Gamma_{g11}^{-1}\left(\Gamma_{g11}S_{g1}(\bar{x}_{1,k})Z_{1,k} - \dot{\hat{W}}_{g1,k}\right) \\ &\quad + \tilde{M}_{1,k}^T\Gamma_{m11}^{-1}\left(\Gamma_{m11}\Phi_1(t)\hat{W}_{f1,k}^T\hat{S}'_{f1,k}Z_{1,k} + \dot{\hat{M}}_{1,k}\right) \\ &\quad - \hat{N}_{1,k}\frac{1}{\Delta_k}Z_{1,k}^2 + \frac{1}{\Delta_k}N_{1,k}Z_{1,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{n11}^{-1}\tilde{N}_{1,k}\dot{\hat{N}}_{1,k} \\ &= Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 \\ &\quad + \tilde{W}_{f1,k}^T\Gamma_{f11}^{-1}\left(\Gamma_{f11}\left(S_{f1}\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\right) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\right)Z_{1,k}\right. \\ &\quad \left. + \dot{\hat{W}}_{f1,k}\right) - \tilde{W}_{g1,k}^T\Gamma_{g11}^{-1}\left(\Gamma_{g11}S_{g1}(\bar{x}_{1,k})Z_{1,k} - \dot{\hat{W}}_{g1,k}\right) \\ &\quad + \tilde{M}_{1,k}^T\Gamma_{m11}^{-1}\left(\Gamma_{m11}\Phi_1(t)\hat{W}_{f1,k}^T\hat{S}'_{f1,k}Z_{1,k} + \dot{\hat{M}}_{1,k}\right) \\ &\quad - \tilde{N}_{1,k}\Gamma_{n11}^{-1}\left(\Gamma_{n11}\frac{1}{\Delta_k}Z_{1,k}^2 - \dot{\hat{N}}_{1,k}\right) + \frac{1}{4}\Delta_k, \end{aligned} \tag{27}$$

where for any $r > 0$ and $mn \le \frac{1}{r}m^2 + \frac{1}{4}n^2 r$, $r = \Delta_k$.

We choose

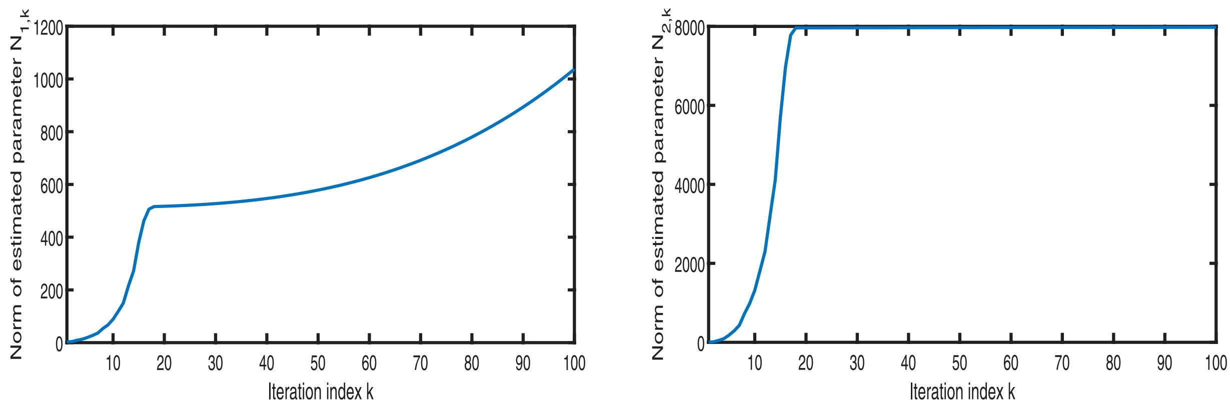

FIGURE 10
Variation in $\|\hat{N}_{1,k}\|$ and $\|\hat{N}_{2,k}\|$ according to the iteration index.

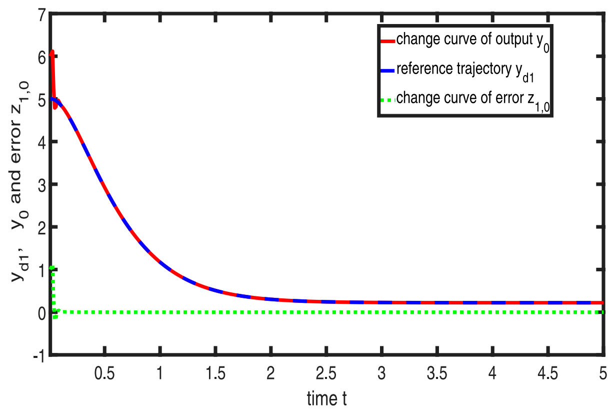

FIGURE 11
Variation in $y_0, y_{d1}, z_{1,0}$ over time without iteration.

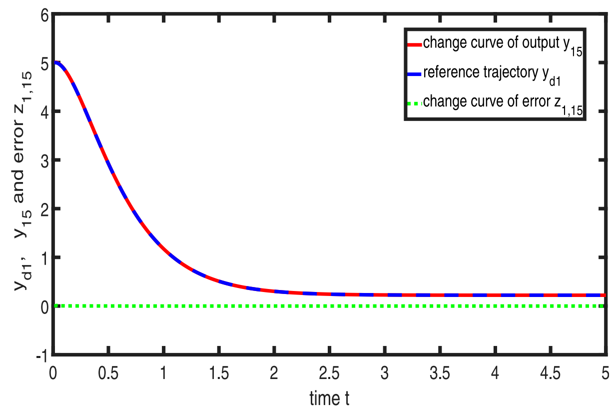

FIGURE 12
Variation in $y_{15}, y_{d1}, z_{1,15}$ over time during the 15th iteration.

$$
\begin{aligned}
\dot{\hat{W}}_{f1,k} &= -\Gamma_{f11}\Big(S_{f1}\Big(\bar{x}_{1,k}, \hat{M}_{1,k}^T\Phi_1(t)\Big) - \hat{S}'_{f1,k}\hat{M}_{1,k}^T\Phi_1(t)\Big)Z_{1,k} \\
\dot{\hat{W}}_{g1,k} &= \Gamma_{g11}S_{g1}(\bar{x}_{1,k})Z_{1,k} \\
\dot{\hat{M}}_{1,k} &= -\Gamma_{m11}\Phi_1(t)\hat{W}_{f1,k}^T\hat{S}'_{f1,k}Z_{1,k} \\
\dot{\hat{N}}_{1,k} &= \Gamma_{n11}\frac{1}{\Delta_k}Z_{1,k}^2,
\end{aligned} \tag{28}
$$

so Equation 27 becomes

$$
\dot{V}_{1,k} \le Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 + \frac{1}{4}\Delta_k. \tag{29}
$$

Step 2: Denote $N_2 = \omega_{M2}^2$, which will be defined later. Due to initial state errors and gradient explosion, we introduce the following error function $Z_{3,k}$ as

$$
\begin{aligned}
Z_{3,k} &= z_{3\phi,k} - \zeta_{3,k} \\
z_{3\phi,k} &= z_{3,k} - \phi_{3,k}(t)sat\left(\frac{z_{3,k}}{\phi_{3,k}(t)}\right) \\
z_{3,k} &= x_{3,k} - \beta_{2,k} \\
\phi_{3,k}(t) &= \epsilon_{3,k}e^{-\eta_3 t}.
\end{aligned} \tag{30}
$$

The derivative of $Z_{2,k}$ is shown as follows:

$$
\begin{aligned}
\dot{Z}_{2,k} &= \dot{z}_{2,k} - sgn\big(z_{2\phi,k}(t)\big)\dot{\phi}_{2,k} - \dot{\zeta}_{2,k} \\
&= z_{3,k} + \beta_{2,k} + f_2(\bar{x}_{2,k}, \theta_2(t)) + g_2(\bar{x}_{2,k}) \\
&\quad -\dot{\beta}_{1,k} - sgn\big(z_{2\phi,k}\big)\dot{\phi}_{2,k} - \dot{\zeta}_{2,k}.
\end{aligned} \tag{31}
$$

Let the error compensation mechanism be defined as follows:

$$
\dot{\zeta}_{2,k} = \beta_{2,k} + \zeta_{3,k} - \eta_2\zeta_{2,k} - \zeta_{1,k} - \alpha_{2,k}. \tag{32}
$$

Using Equation 32, we can find the time derivative of error function as

$$
\begin{aligned}
\dot{Z}_{2,k} &= z_{3,k} - \zeta_{3,k} + \eta_{2,k}\zeta_{2,k} + \zeta_{1,k} + \alpha_{2,k} - \dot{\beta}_{1,k} \\
&\quad + f_2(\bar{x}_{2,k}, \theta_2(t)) + g_2(\bar{x}_{2,k}) - sgn\big(z_{2\phi,k}\big)\dot{\phi}_{2,k}.
\end{aligned} \tag{33}
$$

The uncertain time-varying, nonlinear functions $f_2(\bar{x}_{2,k}, \theta_2(t))$ and $G_2(\bar{x}_{2,k})$ are approximated by FSE-RBFNN and RBFNN, respectively.

$$
\begin{aligned}
f_2(\bar{x}_{2,k}, \theta_2(t)) &= W_{f2}^T S_{f2}(\bar{x}_{2,k}, M_2^T\Phi_2(t)) + \delta_{f2} \\
G_2(\bar{x}_{2,k}) &= W_{g2}^T S_{g2}(\bar{x}_{2,k}) + \delta_{g2},
\end{aligned} \tag{34}
$$

where δ_{f2} and δ_{g2} are reconstructed errors and W_{f2} and W_{g2} are optimal weight vectors.

Let the virtual control be defined as follows:

$$\begin{aligned}\alpha_{2,k} = & -\hat{W}_{f2,k}^T S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{W}_{g2,k}^T S_{g2}(\bar{x}_{2,k}) \\ & -\hat{N}_{2,k}\frac{1}{\Delta_k} Z_{2,k} + \dot{\beta}_{1,k} - \eta_2 z_{2,k} - z_{1\phi,k}.\end{aligned} \tag{35}$$

Substituting Equations 34, 35 into Equation 33, we obtain

$$\begin{aligned}\dot{Z}_{2,k} = & -z_{1\phi,k} + \zeta_{1,k} + z_{3,k} - \phi_{3,k}(t) sat\left(\frac{z_{3,k}}{\phi_{3,k}(t)}\right) - \zeta_{3,k} \\ & + W_{f2}^T S_{f2}(\bar{x}_{2,k}, M_2^T \Phi_2(t)) + \delta_{f2} - \hat{W}_{f2,k}^T S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) \\ & + W_{g2}^T S_{g2}(\bar{x}_{2,k}) + \delta_{g2} - \hat{W}_{g2,k}^T S_{g2}(\bar{x}_{2,k}) \\ & -\hat{N}_{2,k}\frac{1}{\Delta_k} z_{2\phi,k} + \phi_{3,k}(t) sat\left(\frac{z_{3,k}}{\phi_{3,k}(t)}\right) \\ & -\eta_2 z_{2,k} + \eta_2 \zeta_{2,k} - sgn\left(z_{2\phi,k}\right)\dot{\phi}_{2,k} \\ = & -Z_{1,k} + Z_{3,k} - \hat{N}_{2,k}\frac{1}{\Delta_k} z_{2\phi,k} + \phi_{3,k}(t) sat\left(\frac{z_{3,k}}{\phi_{3,k}(t)}\right) \\ & + W_{f2}^T S_{f2}(\bar{x}_{2,k}, M_2^T \Phi_2(t)) + \delta_{f2} - \hat{W}_{f2,k}^T S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) \\ & + W_{g2}^T S_{g2}(\bar{x}_{2,k}) + \delta_{g2} - \hat{W}_{g2,k}^T S_{g2}(\bar{x}_{2,k}) \\ & -\eta_2 z_{2,k} + \eta_2 \zeta_{2,k} - sgn\left(z_{2\phi,k}\right)\dot{\phi}_{2,k},\end{aligned} \tag{36}$$

where $\hat{W}_{f2,k}$, $\hat{W}_{g2,k}$, $\hat{M}_{2,k}$, and $\hat{N}_{2,k}$ are the estimators of W_{f2}, W_{g2}, M_2, and N_2, respectively. $\tilde{W}_{f2,k} = \hat{W}_{f2,k} - W_{f2}$, $\tilde{W}_{g2,k} = \hat{W}_{g2,k} - W_{g2}$, $\tilde{M}_{2,k} = \hat{M}_{2,k} - M_2$, and $\tilde{N}_{2,k} = \hat{N}_{2,k} - N_2$ are estimation errors. It can be proved that the following results are correct.

$$\begin{aligned}-\eta_2 z_{2,k} - sgn\left(z_{2\phi,k}\right)\dot{\phi}_{2,k}(t) + \eta_2 \zeta_{2,k} = & -\eta_2 z_{2\phi,k} - \eta_2 \phi_{2,k}(t) sat\left(\frac{z_{2,k}}{\phi_{2,k}(t)}\right) \\ & -sgn\left(z_{2\phi,k}\right)\dot{\phi}_{2,k}(t) + \eta_2 \zeta_{2,k} \\ = & -\eta_2 z_{2\phi,k} + \eta_2 \zeta_{2,k} \\ & -sgn\left(z_{2\phi,k}\right)\left(\dot{\phi}_{2,k}(t) + \eta_2 \phi_{2,k}(t)\right) \\ = & -\eta_2 z_{2\phi,k} + \eta_2 \zeta_{2,k} \\ = & -\eta_2\left(z_{2\phi,k} - \zeta_{2,k}\right) \\ = & -\eta_2 Z_{2,k}.\end{aligned} \tag{37}$$

Using Equations 7, 37, Equation 36 can be written as

$$\begin{aligned}\dot{Z}_{2,k} = & -Z_{1,k} + Z_{3,k} - \hat{N}_{2,k}\frac{1}{\Delta_k} z_{2\phi,k} - \eta_2 Z_{2,k} \\ & + \tilde{W}_{f2}^T\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right) \\ & + \hat{W}_{f2,k}^T \hat{S}'_{f2,k}\tilde{M}_{2,k}^T \Phi_2(t) - \tilde{W}_{g2}^T S_{g2}(\bar{x}_{2,k}) \\ & + d_2 + \delta_{f2} + \delta_{g2} + \phi_{3,k}(t) sat\left(\frac{z_{3,k}}{\phi_{3,k}(t)}\right).\end{aligned} \tag{38}$$

Let $\omega_2 = d_2 + \delta_{f2} + \delta_{g2} + \phi_{3,k}(t) sat\left(\frac{z_{3,k}(t)}{\phi_{3,k}(t)}\right)$, then Equation 38 becomes

$$\begin{aligned}\dot{Z}_{2,k} = & -Z_{1,k} + Z_{3,k} - \hat{N}_{2,k}\frac{1}{\Delta_k} Z_{2,k} - \eta_2 Z_{2,k} + \omega_2 \\ & + \tilde{W}_{f2,k}^T\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right) \\ & + \hat{W}_{f2,k}^T \hat{S}'_{f2,k}\tilde{M}_{2,k}^T \Phi_2(t) - \tilde{W}_{g2}^T S_{g2}(\bar{x}_{2,k}).\end{aligned} \tag{39}$$

The Lyapunov-like function was chosen as follows:

$$\begin{aligned}V_{2,k} = & V_{1,k} + \frac{1}{2}Z_{2,k}^2 + \frac{1}{2}\tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\tilde{W}_{f2,k} + \frac{1}{2}\tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\tilde{W}_{g2,k} \\ & + \frac{1}{2}\tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\tilde{M}_{2,k} + \frac{1}{2}\Gamma_{n21}^{-1}\tilde{N}_{2,k}^2,\end{aligned} \tag{40}$$

where Γ_{f21}, Γ_{g21}, Γ_{m21}, and Γ_{n21} are adjustable, positive, definite, and symmetric matrices. According to Equation 39, Assumption 3, and Remark 1, $V_{2,k}$ can be expressed as

$$\begin{aligned}\dot{V}_{2,k} = & \dot{V}_{1,k} + Z_{2,k}\dot{Z}_{2,k} + \tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\dot{\hat{W}}_{f2,k} \\ & + \tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\dot{\hat{W}}_{g2,k} + \tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\dot{\hat{M}}_{2,k} + \Gamma_{n21}^{-1}\tilde{N}_{2,k}\dot{\hat{N}}_{2,k} \\ \le & Z_{1,k}Z_{2,k} - \eta_1 Z_{1,k}^2 + \frac{1}{4}\Delta_k - Z_{1,k}Z_{2,k} + Z_{2,k}Z_{3,k} - \eta_2 Z_{2,k}^2 \\ & + \tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\left(\Gamma_{f21}\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right)Z_{2,k}\right. \\ & \left.+ \dot{\hat{W}}_{f2,k}\right) - \tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\left(\Gamma_{g21}S_{g2}(\bar{x}_{2,k})Z_{2,k} - \dot{\hat{W}}_{g2,k}\right) \\ & + \tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\left(\Gamma_{m21}\Phi_2(t)\hat{W}_{f2,k}^T \hat{S}'_{f2,k}Z_{2,k} + \dot{\hat{M}}_{2,k}\right) \\ & -\hat{N}_{2,k}\frac{1}{\Delta_k}Z_{2,k}^2 + \omega_2 Z_{2,k} + \Gamma_{n21}^{-1}\tilde{N}_{2,k}\dot{\hat{N}}_{2,k} \\ \le & -\eta_1 Z_{1,k}^2 + \frac{1}{4}\Delta_k + Z_{2,k}Z_{3,k} - \eta_2 Z_{2,k}^2 \\ & + \tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\left(\Gamma_{f21}\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right)Z_{2,k}\right. \\ & \left.+ \dot{\hat{W}}_{f2,k}\right) - \tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\left(\Gamma_{g21}S_{g2}(\bar{x}_{2,k})Z_{2,k} - \dot{\hat{W}}_{g2,k}\right) \\ & + \tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\left(\Gamma_{m21}\Phi_2(t)\hat{W}_{f2,k}^T \hat{S}'_{f2,k}Z_{2,k} + \dot{\hat{M}}_{2,k}\right) \\ & -\hat{N}_{2,k}\frac{1}{\Delta_k}Z_{2,k}^2 + \frac{1}{\Delta_k}\omega_{M2}^2 Z_{2,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{n21}^{-1}\tilde{N}_{2,k}\dot{\hat{N}}_{2,k} \\ = & Z_{2,k}Z_{3,k} - \sum_{i=1}^{2}\eta_i Z_{i,k}^2 + \frac{1}{4}\Delta_k \\ & + \tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\left(\Gamma_{f21}\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right)Z_{2,k}\right. \\ & \left.+ \dot{\hat{W}}_{f2,k}\right) - \tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\left(\Gamma_{g21}S_{g2}(\bar{x}_{2,k})Z_{2,k} - \dot{\hat{W}}_{g2,k}\right) \\ & + \tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\left(\Gamma_{m21}\Phi_2(t)\hat{W}_{f2,k}^T \hat{S}'_{f2,k}Z_{2,k} + \dot{\hat{M}}_{2,k}\right) \\ & -\hat{N}_{2,k}\frac{1}{\Delta_k}Z_{2,k}^2 + \frac{1}{\Delta_k}N_{2,k}Z_{2,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{n21}^{-1}\tilde{N}_{2,k}\dot{\hat{N}}_{2,k} \\ = & Z_{2,k}Z_{3,k} - \sum_{i=1}^{2}\eta_i Z_{i,k}^2 + \frac{2}{4}\Delta_k \\ & + \tilde{W}_{f2,k}^T \Gamma_{f21}^{-1}\left(\Gamma_{f21}\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right)Z_{2,k}\right. \\ & \left.+ \dot{\hat{W}}_{f2,k}\right) - \tilde{W}_{g2,k}^T \Gamma_{g21}^{-1}\left(\Gamma_{g21}S_{g2}(\bar{x}_{2,k})Z_{2,k} - \dot{\hat{W}}_{g2,k}\right) \\ & + \tilde{M}_{2,k}^T \Gamma_{m21}^{-1}\left(\Gamma_{m21}\Phi_2(t)\hat{W}_{f2,k}^T \hat{S}'_{f2,k}Z_{2,k} + \dot{\hat{M}}_{2,k}\right) \\ & -\tilde{N}_{2,k}\Gamma_{n21}^{-1}\left(\Gamma_{n21}\frac{1}{\Delta_k}Z_{2,k}^2 - \dot{\hat{N}}_{2,k}\right).\end{aligned} \tag{41}$$

We choose

$$\begin{aligned}\dot{\hat{W}}_{f2,k} = & -\Gamma_{f21}\left(S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{S}'_{f2,k}\hat{M}_{2,k}^T \Phi_2(t)\right)Z_{2,k} \\ \dot{\hat{W}}_{g2,k} = & \Gamma_{g21}S_{g2}(\bar{x}_{2,k})Z_{2,k} \\ \dot{\hat{M}}_{2,k} = & -\Gamma_{m21}\Phi_2(t)\hat{W}_{f2,k}^T \hat{S}'_{f2,k}Z_{2,k} \\ \dot{\hat{N}}_{2,k} = & \Gamma_{n21}\frac{1}{\Delta_k}Z_{2,k}^2.\end{aligned} \tag{42}$$

Then, Equation 41 can be changed as

$$\dot{V}_{2,k} \le Z_{2,k}Z_{3,k} - \sum_{i=1}^{2}\eta_i Z_{i,k}^2 + \frac{2}{4}\Delta_k. \tag{43}$$

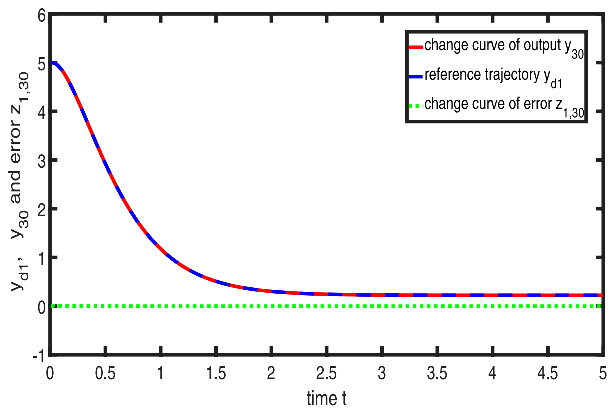

FIGURE 13
Variation in $y_{30}, y_{d1}, z_{1,30}$ over time during the 30th iteration.

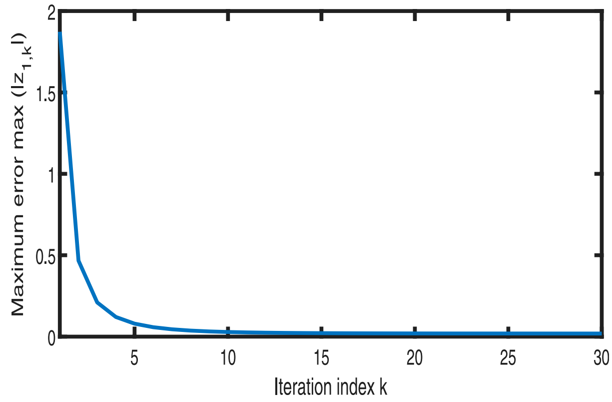

FIGURE 14
Variation in $max\left(|z_{1,k}|\right)$ according to the iteration index.

Step i: $(3 \le i \le n-1)$**.** Denote $N_i = \omega_{Mi}^2$, which will be defined later. Because there exist initial state errors and gradient explosion, the error functions $Z_{i,k}$ and $Z_{i+1,k}$ are defined as

$$\begin{aligned} Z_{i,k} &= z_{i\phi,k} - \zeta_{i,k} \\ z_{i\phi,k} &= z_{i,k} - \phi_{i,k}(t) sat\left(\frac{z_{i,k}}{\phi_{i,k}(t)}\right) \\ z_{i,k} &= x_{i,k} - \beta_{i-1,k} \\ \phi_{i,k}(t) &= \epsilon_{i,k} e^{-\eta_i t}, \end{aligned} \tag{44}$$

$$\begin{aligned} Z_{i+1,k} &= z_{i+1\phi,k} - \zeta_{i+1,k} \\ z_{i+1\phi,k} &= z_{i+1,k} - \phi_{i+1,k}(t) sat\left(\frac{z_{i+1,k}}{\phi_{i+1,k}(t)}\right) \\ z_{i+1,k} &= x_{i+1,k} - \beta_{i,k} \\ \phi_{i+1,k}(t) &= \epsilon_{i+1,k} e^{-\eta_{i+1} t}. \end{aligned} \tag{45}$$

Therefore, $\dot{Z}_{i,k}$ can be deduced as follows:

$$\begin{aligned} \dot{Z}_{i,k} &= \dot{z}_{i,k} - sgn\left(z_{i\phi,k}(t)\right)\dot{\phi}_{i,k} - \dot{\zeta}_{i,k} \\ &= z_{i+1,k} + \beta_{i,k} + f_i\left(\bar{x}_{i,k}, \theta_i(t)\right) + g_i\left(\bar{x}_{i,k}\right) \\ &\quad - \dot{\beta}_{i-1,k} - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k} - \dot{\zeta}_{i,k}. \end{aligned} \tag{46}$$

Let the error compensation mechanism be defined as

$$\dot{\zeta}_{i,k} = \beta_{i,k} + \zeta_{i+1,k} - \eta_i \zeta_{i,k} - \zeta_{i-1,k} - \alpha_{i,k}. \tag{47}$$

Using Equation 47, we can find the time derivative of the error function as

$$\begin{aligned} \dot{Z}_{i,k} &= z_{i+1,k} - \zeta_{i+1,k} + \eta_{i,k}\zeta_{i,k} + \zeta_{i-1,k} + \alpha_{i,k} - \dot{\beta}_{i-1,k} \\ &\quad + f_i\left(\bar{x}_{i,k}, \theta_i(t)\right) + g_i\left(\bar{x}_{i,k}\right) - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k}. \end{aligned} \tag{48}$$

The uncertain time-varying, nonlinear functions $f_i\left(\bar{x}_{i,k}, \theta_i(t)\right)$ and $G_i\left(\bar{x}_{i,k}\right)$ are approximated by FSE-RBFNN and RBFNN, respectively, and reconstruction errors δ_{fi} and δ_{gi} are as given follows:

$$\begin{aligned} f_i\left(\bar{x}_{i,k}, \theta_i(t)\right) &= W_{fi}^T S_{fi}\left(\bar{x}_{i,k}, M_i^T \Phi_i(t)\right) + \delta_{fi} \\ G_i\left(\bar{x}_{i,k}\right) &= W_{gi}^T S_{gi}\left(\bar{x}_{i,k}\right) + \delta_{gi}, \end{aligned} \tag{49}$$

where δ_{fi} and δ_{gi} are the approximation errors and W_{fi} and W_{gi} are ideal weight vectors.

Define $\Delta_k = \frac{a}{k^l}$, where a is any arbitrary number with $a > 0$; meanwhile, $l \ge 2$. Let the virtual control be defined as

$$\begin{aligned} \alpha_{i,k} &= -\hat{W}_{fi,k}^T S_{fi}\left(\bar{x}_{i,k}, \hat{M}_{i,k}^T \Phi_i(t)\right) - \hat{W}_{gi,k}^T S_{gi}\left(\bar{x}_{i,k}\right) \\ &\quad - \hat{N}_{i,k}\frac{1}{\Delta_k} Z_{i,k} + \dot{\beta}_{i-1,k} - \eta_i z_{i,k} - z_{i-1\phi,k}. \end{aligned} \tag{50}$$

By substituting Equations 49, 50 into Equation 48, we obtain

$$\begin{aligned} \dot{Z}_{i,k} &= -z_{i-1\phi,k} + \zeta_{i-1,k} + z_{i+1,k} - \phi_{i+1,k}(t) sat\left(\frac{z_{i+1,k}}{\phi_{i+1,k}(t)}\right) - \zeta_{i+1,k} \\ &\quad + W_{fi}^T S_{fi}\left(\bar{x}_{i,k}, M_i^T \Phi_i(t)\right) + \delta_{fi} - \hat{W}_{fi,k}^T S_{fi}\left(\bar{x}_{i,k}, \hat{M}_{i,k}^T \Phi_i(t)\right) \\ &\quad + W_{gi}^T S_{gi}\left(\bar{x}_{i,k}\right) + \delta_{gi} - \hat{W}_{gi,k}^T S_{gi}\left(\bar{x}_{i,k}\right) \\ &\quad - \hat{N}_{i,k}\frac{1}{\Delta_k} z_{i\phi,k} + \phi_{i+1,k}(t) sat\left(\frac{z_{i+1,k}}{\phi_{i+1,k}(t)}\right) \\ &\quad - \eta_i z_{i,k} + \eta_i \zeta_{i,k} - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k} \\ &= -Z_{i-1,k} + Z_{i+1,k} - \hat{N}_{i,k}\frac{1}{\Delta_k} z_{i\phi,k} + \phi_{i+1,k}(t) sat\left(\frac{z_{i+1,k}}{\phi_{i+1,k}(t)}\right) \\ &\quad + W_{fi}^T S_{fi}\left(\bar{x}_{i,k}, M_i^T \Phi_i(t)\right) + \delta_{fi} - \hat{W}_{fi,k}^T S_{fi}\left(\bar{x}_{i,k}, \hat{M}_{i,k}^T \Phi_i(t)\right) \\ &\quad + W_{gi}^T S_{gi}\left(\bar{x}_{i,k}\right) + \delta_{gi} - \hat{W}_{gi,k}^T S_{gi}\left(\bar{x}_{i,k}\right) \\ &\quad - \eta_i z_{i,k} + \eta_i \zeta_{i,k} - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k}, \end{aligned} \tag{51}$$

where $\hat{W}_{fi,k}$, $\hat{W}_{gi,k}$, $\hat{M}_{i,k}$, and $\hat{N}_{i,k}$ are the estimations of W_{fi}, W_{gi}, M_i, and N_i, respectively. $\tilde{W}_{fi,k} = \hat{W}_{fi,k} - W_{fi}$, $\tilde{W}_{gi,k} = \hat{W}_{gi,k} - W_{gi}$, $\tilde{M}_{i,k} = \hat{M}_{i,k} - M_i$, and $\tilde{N}_{i,k} = \hat{N}_{i,k} - N_i$ are estimation errors. We can rephrase the final three components on the right side of Equation 51 as

$$\begin{aligned} -\eta_i z_{i,k} - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k}(t) + \eta_i \zeta_{i,k} &= -\eta_i z_{i\phi,k} - \eta_i \phi_{i,k}(t) sat\left(\frac{z_{i,k}}{\phi_{i,k}(t)}\right) \\ &\quad - sgn\left(z_{i\phi,k}\right)\dot{\phi}_{i,k}(t) + \eta_i \zeta_{i,k} \\ &= -\eta_i z_{i\phi,k} + \eta_i \zeta_{i,k} \\ &\quad - sgn\left(z_{i\phi,k}\right)\left(\dot{\phi}_{i,k}(t) + \eta_i \phi_{i,k}(t)\right) \\ &= -\eta_i z_{i\phi,k} + \eta_i \zeta_{i,k} \\ &= -\eta_i\left(z_{i\phi,k} - \zeta_{i,k}\right) \\ &= -\eta_i Z_{i,k}, \end{aligned} \tag{52}$$

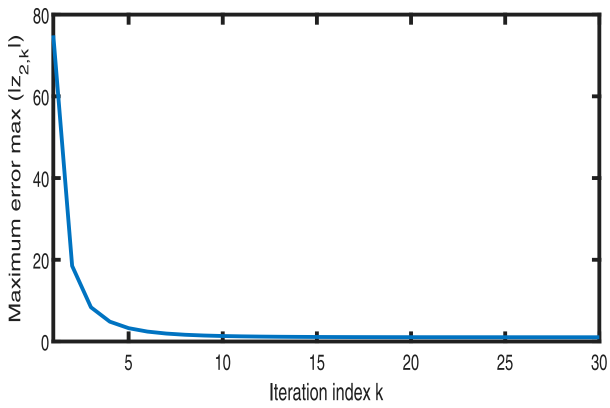

FIGURE 15
Variation in $max(|z_{2,k}|)$ according to the iteration index.

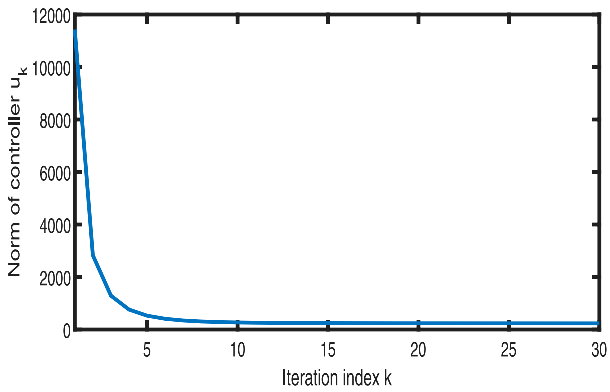

FIGURE 16
Variation in $\|u_k\|$ according to the iteration index.

Using Equations 7, 52, Equation 51 can be reformulated as

$$\begin{aligned}\dot{Z}_{i,k} &= -Z_{i-1,k} + Z_{i+1,k} - \hat{N}_{i,k}\frac{1}{\Delta_k}z_{i\phi,k} - \eta_i Z_{i,k}\\ &+\tilde{W}^T_{fi}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)\\ &+\hat{W}^T_{fi,k}\hat{S}'_{fi,k}\tilde{M}^T_{i,k}\Phi_i(t) - \tilde{W}^T_{gi}S_{gi}(\bar{x}_{i,k})\\ &+d_i + \delta_{fi} + \delta_{gi} + \phi_{i+1,k}(t)sat\Big(\frac{z_{i+1,k}}{\phi_{i+1,k}(t)}\Big).\end{aligned} \tag{53}$$

Let $\omega_i = d_i + \delta_{fi} + \delta_{gi} + \phi_{i+1,k}(t)sat(\frac{z_{i+1,k}(t)}{\phi_{i+1,k}(t)})$, then Equation 53 becomes

$$\begin{aligned}\dot{Z}_{i,k} &= -Z_{i-1,k} + Z_{i+1,k} - \hat{N}_{i,k}\frac{1}{\Delta_k}Z_{i,k} - \eta_i Z_{i,k} + \omega_i\\ &+\tilde{W}^T_{fi,k}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)\\ &+\hat{W}^T_{fi,k}\hat{S}'_{fi,k}\tilde{M}^T_{i,k}\Phi_i(t) - \tilde{W}^T_{gi}S_{gi}(\bar{x}_{i,k}).\end{aligned} \tag{54}$$

Consider the following nonnegative function:

$$\begin{aligned}V_{i,k} &= V_{i-1,k} + \frac{1}{2}Z^2_{i,k} + \frac{1}{2}\tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\tilde{W}_{fi,k} + \frac{1}{2}\tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\tilde{W}_{gi,k}\\ &+\frac{1}{2}\tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\tilde{M}_{i,k} + \frac{1}{2}\Gamma^{-1}_{ni1}\tilde{N}^2_{i,k},\end{aligned} \tag{55}$$

where Γ_{fi1}, Γ_{gi1}, Γ_{mi1}, and Γ_{ni1} are adjustable, positive, definite, and symmetric matrices. According to Equation 54, Assumption 3, and Remark 1, $V_{i,k}$ can be expressed as

$$\begin{aligned}\dot{V}_{i,k} = {}& \dot{V}_{i-1,k} + Z_{i,k}\dot{Z}_{i,k} + \tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\dot{\hat{W}}_{fi,k} + \tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\dot{\hat{W}}_{gi,k} + \tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\dot{\hat{M}}_{i,k}\\ &+\Gamma^{-1}_{ni1}\tilde{N}_{i,k}\dot{\hat{N}}_{i,k} \le Z_{i-1,k}Z_{i,k} - \sum_{j=1}^{i-1}\eta_j Z^2_{j,k} + \frac{i-1}{4}\Delta_k - Z_{i-1,k}Z_{i,k} + Z_{i,k}Z_{i+1,k} - \eta_i Z^2_{i,k}\\ &+\tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\Big(\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)Z_{i,k} + \dot{\hat{W}}_{fi,k}\Big)\\ &-\tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\Big(\Gamma_{gi1}S_{gi}(\bar{x}_{i,k})Z_{i,k} - \dot{\hat{W}}_{gi,k}\Big)\\ &+\tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\Big(\Gamma_{mi1}\Phi_i(t)\hat{W}^T_{fi,k}\hat{S}'_{fi,k}Z_{i,k} + \dot{\hat{M}}_{i,k}\Big) - \hat{N}_{i,k}\frac{1}{\Delta_k}Z^2_{i,k} + \omega_i Z_{i,k}\\ &+\Gamma^{-1}_{ni1}\tilde{N}_{i,k}\dot{\hat{N}}_{i,k} \le -\sum_{j=1}^{i-1}\eta_j Z^2_{j,k} + \frac{i-1}{4}\Delta_k + Z_{i,k}Z_{i+1,k} - \eta_i Z^2_{i,k}\\ &+\tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\Big(\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)Z_{i,k} + \dot{\hat{W}}_{fi,k}\Big)\\ &-\tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\Big(\Gamma_{gi1}S_{gi}(\bar{x}_{i,k})Z_{i,k} - \dot{\hat{W}}_{gi,k}\Big)\\ &+\tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\Big(\Gamma_{mi1}\Phi_i(t)\hat{W}^T_{fi,k}\hat{S}'_{fi,k}Z_{i,k} + \dot{\hat{M}}_{i,k}\Big) - \hat{N}_{i,k}\frac{1}{\Delta_k}Z^2_{i,k} + \frac{1}{\Delta_k}\omega^2_{Mi}Z^2_{i,k}\\ &+\frac{1}{4}\Delta_k + \Gamma^{-1}_{ni1}\tilde{N}_{i,k}\dot{\hat{N}}_{i,k}\\ &= -\sum_{j=1}^{i}\eta_j Z^2_{j,k} + \frac{i-1}{4}\Delta_k + Z_{i,k}Z_{i+1,k} + \tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\Big(\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big)\\ &\quad - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t))Z_{i,k} + \dot{\hat{W}}_{fi,k}\Big) - \tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\Big(\Gamma_{gi1}S_{gi}(\bar{x}_{i,k})Z_{i,k} - \dot{\hat{W}}_{gi,k}\Big)\\ &\quad + \tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\Big(\Gamma_{mi1}\Phi_i(t)\hat{W}^T_{fi,k}\hat{S}'_{fi,k}Z_{i,k} + \dot{\hat{M}}_{i,k}\Big) - \hat{N}_{i,k}\frac{1}{\Delta_k}Z^2_{i,k}\\ &\quad + \frac{1}{\Delta_k}N_{i,k}Z^2_{i,k} + \frac{1}{4}\Delta_k + \Gamma^{-1}_{ni1}\tilde{N}_{i,k}\dot{\hat{N}}_{i,k} = -\sum_{j=1}^{i}\eta_j Z^2_{j,k} + \frac{i}{4}\Delta_k + Z_{i,k}Z_{i+1,k}\\ &\quad +\tilde{W}^T_{fi,k}\Gamma^{-1}_{fi1}\Big(\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)Z_{i,k} + \dot{\hat{W}}_{fi,k}\Big)\\ &\quad -\tilde{W}^T_{gi,k}\Gamma^{-1}_{gi1}\Big(\Gamma_{gi1}S_{gi}(\bar{x}_{i,k})Z_{i,k} - \dot{\hat{W}}_{gi,k}\Big)\\ &\quad +\tilde{M}^T_{i,k}\Gamma^{-1}_{mi1}\Big(\Gamma_{mi1}\Phi_i(t)\hat{W}^T_{fi,k}\hat{S}'_{fi,k}Z_{i,k} + \dot{\hat{M}}_{i,k}\Big)\\ &\quad -\tilde{N}_{i,k}\Gamma^{-1}_{ni1}\Big(\Gamma_{ni1}\frac{1}{\Delta_k}Z^2_{i,k} - \dot{\hat{N}}_{i,k}\Big).\end{aligned} \tag{56}$$

We choose

$$\begin{aligned}\dot{\hat{W}}_{fi,k} &= -\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}^T_{i,k}\Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}^T_{i,k}\Phi_i(t)\Big)Z_{i,k}\\ \dot{\hat{W}}_{gi,k} &= \Gamma_{gi1}S_{gi}(\bar{x}_{i,k})Z_{i,k}\\ \dot{\hat{M}}_{i,k} &= -\Gamma_{mi1}\Phi_i(t)\hat{W}^T_{fi,k}\hat{S}'_{fi,k}Z_{i,k}\\ \dot{\hat{N}}_{i,k} &= \Gamma_{mi1}\frac{1}{\Delta_k}Z^2_{i,k}.\end{aligned} \tag{57}$$

Then, Equation 56 can be written as

$$\dot{V}_{2,k} \le -\sum_{j=1}^{i}\eta_j Z^2_{j,k} + \frac{i}{4}\Delta_k + Z_{i,k}Z_{i+1,k}, \tag{58}$$

Step n: Denote $N_n = \omega^2_{Mn}$, which will be defined later. Because there exist initial state errors and gradient explosion, the function $Z_{n,k}$, denoting the error, is defined as

$$\begin{aligned}Z_{n,k} &= z_{n\phi,k} - \zeta_{n,k}\\ z_{n\phi,k} &= z_{n,k} - \phi_{n,k}(t)sat\Big(\frac{z_{n,k}}{\phi_{n,k}(t)}\Big)\\ z_{n,k} &= x_{n,k} - \beta_{n-1,k}\\ \phi_{n,k}(t) &= \epsilon_{n,k}e^{-\eta_n t}.\end{aligned} \tag{59}$$

The derivative of $Z_{i,k}$ with respect to time is expressed as

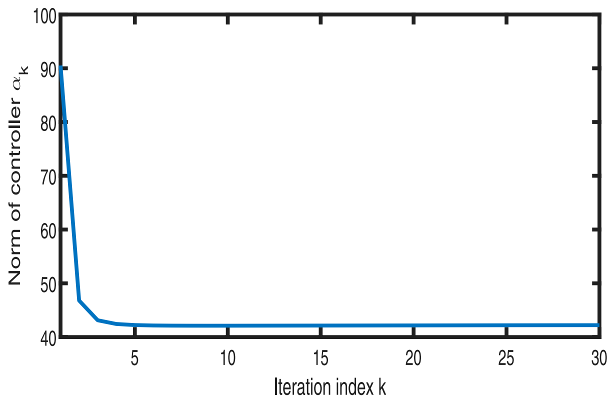

FIGURE 17
Variation in $\|\alpha_k\|$ according to the iteration index.

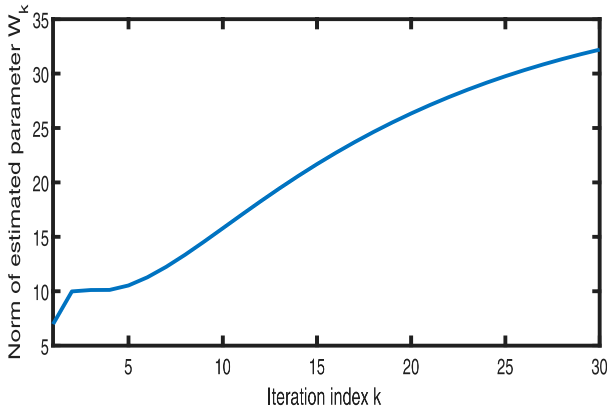

FIGURE 18
Variation of $\|\hat{W}_k\|$ according to the iteration index.

$$\begin{aligned}\dot{Z}_{n,k} &= \dot{z}_{n,k} - sgn\left(z_{n\phi,k}(t)\right)\dot{\phi}_{n,k} - \dot{\zeta}_{n,k}\\ &= u_k + f_n\left(\bar{x}_{n,k}, \theta_n(t)\right) + g_n\left(\bar{x}_{n,k}\right)\\ &\quad -\dot{\beta}_{n-1,k} - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k} - \dot{\zeta}_{n,k}.\end{aligned} \quad (60)$$

Let the error compensation mechanism be defined as

$$\dot{\zeta}_{n,k} = -\eta_n\zeta_{n,k} - \zeta_{n-1,k}. \quad (61)$$

Using Equation 61, we can obtain the time derivative of the error function as

$$\begin{aligned}\dot{Z}_{n,k} &= u_k + \eta_{n,k}\zeta_{n,k} + \zeta_{n-1,k} - \dot{\beta}_{n-1,k}\\ &\quad + f_n\left(\bar{x}_{n,k}, \theta_n(t)\right) + g_n\left(\bar{x}_{n,k}\right) - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k}.\end{aligned} \quad (62)$$

The overall approximation capability of the RBFNN asserts that the unknown nonlinear functions $f_n(\bar{x}_{n,k}, \theta_n(t))$ and $G_n(\bar{x}_{n,k})$ are capable of approximation within a defined scope by FSE-RBFNN and RBFNN, respectively, and reconstruction errors δ_{fn} and δ_{gn} are as follows:

$$\begin{aligned}f_n\left(\bar{x}_{n,k}, \theta_n(t)\right) &= W_{fn}^T S_{fn}\left(\bar{x}_{n,k}, M_n^T\Phi_n(t)\right) + \delta_{fn}\\ G_n\left(\bar{x}_{n,k}\right) &= W_{gn}^T S_{gn}\left(\bar{x}_{n,k}\right) + \delta_{gn},\end{aligned} \quad (63)$$

where δ_{fn} and δ_{gn} are the approximation errors and W_{fn} and W_{gn} are ideal weight vectors.

Define $\Delta_k = \frac{a}{k^l}$,where a is any arbitrary number such that $a > 0$; meanwhile, $l \geq 2$. Let the virtual control be defined as

$$\begin{aligned}u_k &= -\hat{W}_{fn,k}^T S_{fn}\left(\bar{x}_{n,k}, \hat{M}_{n,k}^T\Phi_n(t)\right) - \hat{W}_{gn,k}^T S_{gn}\left(\bar{x}_{n,k}\right)\\ &\quad -\hat{N}_{n,k}\frac{1}{\Delta_k}Z_{n,k} + \dot{\beta}_{n-1,k} - \eta_n z_{n,k} - z_{n-1\phi,k}.\end{aligned} \quad (64)$$

By substituting Equations 63, 64 into Equation 62, we can conclude that

$$\begin{aligned}\dot{Z}_{n,k} &= -z_{n-1\phi,k} + \zeta_{n-1,k} - \eta_n z_{n,k} + \eta_n\zeta_{n,k} - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k}\\ &\quad + W_{fn}^T S_{fn}\left(\bar{x}_{n,k}, M_n^T\Phi_n(t)\right) + \delta_{fn} - \hat{W}_{fn,k}^T S_{fn}\left(\bar{x}_{n,k}, \hat{M}_{n,k}^T\Phi_n(t)\right)\\ &\quad + W_{gn}^T S_{gn}\left(\bar{x}_{n,k}\right) + \delta_{gn} - \hat{W}_{gn,k}^T S_{gn}\left(\bar{x}_{n,k}\right) - \hat{N}_{n,k}\frac{1}{\Delta_k}z_{n\phi,k}\\ &= W_{fn}^T S_{fn}\left(\bar{x}_{n,k}, M_n^T\Phi_n(t)\right) + \delta_{fn} - \hat{W}_{fn,k}^T S_{fn}\left(\bar{x}_{n,k}, \hat{M}_{n,k}^T\Phi_n(t)\right)\\ &\quad + W_{gn}^T S_{gn}\left(\bar{x}_{n,k}\right) + \delta_{gn} - \hat{W}_{gn,k}^T S_{gn}\left(\bar{x}_{n,k}\right) - \hat{N}_{n,k}\frac{1}{\Delta_k}z_{n\phi,k}\\ &\quad - Z_{n-1,k} - \eta_n z_{n,k} + \eta_n\zeta_{n,k} - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k},\end{aligned} \quad (65)$$

where $\hat{W}_{fn,k}$, $\hat{W}_{gn,k}$, $\hat{M}_{n,k}$, and $\hat{N}_{n,k}$ are the estimations of W_{fn}, W_{gn}, M_n, and N_n, respectively. $\tilde{W}_{fn,k} = \hat{W}_{fn,k} - W_{fn}$, $\tilde{W}_{gn,k} = \hat{W}_{gn,k} - W_{gn}$, $\tilde{M}_{n,k} = \hat{M}_{n,k} - M_n$, and $\tilde{N}_{n,k} = \hat{N}_{n,k} - N_n$ are estimation errors. We can rephrase the final three components on the right side of Equation 65 as

$$\begin{aligned}&-\eta_n z_{n,k} - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k}(t) + \eta_n\zeta_{n,k}\\ &= -\eta_n z_{n\phi,k} - \eta_n\phi_{n,k}(t)sat\left(\frac{z_{n,k}}{\phi_{n,k}(t)}\right) - sgn\left(z_{n\phi,k}\right)\dot{\phi}_{n,k}(t) + \eta_n\zeta_{n,k}\\ &= -\eta_n z_{n\phi,k} + \eta_n\zeta_{n,k} - sgn\left(z_{n\phi,k}\right)\left(\dot{\phi}_{n,k}(t) + \eta_n\phi_{n,k}(t)\right)\\ &= -\eta_n z_{n\phi,k} + \eta_n\zeta_{n,k} = -\eta_n\left(z_{n\phi,k} - \zeta_{n,k}\right) = -\eta_n Z_{n,k}.\end{aligned} \quad (66)$$

Using Equations 7, 66, Equation 65 can be reformulated as

$$\begin{aligned}\dot{Z}_{n,k} &= -Z_{n-1,k} - \hat{N}_{n,k}\frac{1}{\Delta_k}z_{n\phi,k} - \eta_n Z_{n,k}\\ &\quad + \tilde{W}_{fn}^T\left(S_{fn}\left(\bar{x}_{n,k}, \hat{M}_{n,k}^T\Phi_n(t)\right) - \hat{S}'_{fn,k}\hat{M}_{n,k}^T\Phi_n(t)\right)\\ &\quad + \hat{W}_{fn,k}^T\hat{S}'_{fn,k}\tilde{M}_{n,k}^T\Phi_n(t) - \tilde{W}_{gn}^T S_{gn}\left(\bar{x}_{n,k}\right)\\ &\quad + d_n + \delta_{fn} + \delta_{gn}.\end{aligned} \quad (67)$$

Let $\omega_n = d_n + \delta_{fn} + \delta_{gn}$, then Equation 67 becomes

$$\begin{aligned}\dot{Z}_{n,k} &= -Z_{n-1,k} - \hat{N}_{n,k}\frac{1}{\Delta_k}Z_{n,k} - \eta_n Z_{n,k} + \omega_n\\ &\quad + \tilde{W}_{fn,k}^T\left(S_{fn}\left(\bar{x}_{n,k}, \hat{M}_{n,k}^T\Phi_n(t)\right) - \hat{S}'_{fn,k}\hat{M}_{n,k}^T\Phi_n(t)\right)\\ &\quad + \hat{W}_{fn,k}^T\hat{S}'_{fn,k}\tilde{M}_{n,k}^T\Phi_n(t) - \tilde{W}_{gn}^T S_{gn}\left(\bar{x}_{n,k}\right).\end{aligned} \quad (68)$$

Assumption 4: The remainder ω_n is bounded with $|\omega_n| \leq \omega_{Mn}$ and $\omega_{Mn} > 0$.

Remark 2: This assumption is reasonable because 1) d_n, δ_{fn}, and δ_{gn} are constrained within the specified area by Equations 6, 8.

Let the following non-negative function be defined as

$$\begin{aligned}V_{n,k} &= V_{n-1,k} + \frac{1}{2}Z_{n,k}^2 + \frac{1}{2}\tilde{W}_{fn,k}^T\Gamma_{fn1}^{-1}\tilde{W}_{fn,k} + \frac{1}{2}\tilde{W}_{gn,k}^T\Gamma_{gn1}^{-1}\tilde{W}_{gn,k}\\ &\quad + \frac{1}{2}\tilde{M}_{n,k}^T\Gamma_{mn1}^{-1}\tilde{M}_{n,k} + \frac{1}{2}\Gamma_{Nn1}^{-1}\tilde{N}_{n,k}^2,\end{aligned} \quad (69)$$

where Γ_{fn1}, Γ_{gn1}, Γ_{mn1}, and Γ_{Nn1} are adjustable, positive, definite, and symmetric matrices. The derivative of $V_{n,k}$ is considered as follows (Equation 68):

$$\begin{aligned}
\dot{V}_{n,k} = &\ \dot{V}_{n-1,k} + Z_{n,k}\dot{Z}_{n,k} + \tilde{W}_{fn,k}^{T}\Gamma_{fn1}^{-1}\dot{\hat{W}}_{fn,k} \\
&+\tilde{W}_{gn,k}^{T}\Gamma_{gn1}^{-1}\dot{\hat{W}}_{gn,k} + \tilde{M}_{n,k}^{T}\Gamma_{mn1}^{-1}\dot{\hat{M}}_{n,k} + \Gamma_{Nn1}^{-1}\tilde{N}_{n,k}\dot{\hat{N}}_{n,k} \\
\le &\ Z_{n-1,k}Z_{n,k} - \sum_{j=1}^{n-1}\eta_j Z_{j,k}^2 + \frac{n-1}{4}\Delta_k - Z_{n-1,k}Z_{n,k} - \eta_i Z_{i,k}^2 \\
&+\tilde{W}_{fn,k}^{T}\Gamma_{fn1}^{-1}\Big(\Gamma_{fn1}\Big(S_{fn}\Big(\bar{x}_{n,k},\hat{M}_{n,k}^{T}\Phi_n(t)\Big) \\
&-\hat{S}'_{fn,k}\hat{M}_{n,k}^{T}\Phi_n(t)\Big)Z_{n,k} + \dot{\hat{W}}_{fn,k}\Big) \\
&-\tilde{W}_{gn,k}^{T}\Gamma_{gn1}^{-1}\Big(\Gamma_{gn1}S_{gn}(\bar{x}_{n,k})Z_{n,k} - \dot{\hat{W}}_{gn,k}\Big) \\
&+\tilde{M}_{n,k}^{T}\Gamma_{mn1}^{-1}\Big(\Gamma_{mn1}\Phi_n(t)\hat{W}_{fn,k}^{T}\hat{S}'_{fn,k}Z_{n,k} + \dot{\hat{M}}_{n,k}\Big) \\
&-\hat{N}_{n,k}\frac{1}{\Delta_k}Z_{n,k}^2 + \omega_n Z_{n,k} + \Gamma_{Nn1}^{-1}\tilde{N}_{n,k}\dot{\hat{N}}_{n,k} \\
\le &\ -\sum_{j=1}^{n-1}\eta_j Z_{j,k}^2 + \frac{n-1}{4}\Delta_k - \eta_n Z_{n,k}^2 \\
&+\tilde{W}_{fn,k}^{T}\Gamma_{fn1}^{-1}\Big(\Gamma_{fn1}\Big(S_{fn}\Big(\bar{x}_{n,k},\hat{M}_{n,k}^{T}\Phi_n(t)\Big) \\
&-\hat{S}'_{fn,k}\hat{M}_{n,k}^{T}\Phi_n(t)\Big)Z_{n,k} + \dot{\hat{W}}_{fn,k}\Big) \\
&-\tilde{W}_{gn,k}^{T}\Gamma_{gn1}^{-1}\Big(\Gamma_{gn1}S_{gn}(\bar{x}_{n,k})Z_{n,k} - \dot{\hat{W}}_{gn,k}\Big) \\
&+\tilde{M}_{n,k}^{T}\Gamma_{mn1}^{-1}\Big(\Gamma_{mn1}\Phi_n(t)\hat{W}_{fn,k}^{T}\hat{S}'_{fn,k}Z_{n,k} + \dot{\hat{M}}_{n,k}\Big) \\
&-\hat{N}_{n,k}\frac{1}{\Delta_k}Z_{n,k}^2 + \frac{1}{\Delta_k}\omega_{Mn}^2 Z_{n,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{Nn1}^{-1}\tilde{N}_{n,k}\dot{\hat{N}}_{n,k} \\
= &\ -\sum_{j=1}^{n}\eta_j Z_{j,k}^2 + \frac{n-1}{4}\Delta_k \\
&+\tilde{W}_{fn,k}^{T}\Gamma_{fn1}^{-1}\Big(\Gamma_{fn1}\Big(S_{fn}\Big(\bar{x}_{n,k},\hat{M}_{n,k}^{T}\Phi_n(t)\Big) \\
&-\hat{S}'_{fn,k}\hat{M}_{n,k}^{T}\Phi_n(t)\Big)Z_{n,k} + \dot{\hat{W}}_{fn,k}\Big) \\
&-\tilde{W}_{gn,k}^{T}\Gamma_{gn1}^{-1}\Big(\Gamma_{gn1}S_{gn}(\bar{x}_{n,k})Z_{n,k} - \dot{\hat{W}}_{gn,k}\Big) \\
&+\tilde{M}_{n,k}^{T}\Gamma_{mn1}^{-1}\Big(\Gamma_{mn1}\Phi_n(t)\hat{W}_{fn,k}^{T}\hat{S}'_{fn,k}Z_{n,k} + \dot{\hat{M}}_{n,k}\Big) \\
&-\hat{N}_{n,k}\frac{1}{\Delta_k}Z_{n,k}^2 + \frac{1}{\Delta_k}N_{n,k}Z_{n,k}^2 + \frac{1}{4}\Delta_k + \Gamma_{Nn1}^{-1}\tilde{N}_{n,k}\dot{\hat{N}}_{n,k} \\
= &\ -\sum_{j=1}^{n}\eta_j Z_{j,k}^2 + \frac{n}{4}\Delta_k \\
&+\tilde{W}_{fn,k}^{T}\Gamma_{fn1}^{-1}\Big(\Gamma_{fn1}\Big(S_{fn}\Big(\bar{x}_{n,k},\hat{M}_{n,k}^{T}\Phi_n(t)\Big) \\
&-\hat{S}'_{fn,k}\hat{M}_{n,k}^{T}\Phi_n(t)\Big)Z_{n,k} + \dot{\hat{W}}_{fn,k}\Big) \\
&-\tilde{W}_{gn,k}^{T}\Gamma_{gn1}^{-1}\Big(\Gamma_{gn1}S_{gn}(\bar{x}_{n,k})Z_{n,k} - \dot{\hat{W}}_{gn,k}\Big) \\
&+\tilde{M}_{n,k}^{T}\Gamma_{mn1}^{-1}\Big(\Gamma_{mn1}\Phi_n(t)\hat{W}_{fn,k}^{T}\hat{S}'_{fn,k}Z_{n,k} + \dot{\hat{M}}_{n,k}\Big) \\
&-\tilde{N}_{n,k}\Gamma_{Nn1}^{-1}\Big(\Gamma_{Nn1}\frac{1}{\Delta_k}Z_{n,k}^2 - \dot{\hat{N}}_{n,k}\Big).
\end{aligned} \tag{70}$$

We choose

$$\begin{aligned}
\dot{\hat{W}}_{fn,k} &= -\Gamma_{fn1}\Big(S_{fn}\Big(\bar{x}_{n,k},\hat{M}_{n,k}^{T}\Phi_n(t)\Big) - \hat{S}'_{fn,k}\hat{M}_{n,k}^{T}\Phi_n(t)\Big)Z_{n,k} \\
\dot{\hat{W}}_{gn,k} &= \Gamma_{gn1}S_{gn}(\bar{x}_{n,k})Z_{n,k} \\
\dot{\hat{M}}_{n,k} &= -\Gamma_{mn1}\Phi_n(t)\hat{W}_{fn,k}^{T}\hat{S}'_{fn,k}Z_{n,k} \\
\dot{\hat{N}}_{n,k} &= \Gamma_{Nn1}\frac{1}{\Delta_k}Z_{n,k}^2.
\end{aligned} \tag{71}$$

Then, Equation 70 can be written as

$$\dot{V}_{n,k} \le -\sum_{j=1}^{n}\eta_j Z_{j,k}^2 + \frac{n}{4}\Delta_k. \tag{72}$$

For the initial state, we rely on the following set of assumed conditions:

Assumption 2: When $t=0$, $\hat{W}_{fi,k}(0) = \hat{W}_{fi,k-1}(T)$, $\hat{W}_{gi,k}(0) = \hat{W}_{gi,k-1}(T)$, $\hat{N}_{i,k}(0) = \hat{N}_{i,k-1}(T)$, and $\hat{M}_{i,k}(0) = \hat{M}_{i,k-1}(T)$ $(i = 1,\ldots,n)$ holds true for all values of k.

3.2 Stability and convergence analysis

Theorem 1: For nonlinear system (1) with assumptions 2, 3, and 4, if we design virtual controllers (21), (35), (50), controller (64), and parameter updating laws (28), (42), (57), (71),then all signals in the closed-loop system are bounded within the interval [0, T]. We obtain

$$lim_{k\to\infty}Z_{j,k}(t) = 0,\quad j = 1,2,\ldots,n. \tag{73}$$

In other words, $lim_{k\to\infty}|z_{1\phi,k}(t)| = lim_{k\to\infty}\|\zeta_{1,k}(t)\| \le \frac{\sqrt{2}\aleph_1}{\eta_1}(1-e^{-\eta_1(t-T)})$, and then $lim_{k\to\infty}|z_{1,k}(t)| \le \phi_{1,\infty}(t) + \frac{\sqrt{2}\aleph_1}{\eta_1}(1-e^{-\eta_1(t-T)})$, where $\aleph_1$ is the boundary of the difference between β_1 and α_1. Let η_1 be chosen sufficiently large, ensuring that $\phi_{1,\infty}(t)$ and $\frac{\sqrt{2}\aleph_1}{\eta_1}(1-e^{-\eta_1(t-T)})$ can be minimized as much as possible throughout the entire time interval [0, T].

Proof: In accordance with Assumption 2, we find that $\|Z_k(0)\|^2 = 0 \le \|Z_k(T)\|^2$. Consider that $V'_{n,k} = V_{n,k}(Z_k(0),\hat{W}_{fk}(T),\ \hat{W}_{gk}(T),\hat{N}_k(T),\hat{M}_k(T))$. Using Equation 69, we obtain $Z_k = [Z_{1,k},Z_{2,k},\ldots,Z_{n,k}]^T$, $\hat{W}_{fk} = [\hat{W}_{f1,k},\hat{W}_{f2,k},\ldots,\ \hat{W}_{fn,k}]^T$,$\hat{W}_{gk} = [\hat{W}_{g1,k},\hat{W}_{g2,k},\ldots,\hat{W}_{gn,k}]^T$, $\hat{M}_k = [\hat{M}_{1,k},\hat{M}_{2,k},\ldots,\hat{M}_{n,k}]^T$, and$\hat{N}_k = [\hat{N}_{1,k},\hat{N}_{2,k},\ldots,\hat{N}_{n,k}]^T$. Using Equation 72,

$$\begin{aligned}
V'_{n,k} \le &\ V_{n,k}\Big(Z_k(0),\hat{W}_{fk}(0),\hat{W}_{gk}(0),\hat{N}_k(0),\hat{M}_k(0)\Big) - \\
&\Sigma_{i=1}^{k}\Sigma_{j=1}^{n}\int_0^T \eta_j\Big(Z_{j,i}\Big)^2 dt + n\Big(\frac{1}{4}\Big)T\Big(\Sigma_{i=1}^{k}\Delta_i\Big).
\end{aligned} \tag{74}$$

Let $V_0(k) = V_{n,1}(Z_1(0),\ \hat{W}_{f1}(0),\hat{W}_{g1}(0),\hat{N}_1(0),\hat{M}_1(0)) + n(\frac{1}{4})T(\Sigma_{i=1}^{k}\Delta_i)$, then Equation 74 can be rewritten as

$$\Sigma_{i=1}^{k}\Sigma_{j=1}^{n}\int_0^T \eta_j\Big(Z_{j,i}\Big)^2 dt \le V_0(k) - V'_{n,k}. \tag{75}$$

Using Equation 9, we obtain $lim_{k\to\infty}V_0(k) \le V_{n,1} + 2an(\frac{1}{4})T$ and $V_0(k)$ is bounded. $V_{n,k}(Z_k(0),\hat{W}_{fk}(T),\hat{W}_{gk}(T),\hat{N}_k(T),\hat{M}_k(T)) \ge 0$, so

$$lim_{k\to\infty}\Sigma_{j=1}^{n}\int_0^T \eta_j\Big(Z_{j,k}\Big)^2 dt = 0. \tag{76}$$

Based on Equation 69, for any given value of k, $V_{n,k}(t) = V_{n,k}(0) + \int_0^t \dot{V}_{m,k}(\tau)d\tau$; substituting Equation 72 obtain

$$V_{n,k}(t) \le V_{n,k}(0) - \Sigma_{j=1}^{n}\int_0^t \eta_j\Big(Z_{j,k}(\tau)\Big)^2 d\tau + tn\Big(\frac{1}{4}\Big)\Delta_k. \tag{77}$$

Based on Equation 76, $\Sigma_{j=1}^{n}\int_0^t \eta_j(Z_{j,k}(\tau))^2 d\tau$ is bounded. According to definition 1, Δ_k is bounded and $t\in[0,T]$, so $tn(\frac{1}{4})\Delta_k$ is also bounded. In addition, $\hat{W}_{fk}(0) = \hat{W}_{f(k-1)}(T)$, $\hat{W}_{gk}(0) = \hat{W}_{g(k-1)}(T)$, $\hat{M}_k(0) = \hat{M}_{k-1}(T)$, and $\hat{N}_k(0) = \hat{N}_{k-1}(T)$; based on Equation 77, for any given value of k,

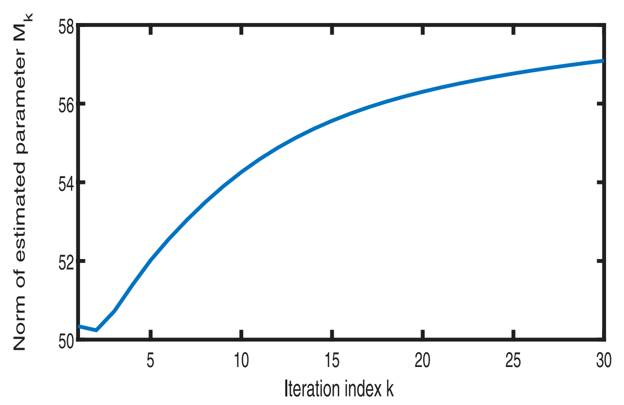

FIGURE 19
Variation in $\|\hat{M}_k\|$ according to the iteration index.

$V_{n,k}(Z_k(0), \hat{W}_{fk}(T), \hat{W}_{gk}(T), \hat{N}_k(T), \hat{M}_k(T))$ is bounded. So, $V_{n,k}(0, \hat{W}_{fk}(0), \hat{W}_{gk}(0), \hat{N}_k(0), \quad \hat{M}_k(0)) = V_{n,k-1}(0, \hat{W}_{f(k-1)}(T), \hat{W}_{g(k-1)}(T), \hat{N}_{k-1}(T), \hat{M}_{k-1}(T))$ is also bounded; from above all, for any given value of k, if $V_{n,k}(t)$ is bounded, then we can deduce that $x_{i,k}$, $\hat{W}_{fk}(t)$, $\hat{W}_{gk}(t)$, $\hat{N}_k(t)$, and $\hat{M}_k(t)$ are bounded. According to Equation 64, u_k is bounded. According to Equation 53, $\dot{Z}_{i,k}$ is bounded, so $Z_{i,k}$ is continuous uniformly. Thus, we can deduce Equation 73.

Then, we need to prove that $\aleph_1$ will converge to a neighborhood that approaches 0. Initially, let $\alpha_{i,k}(t)$ be a signal satisfying $|\alpha_{i,k}(t)| < \bar{\alpha}$ and $|\dot{\alpha}_{i,k}(t)| < \hbar$ for all $t \geq 0$. The compensation error within the compensation system is defined as

$$\varrho_{i,k} = \beta_{i,k} - \alpha_{i,k}. \tag{78}$$

With specified initial conditions, $\beta_{i,0} = \alpha_{i,0}$, i.e., $\varrho_{i,0} = 0, \quad i = 1, 2, \ldots, n-1$. From (11), we obtain

$$\begin{aligned} \dot{\varrho}_{i,k} &= -\xi_{i,k}\left(\beta_{i,k} - \alpha_{i,k}\right) - \alpha_{i,k} \\ &= -\xi_{i,k}\varrho_{i,k} - \dot{\alpha}_{i,k} \\ \varrho_{i,k}(t) &= -\int_0^t \dot{\alpha}_{i,k} e^{-\xi_{i,k}(t-\tau)} d\tau \\ |\varrho_{i,k}(t)| &= |-\int_0^t \dot{\alpha}_{i,k}(\tau) e^{-\xi_{i,k}(t-\tau)} d\tau| \\ &= |\dot{\alpha}_{i,k}(\tau)| |\int_0^t e^{-\xi_{i,k}(t-\tau)} d\tau| \\ &\leq \max |\dot{\alpha}_{i,k}(\tau)| |\int_0^t e^{-\xi_{i,k}(t-\tau)} d\tau| \\ &\leq \frac{\hbar}{\xi_{i,k}}\left(1 - e^{-\xi_{i,k} t}\right) \\ &\leq \frac{\hbar}{\xi_{i,k}} = \aleph_i. \end{aligned} \tag{79}$$

As shown in Equation 79, choosing an appropriate value for $\xi_{i,k}$ confines the error $\varrho_{i,k}$ within a narrow range, approximately equating $\alpha_{i,k}$ to $\beta_{i,k}$. In addition, based on the compensation system, the Lyapunov function is defined on the interval $[0, T]$ as follows:

$$V_{\zeta,k} = \sum_{i=1}^{n} \frac{1}{2}\zeta_{i,k}^2. \tag{80}$$

The derivative of $V_{\zeta,k}$ along systems (78) with respect to time is expressed as

$$\begin{aligned} \dot{V}_{\zeta,k} &= \sum_{i=1}^{n} \zeta_{i,k}\dot{\zeta}_{i,k} \\ &= -\sum_{i=1}^{n} \eta_i \zeta_{i,k}^2 + \sum_{i=1}^{n-1} \zeta_{i,k}\left(\beta_{i,k} - \alpha_{i,k}\right) \\ &\leq -\sum_{i=1}^{n} \eta_i \zeta_{i,k}^2 + \sum_{i=1}^{n-1} |\zeta_{i,k}| |\left(\beta_{i,k} - \alpha_{i,k}\right)| \\ &= -\sum_{i=1}^{n} \eta_i \zeta_{i,k}^2 + \sum_{i=1}^{n-1} |\zeta_{i,k}| |\left(\beta_{i,k} - \alpha_{i,k}\right)| + 0|\zeta_{n,k}| \\ &\leq -\eta_0 \sum_{i=1}^{n} \zeta_{i,k}^2 + \aleph \sum_{i=1}^{n} |\zeta_{i,k}| \\ &\leq -\eta_0 \|\zeta_{i,k}\|^2 + \sqrt{2}\aleph \|\zeta_{i,k}\|, \end{aligned} \tag{81}$$

where $\aleph = \max \aleph_i$, $\eta_0 = \min \eta_i$. To ensure the stability of the compensation system, it is sufficient to satisfy

$$\|\zeta_{i,k}\| \leq \frac{\sqrt{2}\aleph}{\eta_0}\left(1 - e^{-\eta_0(t-T)}\right). \tag{82}$$

Equation 82 leads to the conclusion that $\|\zeta_{i,k}\|$ is bounded. Hence, $\zeta_{i,k}$ is also bounded. Moreover, we can choose a parameter $\xi_{i,k} > 0$ to arbitrarily reduce $\aleph_i$, thereby causing the compensation $\zeta_{i,k}$ of the system to approach 0. In this way, by ensuring that the error Z_k approaches 0, $z_{\phi,k}$ will converge to the neighborhood approaching 0. Thus, we conclude Theorem 1.

4 Illustrative examples

4.1 Number simulation

This section includes an example illustrating the effectiveness of the proposed adaptive iterative learning controller.

The second-order pure-feedback nonlinear system described is considered as follows:

$$\begin{aligned} \dot{x}_{1,k} &= x_{2,k} + \frac{r_1 x_{1,k} + r_1^2 x_{1,k}^2}{1 + r_1^2 x_{1,k}^2} \\ \dot{x}_{2,k} &= u_k + \sin\left(r_2 x_{1,k} x_{2,k}\right) e^{-r_2^2 x_{1,k}^2 x_{2,k}^2} \\ y_k &= x_{1,k}, \end{aligned} \tag{83}$$

where $t \in [0, 5]$, $x_{1,k}$, and $x_{2,k}$ are state variables and u_k is the input variable. Utilizing the widely recognized van der Pol oscillator as the reference model, we obtain

$$\begin{aligned} \dot{x}_{d1} &= x_{d2} \\ \dot{x}_{d2} &= -9x_{d1} - 6x_{d2} + 2 \\ y_{d1} &= x_{d1}, \end{aligned} \tag{84}$$

where x_{d1} and x_{d2} are state variables. The primary control objective is to synchronize the output of systems (82) with the reference trajectory y_{d1} generated by system (84) over the interval [0,5] under the condition $k \to \infty$.

In accordance with Theorem 1, the adaptive iterative learning controller is chosen as

$$\begin{aligned} \alpha_{1,k} &= -\hat{W}_{1,k}^T S_1\left(\bar{x}_{1,k}, \hat{M}_{1,k}^T \Phi_1(t)\right) - \hat{N}_{1,k}\frac{1}{\Delta_k} z_{1\phi,k} + \dot{y}_r - \eta_1 z_{1,k} \\ u_k &= -\hat{W}_{f2,k}^T S_{f2}\left(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\right) - \hat{N}_{2,k}\frac{1}{\Delta_k} Z_{2,k} \\ &\quad + \dot{\beta}_{1,k} - \eta_n z_{2,k} - z_{1\phi,k}. \end{aligned} \tag{85}$$

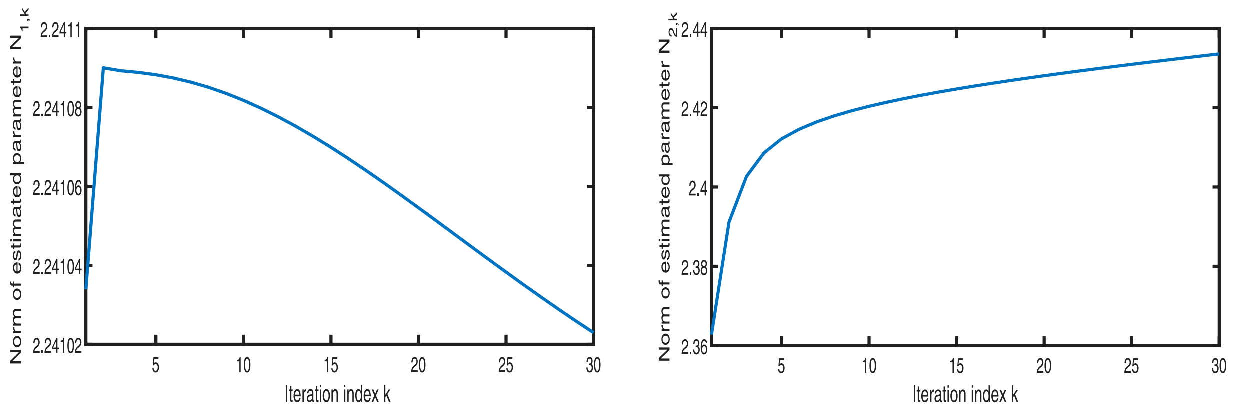

FIGURE 20
Variation in $\|\hat{N}_{1,k}\|$ and $\|\hat{N}_{2,k}\|$ according to the iteration index.

The error compensation mechanism is

$$\begin{aligned} \dot{\zeta}_{1,k} &= \beta_{1,k} + \zeta_{2,k} - \eta_1 \zeta_{1,k} - \alpha_{1,k} \\ \dot{\zeta}_{2,k} &= -\eta_2 \zeta_{2,k} - \zeta_{1,k}, \end{aligned} \tag{86}$$

where $\dot{\beta}_{1,k} = -\xi(\beta_{1,k} - \alpha_{1,k})$.

The parameter adaptive iterative learning laws are provided by (57):

$$\begin{aligned} \dot{\hat{W}}_{fi,k} &= -\Gamma_{fi1}\Big(S_{fi}\Big(\bar{x}_{i,k}, \hat{M}_{i,k}^T \Phi_i(t)\Big) - \hat{S}'_{fi,k}\hat{M}_{i,k}^T \Phi_i(t)\Big) Z_{i,k} \\ \dot{\hat{M}}_{i,k} &= -\Gamma_{mi1}\Phi_i(t)\hat{W}_{fi,k}^T \hat{S}'_{fi,k} Z_{i,k} \\ \dot{\hat{N}}_{i,k} &= \Gamma_{ni1}\frac{1}{\Delta_k} Z_{i,k}^2, \end{aligned} \tag{87}$$

where $i = 1,2$, $c_1 = 5$, $c_2 = 10, \Delta_k = a/k^2$, $a = 50000$, $\Gamma_{11} = diag\{1,1,1,1,1\}$, $\Gamma_{21} = 10$, $\Gamma_{12} = diag\{1,1,1,1,1\}$, $\Gamma_{22} = 1$, and $\xi = 1$.

Figures 1–3 show the tracking performance of the system output and expected output without iteration and at 50th and 100th iterations, respectively. Figures 4, 5 show that as the number of iterations increases, the system error may converge to a small region near the zero point. Furthermore, observations shown in Figures 6–10 confirm that both control signals $\|u_k\|$ and $\|\alpha_k\|$ and estimated parameters, $\|\hat{W}_{1,k}\|$, $\|\hat{W}_{2,k}\|$, $\|\hat{M}_{1,k}\|$, $\|\hat{M}_{2,k}\|$, $\|\hat{N}_{1,k}\|$,and $\|\hat{N}_{2,k}\|$, exhibit bounded behavior within the [0,5] range. The validity of the control strategy presented in this research is reaffirmed by the simulation results shown in Figures 11–20 over the interval $[0, T]$.

4.2 Simulation of a single-joint robotic arm

In this section, we conducted simulation verification on a single degree-of-freedom robotic arm system to assess the performance of the proposed control method. The dynamic equation of a single degree-of-freedom robotic arm is

$$\frac{\partial^2 \theta}{\partial t^2} = -10 \sin\theta - 2\frac{\partial \theta}{\partial t} + u, \tag{88}$$

where θ is the angle between the robotic arm and the reference frame.u is the input of the DC motor.

$$\frac{\partial^2 y_{d1}}{\partial t^2} = -9y_{d1} - 6\frac{\partial y_{d1}}{\partial t} + 2r, \tag{89}$$

where y_{d1} is the output of the reference model. r is the reference input signal. According to Equations 88, 89, the state equation of the system is derived as

$$\begin{aligned} \dot{x}_{1,k} &= x_{2,k} \\ \dot{x}_{2,k} &= -10\sin(x_{1,k}) - 2x(2,k) + u_k \\ y_k &= x_{1,k}, \end{aligned} \tag{90}$$

and its reference model is derived as

$$\begin{aligned} \dot{x}_{d1} &= x_{d2} \\ \dot{x}_{d2} &= -9x_{d1} - 6x_{d2} + 2r \\ y_{d1} &= x_{d1}, \end{aligned} \tag{91}$$

where $x_{1,k}$ equals to θ can be defined as the angle between the robotic arm and the reference frame. $x_{2,k}$ is the time derivative of θ, i.e., $\dot{\theta}$. The primary control objective is to synchronize the output of systems (88) with the reference trajectory y_{d1} generated by system (89) over the interval [0,5] under the condition $k \to \infty$.

In accordance with Theorem 1, the adaptive iterative learning controller is chosen as

$$\begin{aligned} \alpha_{1,k} &= -\hat{N}_{1,k}\frac{1}{\Delta_k} z_{1\phi,k} + \dot{y}_{d1} - \eta_1 z_{1,k} \\ u_k &= -z_{1\phi,k} - c_2 z_{2,k} - \hat{W}_{2,k}^T S_2\Big(\bar{x}_{2,k}, \hat{M}_{2,k}^T \Phi_2(t)\Big) - \hat{N}_{2,k}\frac{1}{\Delta_k} z_{2\phi,k} \\ &+\dot{\beta}_{1,k}. \end{aligned} \tag{92}$$

The error compensation mechanism is

$$\begin{aligned} \dot{\zeta}_{1,k} &= \beta_{1,k} + \zeta_{2,k} - \eta_1 \zeta_{1,k} - \alpha_{1,k} \\ \dot{\zeta}_{2,k} &= -\eta_2 \zeta_{2,k} - \zeta_{1,k}, \end{aligned} \tag{93}$$

where $\beta_{1,k} = -\xi(\beta_{1,k} - \alpha_{1,k})$.

The parameter adaptive iterative learning laws are provided by (57).

$$\dot{\hat{W}}_k = \Gamma_f\Big(S\Big(\bar{x}_{2,k}, \hat{M}_k^T \Phi(t)\Big) - \hat{S}'_k \hat{M}_k^T \Phi_2(t)\Big) z_{2\phi,k}, \tag{94}$$

$$\dot{\hat{N}}_{i,k} = \Gamma_{Ni}\frac{1}{\Delta_k} z_{i\phi,k}^2, \quad i = 1,2, \tag{95}$$

$$\dot{\hat{M}}_k = \Gamma_m \Phi(t)\hat{W}_k^T \hat{S}'_k z_{2\phi,k}, \tag{96}$$

where $c_1 = 50$, $c_2 = 150, \Delta_k = a/k^2$, $a = 50000$, $\Gamma_{11} = diag\{1,1,1,1,1\}$, $\Gamma_{21} = 10$, $\Gamma_{12} = diag\{1,1,1,1,1\}$, $\Gamma_{22} = 1, and \xi = 10$.

Figures 11–13 show the tracking performance of the system output and expected output without iteration and at 15th and 30th iterations, respectively. Figures 14, 15 show that as the number of iterations increases, the system error may converge to a small region near the zero point. Furthermore, observations from Figures 16–20 confirm that both control signals $\|u_k\|$ and $\|\alpha_k\|$ and estimated parameters, $\|\hat{W}_k\|$, $\|\hat{M}_k\|$, $\|\hat{N}_{1,k}\|$, and $\|\hat{N}_{2,k}\|$, exhibit bounded behavior within the [0,5] range. The validity of the control strategy presented in this research is reaffirmed by the simulation results shown in Figures 11–20 over the interval $[0, T]$.

5 Conclusion

This article presents a solution to the complete trajectory, following challenges within a finite time frame for a category of nonlinearly parameterized systems characterized by time-varying disturbed functions and initial state errors. A new FSE neural network is used to learn the time-varying, nonlinearly parameterized term. Based on this and Lyapunov theory, we proposed the new LPF-AILC method. A low-pass filter is used to solve the problem of parameter explosion after obtaining the derivative of the virtual controller. The unmatched uncertainties, nonlinear parameterization, and initial state errors are well considered. Two simulation examples have proven the feasibility of the control approach. This article does not mention time-delay issues, but they often exist in practical systems. Our future work should consider solving the complete tracking problem on a finite time interval for these complex systems with time delays. This is a more interesting issue. In addition, there are two deficiencies in the controller design process: the assumption of time-varying parameters being periodic and the jitter issues caused by the low-pass filter. These challenges will be carefully considered and addressed in our future work.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

CZ: conceptualization, funding acquisition, investigation, methodology, and writing–review and editing. LY: formal analysis, software, writing–original draft, and writing–review and editing. YG: investigation, validation, and writing–original draft. JY: funding acquisition, supervision, and writing–review and editing. FQ: funding acquisition, supervision, and writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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