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He-transform: breakthrough advancement for the variational iteration method

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1 Introduction

The variational iteration method [1] is considered the most powerful tool after Newton's iteration method for solving a wide range of physical problems [2–6]. It has been employed in solving seepage flows with fractional derivatives and nonlinear oscillators, making it a widely used primary mathematical tool for various nonlinear equations. Given its significance, many scholars, including J.H. He [7], S. Momani [8], and Z.M. Odibat [9], have extensively researched this method. A key advantage of VIM over other analytical methods is that it does not require linearization or manipulation of nonlinear terms. By using a suitable initial guess and incorporating a Lagrange multiplier, one can obtain exact or highly accurate solutions for various physical problems. However, the identification of the multiplier can be difficult without a solid understanding of the intricate theory of variational calculus [10–12], which can be challenging for some practitioners. In recent years, the integral transform has been extensively used in numerical simulation due to its rapid convergence and ease of use. It has significant practical implications in addressing various real-world engineering challenges, including electrical, industrial, mechanical, and civil engineering. In many instances, the choice of an appropriate integration transform can simplify the analysis. The choice of transformation becomes very important when we investigate different problems. This short opinion proposes a more accessible and comprehensive method for easily and effectively identifying the multiplier: the introduction of a generalized integral transform. This transform generalizes Fourier series, Laplace transforms, and other transformations, such as the Sumudu transform [13] and the Aboodh transform [14, 15]. This approach is highly appealing and promising and it does not require specialized knowledge of variational calculus. Furthermore, the procedure can be used in all mathematics textbooks.

2 The determination of the lagrange multiplier by the He-transform

Considering a general nonlinear oscillator equation in the following form:

$$\ddot{x}(t) + f(x) = 0 \tag{1}$$

with initial conditions

$$x(0) = A, \dot{x}(0) = 0 \tag{2}$$

We represent Eq. 1 as

$$\ddot{x}(t) + \omega^2 x + g(x) = 0 \tag{3}$$

where ω is unknown frequency, $g(x) = f(x) - \omega^2 x$

According to the variational iteration method (VIM) [1], the correction functional which is essentially a convolution for Eq. 3 can be expressed as

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda(t,\psi)[\ddot{x}_n(\psi) + \omega^2 x_n(\psi) + \tilde{g}(x_n)]d\psi \quad n = 0, 1, 2\cdots \tag{4}$$

where λ is a Lagrange multiplier, and its value can be selectively determined by stationary conditions of Eq. 4 with respect to x_n using variation theory [10–12]. x_n is the n th approximate solution and $\tilde{g}$ is a restricted variation, i.e., $\delta\tilde{g} = 0$.

Next, we present an alternative method for determining the multiplier. Based on the seminal contributions of Abassy [16], Mokhtari [17], and Hesameddini [18], the Laplace transform was initially incorporated into the variational iteration method [19]. It is worth considering whether or not there is a more representative integral transform than the Laplace transform in the context of VIM. J.H. He proposed in 2023 [20] a new generalized integral transform, which not only includes various integral transforms falling under the category of the Laplace transform, but also retains the properties of the Fourier transform as a special case, such as existence and linearity. This new transform offers a new perspective for the identification of Lagrange multipliers with extreme ease [21–23]. In the following, we will use this new generalized integral transform to identify the Lagrange multiplier.

He's integral transform [20] of an integrable function $f(t)$ has the following definition

$$\mathcal{H}\{f(t)\} = \mathcal{H}(s) = p(s)\int_0^\infty e^{-s^n t} f(t)dt$$

Here $\mathcal{H}(s)$ is the image of $f(t)$, $\mathcal{H}$ is the integral transformation operator, and s denotes the transformation variable. The superscript n is from the integer range.

The Lagrange multiplier can usually be expressed as [1].

$$\lambda = \lambda(t - \psi) \tag{5}$$

The correction function given in Eq. 4 is essentially the convolution, so we can easily use the He-transform. By substituting Eq. 5 into Eq. 4 and applying the He-transform to both sides of the resulting equation, we obtain the final transformation of the correction function by employing the linearity theorem and the differentiation theorem [20], as follows:

$$\begin{aligned}\mathcal{H}[x_{n+1}(t)] &= \mathcal{H}[x_n(t)] + \mathcal{H}[\int_0^t \lambda(t-\psi)[\ddot{x}_n(\psi) \\ &\quad + \omega^2 x_n(\psi) + \tilde{g}(x_n)]d\psi] \\ &= \mathcal{H}[x_n(t)] + \mathcal{H}[\lambda(t)^*[\ddot{x}_n(t) + \omega^2 x_n(t) + \tilde{g}(x_n)]] \\ &= \mathcal{H}[x_n(t)] + \frac{1}{p(s)}\mathcal{H}[\lambda(t)]\mathcal{H}[\ddot{x}_n(\psi) + \omega^2 x_n(\psi) + \tilde{g}(x_n)] \\ &= \mathcal{H}[x_n(t)] + \frac{1}{p(s)}\mathcal{H}[\lambda(t)][(s^{2n} + \omega^2)\mathcal{H}[x_n(t)] \\ &\quad - s^n p(s)x(0) - p(s)\dot{x}(0) + \mathcal{H}[\tilde{g}(x_n)]]\end{aligned} \tag{6}$$

The optimal value of λ can be determined by taking Eq. 6 to be stationary with respect to x_n, assuming that. $\frac{\delta}{\delta x_n}\mathcal{H}[g(x_n)] = 0$, $\frac{\delta}{\delta x_n}\mathcal{H}[x_{n+1}(t)] = 0$

$$\begin{aligned}\frac{\delta}{\delta x_n}\mathcal{H}[x_{n+1}(t)] &= \frac{\delta}{\delta x_n}\mathcal{H}[x_n(t)] \\ &\quad + \frac{\delta}{\delta x_n}\frac{1}{p(s)}\mathcal{H}[\lambda(t)][(s^{2n} + \omega^2)\mathcal{H}[x_n(t)] \\ &\quad - s^n p(s)x(0) - p(s)\dot{x}(0) + \mathcal{H}[\tilde{g}(x_n)]] \\ &= \frac{\delta}{\delta x_n}\mathcal{H}[x_n(t)] \\ &\quad + \frac{1}{p(s)}\mathcal{H}[\lambda(t)][(s^{2n} + \omega^2)\frac{\delta}{\delta x_n}\mathcal{H}[x_n(t)] \\ &= \left\{1 + \frac{(s^{2n} + \omega^2)}{p(s)}\mathcal{H}[\lambda(t)]\right\}\frac{\delta}{\delta x_n}\mathcal{H}[x_n(t)] = 0\end{aligned} \tag{7}$$

Eq. 7 leads to the following result

$$\mathcal{H}[\lambda(t)] = \frac{-p(s)}{s^{2n} + \omega^2} \tag{8}$$

Applying the inverse He-transform to Eq. 8 yields the following result

$$\lambda = -\frac{1}{\omega}sin\,\omega t$$

This is the same as that in Ref. [24], showing that the He-transform works more easily and more effectively.

3 Conclusion remark

In this opinion, we elucidate that the He-transform facilitates the identification of the Lagrange multiplier, making the variational iteration method more promising for solving physical problems. We hope that this short opinion can attract a wide audience from various fields, such as mathematics, physics, mechanics, and engineering. As current studies primarily concentrate on solving nonlinear oscillators with an initial value of zero, we will apply the method to solve nonlinear oscillators with generalized initial values in the future.

Author contributions

Q-RS: Writing–original draft, Writing–review and editing. J-GZ: Funding acquisition, Investigation, Project administration, Writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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