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The influence of rotation and viscosity on generalized conformable fractional micropolar thermoelasticity with two temperatures

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This research paper presents the generalized micropolar thermo-visco-elasticity model in an isotropic elastic medium that has two temperatures with conformable fractional order theory. The whole elastic medium rotates at a constant angular velocity. The generalized theory of thermoelasticity with one relaxation time is used to describe this model. We aim to study the effect of conformable fractional derivative, effect of rotation, and the two-temperature coefficients. The normal mode analysis is used to acquire the specific articulations for each component under consideration. Moreover, some specific cases are discussed with regarding to the problem. Numerical findings are gathered and displayed graphically for the variables under consideration. The outcomes were analyzed in terms of the presence or absence of rotation, viscosity, conformable fractional parameter and two temperatures for various values.

KEYWORDS

conformable fractional order theory, rotation, micropolar thermoelasticity, viscosity, one relaxation time, two-temperatures, normal mode analysis

1 Introduction

The study of fractional calculus, a subject that has garnered significant attention from mathematicians, engineers, and physicians, involves the investigation of mathematical analysis utilizing differential operators of any order. The fractional calculus has extended the usual definitions of integer order integrals and derivatives in ordinary differential calculus by transforming them into real-order operators [1–6].

Over the past 40 years, there has been a lot of focus on Theories of thermoelasticity that permit for restricted heat wave speed. These theories, known as general thermo-elasticity theories, are hyperbolic and differ from the traditional combined thermo-elasticity (C-T) theory [7], This predicts an infinite pace of heat propagation and is based on a parabolic heat equation. Lord and Shulman (L-S) [8] were the first to alter the usual Fourier law by including a unique heat conduction law, resulting in a wave type heat equation, on the other hand Green and Lindsay (G-L) [9], introduced the temperature-rate hypothesis of thermoelasticity. Green and Naghdi (G-N) [10], proposed a theory of thermo-elasticity without energy dissipation, this, unlike earlier models, does not account for thermal energy loss. Chandrasekharaia proposed two theories of dual-phase lag thermo-elasticity [11, 12] and Tzou [13]. Eringen developed the general theory of micropolar elasticity [14, 15]. In this form of solid, the vector of displacement and micro-rotation is fully defined, whereas the displacement vector alone defines motion in the case of classical elasticity [16]. The essential equations of the theory of micropolar thermoelasticity linearly get by Tauchert *et al.* [17] and Boschi [18]. Ciarletta [19] proposed a thermoelasticity with micropolar energy dissipation. A Lord-Shulman model of micropolar thermoelasticity dependent linear theory was proposed by Sherief *et al.* [20]. Othman is to blame for various issues with thermoelastic spinning media [21].

Many authors have made significant contributions to resolving the boundary value problem for thermoelasticity [22–27].

Chen and Gurtin discussed two types of temperatures: thermodynamic temperature and conductive temperature [28]. In time-independent settings, the connection between these two temperatures is linked to the heat supply. The two temperatures are similar in the absence of a heat supply and often differ in the presence of a heat supply. The two temperatures and strain are shown to have inputs in the form of a travelling wave and an instantaneous reaction that happens during the body. The waves in the two-temperature thermoelasticity theory were investigated by Warren [29], but no study in the sense of a generalised thermoelasticity theory has been carried out so far. So, along with two temperatures, the theory of two-temperature-generalized thermo-visco-elasticity will be constructed in this work.

Many authors have made significant contributions to resolving the boundary value problem for linear viscoelastic thermal materials [30–33].

The current study seeks to determine the physical quantities, such as viscosity, rotation, conformable fractional parameter, and two-temperatures, in a homogeneous, isotropic, micropolar thermo-elastic material [1, 34–40].

2 Field equations and constitutive relations

We put a system in a generalized micropolar thermo-viscoelastic medium under five theories, two temperatures and rotation as [41, 42]:

(i) The constitutive relations

$$\sigma_{ij} = 2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)e_{ij} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e\delta_{ij} - \gamma_e T_0\left(1+\gamma_0\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \varepsilon\phi_{j,i} + \beta\phi_{i,j}. \tag{2}$$

(ii) Stress equation of motion

$$\begin{aligned}&\left[\lambda_e\left(1+\alpha_o\frac{\partial}{\partial t}\right)+\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\right]\nabla(\nabla\cdot\bar{u}) + \left[\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right]\nabla^2\bar{u} + v(\nabla\times\bar{\phi})\\ &-\gamma_e\left(1+\gamma_o\frac{\partial}{\partial t}\right)\nabla T = \rho\left[\ddot{\bar{u}} + \bar{\Omega}\times(\bar{\Omega}\times\bar{u}) + 2\bar{\Omega}\times\dot{\bar{u}}\right].\end{aligned} \tag{3}$$

(iii) Couple stress equation of motion [43, 44]

$$(\alpha+\beta+\varepsilon)\nabla(\nabla\cdot\bar{\phi}) - \varepsilon\nabla\times(\nabla\times\bar{\phi}) + v(\nabla\times\bar{u}) - 2v\bar{\phi} = \rho j\left(\ddot{\bar{\phi}} + \bar{\Omega}\times\dot{\bar{\phi}}\right). \tag{4}$$

(iv) Heat conduction equation with five theories

$$k\theta_{,ii} = \rho C_E\left(1+\frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\frac{\partial}{\partial t}\right)\dot{T} + \gamma_e T_0\left(1+\gamma_0\frac{\partial}{\partial t}\right)\left(1+\frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\frac{\partial}{\partial t}\right)\dot{e}, \tag{5}$$

With,

$$T = \theta - a\theta_{,ii}. \tag{6}$$

3 Problem formulation

Take a homogenous, isotropic, micropolar-viscoelastic generalised medium with rotation and two temperatures with Cartesian rectangular system of coordinates (x, y, z), getting half-space surface like the plane $z = 0$. Two more terminology for the displacement equation in the rotating frame: the centripetal acceleration $\bar{\Omega}\times(\bar{\Omega}\times\bar{u})$ just because of the time change and the Coriolis acceleration $2\bar{\Omega}\times\dot{\bar{u}}$ because of the moving frame of reference.

Our study was restricted to $x-z$ plane. Then $\bar{u}$, $\bar{\Omega}$ and $\bar{\phi}$ will have the components:

$$\bar{u} = (u, 0, w), \bar{\Omega} = (0, \Omega, 0) \text{ and } \bar{\phi} = (0, \phi, 0) \tag{7}$$

Combination of (3), (4) and (7) provides:

$$\begin{aligned}\rho(\ddot{u} + 2\Omega\dot{w} - \Omega^2 u) = \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2 u + \left(\begin{matrix}\left(1+\alpha_1\frac{\partial}{\partial t}\right)\mu_e\\ +\left(1+\alpha_0\frac{\partial}{\partial t}\right)\lambda_e\end{matrix}\right)e_{,x}\\ - v\phi_{,z} - \gamma_e\left(1+\gamma_0\frac{\partial}{\partial t}\right)T_{,x},\end{aligned} \tag{8}$$

$$\begin{aligned}\rho(\ddot{w} - \Omega^2 w - 2\Omega\dot{u}) = \left(\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right)\nabla^2 w +\\ \left(\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+\lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)\right)e_{,z}\\ + v\phi_{,z} - \gamma_e\left(1+\gamma_0\frac{\partial}{\partial t}\right)T_{,z},\end{aligned} \tag{9}$$

$$\varepsilon\nabla^2\phi + v(u_{,z} - w_{,x}) - 2v\phi = \rho j\ddot{\phi}. \tag{10}$$

From Eqs 1, 2, 7 the stresses can be formulated as follow:

$$\sigma_{xx} = \left(2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right)u_{,x} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e - \gamma_e\left(1+\gamma_0\frac{\partial}{\partial t}\right)T, \tag{11}$$

$$\sigma_{zz} = \left(2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right)w_{,z} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e - \gamma_e\left(1+\gamma_0\frac{\partial}{\partial t}\right)T, \tag{12}$$

$$\sigma_{xz} = \left(\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right)w_{,x} + \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)u_{,z} + v\phi, \tag{13}$$

$$\sigma_{zx} = \left(\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)+v\right)u_{,z} + \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)w_{,x} - v\phi, \tag{14}$$

$$m_{xy} = \varepsilon\phi_{,x}, \quad (15)$$
$$m_{zy} = \varepsilon\phi_{,z}. \quad (16)$$

For simplification, we take the non-dimensional variables:

$$\{x', z'\} = \frac{\varpi}{c_o}\{x_i, z_i\}, \{t', \tau_0', \alpha_0', \alpha_1', \gamma_0'\} = \varpi\{t, \tau_0, \alpha_0, \alpha_1, \gamma_0\},$$
$$\{T', \theta'\} = \frac{\{T, \theta\}}{T_0}, \sigma_{ij}' = \frac{\sigma_{ij}}{\gamma_e T_0}, \Omega' = \frac{\Omega}{c_0^2\eta}, \{u', w'\} = \frac{\rho c_o \varpi}{\gamma_e T_o}\{u, w\},$$
$$m_{ij}' = \frac{\varpi}{\gamma_e T_o c_o} m_{ij}, \ \phi' = \frac{\rho c_o^2}{\gamma_e T_o}\phi. \quad (17)$$

Where, $c_0^2 = \frac{\lambda_e + 2\mu_e + \upsilon}{\rho}, \varpi = \frac{\rho c_o^2 C_E}{k}$.

With regard to the non-dimensional quantities set out in (17), Eqs 8–10, Eqs 5, 6 take the form:

$$\ddot{u} - \Omega^2 u + 2\Omega\dot{w} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)\nabla^2 u + \left[a_2\left(1 + \alpha_o\frac{\partial}{\partial t}\right) + a_3\left(1 + \alpha_o\frac{\partial}{\partial t}\right)\right]e_{,x} - \upsilon_1\phi_{,z} - \left(1 + \gamma_0\frac{\partial}{\partial t}\right)T_{,x}, \quad (18)$$

$$\ddot{w} - \Omega^2 w - 2\Omega\dot{u} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)\nabla^2 w + \left[a_2\left(1 + \alpha_o\frac{\partial}{\partial t}\right) + a_3\left(1 + \alpha_o\frac{\partial}{\partial t}\right)\right]e_{,z} + \upsilon_1\phi_{,x} - \left(1 + \gamma_0\frac{\partial}{\partial t}\right)\left(1 + \upsilon_0\frac{\partial}{\partial t}\right)T_{,z}, \quad (19)$$

$$\nabla^2\phi + a_4(u_{,z} - w_{,x}) - 2a_4\phi = a_5\ddot{\phi}, \quad (20)$$

$$\nabla^2\theta = \left(1 + \frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\frac{\partial}{\partial t}\right)\dot{T} + \varepsilon_o\left(1 + \gamma_0\frac{\partial}{\partial t}\right)\left(1 + \frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\frac{\partial}{\partial t}\right)\dot{e}, \quad (21)$$

$$T = (1 - a^*\nabla^2)\theta. \quad (22)$$

Also, the constitutive relations (11)–(16) reduce to

$$\sigma_{xx} = \left(2a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)u_{,x} + a_2\left(1 + \alpha_0\frac{\partial}{\partial t}\right)e - \left(1 + \gamma_0\frac{\partial}{\partial t}\right)T, \quad (23)$$

$$\sigma_{zz} = \left(2a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)w_{,z} + a_2\left(1 + \alpha_0\frac{\partial}{\partial t}\right)e - \left(1 + \gamma_0\frac{\partial}{\partial t}\right)T, \quad (24)$$

$$\sigma_{xz} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)w_{,x} + a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right)u_{,z} + \upsilon_1\phi, \quad (25)$$

$$\sigma_{zx} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)u_{,z} + a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right)w_{,x} - \upsilon_1\phi, \quad (26)$$

$$m_{xy} = a_6\phi_{,x}, \quad (27)$$
$$m_{zy} = a_6\phi_{,z}, \quad (28)$$

Where,

$$a_1 = \frac{\mu_e}{\rho c_0^2}, a_2 = \frac{\lambda_e}{\gamma_e T_o}, a_3 = \frac{\mu_e}{\gamma_e T_o}, a_4 = \frac{\upsilon c_o^2}{\varepsilon\varpi^2}, a_5 = \frac{\rho j c_o^2}{\varepsilon}, a_6 = \frac{\varepsilon\varpi^2}{\rho c_o^4}, \quad (29)$$
$$\varepsilon_o = \frac{\gamma_e}{\rho C_E}, \ a^* = \frac{a\varpi^2}{c_o^2}, \upsilon_1 = \frac{\upsilon}{\rho c_o^2}.$$

We define $e(x, z, t)$ and $\psi(x, z, t)$ as the displacement potentials recount to u and w

$$e = u_{,x} + w_{,z}, \psi = u_{,z} - w_{,x}. \quad (30)$$

4 Normal mode analysis

In terms of normal modes, the solution of the considered variables can be written as:

$$[u, w, e, \psi, T, \theta, \phi, m_{ij}, \sigma_{ij}](x, z, t) = [\bar{u}, \bar{w}, \bar{e}, \bar{\psi}, \bar{T}, \bar{\theta}, \bar{\phi}, \bar{m}_{ij}, \bar{\sigma}_{ij}](z)\exp(\omega t + ibx). \quad (31)$$

Where, $[\bar{u}, \bar{w}, \bar{e}, \bar{\psi}, \bar{T}, \bar{\theta}, \bar{\phi}, \bar{m}_{ij}, \bar{\sigma}_{ij}](z)$ are the amplitudes of the variables, ω is the complex angular frequency, b is the wave number in the z-direction and $= \sqrt{-1}$.

From Eqs 18, 19 we get

$$\ddot{e} - \Omega^2 e - 2\Omega\dot{\psi} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + a_2\left(1 + \alpha_o\frac{\partial}{\partial t}\right) + a_3\left(1 + \alpha_o\frac{\partial}{\partial t}\right) + \upsilon_1\right)\nabla^2 e - \left(1 + \gamma_0\frac{\partial}{\partial t}\right)\nabla^2 T, \quad (32)$$

$$\ddot{\psi} - \Omega^2\psi + 2\Omega\dot{e} = \left(a_1\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + \upsilon_1\right)\nabla^2\psi - \upsilon_1\nabla^2\phi. \quad (33)$$

Using Eqs 22, 30, 31, Eqs 20, 21, 31 and 32 lead to

$$[b_1(D^2 - b^2) - b_2]\bar{\theta} = b_3\bar{e}, \quad (34)$$
$$[b_4 - b_5(D^2 - b^2)]\bar{e} - b_6\bar{\psi} = -b_7(D^2 - b^2)[1 - a^*(D^2 - b^2)]\bar{\theta}, \quad (35)$$
$$[b_4 - b_8(D^2 - b^2)]\bar{\psi} = -b_6\bar{e} - \upsilon_1(D^2 - b^2)\bar{\phi}, \quad (36)$$
$$(D^2 - b^2)\bar{\phi} + a_4\bar{\psi} - 2a_4\bar{\phi} = \omega^2 a_5\bar{\phi}. \quad (37)$$

Also, the constitutive relations (18)–(21) become

$$\bar{\sigma}_{xx} = (2a_1(1 + \alpha_1\omega) + \upsilon_1)ib\bar{u} + a_2(1 + \alpha_o\omega)\bar{e} - (1 + \gamma_o\omega)\bar{T}, \quad (38)$$
$$\bar{\sigma}_{zz} = (2a_1(1 + \alpha_1\omega) + \upsilon_1)D\bar{w} + a_2(1 + \alpha_o\omega)\bar{e} - (1 + \gamma_o\omega)\bar{T}, \quad (39)$$
$$\bar{\sigma}_{xz} = (a_1(1 + \alpha_1\omega) + \upsilon_1)ib\bar{w} + a_1(1 + \alpha_o\omega)D\bar{u} + \upsilon_1\bar{\phi}, \quad (40)$$
$$\bar{\sigma}_{zx} = (a_1(1 + \alpha_1\omega) + \upsilon_1)D\bar{u} + a_1(1 + \alpha_o\omega)ib\bar{w} - \upsilon_1\bar{\phi}, \quad (41)$$
$$\bar{m}_{xy} = iba_6\bar{\phi}, \quad (42)$$
$$\bar{m}_{zy} = a_6 D\bar{\phi}. \quad (43)$$

Where,

$$D = \frac{d}{dz}, b_1 = a^*\omega\left(1 + \frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\omega\right), b_2 = \varepsilon_o\omega(1 + \gamma_0\omega)\left(1 + \frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\omega\right),$$
$$b_3 = \varepsilon_o\omega(1 + \gamma_0\omega)\left(1 + \frac{\tau_0^\alpha}{\alpha!}t^{1-\alpha}\omega\right), b_4 = \omega^2 - \Omega^2,$$
$$b_5 = (1 + \alpha_1\omega)a_1 + (1 + \alpha_0\omega)a_2 + (1 + \alpha_1\omega)a_3 + \upsilon_1, b_6 = 2\Omega\omega,$$
$$b_7 = (1 + \gamma_o\omega), b_8 = \upsilon_1 + a_1(1 + \alpha_1\omega), b_9 = \omega^2 a_5 + 2a_4.$$

Eliminating $\bar{\psi}(z)$, $\bar{\theta}(z)$ and $\bar{\phi}(z)$ between Eqs 34–37, we get the following eighth order ordinary differential equation satisfied with $\bar{e}(z)$

$$[D^8 - AD^6 + BD^4 - CD^2 + E]\bar{e}(z) = 0. \quad (44)$$

Where,

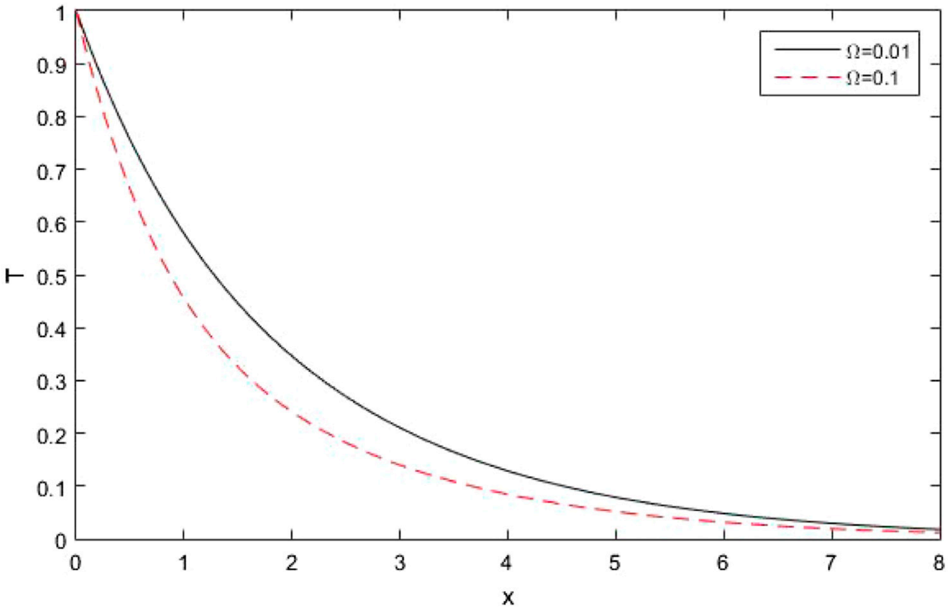

FIGURE 1
Thermodynamic temperature T for various values of rotation.

$$A = \frac{(b_1b_5 + a^* b_3b_7)(a_4v_1 - b_4 - b_8b_9 - 2b^2b_8) - b_8(b_1b_4 + 2b^2b_1b_5 + b_2b_5 + b_3b_7 + 2a^*b^2b_3b_7)}{-b_1b_5b_8 - a^*b_3b_7b_8},$$

$$B = \frac{\begin{array}{c}(-b_1b_5 - a^*b_3b_7)(b_4b_9 + b^2b_4 + b^2b_8b_9 + b^4b_8 - b^2a_4v_1) - b_8(b^2b_1b_4 + b^4b_1b_5 + b_2b_4 + b^2b_2b_5) + (a_4v_1 - b_4 - b_8b_9 - 2b^2b_8)\\ (b_1b_4 + 2b^2b_1b_5 + b_2b_5 + b_3b_7 + 2a^*b^2b_3b_7) - a^*b^4b_3b_7b_8 - b^2b_3b_7b_8 - b_1b_6^2\end{array}}{-b_1b_5b_8 - a^*b_3b_7b_8},$$

$$C = \frac{\begin{array}{c}-(b_4b_9 + b^2b_4 + b^2b_8b_9 + b^4b_8 - b^2a_4v_1)(b_1b_4 + 2b^2b_1b_5 + b_2b_5 + b_3b_7 + 2a^*b^2b_3b_7) + (a_4v_1 - b_4 - b_8b_9 - 2b^2b_8)\\ (a^*b^4b_3b_7 + b^2b_3b_7 + b^2b_1b_4 + b^4b_1b_5 + b_2b_4 + b^2b_2b_5) - b_6^2(b_1b_9 + 2b^2b_1 + b_2)\end{array}}{-b_1b_5b_8 - a^*b_3b_7b_8},$$

$$E = \frac{-(b_4b_9 + b^2b_4 + b^2b_8b_9 + b^4b_8 - b^2a_4v_1)(a^*b^4b_3b_7 + b^2b_3b_7 + b^2b_1b_4 + b^4b_1b_5 + b_2b_4 + b^2b_2b_5) - b_6^2(b^2b_1b_9 + b^4b_1 + b_2b_9 + b^2b_2)}{-b_1b_5b_8 - a^*b_3b_7b_8}.$$

By rewriting Eq. 44, we get

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)\bar{e}(z) = 0. \quad (45)$$

Where, the roots of the characteristic Eq. 44

$$k_n^2 \ (n = 1, 2, 3, 4).$$

The solution of Eq. 44, which bounded as $x \to \infty$, is given by

$$\bar{e}(z) = \sum_{n=1}^{4} M_n e^{-k_n z}. \quad (46)$$

In a similar manner,

$$\bar{\theta}(x) = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \quad (47)$$

$$\bar{\psi}(z) = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}. \quad (48)$$

Substituting from Eqs 46 to Eq. 48 and (31) in Eqs 30, 22 and Eqs 38–43, the thermodynamic temperature, micro-rotation, the displacement, force stresses and the couple stresses components take the form

$$\bar{T}(z) = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \quad (49)$$

$$\bar{\phi}(z) = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n z}, \quad (50)$$

$$\bar{w}(z) = \sum_{n=1}^{4} H_{5n} M_n e^{-k_n z}, \quad (51)$$

$$\bar{u}(z) = \sum_{n=1}^{4} H_{6n} M_n e^{-k_n z}, \quad (52)$$

$$\bar{\sigma}_{xx}(z) = \sum_{n=1}^{4} H_{7n} M_n e^{-k_n z}, \quad (53)$$

$$\bar{\sigma}_{zz}(z) = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z}, \quad (54)$$

$$\bar{\sigma}_{xz}(z) = \sum_{n=1}^{4} H_{9n} M_n e^{-k_n z}, \quad (55)$$

$$\bar{\sigma}_{zx}(z) = \sum_{n=1}^{4} G_{1n} M_n e^{-k_n z}, \quad (56)$$

$$\bar{m}_{xy}(z) = \sum_{n=1}^{4} G_{2n} M_n e^{-k_n z}, \quad (57)$$

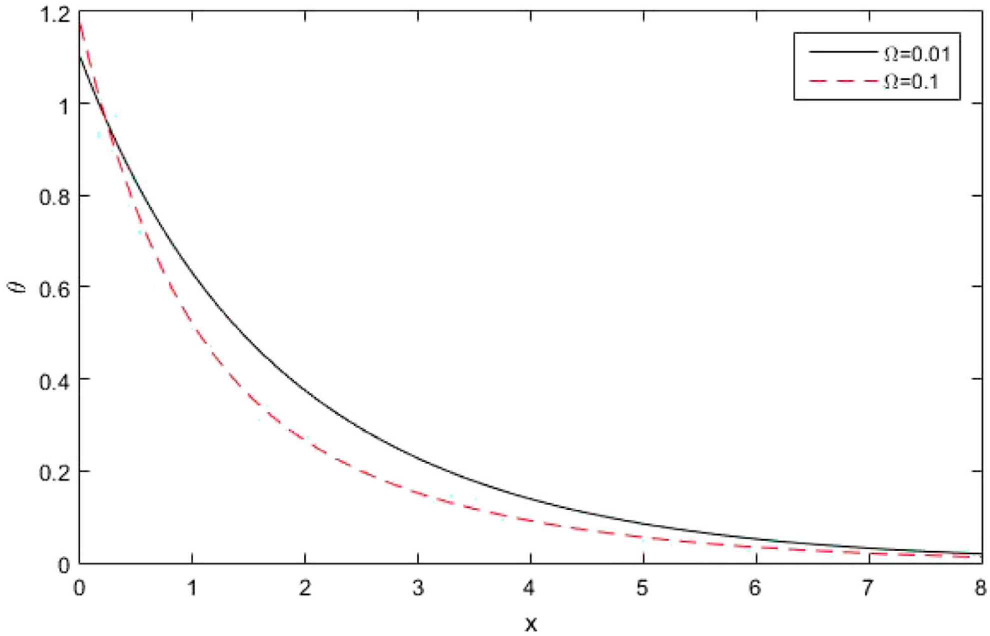

FIGURE 2
Conductive temperature θ for various values of rotation.

$$\bar{m}_{zy}(x) = \sum_{n=1}^{4} G_{3n} M_n e^{-k_n z}. \tag{58}$$

Where,

$$H_{1n} = \frac{b_3}{b_1(k_n^2 - b^2) - b_2}, H_{2n} = \left[1 - a^*(k_n^2 - b^2)\right]H_{1n},$$

$$H_{3n} = \frac{(b_4 - b_5(k_n^2 - b^2)) + H_{2n}(b_7(k_n^2 - b^2))}{b_6},$$

$$H_{4n} = \frac{a_4 H_{3n}}{b_9 - (k_n^2 - b^2)}, H_{5n} = \frac{-k_n - ibH_{3n}}{k_n^2 - b^2}, H_{6n} = \frac{-i(1 + k_n H_{5n})}{b},$$

$$H_{7n} = ibH_{6n}(2a_1(1 + \alpha_1\omega) + \upsilon_1) + a_2(1 + \alpha_o\omega) - H_{2n}(1 + \gamma_o\omega)$$

$$H_{8n} = -k_n H_{5n}(2a_1(1 + \alpha_1\omega) + \upsilon_1) + a_2(1 + \alpha_o\omega) - H_{2n}(1 + \gamma_o\omega),$$

$$H_{9n} = ibH_{5n}(a_1(1 + \alpha_1\omega) + \upsilon_1) + \upsilon_1 H_{4n} - a_1 k_n H_{6n}(1 + \alpha_1\omega),$$

$$G_{1n} = -k_n H_{6n}(a_1(1 + \alpha_1\omega) + \upsilon_1) + iba_1 H_{5n}(1 + \alpha_1\omega) - \upsilon_1 H_{4n},$$

$$G_{2n} = iba_6 H_{4n}, G_{3n} = -a_6 k_n H_{4n}.$$

$M_n(n = 1, 2, 3, 4)$, some coefficients can be calculated based on the boundary conditions

5 The boundary conditions

To obtain the coefficients $M_n(n = 1, 2, 3, 4)$, we use the surface's boundary conditions on the surface $z = 0$ as

$$T(x, 0, t) = f(x, 0, t) = f^* \exp(\omega t + ibx), \tag{59}$$

$$\sigma_{zz}(x, 0, t) = 0, \tag{60}$$

$$\sigma_{zx}(x, 0, t) = 0, \tag{61}$$

$$m_{zy}(x, 0, t) = 0. \tag{62}$$

Where, $f(x, t)$ is an arbitrary function of x, t and f^* is the magnitude of the constant temperature applied to the boundary.

Using Eqs 59–62, the following equations satisfied by the coefficient $M_n(n = 1, 2, 3, 4)$ can be obtained

$$\sum_{n=1}^{4} H_{2n} M_n = f^*, \tag{63}$$

$$\sum_{n=1}^{4} H_{8n} M_n = 0, \tag{64}$$

$$\sum_{n=1}^{4} G_{1n} M_n = 0, \tag{65}$$

$$\sum_{n=1}^{4} G_{3n} M_n = 0. \tag{66}$$

We complete the solution of the problem by solving the system of Eqs 63–66.

6 Numerical results and disccusion

To perform numerical calculations [45] the magnesium crystal value of the related parameters is taken at

$$T_o = 23\ {}^oC$$

$$\lambda_e = 9.4 \times 10^{10} kg\, m^{-1}\, s^{-2}, \varepsilon = 0.779 \times 10^{-9} kg\, m\, s^{-2}, , k = 2.510 w.m^{-1}.k^{-1}, \alpha_t = 2.36 \times 10^{-5} k^{-1}, \rho = 1.74 \times 10^3 kg.m^{-3}, C_E = 9.623 J.kg^{-1}.k^{-1}, \upsilon = 10^{10} kg\, m^{-1} s^{-2}, \mu_e = 4 \times 10^{10} kg\, m^{-1} s^{-2}, j = 0.2 \times 10^{-19} m^2.$$

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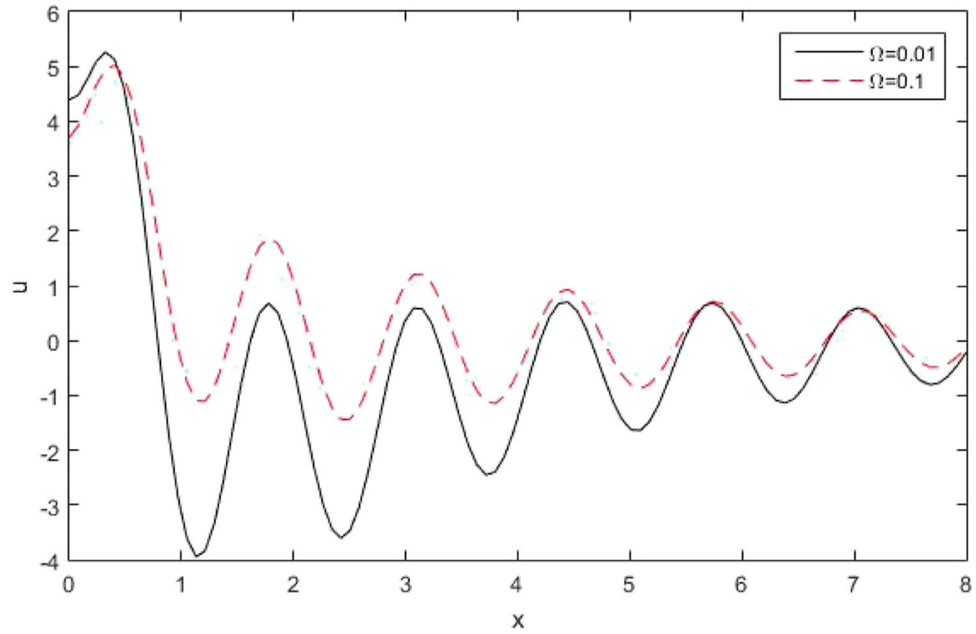

FIGURE 3
Horizontal displacement u for various values of rotation.

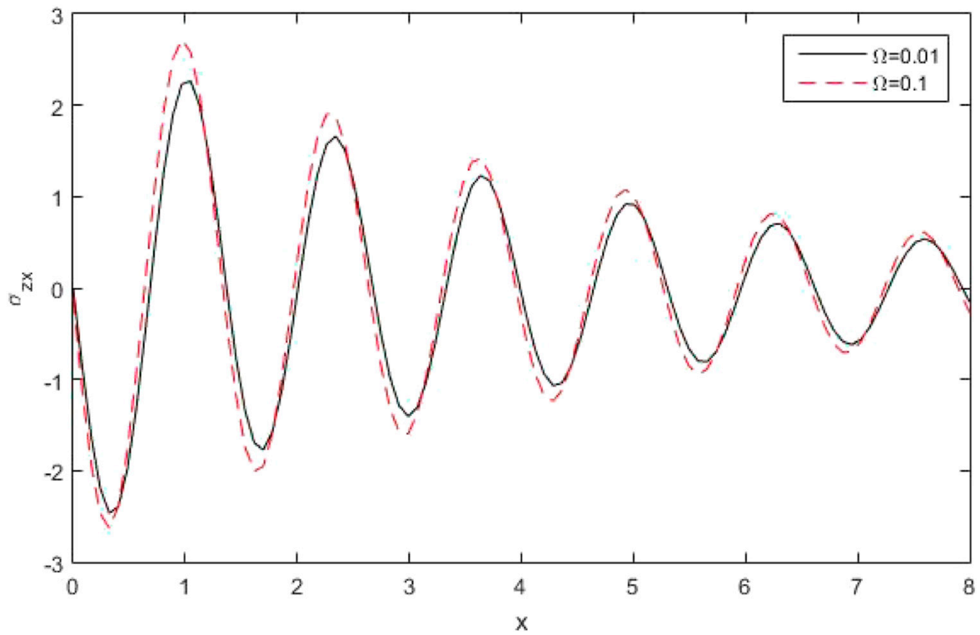

FIGURE 4
Force stress components σ_{zx} for various values of rotation.

The comparison was performed for

$$x = 0.01, f^{*} = 1, \omega = \omega_0 + i\xi, \omega_0 = -2, \xi = 1, b = 0.5, \tau_0 = 0.08,$$
$$\alpha = 0.1a = 0.5, \Omega = 0.1, \alpha_0 = 0.6, \alpha_1 = 0.9.$$

The numerical data given above were used to determine the real part distribution of the displacement component, force stress components, conductive temperature, thermo-dynamic temperature, micro-rotation, and couple stress components in

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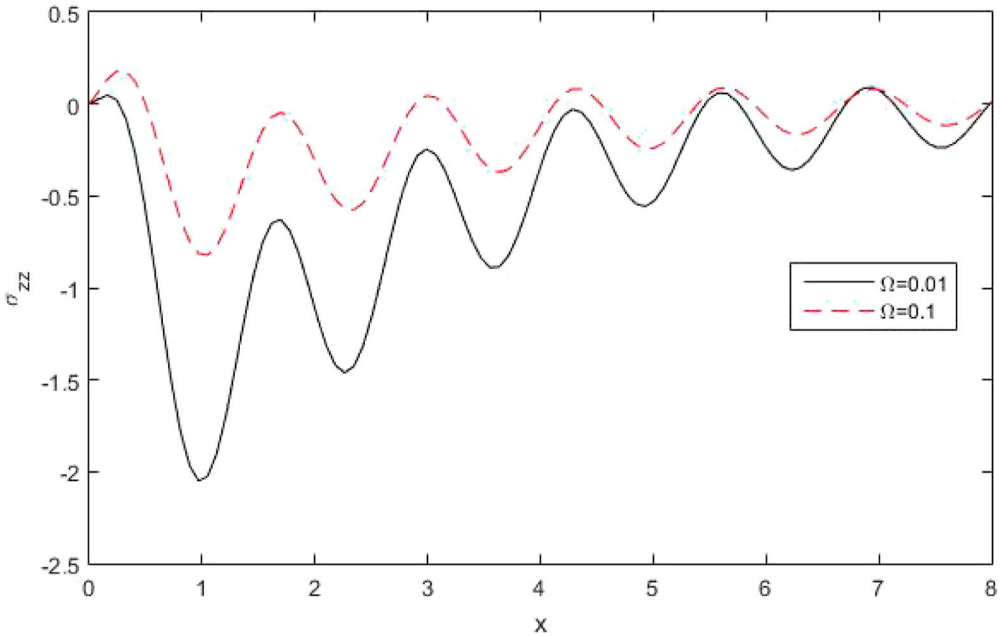

FIGURE 5
Force stress components σ_{zz} for various values of rotation.

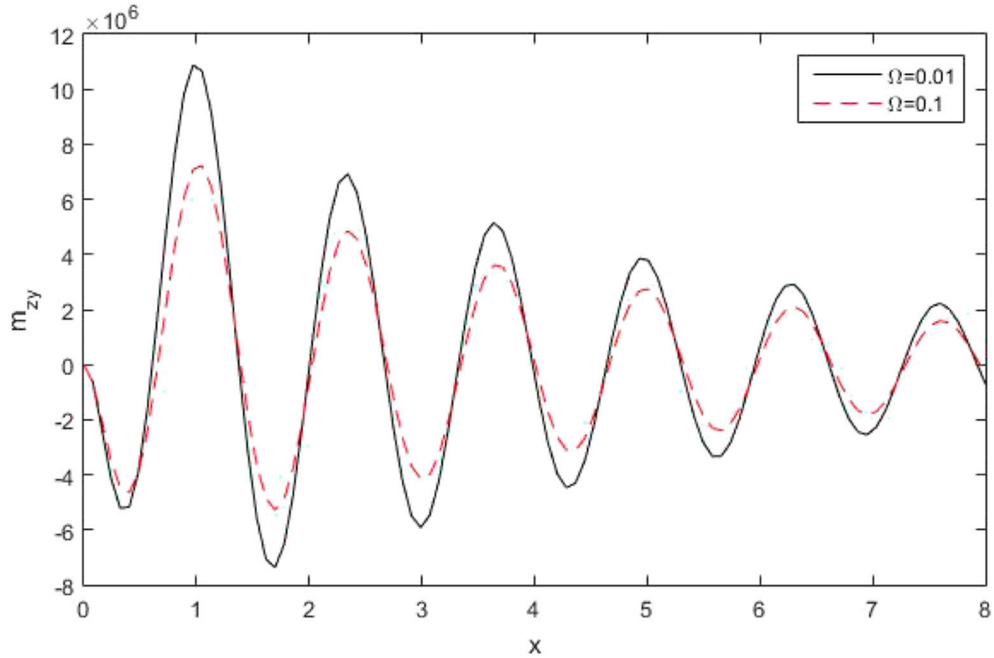

FIGURE 6
Couple stress components m_{zy} for various values of rotation.

relation to the problem under consideration. Figures 1–25 show the results.

From an implementation standpoint, we divided the schematics into four components.

(i) For different values of rotation, the micropolar thermo-visco-elasticity theory with conformable fractional order theory and two-temperatures is concerned. Figures 1–7 i.e.., $(\Omega = 0.1, 0.01)$.

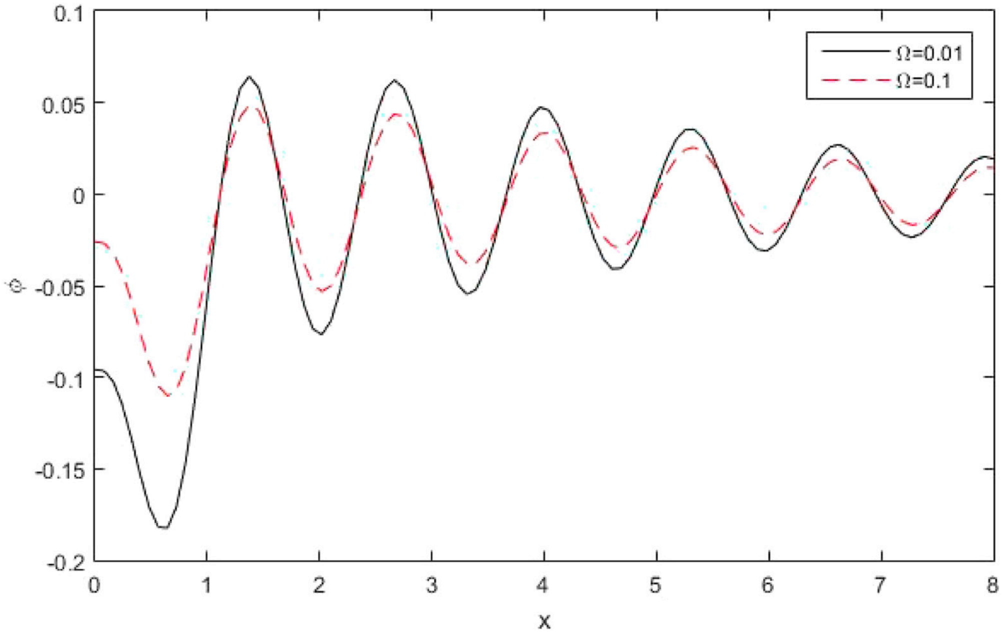

FIGURE 7
Micro-rotation φ for various values of rotation.

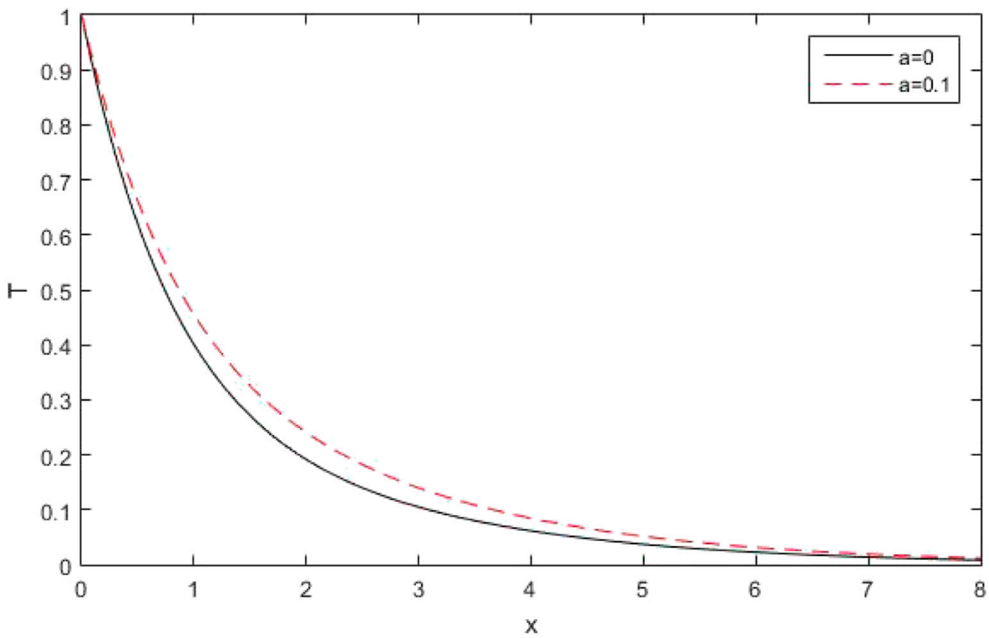

FIGURE 8
Thermodynamic temperature T for two various values of two-temperature parameter.

(ii) For different values of $a\,(a = 0, \mathrm{a} = 0.5)$, the micropolar thermo-visco-elasticity theory with rotation and conformable fractional order theory is concerned. Figures 8–14, where $a = 0$ indicates one-type temperature and $a = 0.5$ indicates two-types of temperature.

(iii) Concerned are micropolar thermoelasticity with rotation, conformable fractional order theory, and two temperatures. Figures 15–21 show variable comparisons for various values of α_0 and α_1 where, $\alpha_0 = \alpha_1 = 0$ indicates a generalized theory of micropolar thermo-elasticity (TE) and $\alpha_0 = 0.6$,

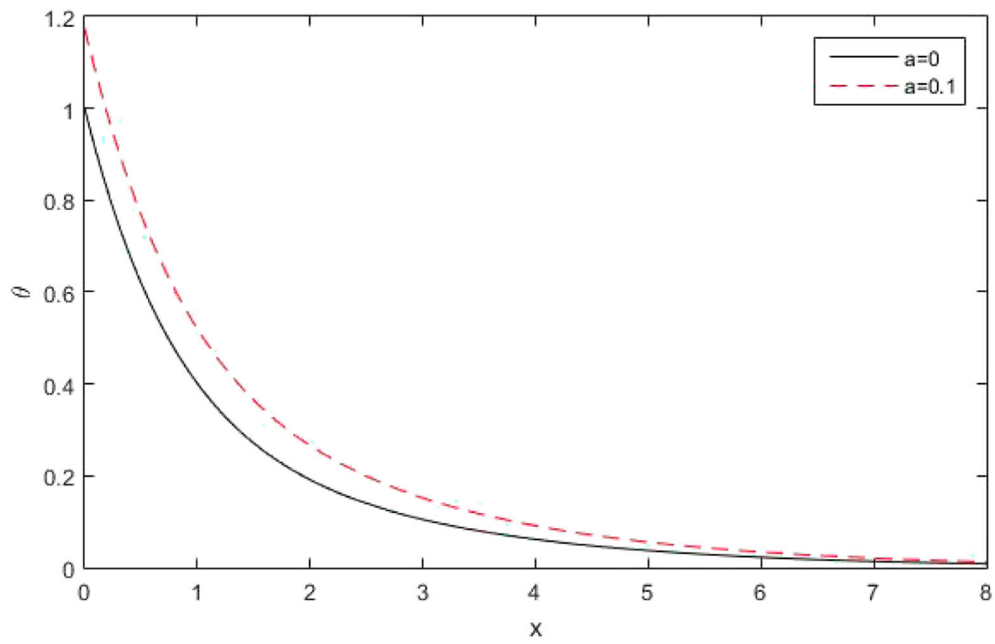

FIGURE 9
Conductive temperature θ for two various values of two - temperature parameter.

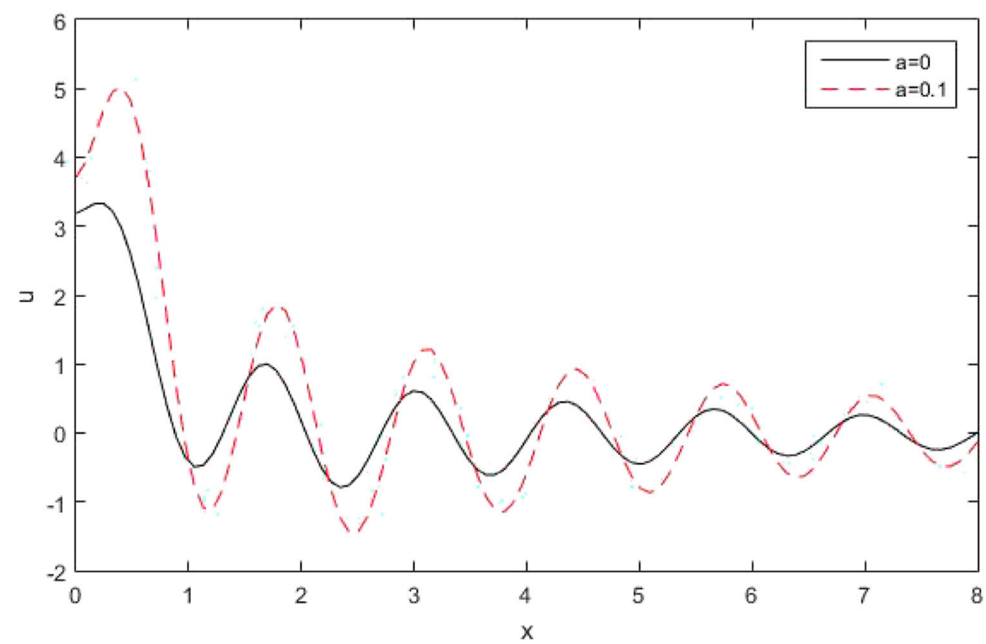

FIGURE 10
Horizontal displacement u for two various values of two-temperature parameter.

$\alpha_1 = 0.9$ indicates generalized theory of micropolar thermo-visco-elasticity (TVE).

(v)The Figures 22–25 clarified four curves predicted by the different theories of thermo-elasticity, such that

- $\alpha = 1$, indicates the generalized micropolar thermoelastic theory with one relaxation time [8].
- $t = 0$, indicates the coupled theory of micropolar thermoelasticity [46].
- $0 < \alpha < 1$, indicates the generalized conformable fractional order theory of micropolar thermo-elasticity.

(i) (Effect of rotation)

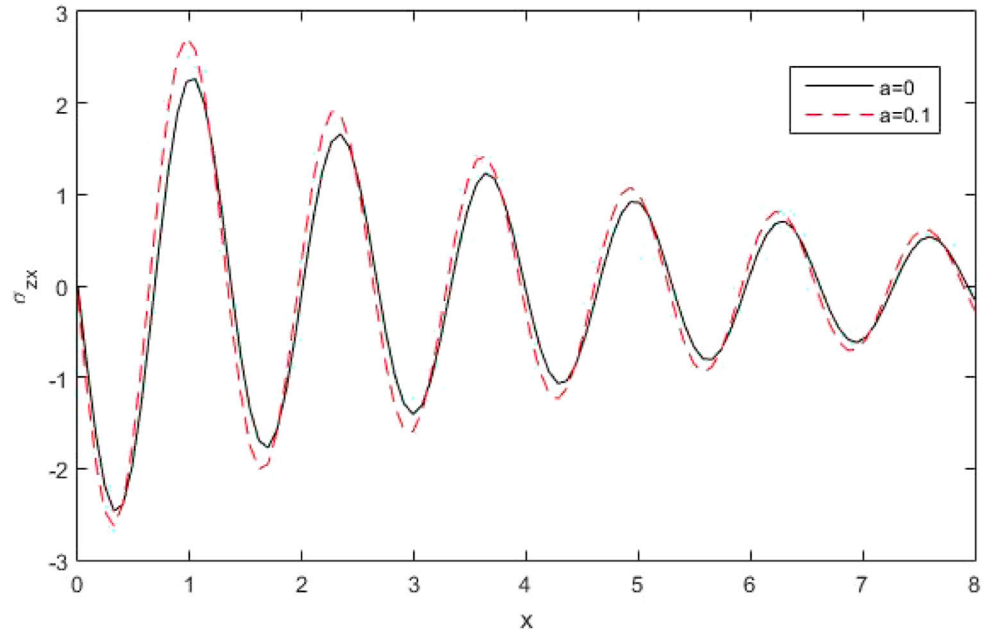

FIGURE 11
Force stress components σ_{zx} for two various values of two-temperature parameter.

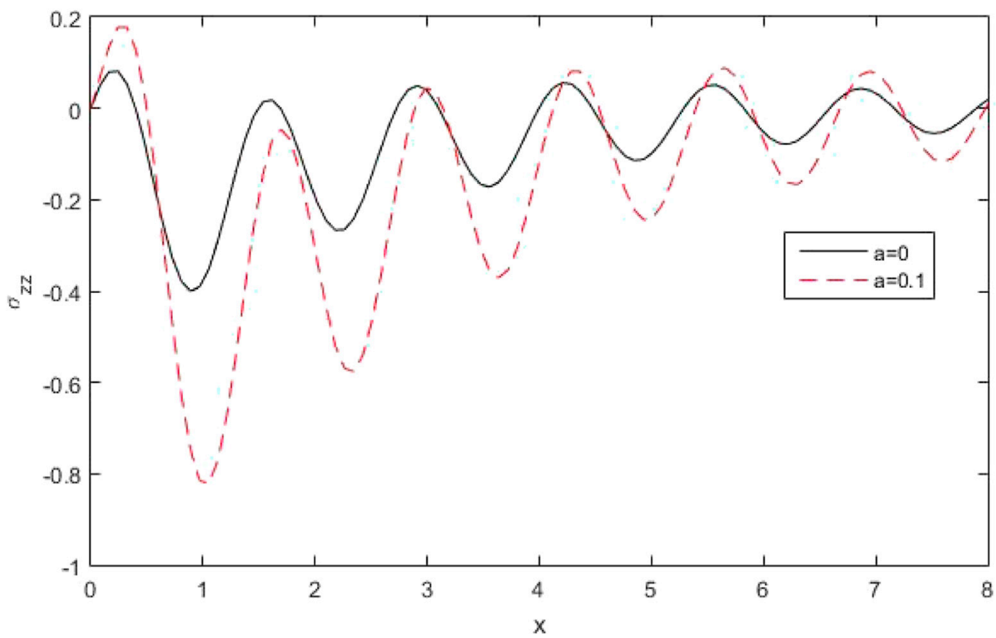

FIGURE 12
Force stress components σ_{zz} for two various values of two-temperature parameter.

Figures 1, 2 depict the distribution of θ and T with distance x with two-temperatures for varying rotational values. It is noticed that curves of θ and T begin from a positive value. Then, it then reduces to zero indefinitely.

Figure 3 shows the variations of u with distance x with two-temperatures for different values of rotation. The rotation has a decreasing influence.

Figures 4, 5 represent the profile of force stress components σ_{zx}, σ_{zz} based for various values of rotation with two-temperature.

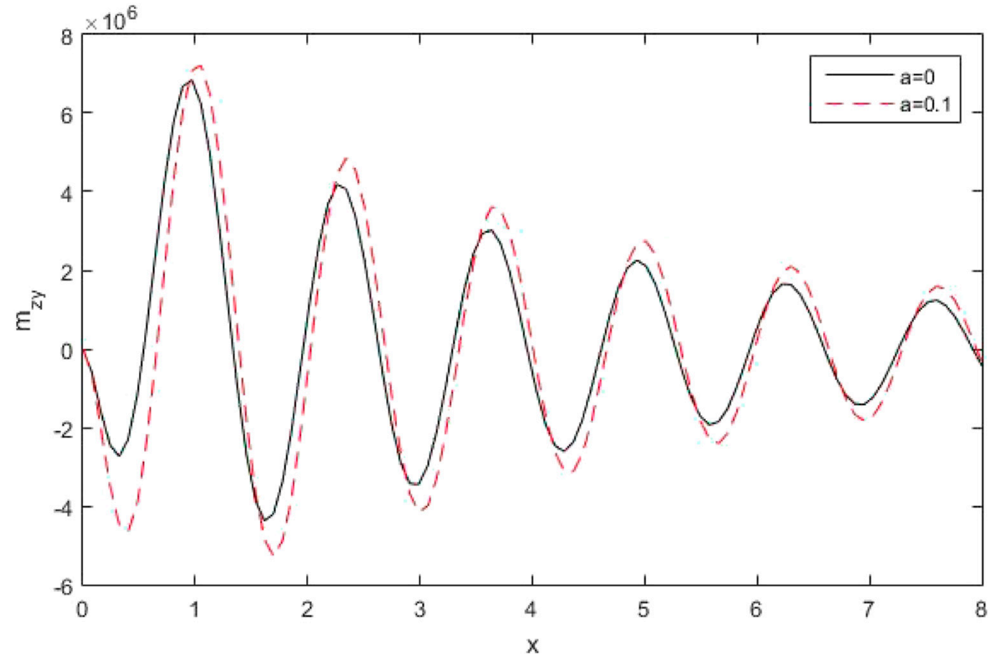

FIGURE 13
Couple stress components m_{zy} for two various values of two-temperature parameter.

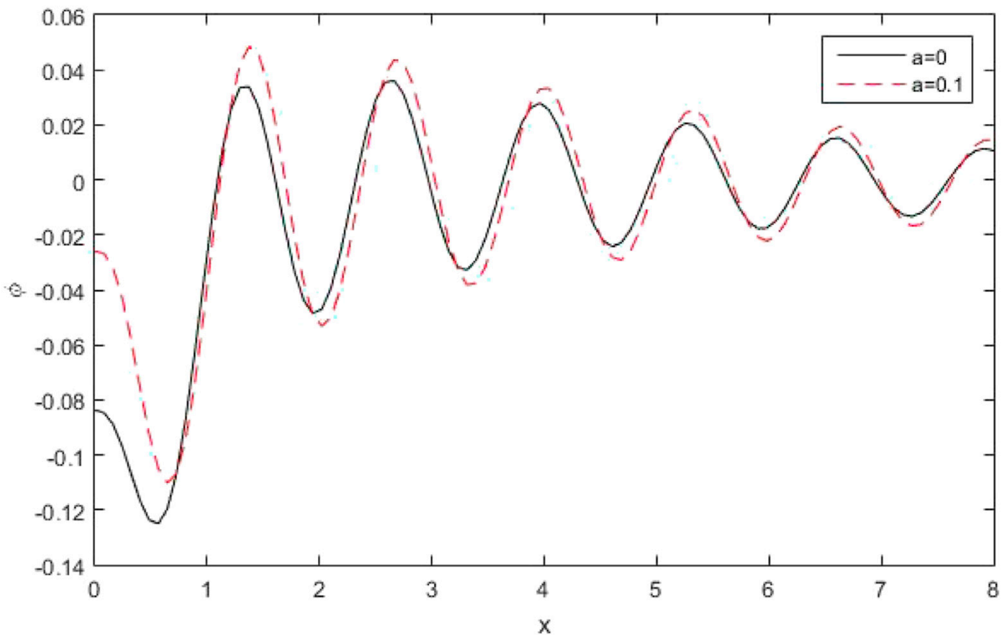

FIGURE 14
Micro-rotation ϕ for two various values of two-temperature parameter.

They start with a zero value that is completely compatible with the limits. The value of σ_{zx}, σ_{zz} for $\Omega = 0.1$ is less as compared to $\Omega = 0.01$, the values of σ_{zx}, σ_{zz} tending to zero.

Figure 6 displays the disparity of the couple stress component m_{zy} with distance x for two-temperatures for varying rotational values. It begins with a zero value entirely consistent with the limit conditions. We notice that the value of m_{zy} for $\Omega = 0.1$ is less as compared to $\Omega = 0.01$, the values of m_{zy} tending to zero.

Figure 7 shows the variation of the micro-rotation ϕ with distance x, with two-temperatures for different values of rotation. The rotation has a decreasing influence.

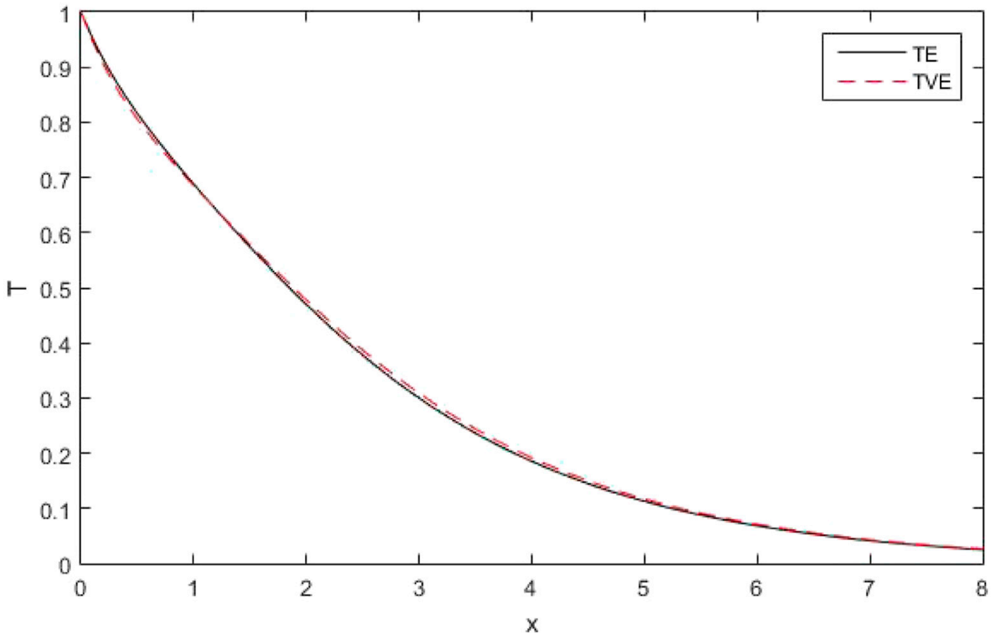

FIGURE 15
Thermodynamic temperature distribution T in the nonexistence and existence of viscosity.

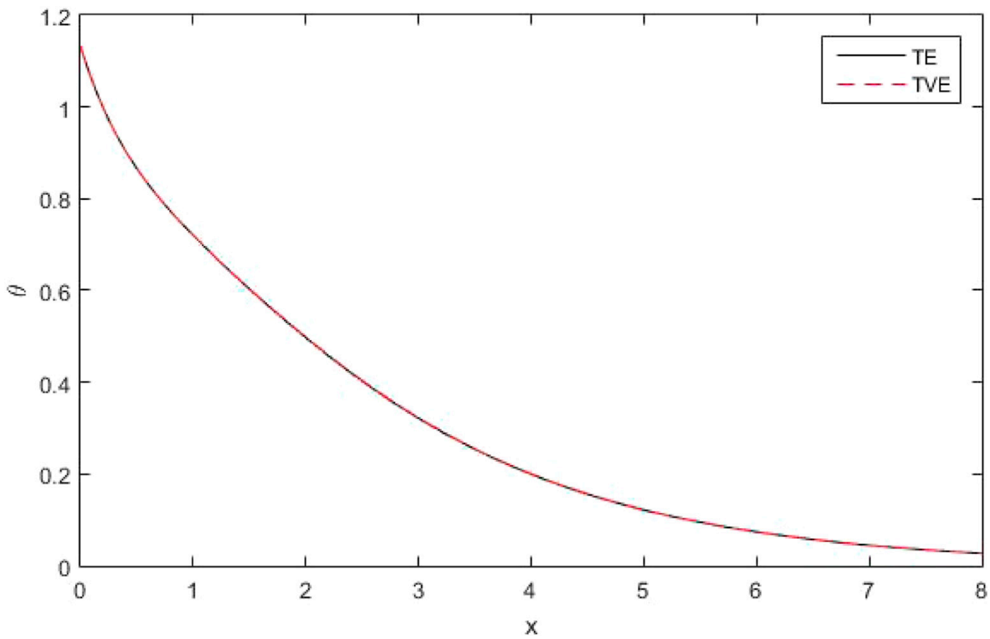

FIGURE 16
Conductive temperature distribution θ in the nonexistence and existence of viscosity.

(ii) Effect of two temperatures

Figure 8 illuminates the supply of the thermodynamic temperature T with distance x for different $a = 0, a = 0.1$. values under the influence of rotation. It begins with $T = 1$ that fully complies with the limit conditions, subsequently decreases continually to zero value. We noticed that the parameter a has an increasing effect

Figure 9 shows the conductive temperature θ with distance x with the effect of rotation. We noticed that the curve of θ begins from a positive value. It then, reduces constantly to zero. We notice that the parameter a has an increasing effect.

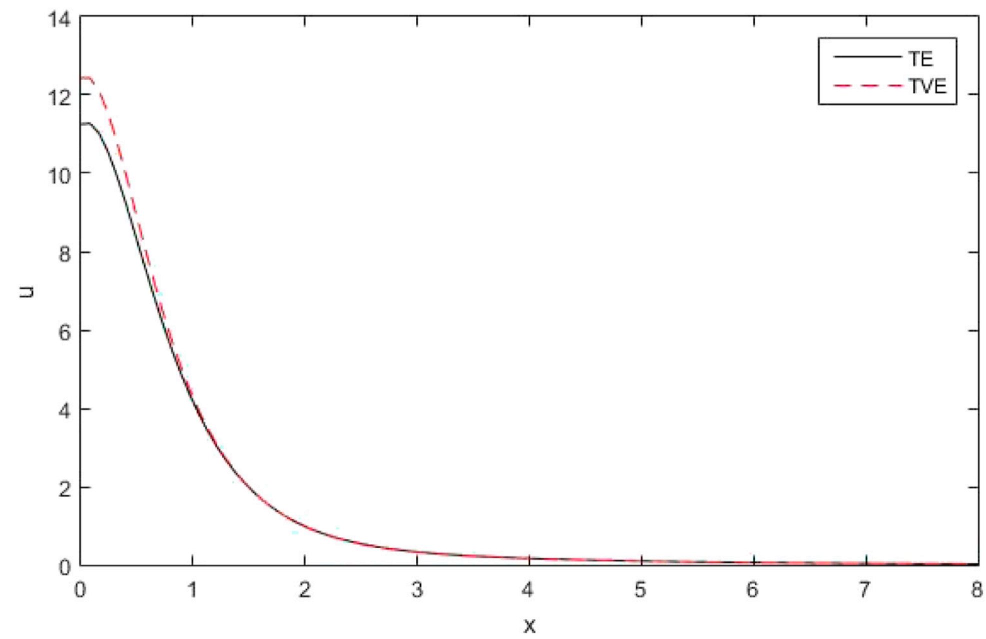

FIGURE 17
Horizontal displacement distribution u in the nonexistence and existence of viscosity.

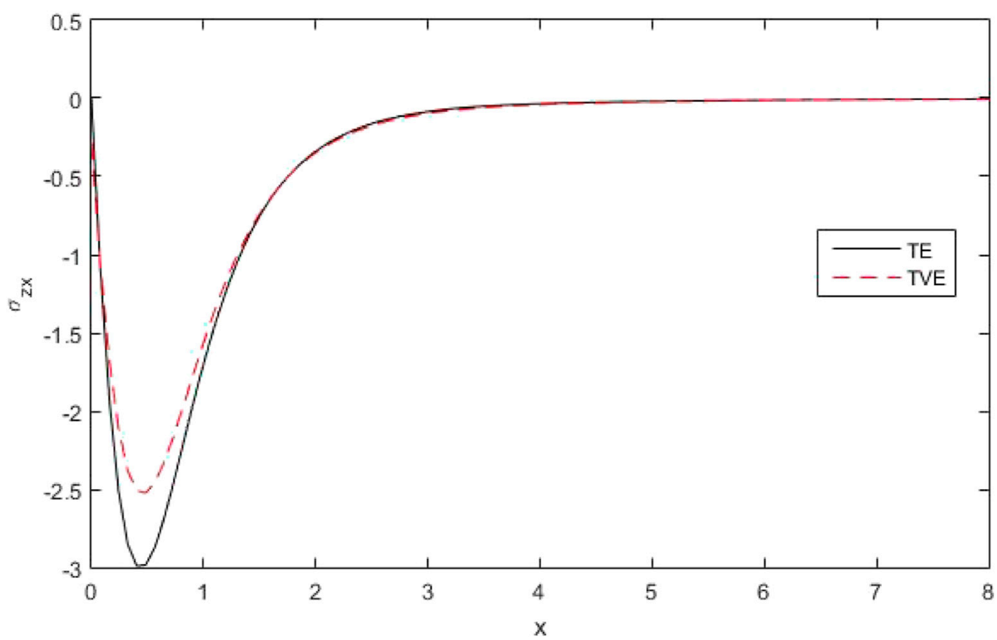

FIGURE 18
Distribution of force stress components σ_{zx} in the nonexistence and existence of viscosity.

Figure 10 displays the variations of u with distance z under the influence of rotation for different values of $a = 0, a = 0.5$. The parameter a is seen to have an increasing impact.

Figures 11, 12 denote the outline of the force stress component σ_{zx}, σ_{zz} under the effect of rotation for changed values of $a = 0, a = 0.5$. It begins with a zero value that is fully consistent with the limits. In this figure, we can see that the values of σ_{zx}, σ_{zz} for $a = 0$ is greater than that at $= 0.5$.

Figure 13 shows the profile of the couple stress component m_{zy} under the influence of rotation for the values of $a = 0, a = 0.5$. This begins with a zero value that is totally compatible with the limits. In this figure, we can see that the

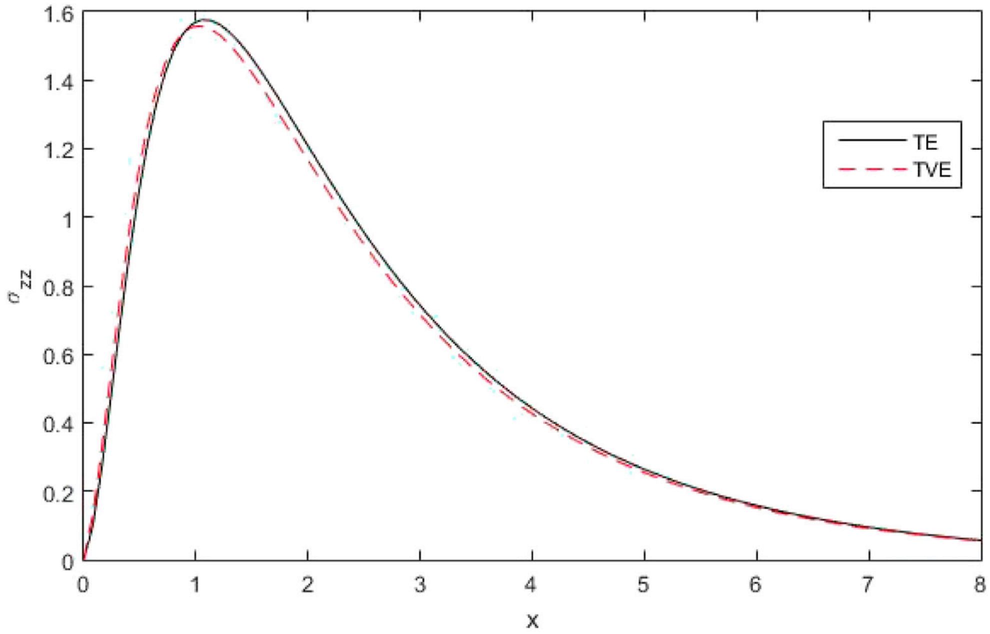

FIGURE 19
Distribution of force stress components σ_{zz} in the nonexistence and existence of viscosity.

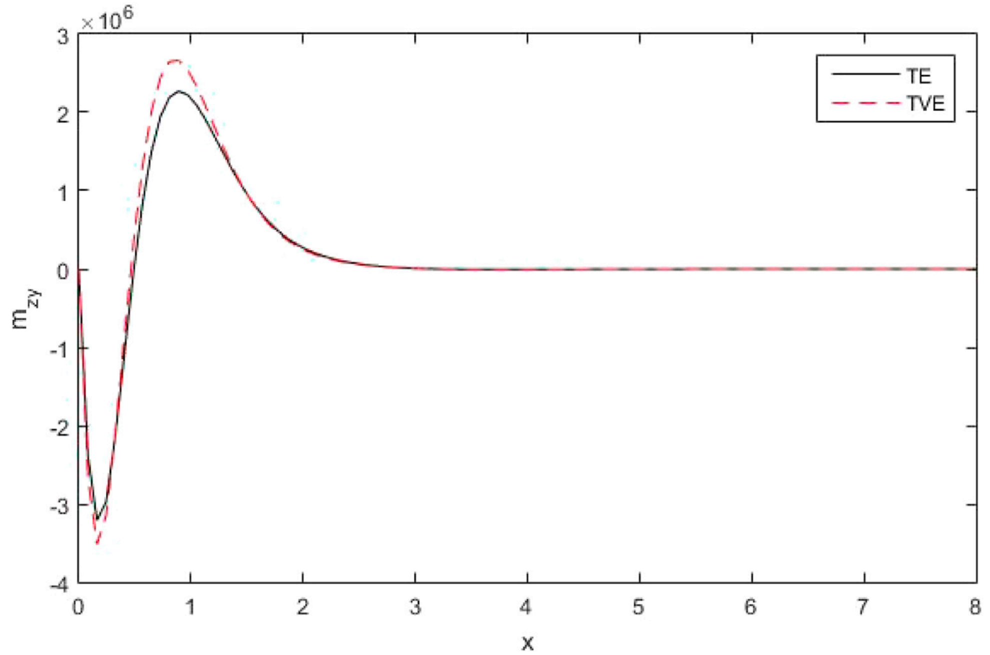

FIGURE 20
Distribution of couple stress components m_{zy} in the nonexistence and existence of viscosity.

value of m_{zy} for $a = 0.5$ is greater than that at as $a = 0$ It is observed that the influence is rising in parameter.

Figure 14 represents the variation of the micro-rotation with distance under the influence of rotation for different values of = 0, $a = 0.5$. In the figure the value of ϕ for $a = 0.5$ is greater than that at $a = 0$. The a parameter is seen to have a growing effect.

(iii) Effect of viscosity

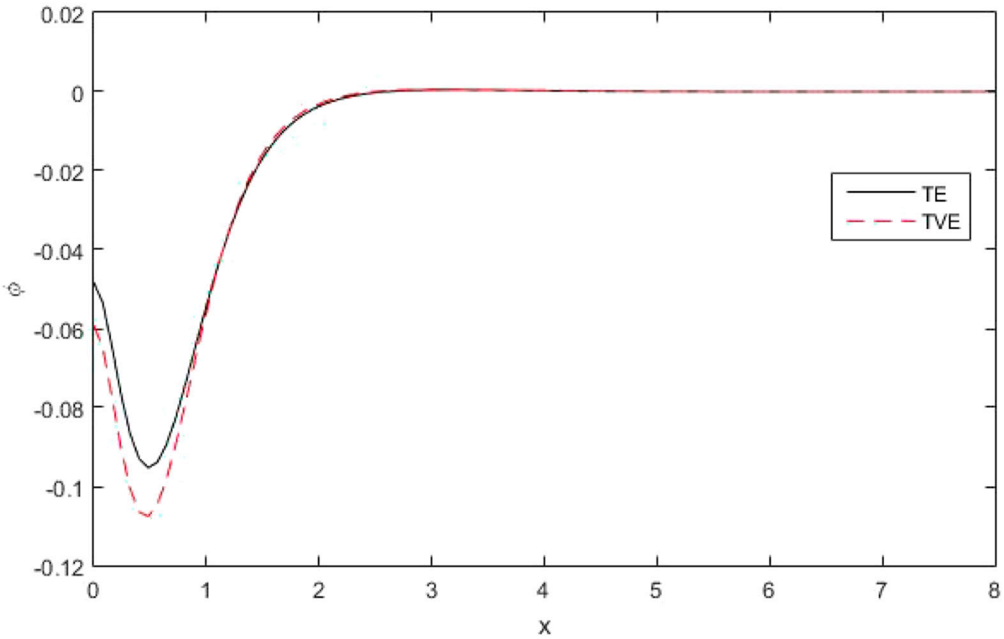

FIGURE 21
Distribution of micro-rotation ϕ in the nonexistence and existence of viscosity.

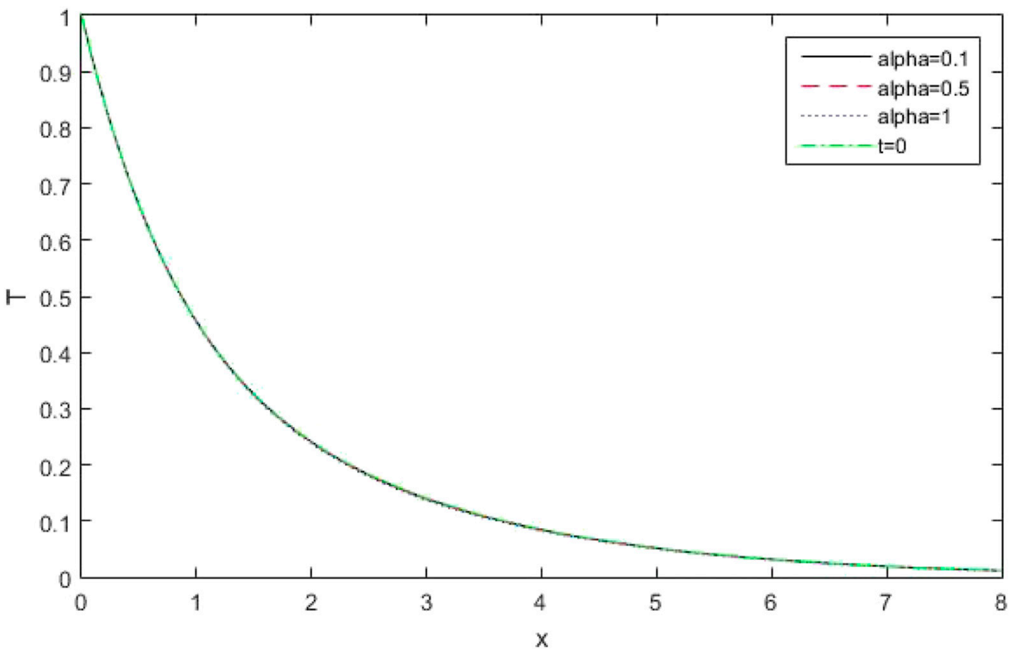

FIGURE 22
Distribution of the temperature T for various values of α.

Figures 15, 16 depict θ and T in two-temperature under the influence of rotation with and without of viscosity. It has been observed that the curves of θ and T begin from a positive value, Then it decreases constantly to zero.

Figure 17 shows the deviation of the horizontal displacement distribution u with distance x under the influence of rotation and two-temperatures with and without viscosity. It can be seen that TE > TVE

Figures 18, 19 represent the profile of stress components σ_{zx}, σ_{zz} under the influence of rotation and two-temperature for TE and TVE.They begin with a zero value that is fully in line with the limit conditions. The value of σ_{zx} for TVE is greater

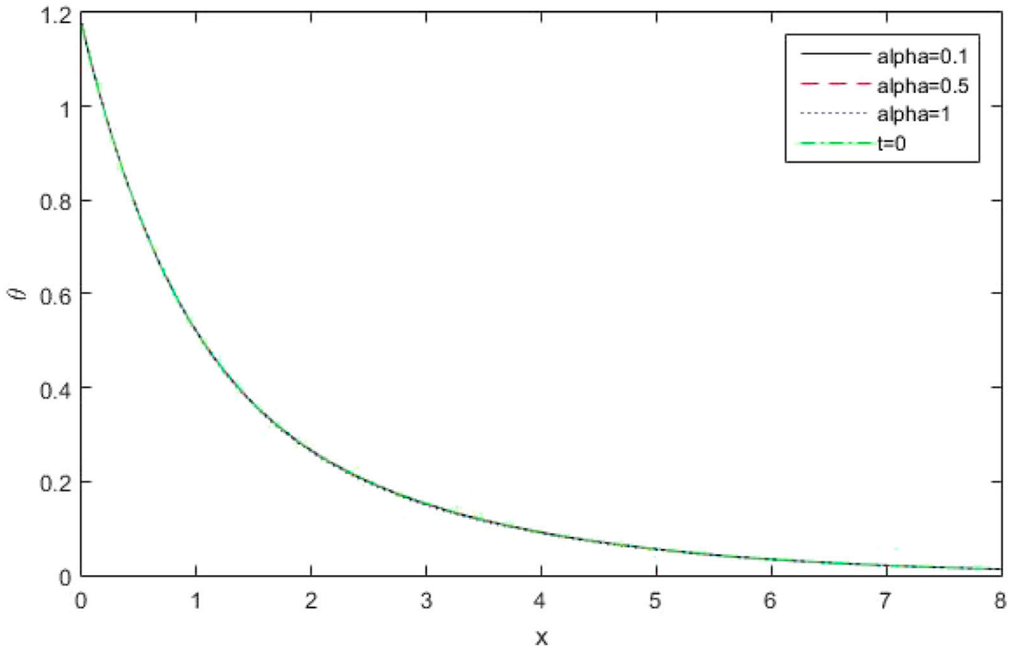

FIGURE 23
Distribution of the temperature θ for various values of α.

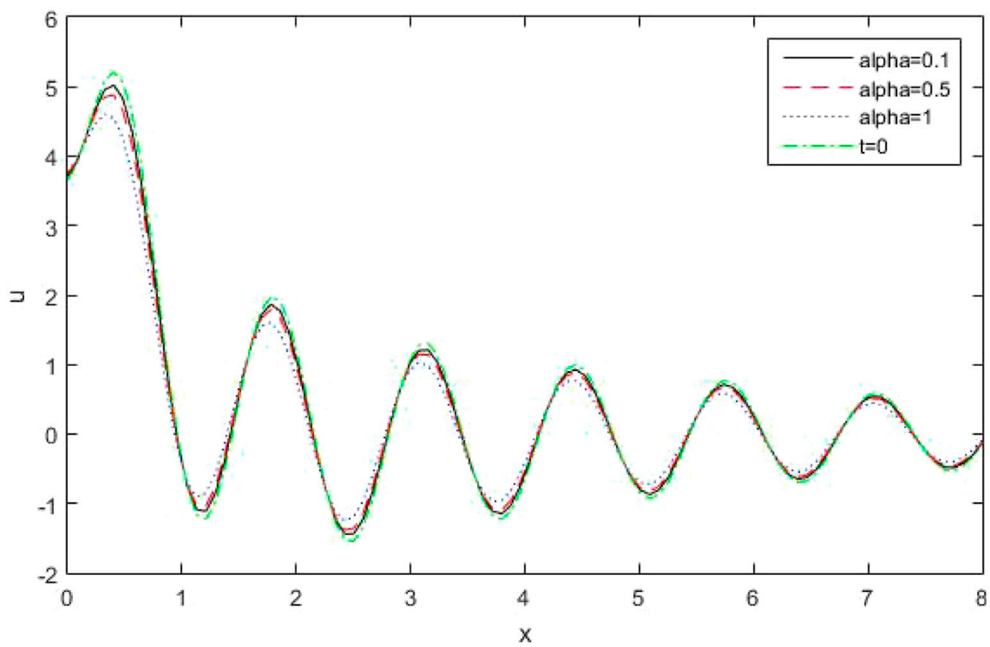

FIGURE 24
Distribution of the horizontal displacement u for various values of α.

than that for TE. the value of σ_{zz} for TE is greater than that for TVE.

Figure 20 depicts the disparity of the couple stress m_{zy} with distance x under the influence of rotation and two-temperatures for TE and TVE.We observe that the value of m_{zy} for TVE is greater than that for TE.

Figure 21 depicts the variation of the micro-rotation ϕ with distance x under the influence of rotation and two-temperature for

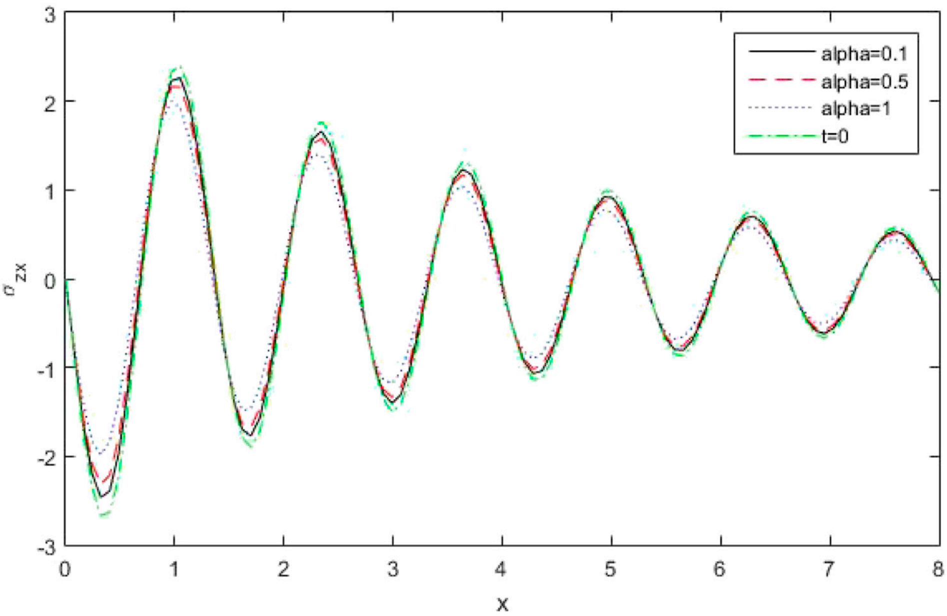

FIGURE 25
Distribution of the stress component σ_{zx} for various values of α.

TE and TVE. We observe that the value of ϕ for TVE is greater than that for TE.

(v) Effect of Conformable fractional parameter

The figures [22–24] clarified four curves predicted by the different theories of thermo-elasticity, such that

- $\alpha = 1$, indicates the generalized thermoelastic theory with one relaxation time [10].
- $t = 0$, indicates the coupled theory of thermoelasticity [9].
- $0 < \alpha < 1$, indicates the generalized conformable fractional order theory of thermo-elasticity.

Figures 22, 23 depict the distribution of the thermo-dynamic temperature T and conductive temperature θ with distance x, under the influence of rotation and two-temperature for various values of α and for $t = 0$. It is observed that the variation of θ starts from a positive value, then decreases continually to zero. It is also visible that for $\alpha = 1$, the result coincides with all results of applications that are taken in the context of the generalized thermoelasticity with one relaxation time, for $t = 0$, the result coincides with all results of applications that are taken in the case of the coupled theory, for $\alpha = 0.1$ and $\alpha = 0.5$ gives new results for the generalized conformable fractional order theory of thermoelasticity From this figure it is spotted that as fractional parameter α increase the measure of the temperature θ rise.

Figure 24 shows the variations of u via distance x under the influence of rotation and two-temperature for various values of α and $t = 0$. For $\alpha = 1$, the result to agree with all results of applications that are taken in the context of the generalized thermoelasticity with one relaxation time, for $t = 0$, the result agree with all results of applications that are taken in the context of the coupled theory, for $\alpha = 1$ and $\alpha = 0.5$ gives new results for the generalized conformable fractional order theory of thermoelasticity.

Figure 25 explains the variations of stress components σ_{zx} with distance x under the influence of rotation and two-temperature for various values of α and for $t = 0$. It can be visible that they beginning with zero value that fully agrees with the boundary conditions.

7 Conclusion

Normal mode analysis was employed to investigate the behavior of the conductive temperature, the thermodynamic temperature, the component of horizontal displacement, the component of force stress, the couple stresses, and the micro-rotation under the effect of rotation, conformable fractional order theory, viscosity, and two-temperatures in a homogeneous, isotropic, generalized micropolar thermo-viscoelastic medium. We aim to study the effect of conformable fractional derivative, effect of rotation, and the two-temperature coefficients. The above analysis leads us to the following conclusions:

- Variations in various fields are clearly constrained to a certain zone in all of the data, and the values vanish outside of the region, which is consistent with the perspective of generalized micropolar thermo-visco- elasticity theory.
- All of the physical values fulfill the boundary criteria.
- The thermodynamic and conductive temperature nature of all models TE and TVE is same.
- The angular velocity and two-temperature parameters have a significant impact on the horizontal displacement component, the force stress components, the couple stresses components, and the micro-rotation. However, there are minor impacts on the conductive temperature and the thermodynamic temperature.

- For two temperature parameter values, θ and T show a virtually identical pattern, and increasing the value of the two-temperature parameter leads the values of those functions to grow. While increasing the two-temperature parameter value leads the values of the horizontal displacement and stress components to drop, increasing the two-temperature parameter value causes the values of these functions to rise, as shown in the figures.

The given model will be valuable for scientists working on micropolar thermoelasticity in understanding the viscoelastic characteristics of human soft tissue and may lead to enhanced diagnostic applications. The findings may be used to both theoretical and empirical wave propagation.

- The fractional parameter α highly influences all variables.
- According to this work, we can treat the theory of conformable fractional order generalized micropolar thermoelasticity as an improvement in studying thermoelastic materials, we have to construct a new ranking for materials according to their fractional parameter α, where this parameter becomes a new conductor of its ability to conduct heat under the effect of thermoelastic properties, we use these properties in the factory of glasses and ceramic.

Author contributions

SE-S: Software, Writing–review and editing. AE-B: Methodology, Writing–original draft. HA: Writing–original draft.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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Appendix

Researchers in the field of thermoelasticity have all employed fractional derivatives to develop the heat conduction equation based on these derivatives

$$k\theta_{,ii} = \rho c_E\left(1 + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\theta_{,t} + \gamma T_0\left(1 + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)e_{,t} - \left(1 + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)Q \tag{A1}$$

They arrived at this equation through the utilization of the following definitions:

(i) Riemann-Liouville definition. For $\alpha \in [n-1, n)$, the α derivatives of f is

$$D_a^\alpha(f)(t) = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}}dx. \tag{A2}$$

(ii) Caputo definition. For $\alpha \in [n-1, n)$, the α derivatives of f is

$$D_a^\alpha(f)(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(x)}{(t-x)^{\alpha-n+1}}dx. \tag{A3}$$

Nevertheless, these two definitions possess certain limitations that can be summarized as follows:

(i) The Riemann-Liouville derivatives do not hold true for

$$D_a^\alpha(1) = 0 \quad \left(D_a^\alpha(1) = 0 \text{ for the caputo derivative}\right), \text{if } \alpha \text{ is not natural.} \tag{A4}$$

(ii) None of the fractional derivatives fulfill the well-known formula for the derivative of the product of two functions:

$$D_a^\alpha(fg) = f\,D_a^\alpha(g) + gD_a^\alpha(f). \tag{A5}$$

(iii) None of the fractional derivatives comply with the established formula for the derivative of the division of two functions:

$$D_a^\alpha(f/g)(t) = \frac{g\,D_a^\alpha(f) - f\,D_a^\alpha(g)}{g^2}. \tag{A6}$$

(iv) None of the fractional derivatives abide by the chain rule.

$$D_a^\alpha(fog)(t) = f^{(\alpha)}(g(t))g^{(\alpha)}(t). \tag{A7}$$

(v) Fractional derivatives do not abide by the general rule $D^\alpha D^\beta f = D^{\alpha+\beta} f$ It was necessary to adopt another definition of fractional derivatives, which is the "Conformable fractional derivatives," in order to overcome the limitations discussed above [47, 48].

The fractional derivatives of the order $\alpha \in (0, 1]$ of the absolutely continuous function $f(t)$ is

$$\frac{d^\alpha f(t)}{dt^\alpha} = t^{1-\alpha}\frac{df(t)}{dt} \tag{A8}$$

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Nomenclature

λ_e, μ_e	Lame elastic constants
ρ	Density
C_E	Specific heat at constant strain
α_t	Coefficient of linear thermal expansion
α_o, α_1	Viscoelastic relaxation times
k	Thermal conductivity
γ_e	$= (3\lambda_e + 2\mu_e)\alpha_t$
t	Time
γ_o	$= (3\lambda_e\alpha_0 + 2\mu_e\alpha_1)\alpha_t/\gamma_e$
λ	$= \lambda_e (1 + \alpha_0 \frac{\partial}{\partial t})$
μ	$= \mu_e (1 + \alpha_1 \frac{\partial}{\partial t})$
γ	$= \gamma_e (1 + \gamma_0 \frac{\partial}{\partial t})$
σ_{ij}	Components of force stress tensor
m_{ij}	Components of couple stress tensor
u_i	Components of displacements vector
Ω	Angular velocity
e_{ij}	Components of strain tensor
e	Cubical dilatation
θ	Conductive temperature
T	Thermodynamic temperature
T_0	References temperature
a	Two- temperature parameter
α	Fractional parameter
τ_0	Relaxation time
δ_{ij}	Kronecker delta
ϕ	Micro rotation vector
j	Micro inertia
$\alpha, \beta, \varepsilon, \upsilon$	Micropolar material constant
ε_{ijk}	Permutation tensor