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Developed method: interactions and their quantum picture

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By developing the previously proposed method of combining continuum mechanics with Einstein's field equations, it has been shown that the classic relativistic description, curvilinear description, and quantum description of the physical system may be reconciled using the proposed Alena Tensor. For a system with an electromagnetic field, the Lagrangian density equal to the invariant of the electromagnetic field was obtained, the vanishing four-divergence of canonical four-momentum appears to be the consequence of the Poynting theorem, and the explicit form of one of the electromagnetic four-potential gauges was introduced. The proposed method allows for further development with additional fields.

KEYWORDS

field theory, Lagrangian mechanics, quantum mechanics, general relativity, unification of interactions

1 Introduction

Over the past decades, great strides have been made in attempts to combine the quantum description of interactions with general relativity [1]. There are currently many promising approaches to connecting the quantum mechanics and general relativity, including perhaps the most promising ones: loop quantum gravity [2–4], string theory [5–7], and noncommutative spacetime theory [8, 9].

There are also attempts made to modify general relativity or find an equally good alternative theory [10–12] that would provide a more general description or would allow for the inclusion of other interactions. A significant amount of work has also been carried out to clear up some challenges related to general relativity and the ΛCMD model [13]. An explanation for the problem of dark energy [14] and dark matter [15] is still being sought, and efforts are still being made to explain the origin of the cosmological constant [16–18].

The author also attempts to bring his own contribution to the explanation of the above physics challenges based on a recently discovered method, as described in [19]. In this article, this method seems very promising and may help clarify at least some of the issues mentioned above. The author's method, which is similar to the approach presented in [20–22], also points to the essential connections between electromagnetism and general relativity; however, the postulated relationship is of a different nature and can be perceived as some generalization of the direction of research proposed in [23–26].

According to the conclusion obtained from [19], the description of motion in curved spacetime and its description in flat Minkowski spacetime with fields are equivalent, and the transformation between curved spacetime and Minkowski spacetime is known because the geometry of curved spacetime depends on the field tensor. This transformation allows for a significant simplification of research because the results obtained in flat Minkowski spacetime can easily be transformed into curved spacetime. The last missing link seems to be the quantum description.

In this article, the author will focus on developing the proposed method for a system with an electromagnetic field in such a manner, as to obtain the convergence with the description of QED and quantum mechanics. In the first section, the Lagrangian density for the system will be derived, allowing to obtain the tensor, as described in [19]. These conclusions will later be used in the article to propose the possible directions of research on combining the GR description with QFT and QM.

The author uses Einstein's summation convention, metric signature (+, −, −, −), and commonly used notations. In order to facilitate the analysis of the article, the key conclusion from [19] is quoted in the subsection below.

1.1 Short summary of the method

According to [19], the stress-energy tensor $T^{\alpha\beta}$ for a system with an electromagnetic field in a given spacetime, described by a metric tensor $g^{\alpha\beta}$, is equal to

$$T^{\alpha\beta} = \varrho U^\alpha U^\beta - \left(c^2\varrho + \Lambda_\rho\right)\left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right), \tag{1}$$

where ϱ_o is for remaining mass density, γ is the Lorentz gamma factor, and

$$\varrho \equiv \varrho_o\gamma, \tag{2}$$

$$\frac{1}{\xi} \equiv \frac{1}{4}\, g_{\mu\nu}\, h^{\mu\nu}, \tag{3}$$

$$\Lambda_\rho \equiv \frac{1}{4\mu_o}\mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} g_{\alpha\beta}, \tag{4}$$

$$h^{\alpha\beta} \equiv 2\, \frac{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}}{\sqrt{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\mu\beta}\, \mathbb{F}_{\alpha\eta}\, g^{\eta\xi}\, \mathbb{F}^{\mu}{}_{\xi}}}. \tag{5}$$

In the above equations, $\mathbb{F}^{\alpha\beta}$ represents the electromagnetic field tensor, and the stress-energy tensor for the electromagnetic filed, which is denoted as $\Upsilon^{\alpha\beta}$, may be represented as follows:

$$\Upsilon^{\alpha\beta} \equiv \Lambda_\rho\left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) = \Lambda_\rho g^{\alpha\beta} - \frac{1}{\mu_o}\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}. \tag{6}$$

Thanks to the proposed amendment toward the continuum mechanics, in the flat Minkowski spacetime occurs

$$\partial_\alpha U^\alpha = -\frac{d\gamma}{dt} \quad \rightarrow \quad \partial_\alpha\, \varrho U^\alpha = 0. \tag{7}$$

Thus, denoting four-momentum density as $\varrho U^\mu = \varrho_o\gamma\, U^\mu$, the total four-force density f^μ operating in the system is

$$f^\mu \equiv \varrho A^\mu = \partial_\alpha \varrho U^\mu U^\alpha. \tag{8}$$

Denoting the remaining charge density in the system as ρ_o and

$$\rho \equiv \rho_o\gamma, \tag{9}$$

the electromagnetic four-current J^α is equal to

$$J^\alpha \equiv \rho\, U^\alpha = \rho_o\gamma\, U^\alpha. \tag{10}$$

The pressure p in the system is equal to

$$p \equiv c^2\varrho + \Lambda_\rho. \tag{11}$$

In the flat Minkowski spacetime, the total four-force density f^α operating in the system calculated from vanishing $\partial_\beta\, T^{\alpha\beta}$ is the sum of electromagnetic $\left(f^\alpha_{EM}\right)$ and gravitational $\left(f^\alpha_{gr}\right)$, and the sum of the remaining $\left(f^\alpha_{oth}\right)$ four-force densities is as follows:

$$f^\alpha = \begin{cases} f^\alpha_{EM} \equiv \partial_\beta\, \Upsilon^{\alpha\beta} \quad (electromagnetic) \\ + \\ f^\alpha_{gr} \equiv \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right)\partial_\beta\, p \quad (gravitational) \\ + \\ f^\alpha_{oth} \equiv \dfrac{\varrho c^2}{\Lambda_\rho} f^\alpha_{EM} \quad (sum\, of\; remaining\; forces). \end{cases} \tag{12}$$

As shown in [19], in curved spacetime ($g_{\alpha\beta} = h_{\alpha\beta}$), presented method reproduces Einstein's field equations with an accuracy of $\frac{4\pi G}{c^4}$ constant and with a cosmological constant Λ that is dependent on the invariant of the electromagnetic field tensor $\mathbb{F}^{\alpha\gamma}$:

$$\Lambda = -\frac{\pi G}{c^4\mu_o}\, \mathbb{F}^{\alpha\mu}\, h_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} h_{\alpha\beta} = -\frac{4\pi G}{c^4}\Lambda_\rho, \tag{13}$$

where $h_{\alpha\beta}$ appears to be the metric tensor of the spacetime in which all motion occurs along geodesics and where Λ_ρ describes the vacuum energy density.

It is worth noting that although in flat Minkowski spacetime Λ_ρ has a negative value due to the adopted metric signature, this does not determine its value in curved spacetime. Therefore, solutions with a negative cosmological constant are also possible, which is an issue discussed in the literature [27–29].

It was also shown that in this solution Einstein's tensor describes the spacetime curvature related to vanishing in curved spacetime four-force densities $f^\alpha_{gr} + f^\alpha_{oth}$ and is, therefore, related to the curvature responsible for gravity only when other forces are being neglected.

The proposed method allows adding additional fields while maintaining its properties. One may define another stress–energy tensor describing the field (e.g., describing the sum of several fields) instead of $\Upsilon^{\alpha\beta}$ and insert it into the stress–energy tensor $T^{\alpha\beta}$ in a manner that is analogous to that presented above. As a result of the vanishing four-divergence of $T^{\alpha\beta}$, one will obtain in the flat spacetime four-force densities related to the new field and in curved spacetime, the equations will transform into EFE with the cosmological constant depending on the invariant of the considered new field strength tensor.

2 Lagrangian density for the system

Since the transition to curved spacetime is known for the considered method, the rest of the article will focus on the calculations in Minkowski spacetime with the presence of an electromagnetic field, where $\eta^{\alpha\beta}$ represents the Minkowski metric tensor.

Using a simplified notation

$$\frac{d\,\ln(p)}{d\tau} = U_\mu\partial^\mu \ln(p) = U_\mu\frac{\partial^\mu p}{p}, \tag{14}$$

it can be seen that the four-force densities resulting from the obtained stress–energy tensor (12) in flat Minkowski spacetime can be written as follows:

$$\begin{cases} f_{gr}^{\alpha} = \left(\eta^{\alpha\beta} - \xi\, h^{\alpha\beta}\right)\partial_\beta p = \dfrac{d\,\ln(p)}{d\tau}\varrho U^\alpha - T^{\alpha\beta}\partial_\beta \ln(p) \\ f_{EM}^{\alpha} = \dfrac{\Lambda_\rho}{p}\left(f^\alpha - f_{gr}^{\alpha}\right) \\ f_{oth}^{\alpha} = \dfrac{\varrho c^2}{p}\left(f^\alpha - f_{gr}^{\alpha}\right), \end{cases} \tag{15}$$

where f_{EM}^{μ} can also be represented in terms of electromagnetic four-potential and four-current. This means that to fully describe the system and derive the Lagrangian density, it is enough to find an explicit equation for the gravitational force or some gauges of electromagnetic four-potential.

Referring to definitions from section 1.1, one may notice that by proposing the following electromagnetic four-potential $\mathbb{A}^\mu$,

$$\mathbb{A}^\mu \equiv -\frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o}U^\mu, \tag{16}$$

one obtains the electromagnetic four-force density f_{EM}^{α} in the form of

$$f_{EM}^{\alpha} = J_\beta\left(\partial^\alpha \mathbb{A}^\beta - \partial^\beta \mathbb{A}^\alpha\right) = \frac{\Lambda_\rho}{p}\left(f^\alpha - \frac{d\,\ln(p)}{d\tau}\varrho U^\alpha + \varrho c^2 \partial^\alpha \ln(p)\right), \tag{17}$$

where J_β is the electromagnetic four-current and where the Minkowski metric property was utilized:

$$U_\beta U^\beta = c^2 \quad \rightarrow \quad U_\beta \partial^\alpha U^\beta = \frac{1}{2}\,\partial^\alpha\left(U_\beta U^\beta\right) = 0. \tag{18}$$

Four-force densities operating in the system may now be described by the following equality:

$$J_\beta\left(\partial^\alpha \mathbb{A}^\beta - \partial^\beta \mathbb{A}^\alpha\right) + \varrho U_\beta\left(\partial^\beta \frac{\varrho c^2}{p}U^\alpha - \partial^\alpha \frac{\varrho c^2}{p}U^\beta\right) = \varrho U_\beta\left(\partial^\beta U^\alpha - \partial^\alpha U^\beta\right) = f^\alpha. \tag{19}$$

Comparing equations (15) and (17), it is seen that the introduced electromagnetic four-potential yields

$$0 = \left(T^{\alpha\beta} - \varrho c^2\, \eta^{\alpha\beta}\right)\partial_\beta \ln(p), \tag{20}$$

which is equivalent to imposing the following condition on the normalized stress–energy tensor

$$0 = \partial_\beta\left(\frac{T^{\alpha\beta}}{\eta_{\mu\gamma}T^{\mu\gamma}}\right) + \partial^\alpha \ln\left(\eta_{\mu\gamma}T^{\mu\gamma}\right), \tag{21}$$

and which yields the gravitational four-force density in Minkowski spacetime in the form of

$$f_{gr}^{\mu} = \varrho\left(\frac{d\,\ln(p)}{d\tau}U^\mu - c^2\partial^\mu \ln(p)\right). \tag{22}$$

Now, one may show that the proposed electromagnetic four-potential leads to correct solutions.

At first, recalling the classical Lagrangian density [30] for electromagnetism, one may show why, in the light of the conclusions from [19] and above, it does not seem to be correct and thus makes it difficult to create a symmetric stress–energy tensor [31]. The classical value of the Lagrangian density for the electromagnetic field, written with the notation used in the article, is

$$-\mathcal{L}_{EMclassic} = \Lambda_\rho + \mathbb{A}^\mu J_\mu. \tag{23}$$

In addition to the obvious doubt that is observed by taking the different gauge of the four-potential $\mathbb{A}^\mu$, one changes the value of the Lagrangian density and one may notice that with the considered electromagnetic four-potential, such Lagrangian density is equal to

$$-\mathcal{L}_{EMclassic} = \Lambda_\rho - \frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o}U^\mu U_\mu \rho_o \gamma = \Lambda_\rho - \frac{\Lambda_\rho \varrho c^2}{p} = \frac{\Lambda_\rho^2}{p}. \tag{24}$$

It is observed that the above Lagrangian density is not invariant under gradients over four-positions, and $\mathbb{A}^\mu$ and J_μ are dependent, what is not taken into account in classical calculation:

$$\frac{\mathbb{A}^\alpha}{\mathbb{A}^\mu \mathbb{A}_\mu} = \frac{J^\alpha}{\mathbb{A}^\mu J_\mu}. \tag{25}$$

The above analysis yields

$$\frac{\partial \ln\left(\frac{1}{\sqrt{\mathbb{A}^\mu \mathbb{A}_\mu}}\right)}{\partial \mathbb{A}_\alpha} = -\frac{J^\alpha}{\mathbb{A}^\mu J_\mu} = \frac{p}{\varrho c^2}\frac{J^\alpha}{\Lambda_\rho}. \tag{26}$$

One may decompose

$$\ln\left(\frac{1}{\sqrt{\mathbb{A}^\mu \mathbb{A}_\mu}}\right) = \ln\left(\frac{p\,\rho_o}{\varrho_o c}\right) - \ln\left(\Lambda_\rho\right), \tag{27}$$

and simplify (26) to

$$\frac{\partial \ln\left(\frac{p\,\rho_o}{\varrho_o c}\right)}{\partial \mathbb{A}_\alpha} - \frac{\partial \ln\left(\Lambda_\rho\right)}{\partial \mathbb{A}_\alpha} = \frac{J^\alpha}{\varrho c^2} + \frac{J^\alpha}{\Lambda_\rho}, \tag{28}$$

where the above equation yields

$$\frac{\partial \ln\left(\Lambda_\rho\right)}{\partial \mathbb{A}_\alpha} = -\frac{J^\alpha}{\Lambda_\rho}, \tag{29}$$

which leads to the conclusion that Λ_ρ acts as the Lagrangian density for the system

$$\frac{\partial \Lambda_\rho}{\partial \mathbb{A}_\alpha} = \partial_\nu\left(\frac{\partial \Lambda_\rho}{\partial\left(\partial_\nu \mathbb{A}_\alpha\right)}\right) = -J^\alpha, \tag{30}$$

which would support the conclusion from [32] and what yields the stress–energy tensor for the system in the form of

$$T^{\alpha\beta} = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma - \Lambda_\rho \eta^{\alpha\beta}. \tag{31}$$

The proof of correctness for the electromagnetic field tensor (noted as $\Upsilon^{\alpha\beta}$) allows seeing this solution as follows:

$$f_{EM}^{\beta} = \partial_\alpha \Upsilon^{\alpha\beta} = J^\gamma \mathbb{F}^\beta{}_\gamma - \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\alpha \mathbb{F}^\beta{}_\gamma, \tag{32}$$

and what yields following property of the electromagnetic field tensor:

$$\mathbb{F}^{\alpha\gamma}\partial_\alpha\partial_\gamma \mathbb{A}^\beta = \mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma. \tag{33}$$

Using the above substitution, one may note that

$$\partial_\alpha \Upsilon^{\alpha\beta} = \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta - \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma - J^\gamma \partial_\gamma \mathbb{A}^\beta - \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma. \tag{34}$$

Therefore, the invariance of Λ_ρ with respect to $\mathbb{A}_\alpha$ and $\partial_\gamma \mathbb{A}_\alpha$ is both a condition on the correctness of the electromagnetic stress–energy tensor and on Λ_ρ in the role of Lagrangian density

$$0 = \partial^\beta \Lambda_\rho = \frac{\partial \Lambda_\rho}{\partial(\partial_\alpha \mathbb{A}_\gamma)}\partial^\beta(\partial_\alpha \mathbb{A}_\gamma) + \frac{\partial \Lambda_\rho}{\partial \mathbb{A}_\gamma}\partial^\beta \mathbb{A}_\gamma = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta\partial_\alpha \mathbb{A}_\gamma - J^\gamma \partial^\beta \mathbb{A}_\gamma = \partial_\alpha \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma, \tag{35}$$

what yields for (34) that

$$\partial_\alpha \Upsilon^{\alpha\beta} = J^\gamma \partial^\beta \mathbb{A}_\gamma - J^\gamma \partial_\gamma \mathbb{A}^\beta = f^\beta_{EM}. \tag{36}$$

Equations (1), (6), and (31) yield

$$\frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta = \varrho U^\alpha U^\beta - \frac{c^2 \varrho}{\Lambda_\rho}\Upsilon^{\alpha\beta}, \tag{37}$$

and what yields the second representation of the stress–energy tensor is

$$T^{\alpha\beta} = \frac{p}{\varrho c^2}\cdot \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^\beta - \frac{\Lambda_\rho}{c^2}U^\alpha U^\beta = \frac{p}{\varrho c^2}\,\partial_\gamma \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\mathbb{A}^\beta. \tag{38}$$

After four-divergence, it provides additional expression for relation between forces and provides useful clues about the behavior of the system when transitioning to the description in curved spacetime.

3 Hamiltonian density and energy transmission

By observing Hamiltonian density as $\mathcal{H}$ from (31), one obtains

$$\mathcal{H} \equiv T^{00} = \frac{1}{\mu_o}\mathbb{F}^{0\gamma}\partial^0 \mathbb{A}_\gamma - \Lambda_\rho. \tag{39}$$

The above Hamiltonian density agrees with the classical Hamiltonian density for the electromagnetic field [33] except that this Hamiltonian density was currently considered for sourceless regions. According to conclusions from previous sections, this Hamiltonian density describes the whole system with an electromagnetic field, including gravity and other four-force densities resulting from the considered stress–energy tensor. Therefore, the above equations may significantly simplify quantum field theory equations ([34]–[36]), which will be shown in this section for the purposes of QED.

At first, one may notice that in transformed (31)

$$-T^{\alpha 0} = -\frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial_\gamma \mathbb{A}^0 + \Upsilon^{\alpha 0}, \tag{40}$$

the first row of the electromagnetic stress–energy tensor $\Upsilon^{\alpha 0}$ is a four-vector, representing the energy density of an electromagnetic field and Poynting vector [37]—the Poynting four-vector. Therefore, the vanishing four-divergence of $T^{\alpha 0}$ must represent the Poynting theorem. Indeed, properties (33) and (35) provide such an equality

$$0 = -\partial_\alpha T^{\alpha 0} = J_\gamma \mathbb{F}^{0\gamma} + \partial_\alpha \Upsilon^{\alpha 0}. \tag{41}$$

Next, one may introduce the auxiliary variable ε with the energy density dimension defined as follows:

$$c\varepsilon \equiv -\frac{1}{\mu_o}\mathbb{F}^{0\mu}\frac{d\mathbb{A}_\mu}{d\tau}, \tag{42}$$

and comparing the result

$$-U_\beta T^{0\beta} = c\varepsilon + c\gamma \Lambda_\rho, \tag{43}$$

between the two tensor definitions (31) and (38), one may notice that it must hold

$$-\frac{p}{\varrho c^2}\cdot \frac{1}{\mu_o}\mathbb{F}^{0\mu}\partial_\mu \mathbb{A}^\beta = -\frac{p}{\varrho c^2 \mu_o}\cdot\left(U^\beta\, \mathbb{F}^{0\mu}\partial_\mu \frac{\mathbb{A}^0}{c\gamma} + \frac{\mathbb{A}^0}{c\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta\right) = \frac{\varepsilon}{c}\,U^\beta - \frac{p}{\varrho c^2}\,\frac{\mathbb{A}^0}{\gamma c \mu_o}\mathbb{F}^{0\mu}\partial_\mu U^\beta \tag{44}$$

because the second component of above vanishes contracted with U_β, due to the property of the Minkowski metric (18). Therefore, (31) and (38) also yield the following:

$$-T^{0\beta} = \varepsilon \frac{\varrho c}{p}\,U^\beta - \frac{c\epsilon_o \mathbb{A}^0}{\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta + \Upsilon^{0\beta}, \tag{45}$$

where ϵ_o is electric vacuum permittivity, and

$$\Upsilon^{0\beta}U_\beta = c\varepsilon \frac{\Lambda_\rho}{p} + c\gamma \Lambda_\rho. \tag{46}$$

Since $\partial^\mu p = \partial^\mu \varrho c^2$, thus from (44), one obtains

$$\varepsilon\gamma = \frac{c\epsilon_o \mathbb{A}^0}{\gamma}\,\mathbb{F}^{0\mu}\partial_\mu \gamma c, \tag{47}$$

and thanks to (44) that was substituted to (38), one also obtains

$$-T^{0\beta} = \frac{\varepsilon + \Lambda_\rho \gamma}{c}\,U^\beta - \frac{p}{\varrho c^2}\,\frac{c\epsilon_o \mathbb{A}^0}{\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta. \tag{48}$$

Since from (1) and (6) for T^{00}, one obtains

$$\mathcal{H} = \varrho c^2 \gamma^2 - \frac{p}{\Lambda_\rho}\Upsilon^{00}. \tag{49}$$

Therefore, comparing the zero-component of (45)

$$\mathcal{H} = -\varepsilon\gamma \frac{\varrho c^2}{p} + \varepsilon\gamma - \Upsilon^{00} = \varepsilon\gamma - \frac{\mathcal{H}}{\Lambda_\rho}\varrho c^2 - \frac{p}{\Lambda_\rho}\Upsilon^{00}, \tag{50}$$

to (49) and comparing that to (48)

$$\mathcal{H} = -\varepsilon\gamma - \Lambda_\rho \gamma^2 + \frac{p}{\varrho c^2}\varepsilon\gamma = \Lambda_\rho\left(\frac{\varepsilon\gamma}{\varrho c^2} - \gamma^2\right), \tag{51}$$

one may notice that

$$\varepsilon\gamma = \varrho c^2 \gamma^2 + \varrho c^2 \tag{52}$$

is a valid solution of the system, which yields

$$\mathcal{H} = \Lambda_\rho, \tag{53}$$

$$\frac{1}{c\gamma}U_\beta T^{0\beta} = -\frac{\varrho_o c^2}{\gamma} - p. \tag{54}$$

There is a whole class of solutions (52) in the form $\varepsilon\gamma = \varrho c^2\gamma^2 + \mathcal{K}\cdot\varrho c^2$; however, $\mathcal{K} <> 1$ would not be consistent with the following conclusions. It is also worth noting that the obtained solution $\mathcal{H} = \mathcal{L} = \Lambda_\rho$ means that there is no potential in the system in the classical sense, and thus, the dynamics of the system depends on itself. This is exactly what would be expected from a description that reproduces general relativity in flat spacetime.

From the analysis of Eq. 45, it may then be concluded that after the integration of $-\frac{1}{c}T^{0\beta}$ with respect to the volume, the total energy transported in the isolated system should be the sum of the four-momentum and four-vectors describing energy transmission related to fields. This would be consistent with the conclusion from [38] that "equations of motion for matter do not need to be introduced separately but follow the field equations." It would mean that the canonical four-momentum density is only a part of the stress–energy tensor.

Therefore, by analogy with the Poynting four-vector $\frac{1}{c}\Upsilon^{0\beta}$, one may introduce a four-vector Z^β that is understood as its equivalent for the remaining interactions and rewrite (45) as

$$-\frac{1}{c}T^{0\beta} = \varrho_o U^\beta + Z^\beta + \frac{1}{c}\Upsilon^{0\beta}, \tag{55}$$

where

$$Z^\beta \equiv \rho_o \mathbb{A}^\beta + \frac{\varrho c^2\gamma^2}{p}\varrho_o U^\beta - \frac{\epsilon_o \mathbb{A}^0}{\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta. \tag{56}$$

The above result ensures that the canonical four-momentum density for the system with the electromagnetic field depends on the four-potential and charge density as expected. This supports the earlier statement about the need to set $\mathcal{K} = 1$ and makes its physical interpretation visible. It is also worth noting that $-\frac{\epsilon_o\mathbb{A}^0}{\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta$, due to its properties, may be associated with some descriptions of the spin.

One may also note that the above solution yields $p < 0$ since the energy density of the electromagnetic field is

$$\Upsilon^{00} = \frac{\Lambda_\rho}{p}\left(\varrho c^2\gamma^2 - \Lambda_\rho\right), \tag{57}$$

where $\Lambda_\rho < 0$ in flat spacetime, due to the adopted metric signature. Thus, Z^β may also be simplified to

$$Z^\beta = \frac{\Upsilon^{00}}{\Lambda_\rho}\varrho_o U^\beta - \frac{\epsilon_o\mathbb{A}^0}{\gamma}\mathbb{F}^{0\mu}\partial_\mu U^\beta. \tag{58}$$

Finally, one may define another gauge $\bar{\mathbb{A}}_\gamma$ of electromagnetic four-potential $\mathbb{A}_\gamma$ in the following manner:

$$\bar{\mathbb{A}}^\gamma \equiv \mathbb{A}^\gamma - \partial^\gamma \mathbb{A}^\beta X_\beta = -X_\beta \partial^\gamma \mathbb{A}^\beta, \tag{59}$$

and it is to be noted that

$$-X_\beta T^{0\beta} = \frac{1}{\mu_o}\mathbb{F}^{0\gamma}\bar{\mathbb{A}}_\gamma + X_\beta \Upsilon^{0\beta}. \tag{60}$$

The four-divergence of $T^{0\beta}$ vanishes, and therefore, (53) indicates that

$$X_\beta \partial^\alpha T^{0\beta} = 0, \tag{61}$$

which yields

$$\partial^\alpha X_\beta T^{0\beta} = T^{0\alpha}. \tag{62}$$

The above equation brings two more important insights as follows:

- $\frac{1}{c}X_\beta T^{0\beta}$ may play a role of the density of Hamilton's principal function;
- Hamilton's principal function may be expressed based on the electromagnetic field only, so in the absence of the electromagnetic field it disappears.

All the above equations also lead to this conclusion that (54) may also act as Lagrangian density in the classic relativistic description based on four-vectors.

One may, thus, summarize all of the above findings and propose a method for the description of the system with the use of classical field theory for point-like particles.

4 Point-like particles and their quantum picture

Initially, it should be noted that the reasoning presented in Section 3 changes the interpretation of what the relativistic principle of least action means. As one may conclude from above, there is no inertial system in which no fields act, and in the absence of fields, the Lagrangian, the Hamiltonian, and Hamilton's principal function vanish. Since the metric tensor (5) for description in curved spacetime depends on the electromagnetic field tensor only, it seems clear that in the considered system, the absence of the electromagnetic field means the actual disappearance of spacetime and the absence of any action.

Then, one may introduce generalized, canonical four-momentum H^μ as four-gradient on Hamilton's principal function S

$$H^\mu \equiv -\frac{1}{c}\int T^{0\mu}\, d^3x \equiv -\partial^\mu S, \tag{63}$$

where

$$-S \equiv H^\mu X_\mu. \tag{64}$$

One may also conclude from previous sections that the canonical four-momentum should be in the form of

$$H^\mu = P^\mu + V^\mu, \tag{65}$$

where

$$V^\mu \equiv \int Z^\mu + \frac{1}{c}\Upsilon^{0\mu}\, d^3x, \tag{66}$$

and where four-momentum P^μ may now be considered just "another gauge" of $-V^\mu$:

$$-\partial^\alpha P^\mu = \partial^\alpha\partial^\mu S + \partial^\alpha V^\mu. \tag{67}$$

Since in the limit of the inertial system, one obtains $P^\mu X_\mu = mc^2\tau$, and therefore, to ensure vanishing Hamilton's principal function in the inertial system, one may expect that

$$V^\mu X_\mu \equiv -mc^2\tau, \tag{68}$$

which would also yield vanishing in the inertial system Lagrangian L for the point-like particle in the form of

$$-\gamma L = U_\mu H^\mu = F^\mu X_\mu, \tag{69}$$

where F^μ is the four-force. Equation 48 yields

$$H^\mu = -\frac{\gamma L}{c^2} U^\mu + \mathbb{S}^\mu, \tag{70}$$

where

$$\mathbb{S}^\beta \equiv \int \frac{\epsilon_o \Lambda_\rho}{\gamma c \rho_o} \mathbb{F}^{0\mu} \partial_\mu U^\beta \, d^3x, \tag{71}$$

and where $\mathbb{S}^\beta U_\beta$ vanishes, what yields from

$$\mathbb{S}^\beta = \frac{dS}{d\tau} \frac{1}{c^2} U^\beta - \partial^\beta S. \tag{72}$$

In the above equation, the Hamilton's principal function, generalized canonical four-momentum, and Lagrangian vanish for the inertial system, as expected.

Since

$$\mathbb{S}^\mu \mathbb{S}_\mu = H^\mu H_\mu - \left(\frac{\gamma L}{c}\right)^2, \tag{73}$$

therefore, to ensure compatibility with the equations of quantum mechanics, it suffices to consider the properties of $\mathbb{S}^\mu \mathbb{S}_\mu$. For instance, if

$$\mathbb{S}^\mu \mathbb{S}_\mu = m^2 c^2 - \left(\frac{\gamma L}{c}\right)^2, \tag{74}$$

then, by introducing quantum wave function Ψ in the form of

$$\Psi \equiv e^{\pm i K^\mu X_\mu}, \tag{75}$$

where K^μ is the wave four-vector related to the canonical four-momentum

$$\hbar K^\mu \equiv H^\mu, \tag{76}$$

from (73), one obtains the Klein–Gordon equation as follows:

$$\left(\Box + \frac{m^2 c^2}{\hbar^2}\right)\Psi = 0. \tag{77}$$

This shows that in addition to the alignment with QFT (39), the first quantization also seems possible which allows for further analysis of the system from the perspective of the quantum mechanics, eliminating the problem of negative energy appearing in solutions [39].

The above representation allows the analysis of the system in the quantum approach, classical approach based on (40), and the introduction of a field-dependent metric in (5) for curved spacetime, which connects the previously divergent descriptions of physical systems.

5 Conclusion and discussion

As shown above, the proposed method summarized in Section 1.1 seems to be a very promising area of further research. In addition to the previous agreement with Einstein's field equations in curved spacetime, by imposing condition (21) on the normalized stress–energy tensor (1) (hereinafter, referred to as the Alena Tensor) in flat Minkowski spacetime, one obtains consistent results, developing the knowledge of the physical system with an electromagnetic field. Gravitational, electromagnetic, and sum of other forces operating in the system may be expressed as shown in (15), where gravitational four-force density is dependent on the pressure p in the system and equal to $f^\mu_{gr} = \varrho(\frac{d\,\ln(p)}{d\tau}U^\mu - c^2\partial^\mu \ln(p))$.

The conclusion from the article can be divided according to their areas of application, as conclusion for QED, QM, and that regarding the combination of QFT with GR.

5.1 Conclusion for QED

Condition (21) leads to the electromagnetic four-potential, for which some gauge may be expressed as $\mathbb{A}^\mu = -\frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o}U^\mu$. It simplifies Alena Tensor (1) to a familiar form $T^{\alpha\beta} = \frac{1}{\mu_o}\mathbb{F}^{\alpha\gamma}\partial^\beta \mathbb{A}_\gamma - \Lambda_\rho \eta^{\alpha\beta}$, and both the Lagrangian and Hamiltonian density for the system with the electromagnetic field appear to be related to the invariant of the electromagnetic field tensor $\mathcal{L} = \mathcal{H} = \Lambda_\rho = \frac{1}{4\mu_o}\mathbb{F}^{\alpha\beta}\mathbb{F}_{\alpha\beta} = \frac{1}{2\mu_o}\mathbb{F}^{0\gamma}\partial^0 \mathbb{A}_\gamma$.

The above would also simplify the Lagrangian density used in QED. Assuming that there is only the electromagnetic field in the system and substituting Λ_ρ for the current Lagrangian density used in QED, one should obtain equations that describe the entire system with the electromagnetic field. Interestingly, such equations would also take into account the behavior of the system related to gravity because according to the model presented here, gravity naturally arises in the system as a consequence of the existence of other fields (more precisely, existence of the energy momentum tensors associated with these fields), and the resulting Lagrangian density takes this into account.

Perhaps, this explains why it is so difficult to identify quantum gravity as a separate interaction within QFT, and it could also explain QED's extremely accurate predictions, assuming that it actually describes the entire system with an electromagnetic field.

5.2 Conclusion for QM

As shown in the article, $H^\beta \equiv -\frac{1}{c}\int T^{0\beta}\, d^3x$ acts as the canonical four-momentum for the point-like particle and the vanishing four-divergence of H^β turns out to be the consequence of the Poynting theorem.

The obtained canonical four-momentum H^μ should satisfy the Klein–Gordon (Eq 77), and it is equal to

$$H^\mu = P^\mu + V^\mu = -\frac{\gamma L}{c^2}U^\mu + \mathbb{S}^\mu, \tag{78}$$

where P^μ is four-momentum and L is Lagrangian for the point-like particle, $\mathbb{S}^\mu$ due to its properties, seems to be some description of the spin, and where V^μ describes the transport of energy due to the field. It may be calculated as

$$V^\mu = q\mathbb{A}^\mu + \frac{\varrho c^2 \gamma^2}{p}P^\beta + \frac{\varrho c^2}{p}\mathbb{S}^\mu + Y^\mu, \tag{79}$$

where $\mathbb{A}^\mu$ is the electromagnetic four-potential and where Y^μ is the volume integral of the Poynting four-vector.

It seems that in such an approach, it would be possible to isolate gravity as a separate interaction, although this would probably require further research on the influence of individual components on the behavior of the particle. It is also not clear how to deal with the interpretation of time in the first quantization; however, a clue is to rely on the possibility of using Geroch's splitting [40], providing (3 + 1) decomposition.

5.3 Conclusion regarding the combination of QFT with GR

It should be noted that the presented solution applies to a system with an electromagnetic field, but it allows for the introduction of additional fields while maintaining the properties of the considered Alena Tensor. Therefore, it appears as a natural direction for further research to verify how the system with additional fields will behave and what fields are necessary to obtain the known configuration of elementary particles and interactions.

For example, considering the previous notation, one may describe the field in the system by some generalized field tensor $\mathbb{W}^{\alpha\beta\gamma}$ part of which is the electromagnetic field (e.g., electroweak). Such a description provides more degrees of freedom compared to the simple example for electromagnetism from Section 1.1 and allows representing the Alena Tensor in flat spacetime as follows:

$$T^{\alpha\beta} = \varrho U^{\alpha}U^{\beta} - \left(\frac{c^2\varrho}{\Lambda_{\rho}} + 1\right)\left(\Lambda_{\rho}\,\eta^{\alpha\beta} - \mathbb{W}^{\alpha\delta\gamma}\,\mathbb{W}^{\beta}{}_{\delta\gamma}\right), \tag{80}$$

where

$$\Lambda_{\rho} \equiv \frac{1}{4}\mathbb{W}^{\alpha\beta\gamma}\mathbb{W}_{\alpha\beta\gamma}, \tag{81}$$

$$\xi\, h^{\alpha\beta} \equiv \frac{\mathbb{W}^{\alpha\delta\gamma}\,\mathbb{W}^{\beta}{}_{\delta\gamma}}{\Lambda_{\rho}}, \tag{82}$$

$$\xi \equiv \frac{4}{\eta_{\alpha\beta}\, h^{\alpha\beta}}. \tag{83}$$

The Alena Tensor that is defined in this manner retains most of the properties described in the previous sections; however, it now describes other four-force densities in the system, related to its vanishing four-divergence.

Further analysis of the above properties using the variational method may lead to future discoveries regarding both the theoretical description of quantum fields and elementary particles associated with them, and the possibility of experimental verification of the obtained results.

As can be seen from the above summary, the conclusion obtained cannot be treated as the final result and requires additional research. These results should be understood as another small step on the path of science, which opens up the possibility of research on the properties of the Alena Tensor in terms of its applications in quantum theories. If these results spark an interest in the scientific community, perhaps further steps can be taken through a concerted effort.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

PO: conceptualization, investigation, writing–original draft, and writing–review and editing.

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