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Application of homotopy perturbation method to solve a nonlinear mathematical model of depletion of forest resources

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Reduction in forest resources due to increasing global warming and population growth is a critical situation the World faces today. As these reserves decrease, it alarms new challenges that require urgent attention. In this paper, we provide a semi-analytical solution to a nonlinear mathematical model that studies the depletion of forest resources due to population growth and its pressure. With the help of the homotopy perturbation method (HPM), we determine an approximate series solution with few perturbation terms, which is one of the essential power of the HPM method. We compare our semi-analytical results with numerical solutions obtained using the Runge-Kutta 4th-order (RK-4) method. Furthermore, we analyze the model's behaviour and dynamics by changing the parametric coefficients that represent the depletion rate of forest resources and the growth rate of population pressure and present these findings using various graphs.

KEYWORDS

semi-analytical solution, system of non-linear differential equations, homotopy perturbation method, depletion of forest resources, mathematical mobel

1 Introduction

The world faces an alarming issue today due to the depletion of forest resources caused by deforestation, fires, illegal logging, and other factors. Many countries will lose their remaining forests by 2030 if this trend continues, according to a recent report [1]. Urgent action is needed to address this challenge, including better coordination and control of the timber industry and communities that depend on the forests [2]. Mathematics provides some powerful tools to tackle such problems with the help of differential equations which can offer a way to solve dynamical systems making them essential to science, engineering and humanity. Some studies have used the mathematical modeling of forest depletion and suggested solutions using various numerical and analytical methods. Gompil et al. [3] proposed numerical and simulated results for a forest depletion model, while Eswari et al. [4] examined the homotopy perturbation method (HPM) to solve the mathematical model for the depletion of forest resources. Nugraheni et al. [5] proposed stability analysis and numerical simulations for a mangrove forest resource dynamical model. Didiharyono and Kasse [6] studied the stability of a mathematical model for deforestation and presented numerical simulations of the system. All these studies offer useful insights into the dynamics of forest resources and propose possible solutions to handle this critical global problem. This paper concentrates on the study of the

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TABLE 1 **Error in** ***B*****(*****t*****),** ***N*****(*****t*****) and** ***P*****(*****t*****) by using HPM and RK 4th order.**

t	$e_{B(t)}$	$e_{N(t)}$	$e_{P(t)}$
0	0	0	0
0.0040	7.02229385$e-11$	3.8795633$e-12$	1.11022302$e-15$
0.0080	2.84295254$e-10$	1.5518253$e-11$	7.77156117$e-15$
0.0120	6.60833165$e-10$	3.4923175$e-11$	2.13162820$e-14$
0.0160	1.24834542$e-09$	6.2129856$e-11$	3.57491813$e-14$
0.0200	2.14010853$e-09$	9.7180929$e-11$	3.15303338$e-14$
0.0240	3.48907391$e-09$	1.4013323$e-10$	2.64233079$e-14$
0.0280	5.52271828$e-09$	1.9115020$e-10$	1.93400850$e-13$
0.032	8.55785131$e-09$	2.5035973$e-10$	5.49560397$e-13$
0.0360	1.30154624$e-08$	3.1803182$e-10$	1.20303766$e-12$
0.0400	1.94354861$e-08$	3.9440806$e-10$	2.29549712$e-12$
0.0440	2.84915344$e-08$	4.7988635$e-10$	4.00479649$e-12$
0.0480	4.10056628$e-08$	5.74907232$e-10$	6.55009380$e-12$
0.0520	5.79630068$e-08$	6.79982292$e-10$	1.01949559$e-11$
0.0560	8.05264832$e-08$	7.95743915$e-10$	1.52524659$e-11$
0.0600	1.10051416$e-07$	9.22938170$e-10$	2.20889972$e-11$
0.0640	1.48100092$e-07$	1.06238928$e-09$	3.11282111$e-11$

depletion of forest resources, employing a mathematical model suggested by Misra, Lata, and Shukla [7]. This mathematical model consists of the cumulative density of forest resources, the density of the population, and population pressure, represented by the variables B, N, and P, respectively. In this model, the connection between forest and population density is considered as a prey-predator logistic model. The forest density decreases as housing and development increase, impacting its growth rate. Population pressure growth is proportional to population density in the model [7]. The authors have investigated existence and uniqueness of the global positive solution and provided numerical simulations to study this model. The cumulative density of forests and population size, are modelled using comprehensive equations with dynamic relations similar to a prey-predator system. The model signifies the depletion of forest resources provoked by population growth, reduction of forest areas for expansion purposes and the depletion by the pressure of the population. In addition, the model considers that the increase in population pressure is proportional to population density. This model consists of dimensionless differential equations. The suggested model [7] can be represented as:

$$\begin{aligned}\frac{dB}{dt} &= sB - hB^2 - \alpha BN - \lambda_2 B^2 P,\\ \frac{dN}{dt} &= rN - jN^2 + \pi\alpha BN, \\ \frac{dP}{dt} &= \lambda N - \lambda_0 P,\end{aligned} \tag{1}$$

where $B(0) \geq 0$, $N(0) \geq 0$, $P(0) \geq 0$ and we define variables and constant coefficients of this model in the following table as.

Notation	Description
B	Cumulative density of forest resources
N	Density of population
P	Population pressure
S	Intrinsic growth rate
$h = \frac{s_0}{L}$	Intraspecific growth rate of forestry resources in absence of population
$j = \frac{r_0}{K}$	Intraspecific growth rate of population in absence of forestry resources
A	Depletion rate of forest resources due to population
Λ	Growth rate of population pressure
λ_0	Natural depletion rate
λ_2	Depletion rate due to population pressure
Π	Growth in population due to forest resources (proportionality constant)
R	Intrinsic growth rate human population

Values for the parameters and coefficients are considered, $s = 0.8$, $s_0 = 0.2$, $L = 50$, $\alpha = 0.0001$, $\lambda = 0.2$, $\lambda_0 = 0.1$, $r = 0.2$, $r_0 = 0.1$, $K = 100$, $\pi = 0.004$, $\lambda_2 = 0.0002$, $h = \frac{s_0}{L}$, $j = \frac{r_0}{K}$ and initial conditions $n_1 = B(0) = 30$, $n_2 = N(0) = 35$, $n_3 = P(0) = 1$ as given in (Misra et al., 2014).

The rate of forest depletion is alarming, mainly driven by illegal logging and land clearing activities. This trend poses a serious threat to our ecosystem and immediate action is needed to mitigate its

TABLE 2 ***B*(*t*) by using HPM with variation of *α*.**

t	$B(t)$ at $\alpha = 0.0001$	$B(t)$ at $\alpha = 0.0002$	$B(t)$ at $\alpha = 0.0004$
0	30	30	30
0.0400	30.8129	30.8123	30.79938896
0.0800	31.64006505	31.63119549	31.61346378
0.1200	32.48303939	32.46937528	32.44206418
0.1600	33.34108369	33.32237882	33.28500024
0.2000	34.21399144	34.18999515	34.14205247
0.2400	35.10152566	35.07198316	35.01297171
0.2800	36.00341892	35.96807153	35.89747924
0.3200	36.91937333	36.8779588	36.7952667
0.3600	37.84906054	37.80131329	37.70599611
0.4000	38.79212173	38.73777318	38.62929989
0.4400	39.74816763	39.68694645	39.56478085
0.4800	40.7167785	40.6484109	40.51201218
0.5200	41.69750417	41.62171415	41.47053744
0.5600	42.68986396	42.60637366	42.43987062
0.6000	43.69334678	43.60187669	43.41949605
0.6400	44.70741106	44.60768033	44.40886848

TABLE 3 ***B*(*t*) by using HPM with variation of *λ*.**

t	$B(t)$ at $\lambda = 0.1$	$B(t)$ at $\lambda = 0.2$	$B(t)$ at $\lambda = 0.3$
0	30	30	30
0.0400	30.81286325	30.81233672	30.81181021
0.0800	31.64226453	31.64006505	31.63786584
0.1200	32.48820566	32.48303939	32.47787449
0.1600	33.35066779	33.34108369	33.33150392
0.2000	34.22961141	34.21399144	34.19838205
0.2400	35.12497633	35.10152566	35.07809694
0.2800	36.03668172	36.00341892	35.97019679
0.3200	36.96462607	36.91937333	36.87418996
0.3600	37.9086872	37.84906054	37.78954498
0.4000	38.86872228	38.79212173	38.71569051
0.4400	39.84456783	39.74816763	39.65201535
0.4800	40.83603966	40.7167785	40.59786848
0.5200	41.84293296	41.69750417	41.55255902
0.5600	42.86502224	42.68986396	42.51535622
0.6000	43.90206134	43.69334678	43.48548951
0.6400	44.95378345	44.70741106	44.46214845

TABLE 4 *B*(*t*) **by using HPM with variation of** λ_2**.**

t	$B(t)$ at $\lambda_2 = 0.0001$	$B(t)$ at $\lambda_2 = 0.0002$	$B(t)$ at $\lambda_2 = 0.0003$
0	30	30	30
0.0400	30.81659405	30.81233672	30.80808055
0.0800	31.64999511	31.64006505	31.63014111
0.1200	32.50021609	32.48303939	32.46588035
0.1600	33.36724938	33.34108369	33.31495729
0.2000	34.2510668	34.21399144	34.17699131
0.2400	35.15161962	35.10152566	35.05156215
0.2800	36.06883856	36.00341892	35.93820992
0.3200	37.00263378	36.91937333	36.83643509
0.3600	37.9528949	37.84906054	37.74569848
0.4000	38.91949097	38.79212173	38.6654213
0.4400	39.9022705	39.74816763	39.59498509
0.4800	40.90106144	40.7167785	40.53373176
0.5200	41.9156712	41.69750417	41.4809636
0.5600	42.94588662	42.68986396	42.43594323
0.6000	43.991474	43.69334678	43.39789368
0.6400	45.05217908	44.70741106	44.36599828

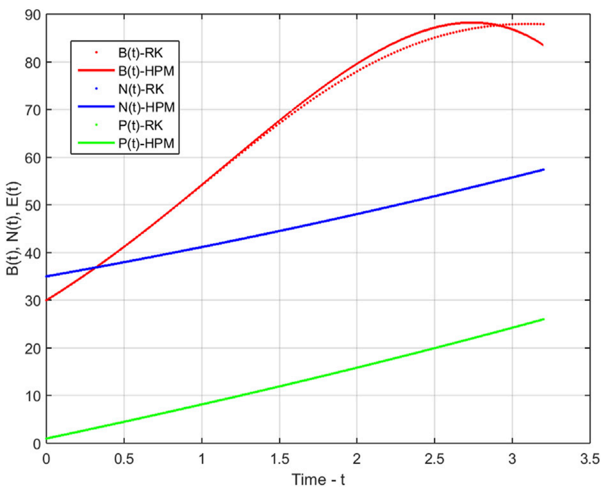

FIGURE 1
Solution for model 1.

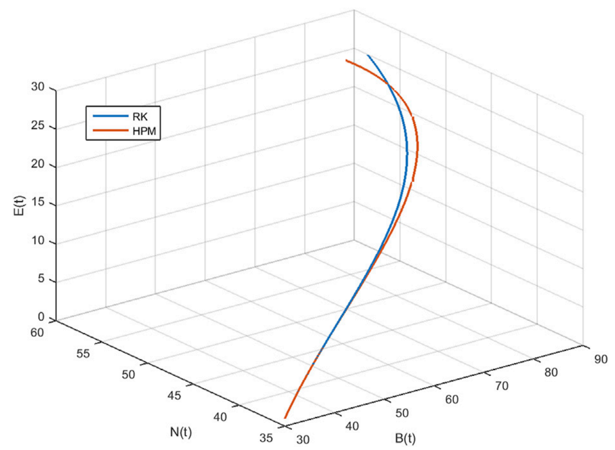

FIGURE 2
Solution for model 1 in 3D.

impact. Without decisive measures, the depletion of forest resources will have long-lasting consequences on the environment and our wellbeing. It is imperative to find sustainable solutions and enforce regulations to curb the rampant destruction of forests and preserve them for future generations.

Many dynamical problems in science and engineering cannot be solved analytically (exactly) and can be approximated numerically. There is another technique named as series solution (semi-analytical techniques) which is more closer to analytical results. For this purpose a wide range of methods have been developed to find approximate solutions that are as close as possible to the exact solutions. Among these methods are the Taylor series method [8], which approximates functions as power series; the Picard method [9], which iteratively computes solutions from initial conditions; the Adomian decomposition method [10], which decomposes a differential equation into simpler sub problems; the variational iteration method [11], which uses Lagrange multipliers to optimize solutions; and the homotopy perturbation method [12–14,14,15,17–19], which constructs a homotopy that gradually deforms the problem into a simpler one

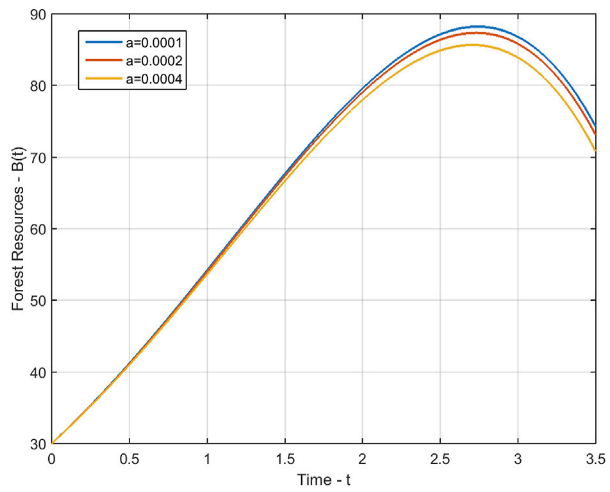

FIGURE 3
Forest resources $B(t)$ and α with the variation of time.

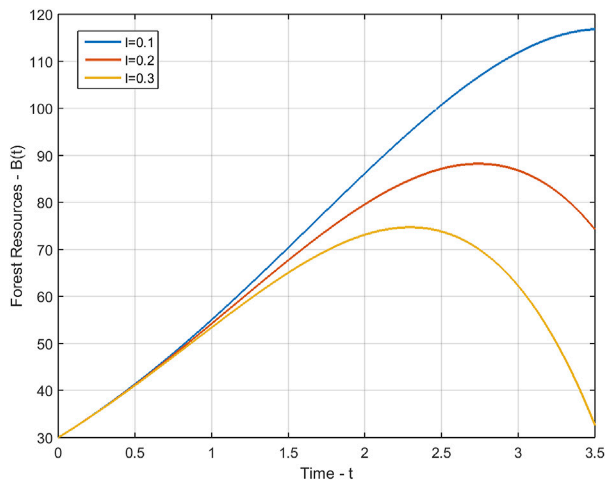

FIGURE 4
Forest resources $B(t)$ and λ with the variation of time.

while adding a perturbation term to the solution. These methods have been applied to a wide range of problems in physics, engineering, various fields and have proven to be highly effective in providing accurate approximations to complex dynamical systems.

Ji Huan He, a mathematician from China proposed a novel semi-analytical method based on homotopy and perturbation techniques in 1999, which was named the homotopy perturbation method (HPM) [12]. He improved and extended the HPM to solve a wide range of problems. In 2004, He used the HPM to non-linear oscillators and asymptotic [13,14]. In 2005, the HPM was applied to solve non-linear wave equations and problems related to limit cycle and bifurcation of non-linear systems [15,16]. In 2008, He employed the HPM to solve boundary value problems [20]. In 2007, Javidi and Golbabai used a revised version of the HPM to solve non-linear Fredholm integral equations [21].Recently, HPM with small variations has been applied to study fractal duffing oscillator problems under arbitrary conditions [22], modified HPM for nonlinear oscillators Anjum and He [23], attachment oscillator arising in nanotechnology [24], conservative nonlinear oscillators [25], non-linear oscillator problems in a fractal space [26] and HPM including Aboodh transformation to solve fractional calculus Tao et al. [27], vibrating magnetic inverted pendulum Moatimid et al. [28], Symmetry-breaking and pull-down motion for the helmholtz–duffing oscillator Niu et al. [29], nonlinear fractional Drinfeld–Sokolov–Wilson Equation Nadeem and Alsayaad [30], trajectory analysis of a zero-pitch-angle e-Sail Niccolai et al. [31], natural convection between two concentric horizontal circular cylinders Abdulameer and Ali Al-Saif [32], nonlocal initial-boundary value problems for parabolic and hyperbolic Al-Hayani and Younis [33], multi-step iterative methods for solving nonlinear equations Saeed et al. [34], telegraph equation Moazzzam et al. [35], triangular linear diophantine fuzzy system of equations Shams et al. [36], condensing coagulation model and Lifshitz-Slyzov equation Arora et al. [37], singular nonlinear system of boundary value problems Pathak et al. [38], rikitake-yype system Ene and Pop [39], heat and mass transfer with 2D unsteady squeezing viscous flow problem Abdul-Ameer and Ali Al-Saif [40], variable Speed Wind Turbine Control Shalbafian and Ganjefar [41], radial thrust problem Niccolai et al. [42], special third grade fluid flow with viscous dissipation effect over a stretching sheet Swain et al. [43], and the frequency–amplitude relationship of a nonlinear oscillator with cubic and quintic nonlinearities He et al. [44]. The HPM has become a widely-used technique to solve a large variety of problems in different fields and many research papers have been published each year using this method as evidenced by a simple search on Google Scholar.

In this paper, we provide an approximate solution of model 1) by using the homotopy perturbation method. The interesting feature of HPM is that it provides the best approximate solution by taking a few numbers of perturbation terms.

2 Homotopy perturbation method

Consider a non-linear differential equation

$$D(\mu) - g(\tau) = 0, \quad \tau \in \mho \tag{2}$$

subject to the boundary condition

$$\beta\left(\mu, \frac{\partial \mu}{\partial \tau}\right) = 0, \quad \tau \in \Gamma \tag{3}$$

where D is a differential operator, β is boundary operator, Γ is the boundary of the domain $\mho$ and $g(\tau)$ is an unknown function. The D, generally consist on two parts, linear and non-linear part, represented as L and N respectively. Therefore, 2) can be written as follows

$$L(\mu) + N(\mu) - g(\tau) = 0, \tag{4}$$

using homotopy method, by taking an embedding parameter q one can construct a homotopy $v(\tau, q)$: $\mho \times [0, 1] \rightarrow R$ for Eq. 4 which satisfies

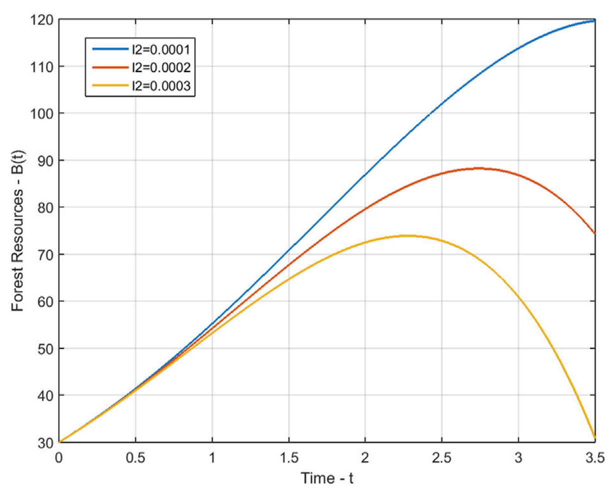

FIGURE 5
Forest resources $B(t)$ and λ_2 with the variation of time.

$$H(w,q) = (1-q)[L(w) - L(\mu_0)] + q[L(w) + N(w) - g(\tau)] = 0, \quad (5)$$

and it is equivalent to

$$H(w,q) = L(w) - L(\mu_0) + qL(\mu_0) + q[N(w) - g(\tau)] = 0, \quad (6)$$

where $q \in [0, 1]$ is an embedding parameter, μ_0 is an initial guess approximation of Eq. 6 which satisfies the initial (or boundary) conditions. It can be written as follows.

$$q = 0, \quad H(w,0) = L(w) - L(\mu_0), \quad (7)$$

$$q = 1, \quad H(w,1) = L(w) + N(w) - g(\tau). \quad (8)$$

We suppose the solution in the form of power series for Eq. 5 by taking an embedding parameter q

$$w = w_0 + qw_1 + q^2 w_2 + q^3 w_3 + \cdots \quad (9)$$

The approximate solution of Eq. 2 can be obtained by setting $q = 1$,

$$\mu = \lim_{q\to 1} w = w_0 + w_1 + w_2 + w_3 + \cdots \quad (10)$$

The convergence of (Eq. 10) has been proved in [12]. The series is convergent for most cases, however, the convergent rate depends upon the nonlinear operator $N(w)$. Furthermore He suggested the following conditions.

1. The second derivative of nonlinear operator $N(w)$ with respect to w must be small, because the parameter q may be relatively large, i.e., $q \to 1$.
2. The norm of $\|L^1 (\frac{\partial N}{\partial w})\|$ must be smaller than one, in order that the series converges.

3 Application of HPM

Now we apply HPM on our model, Eq. 1 of depletion of forest resources (non-linear system of differential equations) as

$$\begin{cases} (1-q)(u' - B_0') + q(u' - su + hu^2 + \alpha uv + \lambda_2 u^2 w) = 0, \\ (1-q)(v' - N_0') + q(v' - rv + jv^2 - \pi\alpha uv) = 0, \\ (1-q)(w' - P_0') + q(w' - \lambda v + \lambda_0 w) = 0. \end{cases} \quad (11)$$

The initial guesses for (11) are constant as defined in [7].

$$\begin{aligned} u_0(t) &= B_0(t) = B(0) = n_1 \\ v_0(t) &= N_0(t) = N(0) = n_2 \\ w_0(t) &= P_0(t) = P(0) = n_3 \end{aligned} \quad (12)$$

and we assume the solution of (11) as,

$$\begin{aligned} u &= u_0 + qu_1 + q^2 u_2 + q^3 u_3 + \cdots, \\ v &= v_0 + qv_1 + q^2 v_2 + q^3 v_3 + \cdots, \\ w &= w_0 + qw_1 + q^2 w_2 + q^3 w_3 + \cdots, \end{aligned} \quad (13)$$

by substituting Eq. 13 in Eq. 11 and collecting the terms of powers of q, we obtain

$$q^0: \begin{cases} u_0' = 0, \ u_0(0) = n_1, \\ v_0' = 0, \ v_0(0) = n_2, \\ w_0' = 0, \ w_0(0) = n_3. \end{cases} \quad (14)$$

$$q^1: \begin{cases} u_1' + u_0(\alpha v_0 - s) + u_0^2(h + \lambda_2 w_0) = 0, \ u_1(0) = 0, \\ v_1' - (r + \alpha\pi u_0)v_0 + jv_0^2 = 0, \ v_1(0) = 0, \\ w_1' - \lambda v_0 + \lambda_0 w_0 = 0, \ w_1(0) = 0. \end{cases} \quad (15)$$

$$q^2: \begin{cases} u_2' + \alpha u_0 v_1 + u_1(\alpha v_0 + 2u_0(h + \lambda_2 w_0) - s) + \lambda_2 u_0^2 w_1, \ u_2(0) = 0, \\ v_2' - \alpha\pi u_1 w_0 - (r + \alpha\pi u_0 - 2jv_0)v_1 = 0, \ v_2(0) = 0, \\ w_2' - \lambda v_1 + \lambda_0 w_1 = 0, \ w_2(0) = 0, \end{cases} \quad (16)$$

$$q^3: \begin{cases} u_3' + \alpha u_0 v_2 + u_1^2(h + \lambda_2 w_0) + u_2(\alpha v_0 + 2u_0(h + \lambda_2 w_0)) \\ \quad + u_1(\alpha v_1 + 2\lambda_2 u_0 w_1) + 2\lambda_2 u_0^2 w_2 = 0, \quad u_3(0) = 0, \\ v_3' - \alpha\pi u_2 v_0 - \alpha\pi u_1 v_1 + jv_1^2 - rv_2 - \alpha\pi u_0 v_2 + 2jv_0 v_2 = 0, \\ \quad v_3(0) = 0, \\ w_3 - \lambda v_2 + \lambda_0 w_2 = 0, \quad w_3(0) = 0. \end{cases} \quad (17)$$

$$q^4: \begin{cases} u_4' + \alpha u_2 v_1 + \alpha u_0 v_3 + u_3(-s + \alpha v_0 + 2u_0(h + \lambda_2 w_0)) + \lambda_2 u_1^2 w_1 \\ \quad + \lambda_2 u_0 u_2 w_1 + u_1(\alpha v_2 + 2u_2(h + \lambda_2 w_0) + 2\lambda_2 u_0 w_2) \\ \quad + \lambda_2 u_0^2 w_4 = 0, \quad u_4(0) = 0, \\ v_4' - \alpha\pi u_3 v_0 - \alpha\pi u_2 v_1 - \alpha\pi u_1 v_2 + 2jv_1 v_2 - rv_3 \\ \quad - \alpha\pi u_0 v_3 + 2jv_0 v_3 = 0, \quad v_4(0) = 0, \\ w_4 - \lambda v_3 + \lambda_0 w_4 = 0, \quad w_4(0) = 0. \end{cases} \quad (18)$$

$$\vdots$$

Now considering $s = 0.8$, $s_0 = 0.2$, $L = 50$, $\alpha = 0.0001$, $\lambda = 0.2$, $\lambda_0 = 0.1$, $r = 0.2$, $r_0 = 0.1$, $K = 100$, $\pi = 0.004$, $\lambda_2 = 0.0002$, $h = \frac{s_0}{L}$, $j = \frac{r_0}{K}$, $n_1 = B(0) = 30$, $n_2 = N(0) = 35$, $n_3 = P(0) = 1$, and simplifying the equations from (Eqs 14–18) we have.

By substituting these values in Eq. 13, we have the solution of model 1) as

$u_0 = 30$	$v_0 = 35$	$w_0 = 1$
$u_1 = 20.115t$	$v_1 = 5.7742t$	$w_1 = 6.9t$
$u_2 = 4.84665t^2$	$v_2 = 0.375578t^2$	$w_2 = 0.232542t^2$
$u_3 = -0.260167t^3$	$v_3 = 0.00519615t^3$	$w_3 = 0.017287t^3$
$u_4 = -0.495765t^4$	$v_4 = -0.000913025t^4$	$w_4 = -0.00017237t^4$

$$B(t) = \lim_{q\to 1} u = 30 + 20.115t + 4.84665t^2 - 0.260167t^3 - 0.495765t^4 - 0.12174t^5 + \cdots, \quad (19)$$

$$N(t) = \lim_{q\to 1} v = 35 + 5.77542t + 0.375578t^2 + 0.00519615t^3 - 0.000913025t^4 - 0.00006t^5 + \cdots, \quad (20)$$

$$P(t) = \lim_{q\to 1} w = 1 + 6.9t + 0.232542t^2 + 0.0172871t^3 - 0.00017237t^4 - 0.000033t^5 + \cdots \quad (21)$$

For $\alpha = 0.0001$, $\alpha = 0.0002$ and $\alpha = 0.0004$, we have.

$B(t)_{\alpha=0.0001} = 30 + 20.115t + 4.84665t^2 - 0.260167t^3 - 0.495765t^4 + \cdots$,

$B(t)_{\alpha=0.0002} = 30 + 20.01t + 4.77438t^2 - 0.274268t^3 - 0.49119t^4 + \cdots$ and.

$B(t)_{\alpha=0.0004} = 30 + 19.8t + 4.63094t^2 - 0.301744t^3 - 0.481988t^4 + \cdots$.

For $\lambda = 0.1$, $\lambda = 0.2$ and $\lambda = 0.3$, we have.

$B(t)_{\lambda=0.1} = 30 + 20.115t + 5.16165t^2 + 0.0854419t^3 - 0.336328t^4 + \cdots$,

$B(t)_{\lambda=0.2} = 30 + 20.115t + 4.84665t^2 - 0.260167t^3 - 0.495765t^4 + \cdots$ and.

$B(t)_{\lambda=0.3} = 30 + 20.115t + 4.53165t^2 - 0.605776t^3 - 0.648587t^4 + \cdots$.

For $\lambda_2 = 0.0001$, $\lambda_2 = 0.0002$ and $\lambda_2 = 0.0003$, we have.

$B(t)_{\lambda_2=0.0001} = 30 + 20.205t + 5.24226t^2 + 0.113954t^3 - 0.334519t^4 + \cdots$,

$B(t)_{\lambda_2=0.0002} = 30 + 20.115t + 4.84665t^2 - 0.260167t^3 - 0.495765t^4 + \cdots$ and.

$B(t)_{\lambda_2=0.0002} = 30 + 20.025t + 4.45157t^2 - 0.629906t^3 - 0.645153t^4 + \cdots$.

3.1 Verification of the solution

To verify the validity of solution, first we check the solution for initial conditions which are satisfied at $t = 0$, secondly we put the solution and its derivatives in the system. If both sides of system are satisfied then we consider the solution is correct or true. For the second condition, we differentiate Eqs 19–21 with respect to t, so we have

$$\frac{dB(t)}{dt} = 20.115 + 9.69329t - 0.780501t^2 - 1.98306t^3 - 0.608698t^4 + \cdots \quad (22)$$

$$\frac{dN(t)}{dt} = 0.77542 + 0.751156t + 0.0155885t^2 - 0.0036521t^3 - 0.000326555t^4 + \cdots \quad (23)$$

$$\frac{dP(t)}{dt} = 6.9 + 0.465084t + 0.0518614t^2 - 0.000689481t^3 - 0.000165368t^4 + \cdots \quad (24)$$

Now using Eqs 19–24 and the values of given parameters in system 1) and we have

$$0. - 1.77636 \times 10^{-15}t + 2.22045 \times 10^{-16}t^3 - 2.22045 \times 10^{-16}t^4 + \cdots = 0 \quad (25)$$

$$0. + 3.46945 \times 10^{-18}t^2 - 8.67362 \times 10^{-19}t^3 + 4.73413 \times 10^{-6}t^5 + \cdots = 0 \quad (26)$$

$$0. - 6.93889 \times 10^{-18}t^2 + 9.75483 \times 10^{-6}t^5 + \cdots = 0 \quad (27)$$

The coefficients of t powers in Eqs 25–27 are around 15 to 19 decimal places correct to zero. So our series solution (5th degree polynomials) satisfies the system up to 4th degree polynomial (where the coefficients are approximately 17 decimal correct to zero). The solution can be improved by taking/adding more terms of power t in it.

3.2 Results and discussions

In this section, we demonstrate the performance of our model 1 through the evaluation of our calculated approximate solutions, $B(t)$, $N(t)$, and $P(t)$. To validate our results, we compare them with the Runge-Kutta 4th-order method and present the absolute error, $e_{B(t)}$, $e_{N(t)}$, and $e_{P(t)}$ in Table 1 for various time steps. The time domain of our Homotopy Perturbation Method (HPM) is divided into sub-intervals and mapped onto $0 \le t \le 400$ with a step size of 0.5 for graphical representation. Our analysis revealed an average absolute error of $6.53290554e-08$, $5.09269781e-10$, and $1.35452205e-11$ for $B(t)$, $N(t)$, and $P(t)$, respectively. In Tables 2–4, we present the cumulative density of forest resources, $B(t)$, for various values of α, λ, and λ_2. These results underscore the versatility and accuracy of our proposed model, which has the potential to contribute significantly to the field of forest resource management. Figure 1 provides a clear illustration of the behaviour of the cumulative density of forest resources $B(t)$, the density of population pressure $P(t)$, and the density of population $N(t)$ as calculated using HPM and RK-4th order method. The solid lines represent the HPM series solution, while the dotted lines show the numerical solution calculated by the RK-4th order method. The graph highlights that the cumulative density of forest resources decreases as the density of population pressure increases. This suggests that controlling population pressure is essential for preserving forests on a large scale. Additionally, Figure 2 depicts the behaviour of model 1 in 3D with respect to HPM and RK method, providing a comprehensive view of the model's behaviour over time. Figure 3, represents the impact of the depletion rate of forest resources due to population, $\alpha = a$, on the cumulative density of forest resources, $B(t)$. It reflects that decreasing the depletion rate of forest resources due to population directs to a growth in the cumulative density of forest resources over time. This emphasizes the significance of controlling the population pressure on forests to control their depletion. In Figure 4, we discuss the impact of the growth rate coefficient of population pressure caused by population $\lambda = l$ on the cumulative density of forest resources $B(t)$. The graph indicates that if we decrease the value of λ, the cumulative density of forest resources increases. Likewise, Figure 5 describes the effect of population pressure λ_2 on $B(t)$. We can see, as the value of λ_2 decreases, the cumulative density of forest resources $B(t)$ increases.

These figures illustrate the significance of controlling population pressure and growth rates to save and preserve forest resources. It also emphasizes the necessity for procedure interventions to control population growth and decrease the depletion of forest resources.

3.3 Technical specification

These calculations are performed on Mathematica® 11.3.0.0 (64-bit) and Matlab® R2015a (8.5.0.197613) 64-bit using a machine Intel(R) Core(TM) i3.2310M CPU @ 2.10 GHz and OS: window 7 Professional (64-bit).

4 Conclusion

In this paper, we used the homotopy perturbation method to obtain a semi-analytical solution for the nonlinear model of the depletion of forest resources. Important characteristic of HPM is that it provides the adequate approximate series solution by taking a few number of perturbation terms which is near to analytical exact solution. Through comparison with the Runge-Kutta method, we established the effectiveness and accuracy of the proposed method. Additionally, we investigated the behaviour of the model by varying the values of the depletion rate of forest resources due to population α, the growth rate coefficient of population pressure caused by population λ, and the depletion rate of its carrying capacity due to population pressure λ_2. The results showed that reducing these coefficients can increase the cumulative density of forest resources $B(t)$. These findings highlight the urgent need for measures to conserve forest resources for the wellbeing of our planet. The presented model and its solution indicate the seriousness of this global issue which needs to be acted upon immediately and effectively to preserve our forest resources. This study suggests that additional investigations and research is needed to build more relevant models for assistance of forest resource experts.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material, further inquiries can be directed to the corresponding authors.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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