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Physically significant solitary wave solutions to the space-time fractional Landau–Ginsburg–Higgs equation via three consistent methods

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The Landau–Ginzburg–Higgs equation (LGHE) is a mathematical model used to describe nonlinear waves that exhibit weak scattering and long-range connections in the tropical and mid-latitude troposphere as interactions between equatorial and mid-latitude Rossby waves. This study assessed the fractional Landau–Ginzburg–Higgs model, previously introduced in truncated M-fractional derivatives utilizing the $(G'/G, 1/G)$, modified (G'/G^2), and new auxiliary equation methods. Using these techniques, different solutions, including unknown parameters, were obtained in trigonometric, hyperbolic, and exponential functions. This study investigated how varying values of the fractional parameter affected the deeds of the solutions obtained for the given conditions. The predicted solutions, obtained under restricted conditions, were visualized through 2D, 3D, and contour plots using appropriate parameter values. The attained results were confirmed for the aforementioned equations using symbolic soft computations. Moreover, the outcomes confirmed that the methods used in this study were effective mathematical tools for discovering exact solitary wave solutions to nonlinear models encountered in various areas of science and engineering.

KEYWORDS

Ginzburg–Higgs equation, truncated M-fractional derivative, the (G′/G,1/G)-expansion method, modified (G′/G²)-expansion method, new auxiliary equation method, exact solitary wave solutions

1 Introduction

Non-linear partial differential equations (NLPDEs) play significant roles in physics, mathematical engineering, and other phenomena such as heat flow, plasma physics, wave propagation, shallow water waves, chemically dispersed electricity, quantum mechanics, fluid dynamics, and reactive materials. NLPDEs also play substantial roles in nonlinear optical fibers and quantum fields, such as nonlinear wave equations, Monge–Ampere equations, Burgers equations, Liouville equations, Fisher equations, and Kolmogorov–Petrovskii–Piskunov equations [1–4]. These equations assist in the implementation of essential parts of the soliton solution. The soliton is stimulated during diffusion by eliminating the effects of diffusion. Now, soliton assessment is very common [5]. Solitons are solutions to large, weakly detached partial differential equations (PDEs) for physical structures. Nowadays, many models are considered for computing the soliton solutions (SS) [6–8]. Among these, the Landau–Ginzburg–Higgs (LGH) model [9, 10] is one of the most considered in recent years, as follows:

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} - g^2 v + h^2 v^3 = 0, \quad (1)$$

where $v = v(x,t)$ is the ion-cyclotron wave electrostatic potential g and h are real parameters and x, t indicate the nonlinearized spatial and temporal coordinates. Lev Davidovich Landau and Vitaly Lazarevich Ginzburg designed the LGHE (1) to describe superconductivity and drift cyclotron waves in radially inhomogeneous plasmas of integrated ion cyclotrons [11]. Numerous methods have been used to determine the distinctive SS of the integrable nonlinear evolution equation (NLEE) (1). Bekir and Unsal [12] provided exponential function solutions by using the first integral method for NLEE (1). Iftikhar et al. [13] utilized the $(G'/G, 1/G)$-expansion method and inspected a variety of analytical solutions for NLEE (1). They also determined general and kinked shape soliton solutions for different parameter selections. Barman et al. [14, 15] obtained various analytical solutions using the Kudryashov technique comprising the undisclosed parameters of Eq. 1. In addition, they employed the tanh function to create solutions with soliton-like shapes, such as dark solitons, bright solitons, peakons, compactons, and periodic solutions, among others. These solutions can be utilized to investigate the propagation of various waves, such as tidal and tsunami waves, ion-acoustic waves, and magneto-sound waves in plasma. Islam and Akbar [16] used the IBSEF and presented innumerable stable solutions. The results provided several soliton shapes, which considered one-way wave propagation with diffuse systems in nonlinear science.

For two centuries, fractional calculus has fascinated many intellectuals' curiosity. Use them to develop many nonlinear aspects, inclosing bioprocesses, chemical processes, fluid mechanics, etc. In the traditional integer order, the fractional-order PDEs are used to generalize PDEs. Several definitions of the fractional derivative exist in the literature, such as Riemann–Liouville [17], Caputo [18], Caputo–Fabrizio [19], conformable fractional derivative (FD) [20], and beta-derivative [21] to solve non-integer-order models. Studies have shown that these definitions of FD do not meet some of the basic assets of derivatives, such as product and chain rules. Sousa and Oliveira [22] developed a novel truncated-M fractional derivative that meets numerous properties considered to be the FD' boundary. This derivative has interesting results in different areas, such as chaos theory, biological modeling, circuit analysis, optical physics, and disease analysis.

The core aim of this study was to explore the space-time fractional LGH model [23], symbolized as

$$D_{M,t}^{2\alpha,\beta} v - D_{M,x}^{2\alpha,\beta} v - g^2 v + h^2 v^3 = 0, 0 < \alpha < 1, \beta > 0, \quad (2)$$

where α and β are the fractional parameters representing the fractional time derivative's order.

The fundamental consideration of this exploration was to take advantage of the novel indication of fractional-order derivatives, called truncated truncated-M fractional derivatives [22, 24, 25], for space-time fractional LGHE [23], and to use the $(G'/G, 1/G)$, modified (G'/G^2), and new auxiliary equation methods (NAEMs) [23, 26, 27] to obtain new inclusive solitary solutions in the form of solutions of bright, dark, single solitons, and periodic isolated waves. Up to now, the results have different corporate and diverse forms, which have not been reported previously [23].

Moreover, the planned technique has been used to solve various models. For instance, Hafiz [28] employed the $(G'/G, 1/G)$-expansion method to determine the closed-form solutions of the generalized fractional reaction Duffing model and the density-dependent fractional diffusion-reaction equation. Li et al. [29] discovered the traveling wave solutions of the Zakharov equation, and Zayed et al. [30] established solutions to the nonlinear Kdv–mKdv equation. Uddin [31] and Wazwaz [32] provided general solutions for the fifth-order NLEEs and the Burger KP-equation, respectively. Sirisubtawee [33] found exact traveling wave solutions for nonlinear fractional evolution equations. Traveling wave solutions for the nonlinear Schrodinger equation with third-order dispersion were obtained using the modified (G'/G^2)-expansion model [34]. The Fokas–Lenells equations were solved using this technique to regulate different traveling wave solutions [35]. Aljahdaly [36] extended the NLEEs and described the general exact traveling wave solutions. Dragon and Donmez [37] discovered solutions in the form of traveling waves for the Gardner equation and then used these solutions to address different plasma-related issues. The Sharma–Tasso–Olver (STO) equations were also solved, and exact nonlinear and super nonlinear traveling wave solutions were obtained [38]. Jhangeer et al. [39] used the new auxiliary equations method to find innovative soliton solutions for the fractional Caudrey–Dodd–Gibbon–Sawada–Kotera equation. Raza et al. [40] obtained the new optical solitary wave solitons of the three-dimensional Fractional Wazwaz–Benjamin–Bona–Mahony (WBBM) equation. Furthermore, Riaz et al. [41] scrutinized the various forms of solitary wave solutions for the modified equal-width wave equation.

This work is structured into six sections. Section 2 presents the truncated M-fractional derivative and its properties, which is the foundation of the proposed methods. The methodologies of the three proposed approaches are discussed in Section 3, where we explain how to use the truncated M-fractional derivative to solve mathematical models. Section 4 involves a mathematical examination of the models we have presented and the solutions we have obtained using the proposed methods. We compare them with existing methods in the literature. Section 5 provides a graphical representation of the obtained solutions for each analyzed model. Finally, Section 6 provides the study conclusion by summarizing the key findings and their implications.

2 Truncated M-fractional derivative and its properties

The following section will discuss the truncated M-fractional derivative (TMFD) of order α with its properties.

Definition 2.1. Let $f: (0,\infty) \rightarrow R$, then, the TMFD of a function f of order α is determined as

$$D_M^{\alpha,\beta} f(t) = \lim_{\varepsilon\rightarrow 0} \frac{f\left(t\epsilon_\beta\left(\varepsilon t^{1-\alpha}\right)\right) - f(t)}{\varepsilon}, \text{for all } t>0, 0<\alpha<1, \beta>0,$$

where $\alpha\epsilon_\beta(\cdot)$ is a truncated Mittag–Leffler function of one parameter [22].

Properties 2.2. Let $\alpha \in (0,1], \beta > 0$ and $f = f(t), g = g(t)$ be α-differentiable at a point $t > 0$, then:

1. $D_M^{\alpha,\beta}(af + bg) = aD_M^{\alpha,\beta} f + bD_M^{\alpha,\beta} g,\ \forall\ a,b \in R.$
2. $D_M^{\alpha,\beta}(c) = 0$, where $f(t) = c$, is a constant.
3. $D_M^{\alpha,\beta}(f\cdot g) = D_M^{\alpha,\beta} f + D_M^{\alpha,\beta} g.$
4. $D_M^{\alpha,\beta}\left(\frac{f}{g}\right) = \frac{gD_M^{\alpha,\beta} f - fD_M^{\alpha,\beta} g}{g^2}.$
5. If f is differentiable, then

$$D_M^{\alpha,\beta} f(t) = \frac{t^{1-\alpha}}{\Gamma(\beta+1)}\frac{df}{dt}. \tag{3}$$

6. $D_M^{\alpha,\beta}(f\circ g)(t) = f'(g(t))D_M^{\alpha,\beta} g(t)$, for f differentiable at $g(t)$.

3 General form of the methods

3.1 $(G'/G, 1/G)$-expansion method

The core steps of the $(G'/G, 1/G)$-expansion model [24, 28] for discovering traveling wave solutions to nonlinear evolution equations are outlined in this section. We begin by examining the second-order linear ordinary differential equation (ODE):

$$G''(\eta) + \lambda G(\eta) = \mu, \tag{4}$$

where $\phi = G'/G$ and $\psi = 1/G$, then

$$\phi' = -\phi^2 + \mu\psi - \lambda, \psi' = -\phi\psi. \tag{5}$$

Case 1: When $\lambda < 0$, the general solutions of Eq. 4 is given as

$$G(\eta) = A_1 \sinh\left(\sqrt{-\lambda}\,\eta\right) + A_2\cosh\left(\sqrt{-\lambda}\,\eta\right) + \frac{\mu}{\lambda}, \tag{6}$$

and we have

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma + \mu^2}\left(\phi^2 - 2\mu\psi + \lambda\right), \tag{7}$$

where A_1 and A_2 are arbitrary integration constants and $\sigma = A_1{}^2 - A_2{}^2$.

Case 2: When $\lambda > 0$, the general solution of Eq. 4 is clearly

$$G(\eta) = A_1 \sin\left(\sqrt{\lambda}\,\eta\right) + A_2\cos\left(\sqrt{\lambda}\,\eta\right) + \frac{\mu}{\lambda}, \tag{8}$$

and we have

$$\psi^2 = \frac{\lambda}{\lambda^2\sigma - \mu^2}\left(\phi^2 - 2\mu\psi + \lambda\right), \tag{9}$$

where A_1 and A_2 are arbitrary integration constants and $\sigma = A_1{}^2 + A_2{}^2$.

Case 3: When $\lambda = 0$, the general solutions of Eq. 4 is

$$G(\eta) = \frac{\mu}{2}\eta^2 + A_1\eta + A_2, \tag{10}$$

and we have

$$\psi^2 = \frac{1}{A_1{}^2 - 2\mu A_2}\left(\phi^2 - 2\mu\psi\right), \tag{11}$$

where A_1 and A_2 are arbitrary integration constants.

Consider the NLPDE, such as

$$Q(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0. \tag{12}$$

The unfamiliar function $u = u(x,t)$ is represented by a Q polynomial of the variable and its partial derivatives. The key phases involved in the $(G'/G, 1/G)$-expansion model are as follows:

Step 1: By coordinate transformation

$$\eta = x - ct, u(x,t) = v(\eta). \tag{13}$$

where c is the speed of the traveling wave.

The wave variable allows us to reduce Eq. 12 into a nonlinear ODE for $v = v(\eta)$:

$$R(v, v', v'', v''', \ldots) = 0, \tag{14}$$

where R is a polynomial of $v(\eta)$ and its total derivatives concerning η.

Step 2: Assume that a polynomial can express the solutions of Eq. 14 in two variables ϕ and ψ as

$$v(\eta) = \sum_{i=0}^{m} a_i\phi^i + \sum_{i=0}^{m} b_i\phi^{i-1}\psi. \tag{15}$$

To determine the values of the constants $a_i\,(i = 0, 1, \ldots, m)$ and $b_i\,(i = 1, \ldots, m)$ and the positive integer m, a homogenous imbalance is used among the highest-order derivatives and the nonlinear terms in the given ODE Eq. 14.

Step 3: Substitute Eq. 15 into Eq. 14 along with Eqs 5 and 7, reducing the left-hand side of the ODE into a polynomial in terms of ϕ and ψ, with a maximum degree of 1 for ψ. A system of algebraic equations is obtained by setting each coefficient of the polynomial to zero, which can be solved with the aid of Mathematica software to obtain the values for $a_i\,(i = 0, 1, \ldots, m), b_i\,(i = 1, \ldots, m), c, \mu, \lambda\,(\lambda < 0), A_1 \text{and} A_2$.

Step 4: Substitute the values obtained for $a_i\,(i = 0, 1, \ldots, m)$, $b_i\,(i = 1, \ldots, m)$, $c, \mu, \lambda(\lambda<0)$, A_1 and A_2 in Eq. 15 to determine the traveling wave solutions in terms of hyperbolic functions, as expressed in Eq. 14.

Step 5: Similarly, substitute Eq. 15 into Eq. 14 along with Eq. 5 and either Eq. 9 or Eq. 11 to obtain exact traveling wave solutions expressed in terms of trigonometric or rational functions, respectively.

3.2 The modified (G'/G^2)-expansion method

We outline the fundamental steps of the modified (G'/G^2)-expansion method [24, 29] as follows:

Step 1: Start by considering Eqs 12–14.

Step 2: Extend the solutions to Eq. 14 as follows:

$$v(\eta) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G^2}\right)^i, \tag{16}$$

where $a_i\,(i = 0, 1, 2, 3, \ldots, m)$ are constants and found later. It is important that $a_i \neq 0$.

The function $G = G(\eta)$ satisfies the following Riccati equation:

$$\left(\frac{G'}{G^2}\right)' = \lambda_1 \left(\frac{G'}{G^2}\right)^2 + \lambda_0, \tag{17}$$

where λ_0 and λ_1 are constants.

We can obtain the following solutions to Eq. 17 under different conditions λ_0:

When $\lambda_0\lambda_1 < 0$,

$$\left(\frac{G'}{G^2}\right) = -\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1} + \frac{\sqrt{|\lambda_0\lambda_1|}}{2}\left[\frac{C_1 \sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2 \cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}{C_1 \cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2 \sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}\right]. \tag{18}$$

When $\lambda_0\lambda_1 > 0$,

$$\left(\frac{G'}{G^2}\right) = \sqrt{\frac{\lambda_0}{\lambda_1}}\left[\frac{C_1 \cos\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2 \sin\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}{C_1 \sin\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) - C_2 \sin\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}\right]. \tag{19}$$

When $\lambda_0 = 0$ and $\lambda_1 \neq 0$,

$$\left(\frac{G'}{G^2}\right) = -\frac{C_1}{\lambda_1 (C_1\eta + C_2)}, \tag{20}$$

where C_1 and C_2 are arbitrary constants.

Step 3: If we substitute Eq. 16 and Eq. 17 into Eq. 14 and equate the coefficients of each power of $(\frac{G'}{G^2})^i$ to zero, a set of algebraic equations can be obtained. These equations can then be solved to determine the values of $a_i, \lambda_0, \lambda_1, c$, and other parameters.

Step 4: Replacing Eq. 16 of which α_i, c, and other parameters are found in step 3 in Eq. 13, we obtain the solutions for Eq. 12.

3.3 The new auxiliary equation method

Now, we will designate the elementary steps of the new auxiliary equation method [39, 40].

Step 1: Consider Eqs 12–14.

Step 2: Subsequently determine the solutions of Eq. 14:

$$v(\eta) = \sum_{i=0}^{m} a_i \gamma^{if(\eta)}, \tag{21}$$

which satisfies the auxiliary equation:

$$f'(\eta) = \frac{1}{\ln(\gamma)}\left(\mu\gamma^{-f(\eta)} + \lambda + \zeta\gamma^{f(\eta)}\right), \tag{22}$$

where $a_0, a_1, a_2, \ldots, a_m$ are coefficients to be solved such that $a_m \neq 0$. We then utilized the balancing principle to obtain the value of m, which states that we can find m by equating the nonlinear term of Eq. 14 with the highest-order derivative.

For Eq. 22, the family of solutions can be attained as follows:

Family-1 When $\lambda^2 - 4\mu\zeta < 0$ and $\zeta \neq 0$,

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\zeta} + \frac{\sqrt{4\mu\zeta - \lambda^2}}{2\zeta}\tan\left(\frac{\sqrt{4\mu\zeta - \lambda^2}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\zeta} - \frac{\sqrt{4\mu\zeta - \lambda^2}}{2\zeta}\cot\left(\frac{\sqrt{4\mu\zeta - \lambda^2}}{2}\eta\right).$$

Family-2 When $\lambda^2 - 4\mu\zeta > 0$ and $\zeta \neq 0$,

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\zeta} - \frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2\zeta}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\zeta} - \frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2\zeta}\coth\left(\frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2}\eta\right).$$

Family-3 When $\lambda^2 + 4\mu^2 < 0, \zeta \neq 0$ and $\zeta = -\mu$,

$$\gamma^{f(\eta)} = \frac{\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 - \lambda^2}}{2\mu}\tan\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{\lambda}{2\mu} + \frac{\sqrt{-4\mu^2 - \lambda^2}}{2\mu}\cot\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right).$$

Family-4 When $\lambda^2 + 4\mu^2 > 0, \zeta \neq 0$ and $\zeta = -\mu$,

$$\gamma^{f(\eta)} = \frac{\lambda}{2\mu} + \frac{\sqrt{4\mu^2 + \lambda^2}}{2\mu}\tanh\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{\lambda}{2\mu} + \frac{\sqrt{4\mu^2 + \lambda^2}}{2\mu}\coth\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right).$$

Family-5 When $\lambda^2 - 4\mu^2 < 0$ and $\zeta = \mu$,

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\mu} + \frac{\sqrt{4\mu^2 - \lambda^2}}{2\mu}\tan\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\mu} - \frac{\sqrt{4\mu^2 - \lambda^2}}{2\mu}\cot\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right).$$

Family-6 When $\lambda^2 - 4\mu^2 > 0$ and $\zeta = \mu$,

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 + \lambda^2}}{2\mu}\tanh\left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2}\eta\right),$$

$$\gamma^{f(\eta)} = \frac{-\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 + \lambda^2}}{2\mu}\coth\left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2}\eta\right).$$

Family-7 When $\lambda^2 = 4\mu\zeta$,

$$\gamma^{f(\eta)} = -\frac{2 + \lambda\eta}{2\zeta\eta}.$$

Family-8 When $\mu\zeta < 0, \lambda = 0$ and $\zeta \neq 0$,

$$y^{f}(\eta) = -\sqrt{\frac{-\mu}{\zeta}}\tanh\left(\sqrt{-\mu\zeta}\,\eta\right),$$

$$y^{f}(\eta) = -\sqrt{\frac{-\mu}{\zeta}}\coth\left(\sqrt{-\mu\zeta}\,\eta\right).$$

Family-9 When $\lambda = 0$ and $\mu = -\zeta$,

$$y^{f}(\eta) = \frac{1 + e^{-2\zeta\eta}}{-1 + e^{-2\zeta\eta}}.$$

Family-10 When $\mu = \zeta = 0$,

$$y^{f}(\eta) = \cosh(\lambda\eta) + \sinh(\lambda\eta).$$

Family-11 When $\mu = \lambda = K$ and $\zeta = 0$,

$$y^{f}(\eta) = e^{K\eta} - 1.$$

Family-12 When $\zeta = \lambda = K$ and $\mu = 0$,

$$y^{f}(\eta) = \frac{e^{K\eta}}{1 - e^{K\eta}}.$$

Family-13 When $\lambda = \mu + \zeta$,

$$y^{f}(\eta) = -\frac{1 - \mu e^{(\mu-\zeta)\eta}}{1 - \zeta e^{(\mu-\zeta)\eta}}.$$

Family-14 When $\lambda = -(\mu + \zeta)$,

$$y^{f}(\eta) = \frac{\mu - e^{(\mu-\zeta)\eta}}{\zeta - e^{(\mu-\zeta)\eta}}.$$

Family-15 When $\mu = 0$,

$$y^{f}(\eta) = \frac{\lambda e^{\lambda\eta}}{1 - \zeta e^{\lambda\eta}}.$$

Family-16 When $\lambda = \mu = \zeta \neq 0$,

$$y^{f}(\eta) = \frac{1}{2}\left[\sqrt{3}\tan\left(\frac{\sqrt{3}}{2}\mu\eta\right) - 1\right].$$

Family-17 When $\lambda = \zeta = 0$,

$$y^{f}(\eta) = \mu\eta.$$

Family-18 When $\lambda = \mu = 0$,

$$y^{f}(\eta) = -\frac{1}{\zeta\eta}.$$

Family-19 When $\mu = \zeta$ and $\lambda = 0$,

$$y^{f}(\eta) = \tan(\mu\eta).$$

Family-20 When $\zeta = 0$,

$$y^{f}(\eta) = e^{\lambda\eta} - \frac{m}{n}.$$

4 Mathematical analyses of the models and their solutions

Assuming the transformations:

$$v(x,t) = v(\eta), \eta = \frac{\Gamma(\beta+1)}{\alpha}(kx^{\alpha} - ct^{\alpha}), \tag{23}$$

where k and c are constants. Using Eq. 8 in Eq. 2, we acquire the subsequent ODE

$$(c^2 - k^2)v'' - g^2 v + h^2 v^3 = 0. \tag{24}$$

The subsequent sections employ the planned techniques to obtain the desired solutions.

4.1 Solutions with the $(G'/G, 1/G)$-expansion method

Using the homogenous balance technique to the highest-order derivative with the nonlinear term in Eq. 24, we get $m = 1$. For $m = 1$, Eq. 15 has the form:

$$v(\eta) = a_0 + a_1\phi(\eta) + b_1\psi(\eta), \tag{25}$$

where a_0, a_1 and b_1 are unknown parameters.

Case 1: The obtained Eq. 25 is substituted into Eq. 24 with the use of Eqs 5 and 7 to result in a polynomial equation. A system of algebraic equations is obtained by setting each polynomial coefficient to zero $a_0, a_1, b_1, \mu, \sigma, \lambda, c,$ and k. This system of algebraic equations can be solved using symbolic computation software such as MATHEMATICA, which provides the following results:

$$a_0 = 0, a_1 = \frac{\sqrt{k^2 - c^2}}{\sqrt{2}h}, b_1 = \frac{\sqrt{k^2 - c^2}\sqrt{\lambda}\sqrt{\sigma}}{\sqrt{2}h}, g = \frac{\sqrt{c^2 - k^2}\sqrt{\lambda}}{\sqrt{2}}, \mu = 0. \tag{26}$$

The hyperbolic traveling wave solutions of Eq. 24 can be obtained by substituting Eq. 26 into Eq. 25:

$$v(x,t) = \frac{\sqrt{k^2 - c^2}}{\sqrt{2}h}\left(\frac{A_1\sqrt{-\lambda}\cosh(\sqrt{-\lambda}\,\eta) + A_2\sqrt{-\lambda}\sinh(\sqrt{-\lambda}\,\eta)}{A_1\sinh(\sqrt{-\lambda}\,\eta) + A_2\cosh(\sqrt{-\lambda}\,\eta) + \frac{\mu}{\lambda}}\right),$$
$$+\frac{\sqrt{k^2 - c^2}\sqrt{\lambda\sigma}}{\sqrt{2}h}\left(\frac{1}{A_1\sinh(\sqrt{-\lambda}\,\eta) + A_2\cosh(\sqrt{-\lambda}\,\eta) + \frac{\mu}{\lambda}}\right), \tag{27}$$

where $\sigma = {A_1}^2 - {A_2}^2$.

Family 1.1: If $A_1 = 0, A_2 \neq 0$, and $\mu = 0$ in Eq. 27, then we obtain the subsequent hyperbolic traveling wave solution:

$$v(x,t) = -\frac{\sqrt{c^2 - k^2}\sqrt{\lambda}}{\sqrt{2}h}\left(\tanh\left(\sqrt{-\lambda}\,\eta\right) - \sqrt{\sigma}\;\frac{1}{A_2}\operatorname{sech}\left(\sqrt{-\lambda}\,\eta\right)\right). \tag{28}$$

Family 1.2: If $A_1 \neq 0, A_2 = 0$ and $\mu = 0$ in Eq. 27, we obtain the following hyperbolic traveling wave solution:

$$v(x,t) = -\frac{\sqrt{c^2-k^2}\sqrt{\lambda}}{\sqrt{2}h}\left(\coth\left(\sqrt{-\lambda}\,\eta\right) - \sqrt{\sigma}\,\frac{1}{A_1}\operatorname{cosech}\left(\sqrt{-\lambda}\,\eta\right)\right). \tag{29}$$

Case 2: By substituting Eq. 25 into Eq. 24 along with Eqs 5 and 9 for $\lambda > 0$, we can obtain a polynomial equation. Setting each polynomial coefficient to zero generates a system of algebraic equations for $a_0, a_1, b_1, \mu, \sigma, \lambda, c,$ and k. By solving this system of algebraic equations using software such as Mathematica, we can obtain the following outcomes:

$$a_0 = 0, a_1 = \frac{\sqrt{k^2-c^2}}{\sqrt{2}h}, b_1 = -\frac{\sqrt{k^2-c^2}\sqrt{\lambda}\sqrt{\sigma}}{\sqrt{2}h}, g = \frac{\sqrt{c^2-k^2}\sqrt{\lambda}}{\sqrt{2}}, \mu = 0. \tag{30}$$

The periodic trigonometric traveling wave solution of Eq. 24 can be obtained by substituting Eq. 30 into Eq. 25, as follows:

$$v(x,t) = \frac{\sqrt{k^2-c^2}}{\sqrt{2}h}\left(\frac{A_1\sqrt{\lambda}\cos(\sqrt{\lambda}\,\eta) - A_2\sqrt{\lambda}\sin(\sqrt{\lambda}\,\eta)}{A_1\sin(\sqrt{\lambda}\,\eta) + A_2\cos(\sqrt{\lambda}\,\eta) + \frac{\mu}{\lambda}}\right), -\frac{\sqrt{k^2-c^2}\sqrt{\lambda\sigma}}{\sqrt{2}h}\left(\frac{1}{A_1\sin(\sqrt{\lambda}\,\eta) + A_2\cos(\sqrt{\lambda}\,\eta) + \frac{\mu}{\lambda}}\right), \tag{31}$$

where $\sigma = {A_1}^2 + {A_2}^2$.

Family 2.1: If $A_1 = 0, A_2 \neq 0,$ and $\mu = 0$ in Eq. 31, we obtain the following trigonometric traveling wave solution:

$$v(x,t) = -\frac{\sqrt{k^2-c^2}\sqrt{\lambda}}{\sqrt{2}h}\left(\tan\left(\sqrt{\lambda}\,\eta\right) - \sqrt{\sigma}\,\frac{1}{A_2}\sec\left(\sqrt{\lambda}\,\eta\right)\right), \tag{32}$$

$$v(x,t) = \frac{\sqrt{k^2-c^2}\sqrt{\lambda}}{\sqrt{2}h}\left(\cot\left(\sqrt{\lambda}\,\eta\right) - \sqrt{\sigma}\,\frac{1}{A_1}\cos ec\left(\sqrt{\lambda}\,\eta\right)\right). \tag{33}$$

4.2 Solutions with the modified (G'/G^2)-expansion method

Using the homogenous balance technique to the highest order derivatives with the nonlinear term in Eq. 24, we get $m = 1$. For $m = 1$, Eq. 16 has the form:

$$v(\eta) = a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{34}$$

where a_0 and a_1 are unknown parameters. We can then substitute Eq. 34 and Eq. 17 into Eq. 24 and sum all coefficients of the same order. (G'/G^2) yields a set of algebraic equations involving a_0, a_1, and other parameters. The set of algebraic equations is then solved using the symbolic computation software Mathematica, resulting in specific values for the unknown parameters:

$$a_0 = 0, a_1 = \pm\frac{ig\sqrt{\lambda_1}}{h\sqrt{\lambda_0}}, k = \pm\frac{\sqrt{-g^2+2c^2\lambda_0\lambda_1}}{\sqrt{2\lambda_0\lambda_1}}. \tag{35}$$

By substituting Eqs 35, 18, and 19 into Eq. 34 and considering the following cases, if $\lambda_1 < 0$, then

$$v_1(x,t) = -\frac{ig\sqrt{|\lambda_0\lambda_1|}}{h\sqrt{\lambda_0\lambda_1}}\left(1 - \frac{\lambda_1}{2}\left[\frac{C_1\sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2\cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}{C_1\cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2\sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}\right]\right), \tag{36}$$

$$v_2(x,t) = \frac{ig}{h}\left(\frac{C_1\sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2\cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}{C_1\cosh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right) + C_2\sinh\left(\sqrt{\lambda_0\lambda_1}\,\eta\right)}\right). \tag{37}$$

4.3 Solutions with the new auxiliary equation method

Using the homogenous balance technique to the highest order derivative with the nonlinear term in Eq. 24, we obtain $m = 1$. For $m = 1$, Eq. 24 has the form:

$$v(\eta) = a_0 + a_1\gamma^{f(\eta)}, \tag{38}$$

where a_0 and a_1 are unknown parameters.

Switching Eq. 10 into Eq. 24 with Eq. 22, we obtain the algebraic equations involving a_0, a_1, and other parameters by equating all coefficients of different powers $\gamma^{f(\eta)}$ to zero:

$$\begin{aligned} &f^0(\eta):\ -a_0g^2 + {a_0}^3h^2 - a_1k^2\lambda\mu + a_1c^2\lambda\mu = 0,\\ &f^1(\eta):\ -a_1g^2 + 3{a_0}^2a_1h^2 - a_1k^2\lambda^2 + a_1c^2\lambda^2 - 2a_1k^2\zeta\mu + 2a_1c^2\zeta\mu = 0,\\ &f^2(\eta):\ 3a_0{a_1}^2h^2 - 3a_1k^2\zeta\lambda + 3a_1c^2\zeta\lambda = 0,\\ &f^3(\eta):\ {a_1}^3h^2 - 2a_1k^2\zeta^2 + 2a_1c^2v^2 = 0. \end{aligned} \tag{39}$$

Using mathematical software (Mathematica) to solve the aforementioned system of algebraic equations, we obtain the subsequent solution:

$$a_0 = \lambda\Lambda, a_1 = 2\zeta\Lambda, g = -\frac{\sqrt{k^2-c^2}\sqrt{\lambda^2-4v\mu}}{\sqrt{2}}, \tag{40}$$

where $\Lambda = \frac{\sqrt{k^2-c^2}}{\sqrt{2}h}$.

Substituting the attained solution Eq. 40 into Eq. 38, we obtain the following:

$$v(\eta) = \Lambda\left\{\lambda + 2\zeta\gamma^{f(\eta)}\right\}. \tag{41}$$

Substituting the solution stated by Eq. 22 into Eq. 41, the solutions regained are:

For Family 1: When $\lambda^2 - 4\mu\zeta < 0$ and $\zeta \neq 0$,

$$v_{1,1}(x,t) = \Lambda\left[\sqrt{4\mu\zeta-\lambda^2}\tan\left(\frac{\sqrt{4\mu\zeta-\lambda^2}}{2}\eta\right)\right], \tag{42}$$

$$v_{1,2}(x,t) = -\Lambda\left[\sqrt{4\mu\zeta-\lambda^2}\cot\left(\frac{\sqrt{4\mu\zeta-\lambda^2}}{2}\eta\right)\right]. \tag{43}$$

For Family 2: When $\lambda^2 - 4\mu\zeta > 0$ and $\zeta \neq 0$,

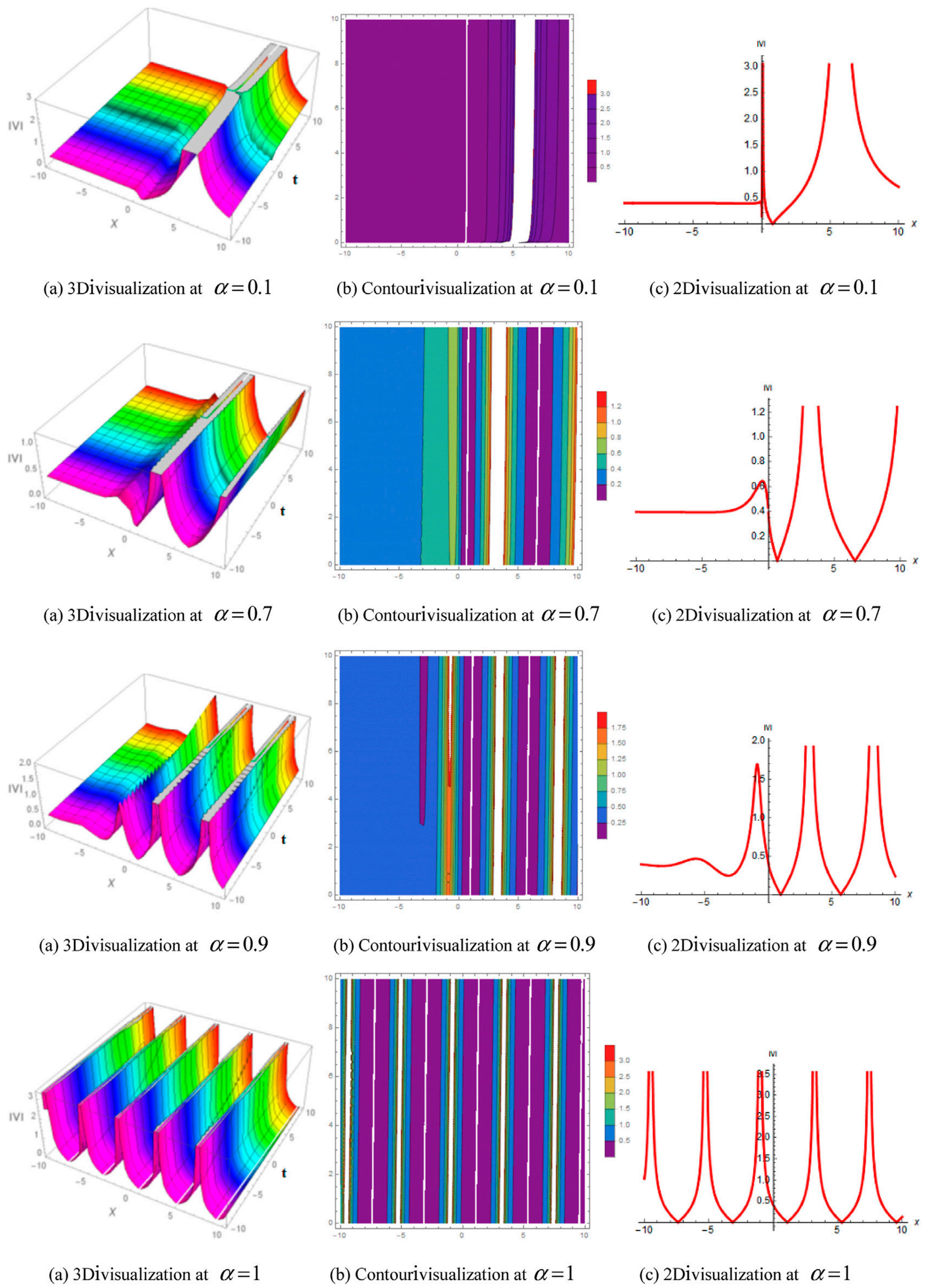

FIGURE 1
Influence of fractional order by 2D, 3D, and corresponding contours of Eq. 32 for $k = 2, h = 0.6, \lambda = 0.3, \beta = 0.5, A_2 = 2, \sigma = 4, c = 0.05, t = 1$. **Family 2.2:** If $A_1 \neq 0, A_2 = 0$ and $\mu = 0$ in Eq. 31, we obtain the following trigonometric traveling wave solution.

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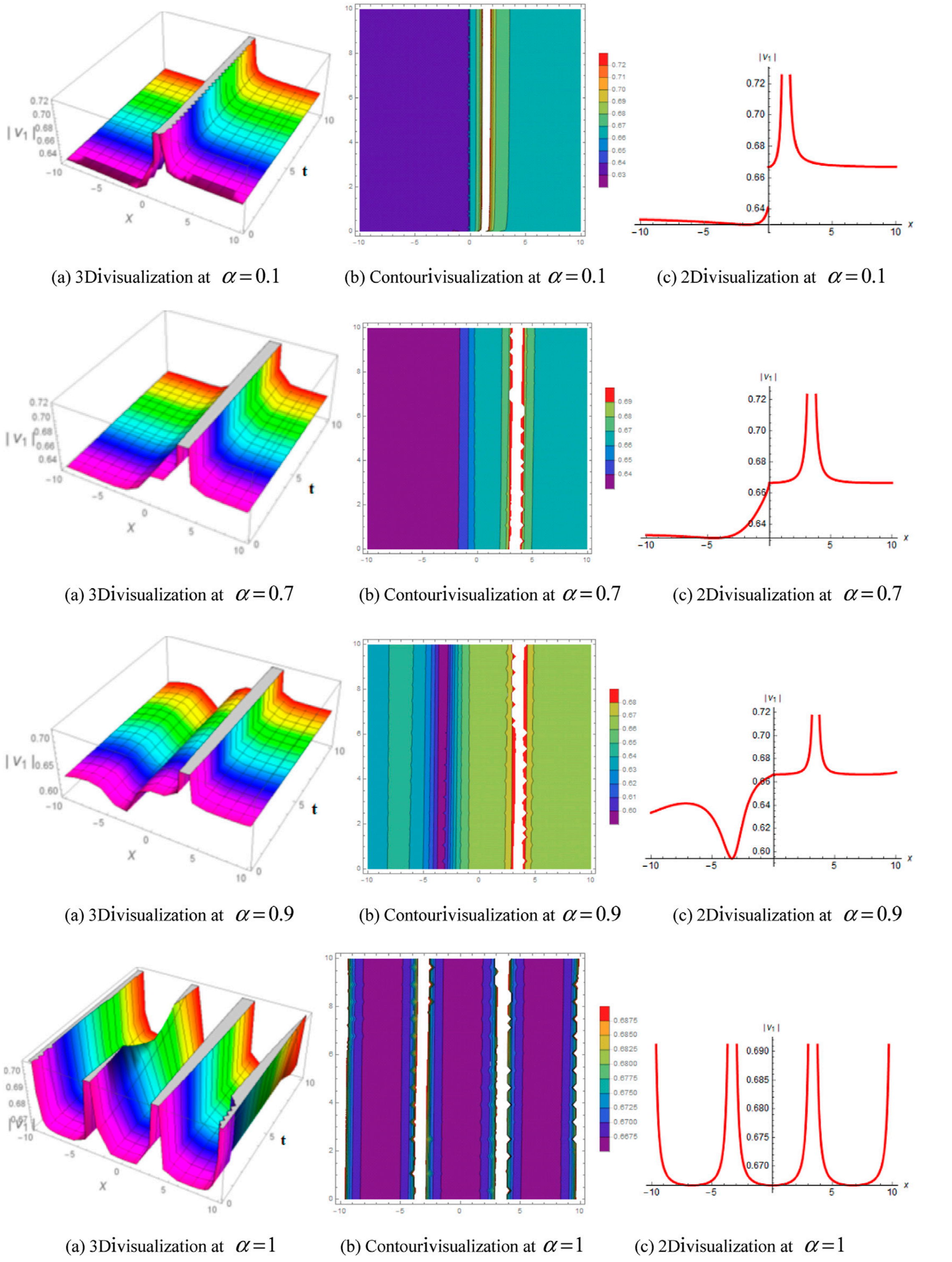

FIGURE 2
Influence of fractional order by 2D, 3D, and corresponding contours of Eq. 36 for $k = 2, h = 0.6, \lambda_0 = 0.4, \lambda_1 = -0.2, \beta = 0.5, c = 0.05, t = 1$. If $\lambda_0\lambda_1 > 0$, then

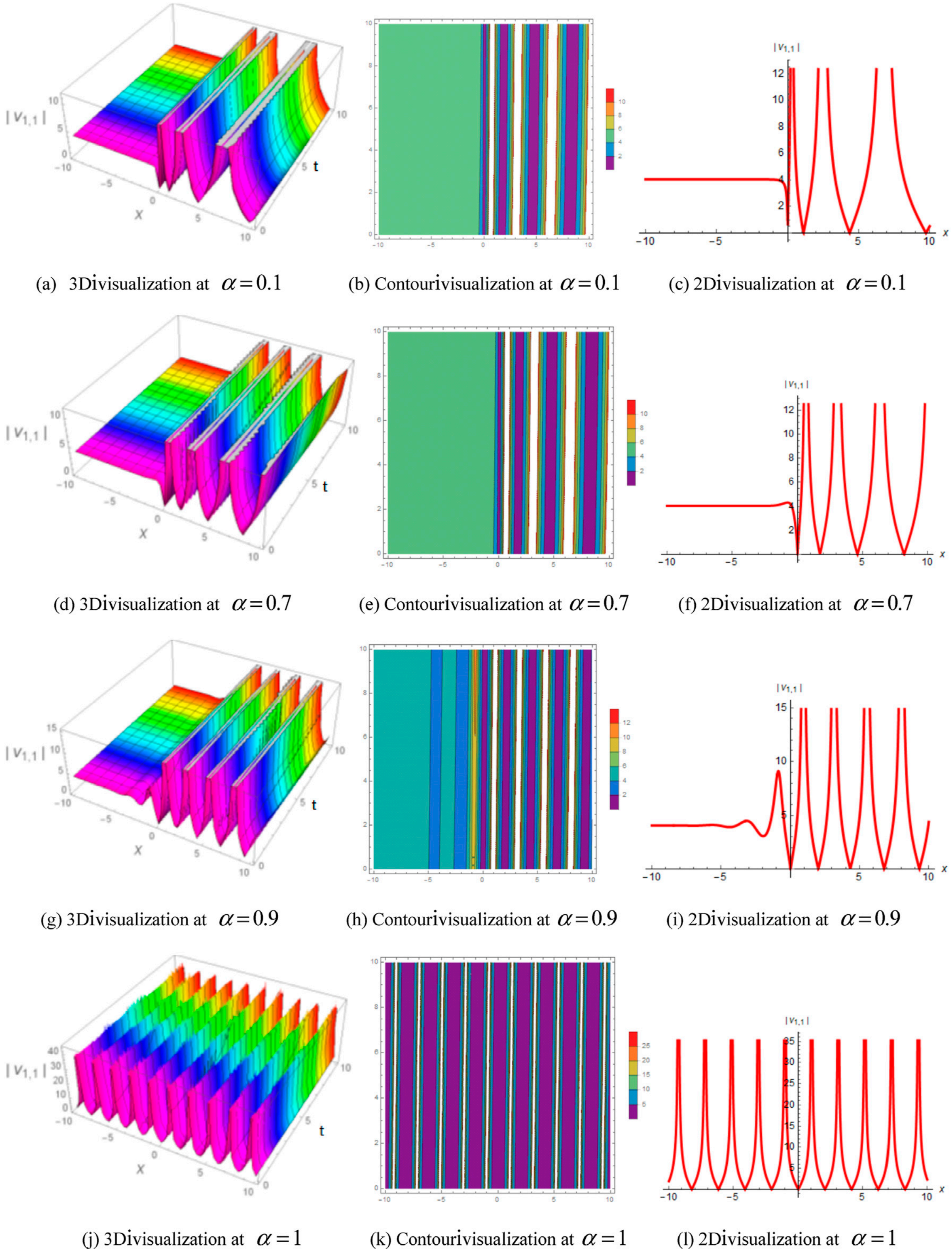

FIGURE 3
Influence of fractional order by 2D, 3D, and corresponding contours of Eq. 42 for $k = 2, h = 0.6, \lambda = 0.5, \beta = 0.5, \mu = 0.8, \zeta = 1, c = 0.05, t = 1$.

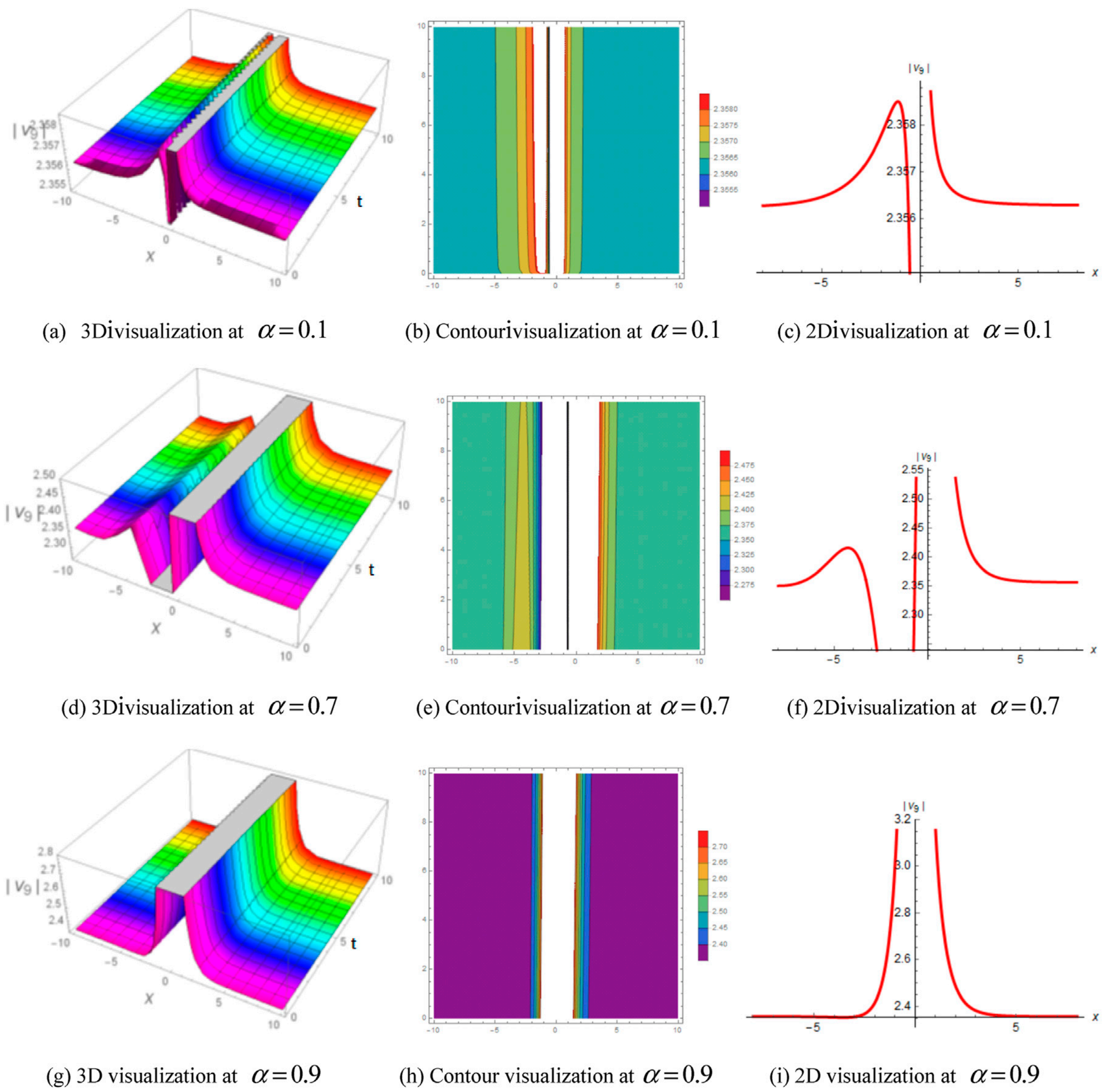

FIGURE 4
Influence of fractional order by 2D, 3D, and corresponding contours of Eq. 57 for $k = 2, h = 0.6, \lambda = 0.5, \beta = 0.5, \mu = 0.8, \zeta = 1, c = 0.05, t = 1$. For Family 12: When $\zeta = \lambda = K$ and $\mu = 0$,

$$v_{2,1}(x,t) = -\Lambda\left[\sqrt{\lambda^2 - 4\mu\zeta}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2}\eta\right)\right], \tag{44}$$

$$v_{2,2}(x,t) = -\Lambda\left[\sqrt{\lambda^2 - 4\mu\zeta}\coth\left(\frac{\sqrt{\lambda^2 - 4\mu\zeta}}{2}\eta\right)\right]. \tag{45}$$

For Family 3: When $\lambda^2 + 4\mu^2 < 0, \zeta \neq 0$ and $\zeta = -\mu$,

$$v_{3,1}(x,t) = \Lambda\left[\sqrt{-4\mu^2 - \lambda^2}\tan\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right)\right], \tag{46}$$

$$v_{3,2}(x,t) = -\Lambda\left[\sqrt{-4\mu^2 - \lambda^2}\cot\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right)\right]. \tag{47}$$

For Family 4: When $\lambda^2 + 4\mu^2 > 0, \zeta \neq 0$ and $\zeta = -\mu$,

$$v_{4,1}(x,t) = -\Lambda\left[\sqrt{4\mu^2 + \lambda^2}\tanh\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right)\right], \tag{48}$$

$$v_{4,2}(x,t) = -\Lambda\left[\sqrt{4\mu^2 + \lambda^2}\coth\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right)\right]. \tag{49}$$

For Family 5: When $\lambda^2 - 4\mu^2 < 0$ and $\zeta = \mu$,

$$v_{5,1}(x,t) = \Lambda\left[\sqrt{4\mu^2 - \lambda^2}\tan\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right)\right], \tag{50}$$

$$v_{5,2}(x,t) = -\Lambda\left[\sqrt{4\mu^2 - \lambda^2}\cot\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right)\right]. \tag{51}$$

For Family 6: When $\lambda^2 - 4\mu^2 > 0$ and $\zeta = \mu$,

$$v_{6,1}(x,t) = -\Lambda\left[\sqrt{\lambda^2 - 4\mu^2}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu^2}}{2}\eta\right)\right], \tag{52}$$

$$v_{6,2}(x,t) = -\Lambda\left[\sqrt{\lambda^2 - 4\mu^2}\coth\left(\frac{\sqrt{\lambda^2 - 4\mu^2}}{2}\eta\right)\right]. \tag{53}$$

For Family 7: When $\lambda^2 = 4\mu\zeta$,

$$v_7(x,t) = \frac{-2\Lambda}{\eta}. \tag{54}$$

For Family 8: When $\mu\zeta < 0, \lambda = 0$ and $\zeta \neq 0$,

$$v_{8,1}(x,t) = -\Lambda\left[2\sqrt{-\mu\zeta}\tanh\left(\sqrt{-\mu\zeta}\,\eta\right)\right], \tag{55}$$

$$v_{8,2}(x,t) = -\Lambda\left[2\zeta\sqrt{-\mu\zeta}\coth\left(\sqrt{-\mu\zeta}\,\eta\right)\right]. \tag{56}$$

For Family 9: When $\lambda = 0$ and $\mu = -\zeta$,

$$v_9(x,t) = \Lambda\left[2\zeta\left(\frac{e^{-2\zeta\eta} + 1}{e^{-2\zeta\eta} - 1}\right)\right], \tag{57}$$

$$v_{12}(x,t) = \Lambda\left[K + 2K\left(\frac{e^{K\eta}}{1 - e^{K\eta}}\right)\right]. \tag{58}$$

For Family 13: When $\lambda = \mu + \zeta$,

$$v_{13}(x,t) = \Lambda\left[\mu + \zeta - 2\zeta\left(\frac{1 - \mu e^{(\mu-\zeta)\eta}}{1 - \zeta e^{(\mu-\zeta)\eta}}\right)\right]. \tag{59}$$

For Family 14: When $\lambda = -(\mu + \zeta)$,

$$v_{14}(x,t) = -\Lambda\left[\mu + \zeta - 2\zeta\left(\frac{\mu - e^{(\mu-\zeta)\eta}}{\zeta - e^{(\mu-\zeta)\eta}}\right)\right]. \tag{60}$$

For Family 15: When $\mu = 0$,

$$v_{15}(x,t) = \Lambda\left[\lambda + 2\zeta\left(\frac{\lambda e^{\lambda\eta}}{1 - \zeta e^{\lambda\eta}}\right)\right]. \tag{61}$$

For Family 16: When $\lambda = \mu = \zeta \neq 0$,

$$v_{16}(x,t) = \Lambda\left[\lambda + \zeta\left(\sqrt{3}\,\tan\left(\frac{\sqrt{3}}{2}\mu\eta\right) - 1\right)\right]. \tag{62}$$

For Family 18: When $\lambda = \mu = 0$,

$$v_{18}(x,t) = -\frac{2\Lambda}{\eta}. \tag{63}$$

For Family 19: When $\mu = \zeta$ and $\lambda = 0$,

$$v_{19}(x,t) = 2\zeta\Lambda\tan(\mu\eta). \tag{64}$$

5 Graphical demonstration and explanation

To demonstrate the dynamics and behavior of our solutions, we used Eqs 32, 36, 42, and 17 to graphically represent the solutions in 3D, 2D, and contour graphs, which are shown in Figures 1–4. To illustrate the variation over time or to compare multiple wave items, 3D plots are often used. In this study, the wave points were arranged in a series with evenly spaced breaks and connected by a line to emphasize their relationships. In contrast, 2D line plots demonstrate very high and low frequency and amplitude. The authors note that the plots show the different natures of the solutions, such as periodic, singular-kink type, singular-bell shaped, and bright singular wave solutions. Furthermore, the authors emphasize that the correct physical description of the solutions can be generated by choosing distinct values for the fractional parameter α.

6 Conclusion

In this work, we applied the $(G'/G, 1/G)$-expansion, modified the (G'/G^2)-expansion, and provided new auxiliary equations methods in a satisfactory way to determine the novel soliton solutions of the space-time fractional LGHE by considering the truncated M-fractional derivative. These methods restored the periodic, singular-kink type, singular-bell shaped, and bright singular wave solutions dark, bright-singular, exponential, trigonometric, and rational solitons. Mathematica was utilized to perform the algebraic computations and generate graphical representations of the obtained solutions at different parameter values. Compared with other works [10, 23], our solutions have not been reported in the previous literature. These techniques are highly effective and robust for discovering soliton solutions for nonlinear fractional differential equations. Furthermore, the solutions obtained can provide deeper insights into the nonlinear dynamics of optical soliton propagation.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material. Further inquiries can be directed to the corresponding author.

Author contributions

RZ, W-XM, SA, and IS contributed to the study conception and design. IS and AH organized the database. AH and SA performed the statistical analysis. RZ and KM wrote the first draft of the manuscript. W-XM, IS, and AH wrote sections of the manuscript. SA and AH writing-review and editing. SA is the project administrative. All authors contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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