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A routing strategy for spatial networks based on harmonic centrality

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With the rapid development of networks, the traffic in the networks has increased sharply, resulting in frequent congestion, especially in spatial networks, such as the railway network, aviation network, and sensor network, and congestion not only affects the user's experience but also causes serious economic losses. Therefore, in this paper, we effectively identify the high-load nodes in spatial networks by considering harmony centrality and degree. On this basis, we design the *HD* routing strategy by avoiding these key nodes, which can enhance the traffic throughput of spatial networks efficiently. The results provide new ideas and directions for the design of routing strategies for spatial networks.

KEYWORDS

spatial networks, routing strategy, harmony centrality, congestion, complex network

1 Introduction

At the end of the 20th century, the discoveries of the small-world phenomenon [1] and scale-free property [2] attracted much attention. Since then, complex networks have become a research hotspot in many fields, including communication, transportation, power grids, and social relations [3–13]. Many complex systems can be modeled into complex networks, for example, in some complex systems with transmission as their main function, such as communication networks, internet, and transport network. The constituent elements of these systems can be abstracted as nodes, and links can be used to describe the interrelationships between different elements, which can help us analyze traffic dynamics on these systems effectively. Among them, congestion is the most critical problems of such complex networks, which is related to network topology [14] and routing strategy [15]. However, it is too expensive to modify the network structure. Therefore, optimizing the routing strategy seems to be more practical to improve network transmission performance.

The shortest path (*SP*) routing strategy is the most common routing strategy, which is extensively adopted in various complex systems. However, it is easy to cause congestion at some hub nodes. To solve this problem, many efficient routing strategies have been proposed to avoid these hub nodes [16–26]. Yan et al. [24] realized the influence of network topology on traffic dynamics. They focused on the node degree and proposed an efficient routing strategy, which can enhance the network throughput more than 10 times. Jiang et al. [25] found that nodes with the largest betweenness are most likely to be congested, so they designed an *IE* strategy. The load can avoid these high-betweenness nodes during transmission, which can achieve high network throughput. Zhang et al. [26] considered both static and dynamic information and proposed an adaptive routing strategy. The load can select the appropriate path for transmission based on the waiting time and degree, which can reduce congestion effectively.

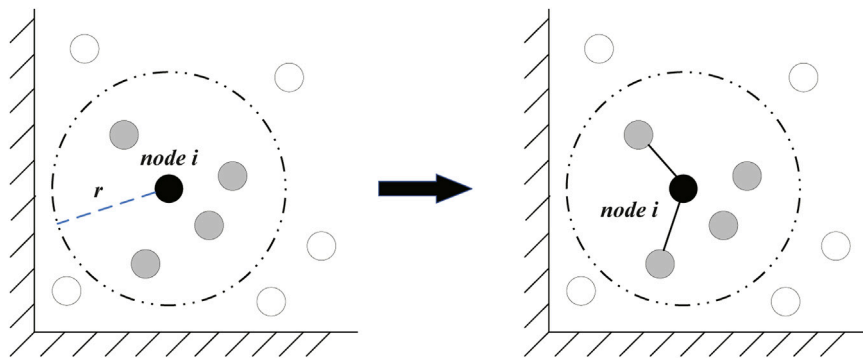


FIGURE 1
Node i build links with nodes in the set Ω_i with probability p .

However, most current works ignore the influence of a space factor. In fact, aviation networks [27], transportation networks [28], wireless sensor networks [29], and swarming networks [30] are all limited by spatial locations. In these networks, each node has a fixed spatial location, and link length is limited. We generally call these networks as spatial networks, which is a significant class of complex networks [31]. Due to the limitation of link length, the topological structure in spatial networks is quite different with topological networks. At the same time, the traffic dynamics on spatial networks will also vary due to distinct structures [32]. For example, in the topological networks, the node degrees often exhibit an obvious power-law relationship with the loads when adopting the *SP* routing strategy. However, Lin et al. [33] found that this power-law relationship is not obvious in spatial networks. Most of the existing routing strategies cannot achieve great results in spatial networks. Thus, there is an urgent need for an efficient routing strategy to alleviate congestion of spatial networks.

In this paper, we focus on traffic dynamics for spatial networks. Based on the harmony centrality and degree, we redefine the key nodes in the transmission. We find that the node with the high harmony centrality index and degree usually deals with more loads. Therefore, we design a harmony-degree (*HD*) routing strategy to bypass these key nodes. All simulations are made on the local-area and energy-efficient (*LAEE*) evolution model and improved the random geometric graph (*IRGG*) model, which are two spatial networks with different structures. According to the results of simulation, our *HD* routing strategy can help spatial networks obtain greater traffic throughput.

The outline of this paper is as follows. In Section 2, we describe the generation of spatial networks. In Section 3, we explain the traffic dynamic model. In Section 4, we introduce our *HD* routing strategy. Simulation results and discussions are given in Section 5. In Section 6, we summarize the conclusion of this paper.

2 Network models

It has been shown that the network structure is an important factor affecting load transmission. For an efficient routing strategy, it

should ensure high performance on different spatial networks. Therefore, we will test the performance of the proposed routing strategy on two spatial networks with homogeneous and heterogeneous properties, respectively. The generation of these two models is as follows.

2.1 IRGG model

The *IRGG* model is a simple homogeneous spatial network, which has a uniform degree distribution.

The generation process of the *IRGG* model is as follows:

Step 1: N nodes are distributed randomly in the 1×1 square area S .

Step 2: We set a connection radius r for each node. As shown in Figure 1, any node i only can establish links with nodes located in its circular connected area. The set of these nodes can be represented by Ω_i .

Step 3: We set a connection probability p . Node i establishes links with the nodes in the set Ω_i with probability p .

Step 4: We repeat Step 3 until all nodes follow this rule to build links with nodes within their respective connected areas.

2.2 LAEE model

The study found that many practical networks follow the scale-free property. To explore the applicability of our proposed routing strategy on heterogeneous spatial networks, we adopt the *LAEE* evolution model proposed by Jiang et al. [34], which builds a spatial network with a power-law degree distribution.

The generation process of the *LAEE* evolution model is as follows [34]:

Step 1: N nodes are distributed randomly in the 1×1 square region S .

Step 2: We define the node closest to the origin as the sink node. At this point, all nodes are isolated. We set every node with the same connection radius r . If the scatter node A lies within the connection radius of node B , we call the scatter node A as the potential neighbor node of the node B .

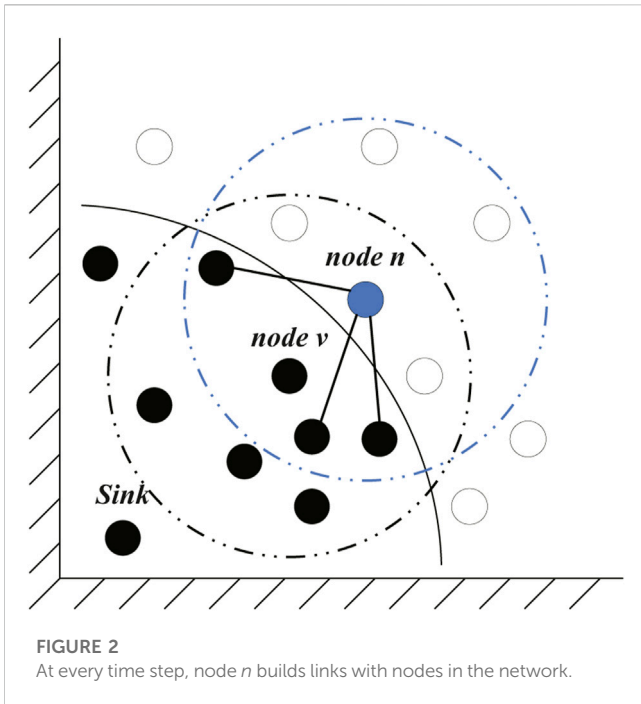


FIGURE 2
At every time step, node *n* builds links with nodes in the network.

Step 3: Sink node builds link with m_0 potential neighbor nodes to form the initial network.

Step 4: At time step i , we calculate the number of potential neighbor nodes owned by different nodes in the network separately. Next, we select the one with the most network and name it as node v . Then, we select one of its potential neighbor nodes randomly and call it as node n .

Step 5: As shown in Figure 2, the node n establishes links with m nodes in the network with the priority probability Π_i :

$$\Pi_i = \Pi'_i (i \in local - area) \frac{\varphi(E_i)k_i}{\sum_{local-area} \varphi(E_j)k_j - qk_{max}}, \quad (1)$$

where the local-area is the set of node n 's potential neighbor nodes. k_{max} represents the upper limit that the node's degree is allowed to reach, q indicates the number of nodes whose degrees arrive at k_{max} and $\varphi(E)$ is a function. In this paper, we set $\varphi(E_i) = 1$ and $\varphi(E_j) = 1$.

Step 6: We repeat Step 4 and Step 5 until all nodes are connected to the network.

Based on the aforementioned generation methods, we can generate these two networks and observe the degree distribution. As shown in Figure 3, the degree distribution of the IRGG model is relatively even, but the LAEE model presents power-law distribution. Then, the IRGG model is a homogeneous spatial network and LAEE belongs to a heterogeneous spatial network.

3 Traffic dynamics

At every time step, every node can handle at most C units of load, and R units of load are generated in the whole system. We can randomly select pairs of nodes as the sources and destinations. At each time step, we select a neighbor node as the next hop for the load according to the routing strategy. Once the load arrives at its destination, it will disappear automatically. To better understand congestion, we shall introduce the order parameter [35] as follows:

$$\eta(R) = \lim_{t \rightarrow \infty} \frac{C}{R} \frac{\langle W(t + \Delta t) - W(t) \rangle}{\Delta t}, \quad (2)$$

where $W(t)$ is denoted as the units of load in the network at time t and $\langle \dots \rangle$ indicates the average over time windows of the width Δt .

When R is small, the inflow and outflow of a node are balanced and no congestion occurs in the network, so $\eta(R) = 0$. We generally call this state as the free-flow state. However, with the increase of R , some load cannot be processed in time. At that time, $\eta(R) > 0$, and the congestion occurs. We always use the critical value R_c to describe this phase transition. When $R < R_c$, the network is in a free-flow state; when $R > R_c$, congestion occurs. We call R_c as the maximum network throughput. In this paper, our main work is to design an efficient routing strategy to help the spatial network obtain a higher value of throughput.

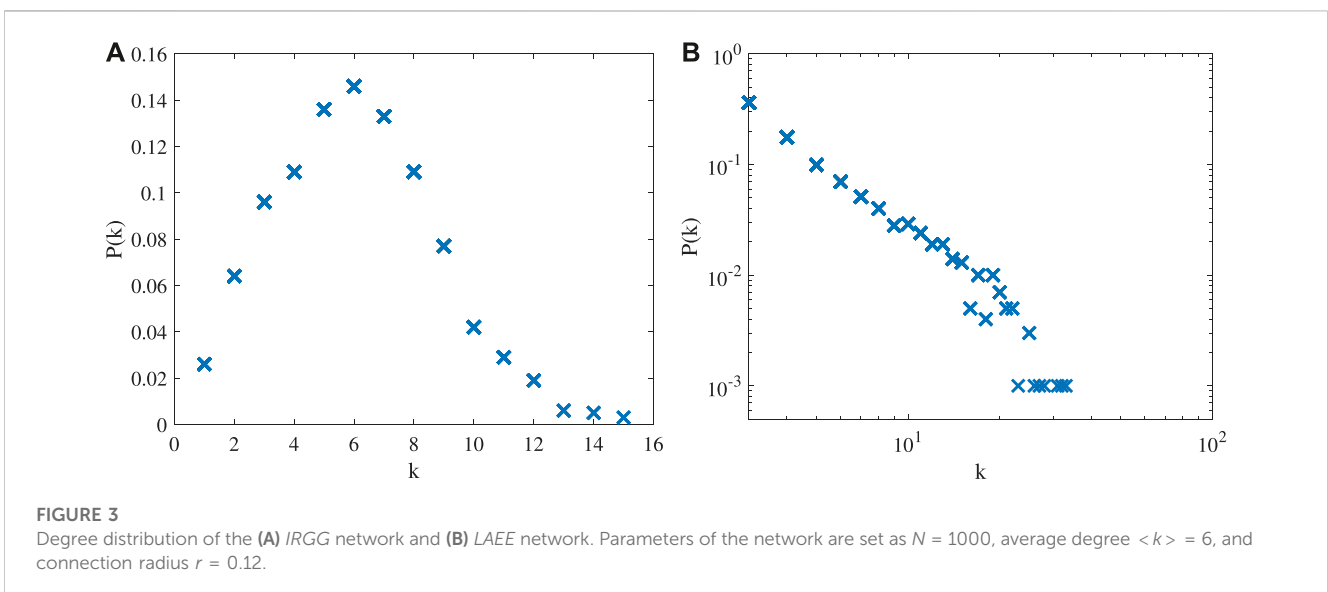


FIGURE 3
Degree distribution of the (A) IRGG network and (B) LAEE network. Parameters of the network are set as $N = 1000$, average degree $\langle k \rangle = 6$, and connection radius $r = 0.12$.

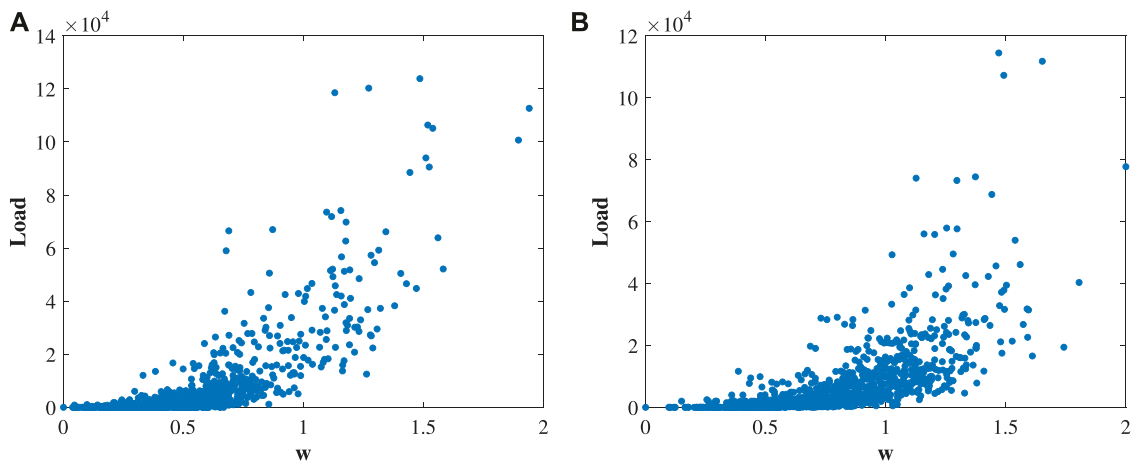


FIGURE 4 Load versus w of nodes under the shortest path (SP) routing strategy for the (A) LAEE network and (B) IRGG network with $\alpha = 1$ and $\beta = 1$, respectively. Parameters of the network are set as $N = 1000$, average degree $\langle k \rangle = 4$, and connection radius $r = 0.12$.

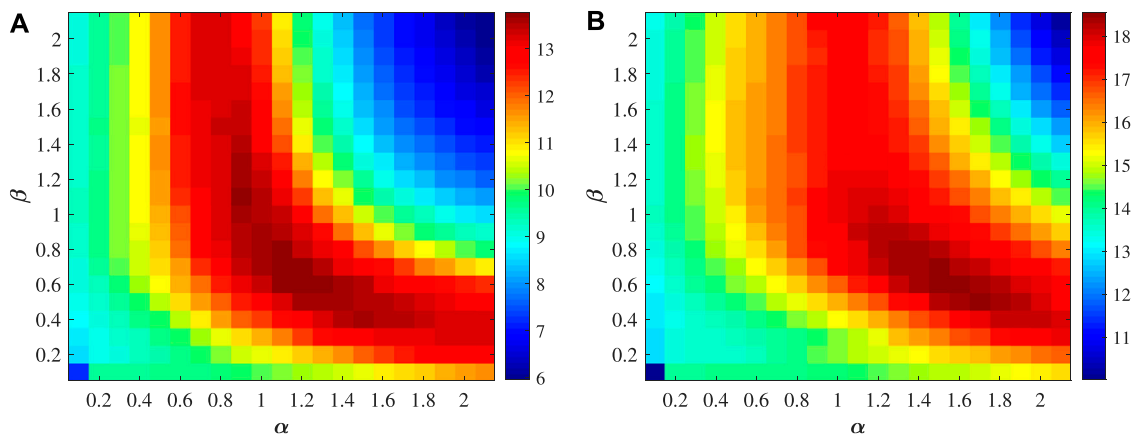


FIGURE 5 Network throughput R_c , determined by adjustable exponents α and β in the (A) LAEE network; (B) IRGG network. Parameters of the network are set as $N = 1000$, average degree $\langle k \rangle = 4$, and connection radius $r = 0.12$. Experimental results are the averages over 10 independent simulations.

Betweenness is a significant indicator to describe the load of nodes. The betweenness of node n is calculated as follows:

$$b(v) = \sum_{s \neq t} \frac{\sigma_{st}(n)}{\sigma_{st}}, \tag{3}$$

where σ_{st} is the number of paths from node s to node t according to the adopted routing strategy and $\sigma_{st}(n)$ represents the number of paths from node s to node t through node n . Traffic congestion first occurs at the node with maximum betweenness. In addition, the throughput of the network can be calculated by

$$R_c = \frac{N(N-1)C}{b_{max}}, \tag{4}$$

where b_{max} is the maximum betweenness in the network. Therefore, in order to enhance the throughput of the network, we should minimize the value of b_{max} .

4 Routing strategy

For any pair of nodes $\{s, t\}$, the path between them is defined as follows:

$$P(s \rightarrow t): s \equiv x^{(0)}, x^{(1)}, \dots, x^{(n)} \equiv t. \tag{5}$$

An efficient routing strategy attempts to find the optimal path to achieve a high network throughput R_c .

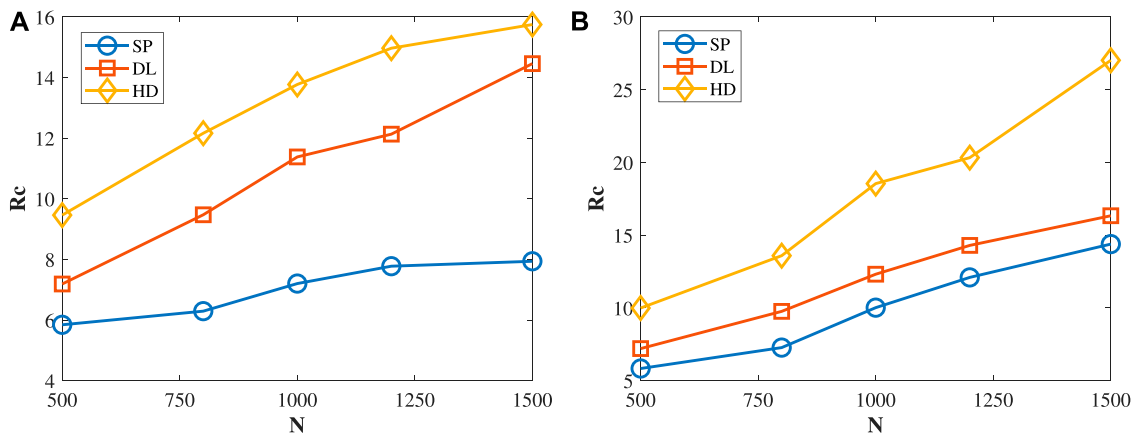


FIGURE 6 R_c of the three routing strategies on the (A) LAEE network and (B) IRGG network with different node sizes N . Parameters of the network are set as average degree $\langle k \rangle = 4$ and connection radius $r = 0.12$. α and β are set to be optimal to maximize R_c . Experimental results are the averages over 10 independent simulations.

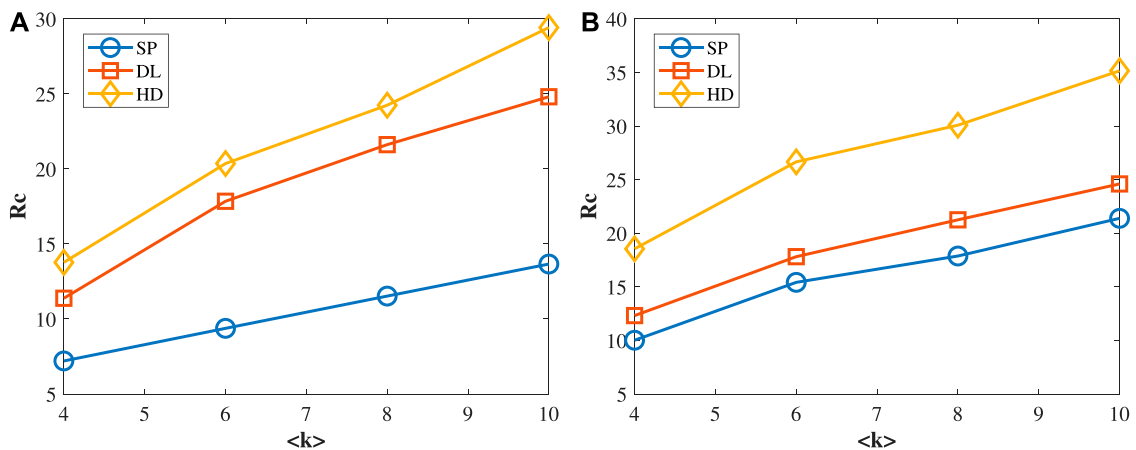


FIGURE 7 R_c of the three routing strategies on the (A) LAEE network and (B) IRGG network with different average degree $\langle k \rangle$. Parameters of the network are set as $N = 1000$ and connection radius $r = 0.12$. α and β are set to be optimal to maximize R_c . Experimental results are the averages over 10 independent simulations.

4.1 SP routing strategy

The shortest path means the path with the minimum number of links between two nodes. In the SP routing strategy, load can be transmitted from its source to its destination with fewer hops. However, it easily causes congestion at hub nodes and the network has a low throughput.

4.2 Degree-location routing strategy

In the topological network, Yan et al. [24] found that the node with the larger degree always has to deal with more loads. However, in the spatial network, this relationship is not that obvious. Lin et al.

[33] investigated the influence of network topology on load transmission. They found that the nodes with larger links and closer to the regional center usually process more loads. Based on this idea, they proposed the degree-location (DL) routing strategy, which is presented as follows.

Nodes are distributed in a two-dimensional region. Each node has its own coordinates. We set the center of the region as C , whose coordinate is (x^c, y^c) . For any node v , we can calculate its Euclidean distance from the center C as

$$L_v = \sqrt{(x_v - x^c)^2 + (y_v - y^c)^2}, \tag{6}$$

where (x_v, y_v) represents the 2D coordinate of node v . Next, we normalize L_v to L'_v :

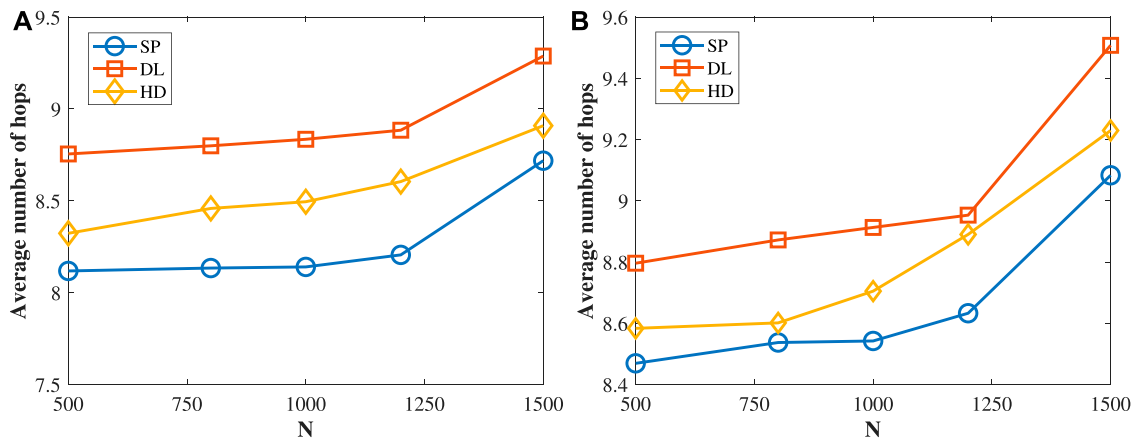


FIGURE 8

Average number of hops vs. N under three routing strategies on the (A) LAEE network and (B) IRGG network. Parameters of the network are set as average degree $\langle k \rangle = 4$ and connection radius $r = 0.12$. α and β are set to be optimal to maximize R_c . Experimental results are the averages over 10 independent simulations.

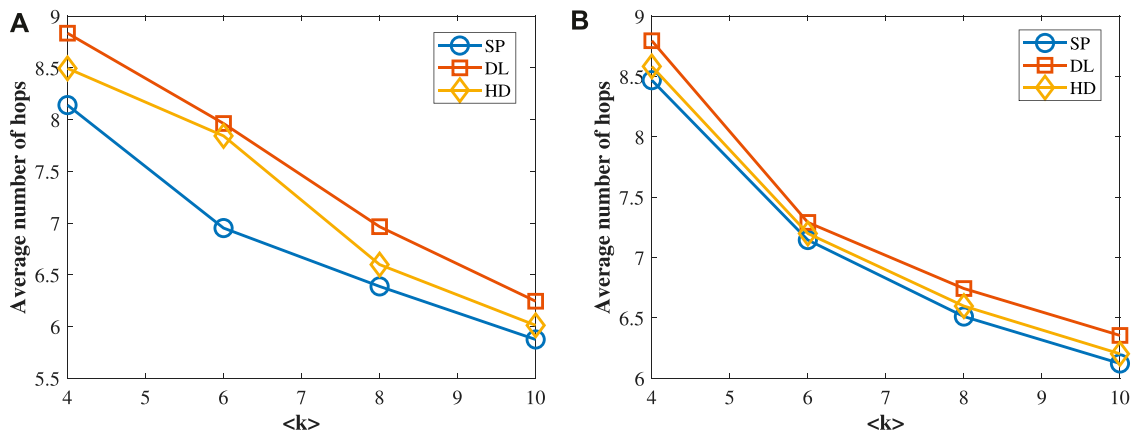


FIGURE 9

Average number of hops vs. $\langle k \rangle$ under three routing strategies on the (A) LAEE network and (B) IRGG network. Parameters of the network are set as $N = 1000$ and connection radius $r = 0.12$. α and β are set to be optimal to maximize R_c . Experimental results are the averages over 10 independent simulations.

$$L'_v = 1 - \frac{L_v - \min(L)}{\max(L) - \min(L)} \tag{7}$$

Similarly, the normalized degree is defined as

$$k'_v = \frac{k_v - \min(k)}{\max(k) - \min(k)} \tag{8}$$

The weight of node v is denoted as follows:

$$Q_v = k'^{\alpha}_v + L'^{\beta}_v \tag{9}$$

where α and β are two adjustable exponents, corresponding to degree and location, respectively.

Considering the node with a high Q value should have large load, the DL routing strategy tends to bypass these busy nodes to improve

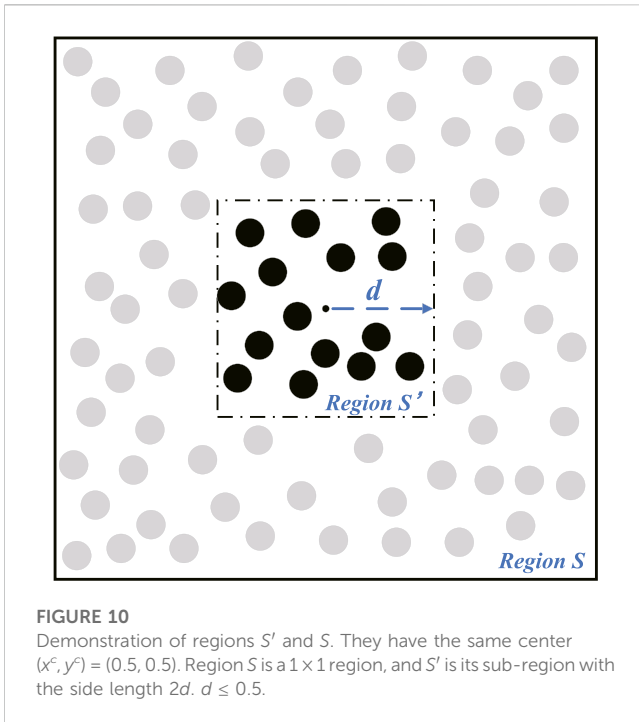
the network throughput. To perform that, the DL routing strategy attempts to use the path with the smallest sum of the Q value, i.e.,

$$G_Q(P(s \rightarrow t)) = \min \sum_{m=0}^n Q_{x(m)} \tag{10}$$

4.3 Harmony-degree routing strategy

In 2000, Marchiori and Latora proposed the harmonic centrality [36] denoted as

$$H_i = \sum_{j \neq i} \frac{1}{d(j, i)} \tag{11}$$



where $d(i, j)$ represents the number of hops from node i to node j . If there is no path between node i and node j , then $d(i, j) = \infty$. We normalize H_i as follows:

$$H'_i = \frac{H_i - \min(H)}{\max(H) - \min(H)} \quad (12)$$

According to Eq. 11, if node i has a high harmonic centrality index, it means that this node establishes contact with other nodes through fewer hops. Therefore, this node is busy in load transmission. At the same time, degree is also a significant factor to identify high-load nodes. With the consideration of the aforementioned two factors, the node with a high harmonic centrality index and high degree should deal with high load. Next, we attempt to combine these two factors to form a new measurement as follows:

$$w_i = k_i^\alpha + H_i^\beta, \quad (13)$$

where α and β are two exponents, corresponding to degree and harmonic centrality, respectively.

Next, we investigate the load distribution in spatial networks. Figure 4 shows that this new measurement can help us easily identify the high-load nodes in the spatial networks. If the node has a high value of w , this node needs to process more loads.

In order to enhance the transmission efficiency of spatial networks, we redistribute the load from the nodes with high w to these with low. Since w depends on the harmonic centrality and degree of the node, this strategy is named as the *HD* routing strategy, which is defined as follows:

$$G_w(P(s \rightarrow t)) = \min \sum_{m=0}^n w_{x(m)}. \quad (14)$$

That is, the *HD* routing strategy attempts to seek a path with the minimal sum of the w value along the path.

5 Simulation results

To verify the effectiveness of *HD* routing strategies, we adopt R_c to measure the traffic capacity of spatial networks. If the network carries a large R_c , the congestion hardly occurs. All simulations are carried out on heterogeneous and homogeneous spatial networks, corresponding to the *LAEE* model and *IRGG* model, respectively, mentioned previously. Without the loss of generality, we set $C = 1$ in the following simulations.

The *HD* routing strategy is based on the harmonic centrality and degree, and we can use α and β to adjust their weights, respectively. Figure 5 shows the variation of R_c under different α and β . In the *LAEE* network, the peak value of R_c is observed at $\alpha = 1$ and $\beta = 0.6$; in the *IRGG* network, the maximum value is observed at $\alpha = 1.4$ and $\beta = 0.5$.

In order to further verify the efficiency of our routing strategy, we observe the change of R_c when adjusting the network size and average degree. In addition, we also compare with *SP* and *DL* routing strategies. Figure 6 shows the results of R_c in different scale networks. When N increases, R_c also increases. The *HD* strategy is always better than the *SP* and *DL* strategies in the *LAEE* and *IRGG* models.

Figure 7 shows the R_c increases almost linearly with the average degree $\langle k \rangle$. With the growth of the $\langle k \rangle$, the network connections become denser and the load has more path options during transmission. Therefore, the load distribution becomes more even, which also directly leads to the increase of R_c . R_c of the *HD* routing strategy is higher than that in the *SP* and *DL* routing strategies. For the aforementioned results, we observe that spatial networks can carry the highest network throughput under our *HD* routing strategy in all cases.

The number of hops is also an important factor to evaluate the performance of the routing strategy. As we all know, the aim of a good routing strategy is not only to enable the network to carry more load but also to allow loads to be transmitted from its source to its destination quickly. Therefore, we expect a smaller number of hops under an efficient routing strategy. Figure 8 shows the relationship between node size N and average number of hops under different routing strategies. As the name suggests, the *SP* routing strategy always has the minimum number of hops. In the *DL* strategy, the load tends to be transferred around the edge of the region. Therefore, it needs the highest number of hops.

Similarly, Figure 9 shows the effect of $\langle k \rangle$ on the average number of hops under different routing strategies. No matter how the average degree $\langle k \rangle$ changes, the *SP* routing strategy always has the smallest average number of hops, and the average number of hops under the *DL* routing strategy is always higher under than the *HD* strategy.

In order to better understand the performance of these routing strategies, we observe the average load per node under these routing strategies in different regions. As shown in Figure 10, all nodes are placed in the 1×1 square region S , and the region center $(x^c, y^c) = (0.5, 0.5)$. S' is a square sub-area of region S , which has the same center as the region S and side length $2d$.

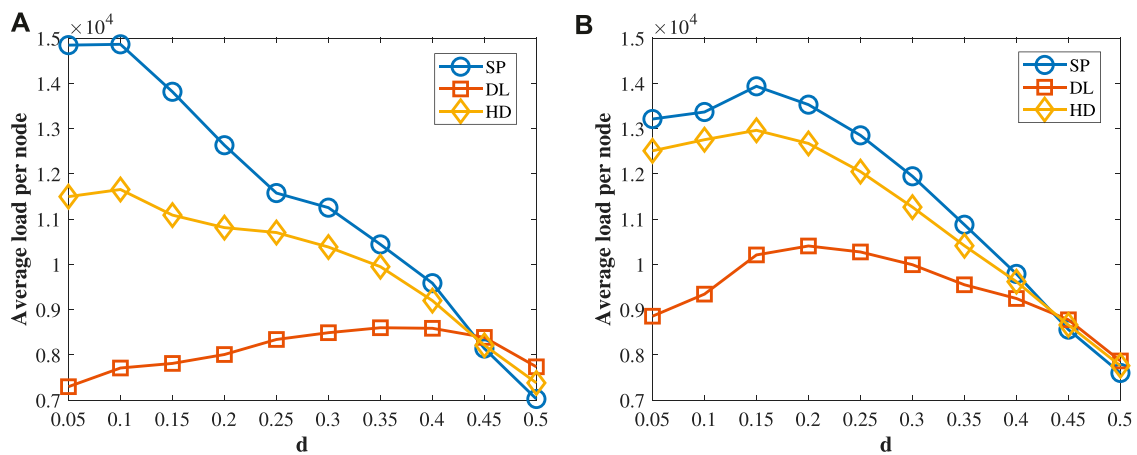


FIGURE 11

Average load per node vs. d under three routing strategies on the (A) LAEE network and (B) IRGG network. $N = 1000$ and $r = 0.12$. Center of a region is (0.5,0.5). Experimental results are the averages over 10 independent simulations.

Figure 11 shows the variation of the average load per node in the region S' . When d is small, nodes in region S' are close to the center. In this case, these nodes have high average load under the SP routing strategy. This is because the SP routing strategy finds the shortest path, which often goes through the central region. The high load leads to congestion at these nodes in the central region easily. That is why R_c is low under the SP routing strategy. On the contrary, when d is small, the average load is the lowest under the DL routing strategy. However, when d is large, the nodes need to deal with more loads under the DL routing strategy. Evidently, the load is more inclined to be transmitted along the edge of the region S . This load distribution makes the DL routing strategy always has the largest number of hops, as shown in Figures 8, 9. A high-detour cost becomes the main factor restricting the network throughput in the DL routing strategy. Compared with these two routing strategies, the load distribution is more even under the HD routing strategy. That is the main reason why the HD routing strategy performs better in spatial networks than other two routing strategies.

6 Conclusion

In this paper, we design an efficient routing strategy for spatial networks. To alleviate congestion, we attempt to find high-load nodes in spatial networks. Our routing strategy redefines the high-load nodes by considering both the harmony centrality and degree to improve the throughput significantly. Moreover, it not only ensures that the network can carry more load but also ensures that the load can be quickly transmitted to the destination. Therefore, our strategy can be

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adopted in spatial networks to help alleviate the congestion and provide a new thought for the design of routing strategies.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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