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Enhancement of the two-photon blockade effect *via* Van der Waals interaction

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We theoretically investigate the influence of the Van der Waals interaction on the two-photon blockade phenomenon with the corresponding photon correlation functions $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$ in a two-atom cavity-QED system, where two three-level ladder-type atoms are coherently driven by a pumping field and a coupling field simultaneously. Choosing a specific frequency of the coupling field, we show the energy splitting phenomenon caused by electromagnetically induced transparency. Correspondingly, the two-photon blockade phenomenon can be achieved near the two-photon resonant frequency. Using the Van der Waals interaction between the Rydberg states of the two atoms, we also show that the two-photon blockade can be improved when two atoms radiate in-phase or out-of-phase. As a result, two photons leak from the cavity simultaneously, but the third photon is blockaded. These results presented in this study hold potential applications in manipulating photon states and generating nonclassical light sources.

KEYWORDS

quantum optics, photon blockade, cavity-QED system, Rydberg atoms, Van der Waals interaction

1 Introduction

Creation and manipulation of single photons are pivotal requirements in quantum science and technology. Photon blockade (PB), proposed by Imamoḡlu et al. [1], is one of the effective methods for realizing single-photon sources. In the process of single PB, the absorption of the first photon blocks the absorption of the next photon until the previous photon is released from the system. As a consequence, the photons are emitted from the system one by one and antibunched photons with sub-Poissonian distribution can be observed [2].

To date, this single PB has been experimentally implemented in various quantum systems, such as the atom-cavity quantum electrodynamics (QED) system [3–6], semiconductor cavity-QED system [7], photonic crystal system [8], circuit cavity-QED system [9], and so on. Recently, the realization of two-photon blockade (two-PB) via the quantum interference effect

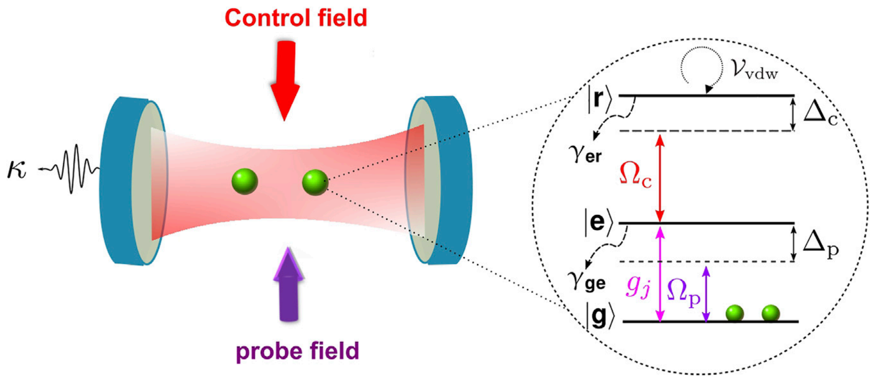

FIGURE 1
(Color online) Schematic diagram of a single-mode cavity coupled with two three-level ladder-type atoms. Levels $|g\rangle$ and $|e\rangle$ are coupled to the cavity field with the coupling strength g_j. Moreover, the control field of frequency ω_c with Rabi frequency Ω_C couples the transition $|e\rangle$ to $|r\rangle$ and a probe field with frequency ω_p and strength Ω_p coupled the state $|g\rangle$ and $|e\rangle$, respectively. γ_{er} and γ_{ge} are the spontaneous emission rates from states $|r\rangle$ and $|e\rangle$, respectively, and κ is the cavity decay. The two-atom system with Van der Waals potential between the Rydberg states creates an interaction $\mathcal{V}_{\rm vdw}$ dependent on the atom separation.

has been proposed [10, 11] and demonstrated in quantum dot cavity-QED systems [12]. Moreover, the two-PB phenomenon also increases great research interest, where two photons go out of the cavity together, but the third photon is blockaded [13–15].

Two-PB can occur if the driving field frequency is equal to the two-photon dressed-state resonance frequency while far from one-photon resonance and three-photon absorption [16]. Thus, the observation of two-PB requires strong nonlinearities with respect to the decay rate of the system, e.g., Kerr nonlinearity magnitudes [1] or atom-cavity coupling strength [3] to be at least larger than the cavity mode linewidth. In the strong coupling regime, two-PB has been predicted in a few systems, such as a cavity with a Kerr-type medium coherently driven by a laser field [17] or a sequence of Gaussian pulses [18], and coupled nanomechanical resonator systems [19]. Recently, the two-PB phenomenon has been realized experimentally in an atom-driven cavity-QED system [20].

In this study, we study the two-PB behavior in a single-mode cavity-QED system with two three-level ladder-type atoms coupled via the Rydberg interaction. Unlike using an effective Kerr-type interaction to achieve two-PB in Ref. [19], we show that the Van der Waals interaction (Vdw) between two Rydberg states will significantly enhance the optical nonlinearity [21] in the cavity-QED system, leading to a significant improvement of the two-PB. Moreover, the mean photon number can also be enhanced in the presence of the Vdw interaction.

2 Physical model

As shown in Figure 1, two ladder-type three-level atoms are trapped inside a single model optical cavity with wavelength $\lambda_{\rm cav}$, and the distance between the two atoms is ΔZ. The corresponding energy levels and energies of each atom are labeled by $|i\rangle$ and $\hbar\omega_i$ ($i = g, e, r$), respectively. A weak probe field with the Rabi frequency $\Omega_{\rm p}$ and angular frequency ω_p drives the $|g\rangle \leftrightarrow |e\rangle$ transition, and a strong coupling field with the Rabi frequency Ω_c and angular frequency ω_c drives the $|e\rangle \leftrightarrow |r\rangle$ transition. In our system, the atomic state $|r\rangle$ is chosen as the Rydberg state so that the decay rate of state $|r\rangle$ is much smaller than that of the state $|e\rangle$. Assuming the cavity-mode frequency $\omega_{\rm cav}$ is equal to ω_e for mathematical simplicity and using the electric dipole and rotating-wave approximations, the Hamiltonian of the system can be expressed as (setting $\hbar = 1$)

$$H = \Delta_{\rm p} a^\dagger a + \sum_{j=1}^{2}\left[\Delta_{\rm p}\sigma_{ee}^{(j)} + \Delta_r \sigma_{rr}^{(j)}\right] + \sum_{j=1}^{2}\left[g_j a^\dagger \sigma_{ge}^{(j)} + \Omega_c \sigma_{er}^{(j)} + \Omega_{\rm p}\sigma_{me}^{(j)} + {\rm H.c}\right] + \mathcal{V}_{\rm vdw}\sigma_{rr}^{(1)}\sigma_{rr}^{(2)}, \quad (1)$$

where $\Delta_{\rm p} = \omega_{\rm cav} - \omega_p$ and $\Delta_r = \Delta_{\rm p} + \Delta_c$ with $\Delta_c = \omega_r - \omega_e - \omega_c$. Here, a ($a^\dagger$) is the annihilation (creation) operator of the cavity mode, while $\sigma_{mn}^{(j)} = |m^{(j)}\rangle\langle n^{(j)}|$ ($m, n = g, e, r$ and $j = 1$–2) with z_j is the atomic operator for the j-th atom. The position-dependent atom cavity coupling strength for the j-th atom is given by $g_j = g_0$ cos $(2\pi z_j)/\lambda_{\rm cav}$ being the position of the j-th atom [22], and $g_0 = \mu_{eg}\sqrt{\omega_{\rm cav}/2\hbar\varepsilon_0 V}$ being the amplitude of the coupling strength. Assuming that one atom is located at the antinode of the cavity mode, then the coupling strengths can be expressed as $g_1 = g_0$ and $g_2 = g_0 \cos(\Phi_z)$ with $\Phi_z = 2\pi\Delta Z/\lambda_{\rm cav}$ and $\Delta Z = z_2 - z_1$. The last term of Eq.1 describes the Vdw interaction between of two atoms with interaction strength $\mathcal{V}_{\rm vdw}$, which results in an energy shift of state $|rr\rangle$ and prevents the simultaneous excitation of two atoms.

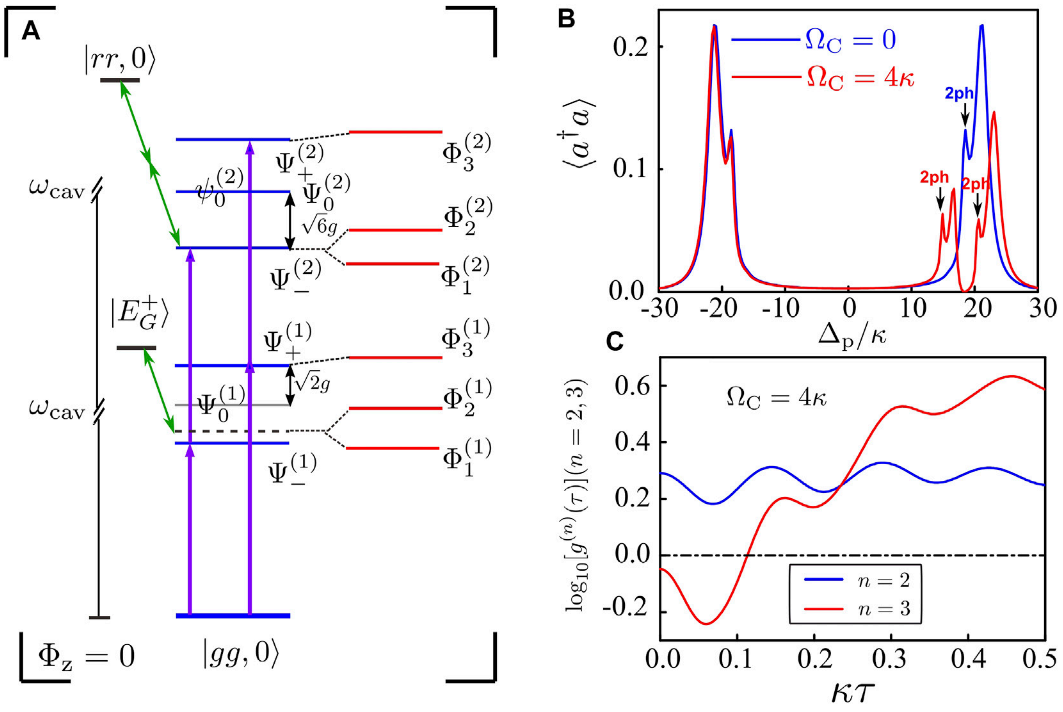

FIGURE 2
(Color online) **(A)** Schematic energy-level structure diagram explaining the energy splitting induced by Ω_C in the case of $\Phi_z = 0$. **(B)** Mean photon number $\langle a^\dagger a\rangle$ as a function of normalized detuning Δ_p/κ for the $\Omega_C = 0$ (blue solid curve) and $\Omega_C = 4\kappa$ (pink solid curve). **(C)** Time-dependent field correlation functions $\log_{10}[g^{(2)}(\tau)]$ and $\log_{10}[g^{(3)}(\tau)]$ versus the normalized delay time $\kappa\tau$ with $\Delta_p = 21.3\kappa$. The strength of the probe field is $\Omega_p = 0.8\kappa$ in panel **(B)** and $\Omega_p = 1.5\kappa$ in panel **(C)**. Other system parameters are chosen as $g_0 = 15\kappa$, $\gamma_{er} = 1\kappa$, and $\gamma_{ge} = \gamma_{er}/200$.

The master equation for the density matrix operator of the system is then given by

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + \frac{\kappa}{2}\mathcal{L}[a] + \sum_{j=1}^{2}\left[\frac{\gamma_{ge}}{2}\mathcal{L}\left(\sigma_{ge}^{(j)}\right) + \frac{\gamma_{er}}{2}\mathcal{L}\left(\sigma_{er}^{(j)}\right)\right], \tag{2}$$

where $\mathcal{L}(O) = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$ is the Lindblad superoperator with respect to the operate O. Here, κ and γ_{ij} are the cavity decay rate and the atomic decay rate, respectively, from state $|j\rangle$ to state $|i\rangle$.

Numerically solving Eq. 2, one can evaluate the equal-time second-order correlation function $g^{(2)}(0) = \langle a^\dagger a^\dagger aa\rangle/\langle a^\dagger a\rangle^2$ and the equal-time third-order correlation function $g^{(3)}(0) = \langle a^\dagger a^\dagger a^\dagger aaa\rangle/\langle a^\dagger a\rangle^3$, which described the statistical properties of the photons leaking from the cavity. Generally, $g^{(2)}(0) < 1$ represents the photon antibunching phenomenon, implying that photons leave the cavity one by one with sub-Poisson statistics. In this work, we focus on another case, where two photons leave the cavity together but the third photon is blockaded. Likewise, this phenomenon can also be described by exploring the photon correlation functions, which requires $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$.

3 Two-PB for in phase radiations

First, we turn off the coupling field and consider the case of in-phase radiation, i.e., $\Phi_Z = 0$, resulting in $g_1 = g_2 = g_0$. In this case, the dressed states in absence of the coupling field (equivalent Jaynes–Cummings model) are given in Figure 2A(see the blue states). In panel (b), there exist four peaks in the cavity excitation spectrum, where two large side peaks correspond to the single photon excitations with $\Delta_p = \pm\sqrt{2}g_0$, while two small peaks correspond to the two-photon excitations with $\Delta_p = \pm\sqrt{6}g_0/2$[see Figure 2B, blue arrow]. As shown in Ref. [13], it is difficult to observe the two-PB phenomenon by increasing the probe field intensity in this case, because the increasing of Ω_P will result in the broadening of the dressed states, which compensates the inharmonic energy splitting and prohibits the observation of the two-PB [20].

To overcome this energy broadening, we turn on the coupling field but neglect the Vdw interaction so that the physical mechanism can be clearly demonstrated. Choosing $\Delta_c = -\sqrt{6}g_0/2$, the coupling field resonantly couples states $|rr, 0\rangle$ and $\Psi_-^{(2)}$ so that the state $\Psi_-^{(2)}$ is split into a doublet [see Figure 2A, the red states], which is well known as the

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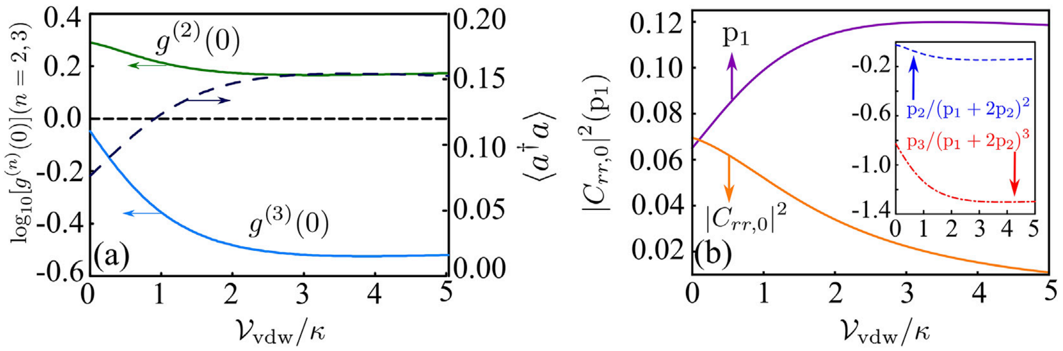

FIGURE 3
(Color online) **(A)** $\log_{10}$ [$\langle a^\dagger a\rangle$] (blue-black dashed curve), $\log_{10}$ [$g^{(2)}(0)$] (green solid curve), and $\log_{10}$ [$g^{(3)}(0)$] (wathet blue solid curve) are plotted as functions of the Vdw interaction strength $\mathcal{V}_{vdw}/\kappa$ for $\Phi_z = 0$. **(B)** Probability of the product state $|rr, 0\rangle$ and one-photon distributions p_1 versus the $\mathcal{V}_{vdw}/\kappa$. The inset is the logarithmic plot (of base 10) of the proportional relationship which is described in Eq. 3. System parameters used here are the same as in Figure 2C.

electromagnetically induced transparency (EIT) effect. Correspondingly, the right peaks in the cavity excitation spectrum are split into two doublets, where two large peaks originate from the one-photon excitations, but the other two small peaks result from the two-photon excitations [see Figure 2B, red curves].

In Figure 2C, we plot the time-dependent second-order correlation $g^{(2)}(\tau) = \langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle/\langle a^\dagger(t)a(t)\rangle_{ss}^2$ and the third-order correlation $g^{(3)}(\tau) = \langle a^\dagger(t)a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)a(t)\rangle/\langle a^\dagger(t)a(t)\rangle_{ss}^3$ at two-photon resonance $|gg,0\rangle\to\Phi_1^{(2)}$ frequency $\Delta_p = 21.3\kappa$ with $\Omega_p = 1.5\kappa$. It is clear that the two-PB with $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$ can be achieved under this condition, where two bunching photons leak from the cavity with the super Poissonian statistical distribution, but the third photon is blockaded due to the energy splitting induced by the coupling field.

Now, let us consider the influence of the Vdw interaction on the two-PB. As shown in Figure 3A, the $g^{(2)}(0)$ is always larger than unity, but the $g^{(3)}(0)$ decreases as the Vdw interaction [23–26] strength increases. Thus, the two-PB can be improved by increasing the Vdw interaction strength. At the same time, the mean photon number is also slightly increased (see blue dashed curve). This phenomenon can be explained by exploring the analytical expression of the photon correlation functions. It should be noted that the following analytical expressions are valid only for weak driving fields. Truncating the Hilbert space by the three-photon space, $g^{(2)}(0)$ and $g^{(3)}(0)$ can be expressed analytically as functions of the photon number distributions p_n under the weak probe field regime,

$$g^{(2)}(0) = \frac{\sum_n (n-1)p_n}{[\sum_n np_n]^2} \approx \frac{2p_2}{(p_1+2p_2)^2}, \tag{3a}$$

$$g^{(3)}(0) = \frac{\sum_n n(n-1)(n-2)p_n}{[\sum_n np_n]^3} \approx \frac{6p_3}{(p_1+2p_2)^3}, \tag{3b}$$

where $p_1 \approx |C_{gg,1}|^2 + |C_{+,1}|^2 + |C_{+',1}|^2$ and $p_2 \approx 2|C_{gg,2}|^2$, and $p_3 \approx 3|C_{gg,3}|^2$. Here, $|C_{\alpha,n}|^2$ is the probability of the product state $|\alpha, n\rangle$, involving $|+,1\rangle = 1/\sqrt{2}(|eg,1\rangle + |ge,1\rangle)$ $[|+',1\rangle = 1/\sqrt{2}(|rg,1\rangle + |gr,1\rangle)]$. Due to the dressing of atoms by the Vdw interaction, the bare state $|rr, 0\rangle$ is shifted so that the probability $|C_{rr,0}|^2$ drops quickly as shown in Figure 3B. On the other hand, the probability of one photon increases quickly since the Vdw interaction does not affect the states in one-photon space, resulting in the drops in the values of $p_2/[p_1+2p_2]^2$ and $p_3/[p_1+2p_2]^3$[the inset figure of Figure 3B]. Moreover, provided that only the lowest several photon states can be excited, the mean photon number can be approximately expressed as $\langle a^\dagger a\rangle \approx p_1 + 2p_2 \approx p_1$. Therefore, the increased Vdw interaction strength increases the mean photon number in the cavity.

4 Two-PB for out-of-phase radiations

Now, we study the out-of-phase configuration $\Phi_z = \pi$, yielding $g_1 = -g_2 = g_0$. In the absence of the control field, one can easily obtain the dressed states of the system as shown in Figure 4A (see the blue states). Detailed expressions of the eigenvalues and eigenstates can be found in Ref. [13]. Contrary to the in-phase case, the $|gg,0\rangle\to\Psi_\pm^{(1)}$ transitions are forbidden since the driving field only couples the symmetric state (i.e., $|+,n\rangle$ state). Thus, one can observe three peaks in the cavity excitation spectrum [see Figure 4A, blue curve]. Here, the large central peak corresponds to the multi-photon excitation process, i.e., $|gg,0\rangle\to\Psi_0^{(1)}\to\Psi_0^{(2)}\cdots\to\Psi_0^{(n)}$ transition. The other two small side peaks correspond to the two-photon excitation processes, i.e., $|gg,0\rangle\to\Psi_\pm^{(2)}$ transitions.

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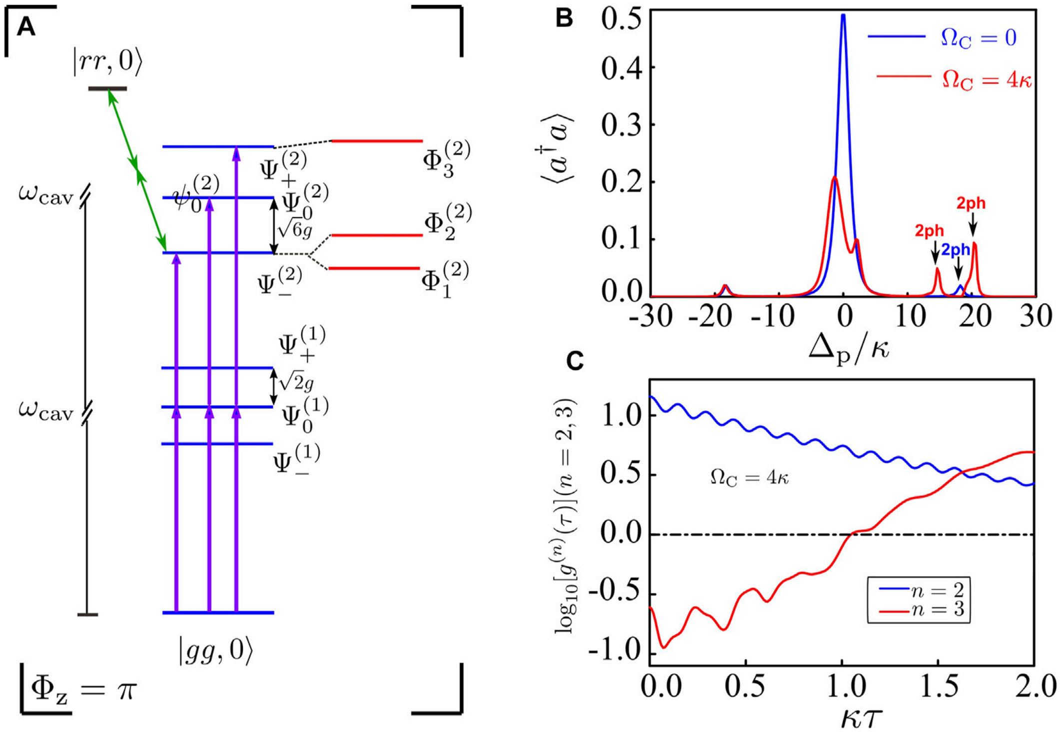

FIGURE 4
(Color online) **(A)** Schematic energy-level structure illustrating the energy splitting induced by Ω_C in the case of $\Phi_z = \pi$. **(B)** Numerical results for $\langle a^{\dagger}a\rangle$ as functions of normalized detuning Δ_p/κ with $\Omega_C = 0$ (blue solid line) and $\Omega_C = 4\kappa$ (red solid line). **(C)** Time evolution of $\log_{10}[g^{(2)}(\tau)]$ (reseda solid curve) and $\log_{10}[g^{(3)}(\tau)]$ (wathet blue solid curve) with $\Delta_p = 21.3\kappa$. In (b) $\Omega_P = 1.5\kappa$ and in (c) $\Omega_P = 2.0\kappa$, other system parameters are the same as in Figure 2.

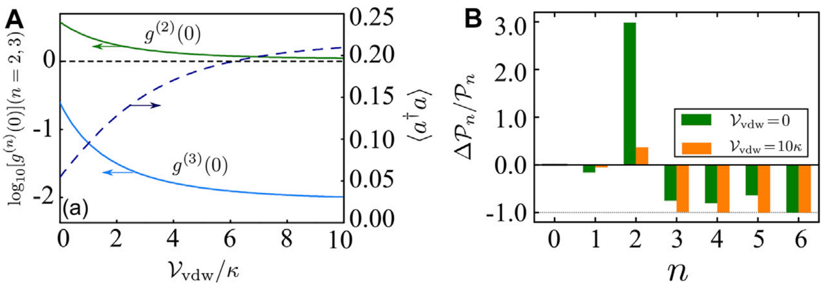

FIGURE 5
(Color online) **(A)** $\log_{10}[\langle a^{\dagger}a\rangle]$ (blue-black dashed curve), $\log_{10}[g^{(2)}(0)]$ (green solid curve), and $\log_{10}[g^{(3)}(0)]$ (wathet blue solid curve) for different values of $\mathcal{V}_{vdw}/\kappa$ in the case of $\Phi_z = \pi$ with $\Omega_C = 8\kappa$. **(B)** Relative deviations of photon number distribution with respect to the Poisson distribution. Other system parameters used here are the same as in Figure 4C.

In the presence of the control field but without the Vdw interaction, the energy levels (blue lines in Figure 4A) will experience shifting and splitting due to the EIT effect. To improve the two-PB blockade, one requires that the state in two-photon space is resonantly coupled by the pump field via the two-photon process, but other states in N-photon ($N > 3$) states are far off-resonance. Thus, we choose the control field detuning $\Delta_c = -\sqrt{6}g_0/2$ so that the control field resonantly couples state $\Psi_{-}^{(2)}$ and state $|rr, 0\rangle$. Then, the state $\Psi_{-}^{(2)}$ is split into a doublet as shown in Figure 4A, and the states in high-order photon space are shifted. In Figure 4B, we demonstrate this energy splitting by plotting the mean photon number against the pump field detuning. Choosing the probe field frequency to $\Delta_p = 21.3\kappa$, two-PB can be achieved and the time-dependent correlation functions $g^{(2)}(0) \approx 15.3$ and $g^{(3)}(0) \approx 0.32$ are shown in Figure 4C. Apparently, the two-PB phenomenon can be improved in the case of out-of-phase radiations.

Finally, we investigate the influence of the Vdw interaction on the two-PB when two atoms radiate out of phase. In Figure 5A, we show the dependence of the correlation functions $\log_{10}[g^{(2)}(0)]$, $\log_{10}[g^{(3)}(0)]$, and the mean photon number $\langle a^\dagger a\rangle$ on the Vdw interaction strength. Here, we choose the system parameters as $g = 15\kappa$, $\gamma_{ge} = \kappa$, $\gamma_{er} = \gamma_{ge}/200$, $\Omega_p = 2.0\kappa$, $\Delta_p = 21.3\kappa$, $\Delta_c = -\sqrt{6}g_0/2$, and $\Omega_C = 8\kappa$. It is clear that the values of $g^{(3)}(0)$ decrease quickly with the increase of the Vdw interaction strength. Taking $\mathcal{V}_{vdw} = 10\kappa$, we find $g^{(3)}(0) \approx 0.01$, which is 25 times smaller than the value at $\mathcal{V}_{vdw} = 0\kappa$. At the same time, the value of $g^{(2)}(0)$ is always larger than unity, and the mean photon number increases slightly. In Figure 5B, we show the deviations of photon number distribution p_n with respect to the Poisson distribution $\mathcal{V}_{vdw}/\kappa$, which is defined as $(p_n - \mathcal{P}_n)/\mathcal{P}_n$. It is clear that multi-photon transitions for $\mathcal{V}_{vdw} = 10\kappa$ are suppressed compared with the case of $\mathcal{V}_{vdw} = 0$. Thus, a significantly improved two-PB can be accomplished in the presence of the Vdw interaction.

5 Conclusion

In summary, we have proposed a model to realize strong two-PB ($g^{(2)}(0) > 1$ and $g^{(3)}(0) < 0.01$) by trapping two Rydberg atoms with EIT configuration inside an optical cavity. We show that the external control field can be used to effectively control the collective states of two atoms and the cavity mode, yielding two-PB for in-phase and out-of-phase radiations, respectively. Moreover, in the presence of the Vdw interaction, the mean photon number can be increased, and the two-PB phenomenon can be strongly improved in the case of out-of-phase radiations. With the development of atom trapping and cooling techniques, our scheme can be implemented not only in classical cavity-QED systems, as in experiments of Meschede [27] and Rempe groups [28], but also in a 1D topological photonics system involving an asymmetric edge mode [29]. Thus, our scheme has potential applications in manipulating photon states and generating nonclassical light, such as the two-photon source.

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Author contributions

YY and CZ took the lead on the research work by running the simulations, performing most of the analysis, and producing all the figures. KH contributed to the code development. KH, ZZ, CZ, and YY wrote substantial parts of the manuscript. All authors contributed equally to the discussions, read the manuscript, and provided critical feedback.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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