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# Numerical analysis of hydrothermal flow and heat transfer inside a cavity formed due to faults causing earthquakes

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This article studies fluid dynamics and convection of the geothermal system. Earthquakes cause faults. Fault zones come up with the pathways for fluid convection. These paths have different characteristics and space distribution, causing the challenge to investigate the geothermal system. The cavity considered in the study is normally found in energy reservoirs. An unsteady, incompressible, and laminar flow along with convection is studied. The finite element method (FEM) is operated to study the flow and heat transfer governed by continuity equations, Navier-Stokes equations, and temperature equations. These equations are tackled with the finite element method. The streamlines and isothermal contours for the problem under discussion are displayed in the Results section. It is observed that the Nusselt number and velocity of the fluid increase with the increased Grashof number.

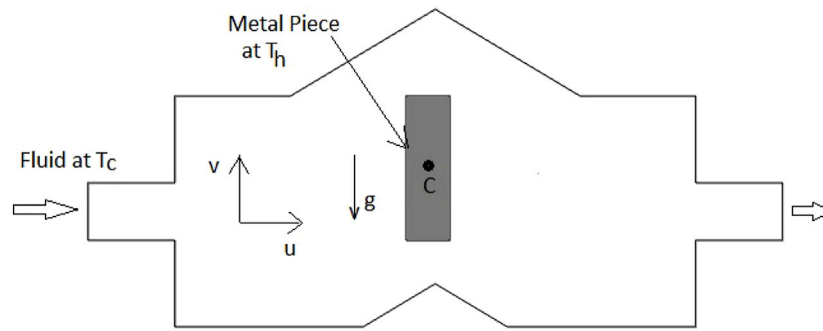
## KEYWORDS

hydrothermal flow, heat transfer, finite element method, cavity access, Nusselt number (Nu)

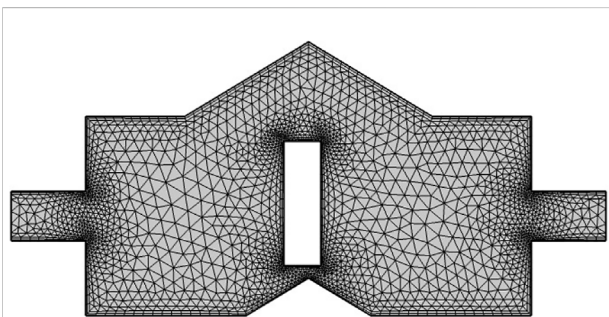
## Introduction

According to geophysics, earthquakes cause faults. Faults can be considered as planar or curved fractures in rock due to which the rock displaces from its place to the other side of the fracture. Their length starts from centimeters and varies up to kilometers. They move either slowly or rapidly. Faults moving rapidly release more energy. The conglomeration of parallel faults forms a fault zone. Faults can be classified into three types: (I) tensional faults, in which the block of rock moves vertically upwards to the fault; (II) compressional faults, in which the block of rock moves vertically downwards to the fault; and (III) strike-slip faults, in which the block of rock moves along the fault horizontally [1–3].

The energy accumulation of radioactive material disintegration results in the expansion of material in the earth's interior with the increase of temperature. The



**FIGURE 1**  
Geometry of cavity formed by fault.



**FIGURE 2**  
Meshing of the cavity.

expansion may produce tensional fault zones. Faults without magma are filled with mantle gas and supercritical fluid. Fluids in their supercritical states have very high molar volume, and very low density and conductivity. These fluids exhibit strong fluidity and low viscosity. According to hydrothermal ore-forming theory, hydrothermal fluid in the middle-lower crust can move to the upper crust where it changes to its normal state. Also  $\text{SiO}_2$ ,  $\text{CaCO}_3$ , some metals, and non-metals add to this hydrothermal fluid. This movement of fluid creates a cavity in the middle and lower crust. This cavity cannot be filled because sedimentation cannot occur in a supercritical state [4, 5]. The analysis of heat transfer and flow properties of hydrothermal fluid through such cavities is very influential and has several applications. The cavity found in the energy reservoirs is considered in this study.

Energy that transfers from system to system as a consequence of temperature difference is called heat. Heat transfer is a process to determine such energy transfers. It is very significant for physicists, mathematicians, engineers, and researchers. It affects the fluid flow and due to its remarkable applications in different engineering fields, biological processes, industrial

mechanisms, and geothermal studies, its study is of great significance [6–17].

Conduction, convection, and radiation are three different ways in which heat transfer is classified. Heat transfer through molecular collision is called conduction, heat transportation between a fluid adjacent to a surface and that surface is called convection, and heat transfer through electromagnetic waves is called radiation. Convection is either natural convection or forced convection. Forced convection is driven by external body forces, and natural convection is simply a result of buoyancy force [18–30].

Fluid flow is an analysis to study the behavior of fluids and their interaction with their surroundings. The flow of a fluid may be turbulent or laminar. In laminar flow, parallel layers form during the flow such that these layers do not disturb each other, while flows are more chaotic in the case of turbulent flows. On the other hand, flow is steady if fluid properties do not depend on time; otherwise they are unsteady. Fluid flow is visualized using streamlines, streaklines, and pathlines. Most problems in fluid dynamics are too complex to solve analytically. To solve such complex problems, numerical methods are implemented with the help of computer simulation. This field of study is called computational fluid dynamics (CFD). In this paper, the subsurface hydrothermal fluid flow has been studied using computational techniques [31–43].

A cavity having an inlet and an outlet is considered to be in the fault zone. A viscous and incompressible fluid enters through the inlet on one side and leaves the cavity on the other side. A metal piece in a rectangular shape is considered inside the cavity. The article studies unsteady and laminar fluid flow through the cavity and heat transfer during natural convection between the metal piece and the fluid.

The prevailing equations are worked out followed by Boussinesq assumption. Numerical results are acquired by the use of the Galerkin finite element method (FEM) [44–54]. The influences of changing the involved parameters in the study on velocity and isothermal contours are presented. The physical

TABLE 1 Grid independent test.

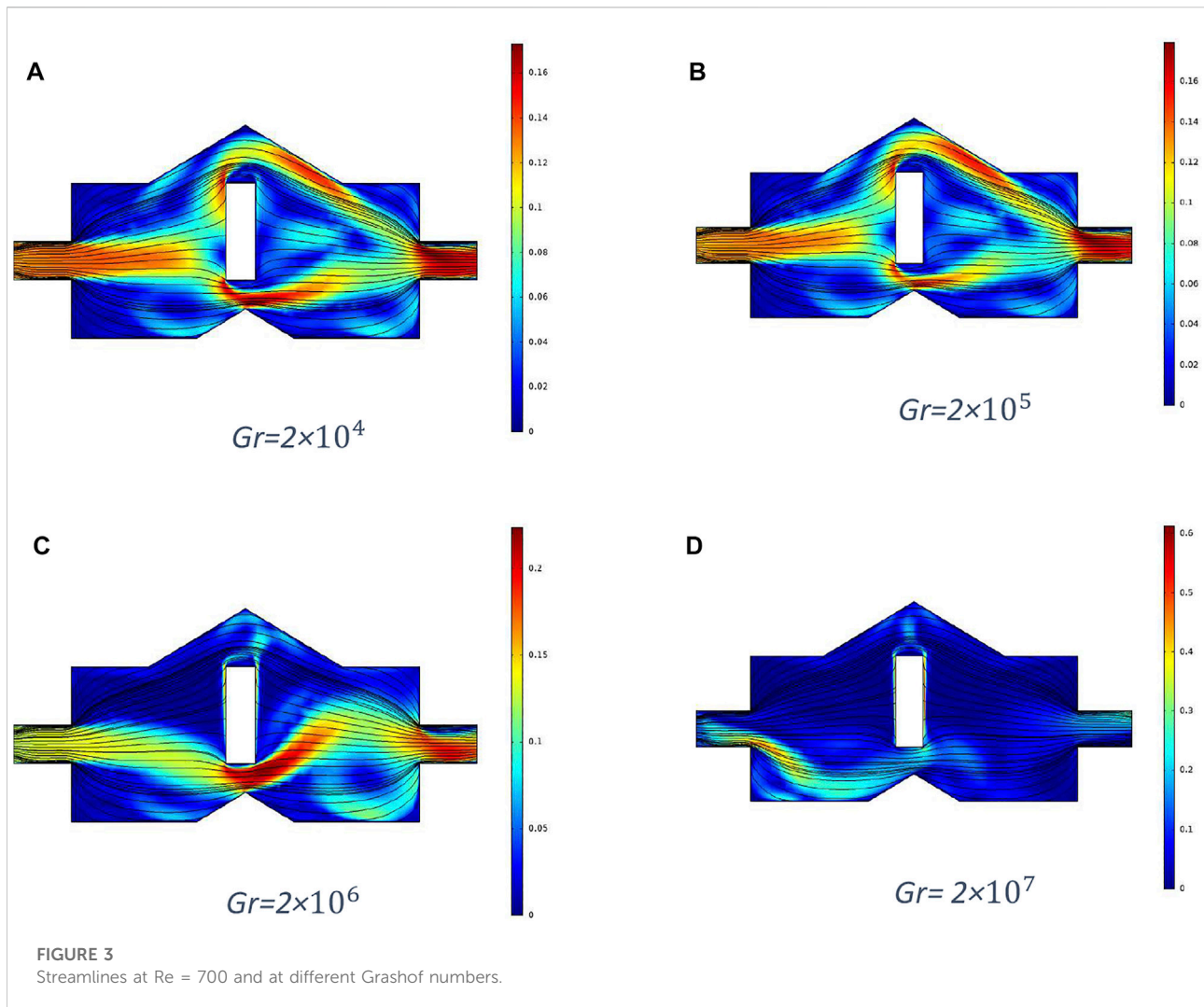
Number of triangular elements	Average Nusselt number (Nu)	Percentage error
1,236	2.8590	---
2,348	2.8639	0.17%
2,879	2.8655	0.05%
3,001	2.8661	0.02%
3,930	2.8664	0.01%

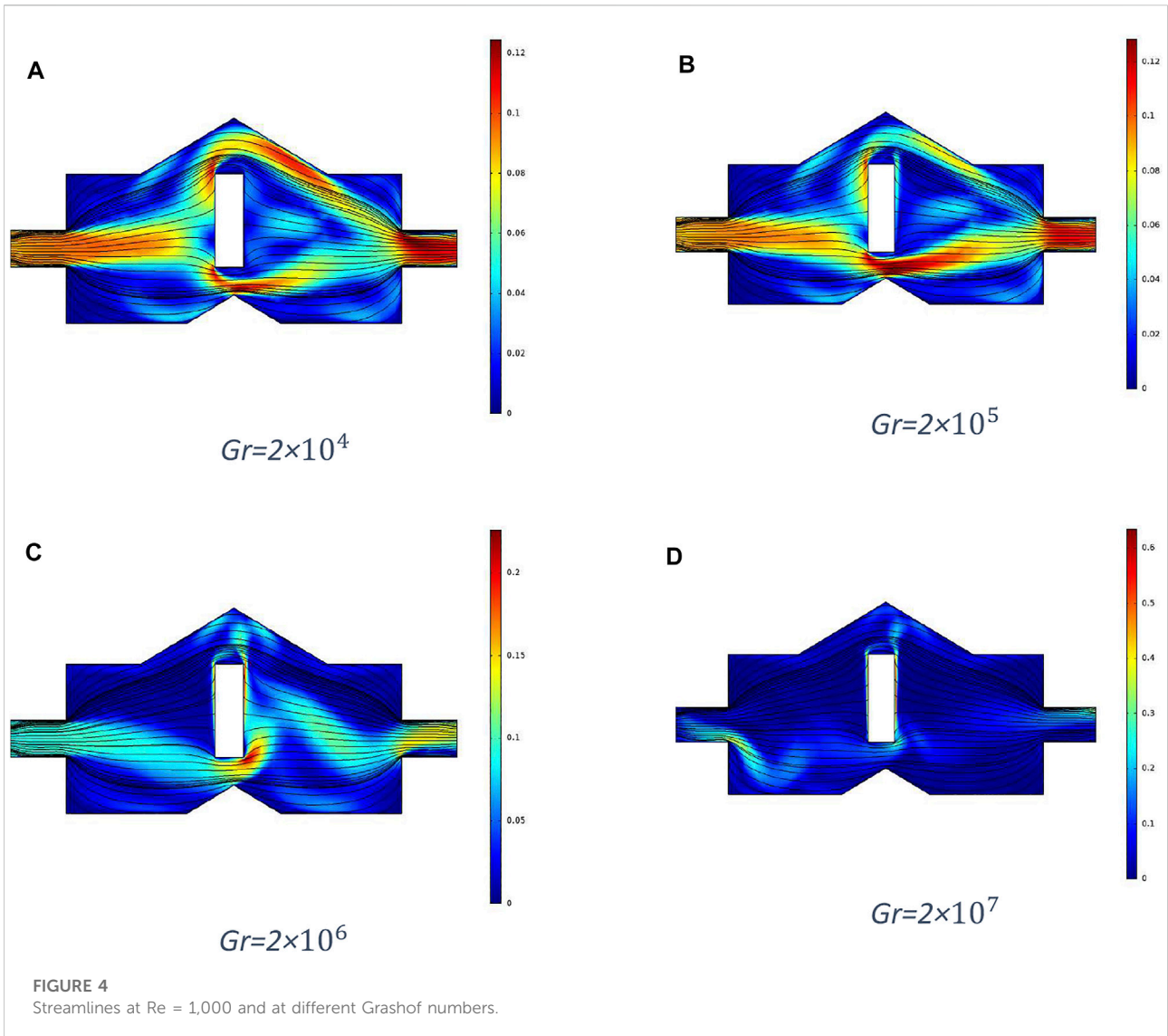
properties of heat transfer and flow are exhibited using isotherms, temperature distribution, and streamlines. Further, the average Nusselt number is evaluated which analyzes heat transfer rate from the rod.

### Mathematical modeling

A 2D unsteady, incompressible, laminar, and natural convective flow inside a cavity is carried out in the study. A metal piece is placed at the middle of the considered cavity. The walls of the cavity are thermally active and are at a higher temperature  $T_h$ , while cavity walls are thermally insulated. The fluid is considered to be at low temperature  $T_c$ . Figure 1 exhibits the illustrative diagram for the considered cavity. The center C of the metal piece is taken to be the origin of the coordinate system.

The fluid’s thermophysical characteristics are defined to be fixed. For natural convection, the density can be defined according to the Boussinesq approximation; this variation is expressed in the momentum equation (Eq. 4 below). The density can be related to temperature linearly by the relation defined in Eq. 1:





$$\rho = \rho_o [1 - \beta(T - T_o)], \tag{1}$$

In the energy equation, the radiation heat transfer and joule heating are disregarded. The governing equations [55, 56] depending on the above assumptions taken are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \tag{3}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c), \tag{4}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \tag{5}$$

Initial conditions for the under-discussion problem are as follows:

For  $t = 0$ :

$$u = v = 0, T = T_c, \text{ (for the fluid inside the cavity)}, \tag{6}$$

$$T = T_h, \text{ (for the walls of metal rod)}. \tag{7}$$

Boundary conditions are as follows:

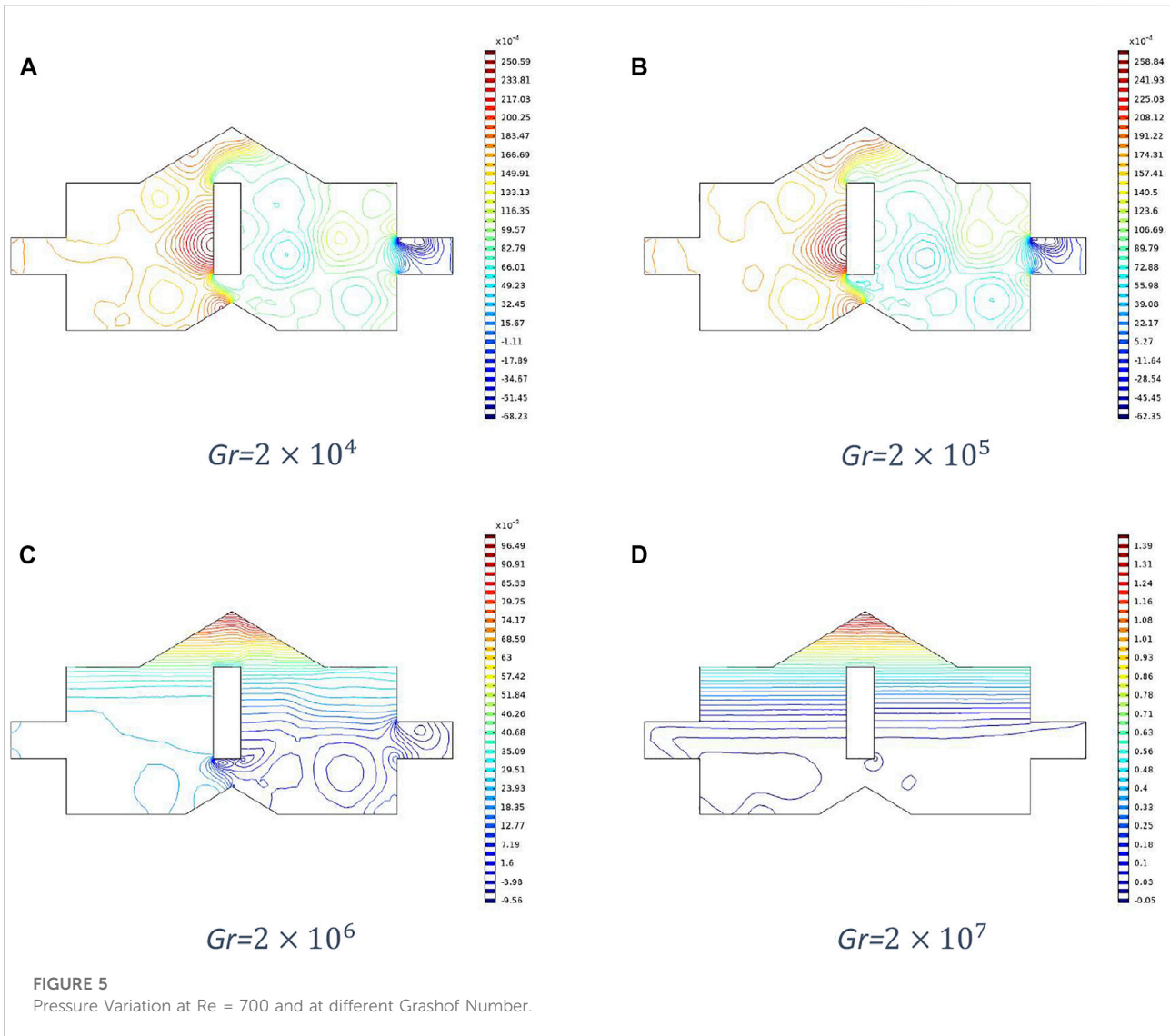
For  $t > 0$ :

$$u = U_0, v = 0 \text{ (At inlet)} \tag{8}$$

$$T = T_h \text{ (for the walls of metal rod)}, \tag{9}$$

$$\frac{\partial T}{\partial n} = 0 \text{ (for walls of the cavity)}, \tag{10}$$

where  $n$  is normal to the surface.



### Reduction to dimensionless form

The system of governing equations given by Eqs. 2–10 is reduced to dimensionless form by the use of dimensionless variables given as follows:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0},$$

$$\hat{t} = \frac{t\mu}{\rho L^2}, \quad P = \frac{p}{\rho U_0^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}.$$

After this parametrization, Eqs. 2–5 become the following:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{11}$$

$$\frac{\partial U}{\partial \hat{t}} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \tag{12}$$

$$\frac{\partial V}{\partial \hat{t}} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Gr}{Re^2} \theta, \tag{13}$$

$$\frac{\partial \theta}{\partial \hat{t}} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re \times Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right). \tag{14}$$

with initial conditions as the following:

For  $\hat{t} = 0$ :

$$U = V = 0, \quad \theta = 0, \quad (\text{for the fluid inside the cavity}), \tag{15}$$

$$\theta = 1, \quad (\text{for the walls of metal rod}), \tag{16}$$

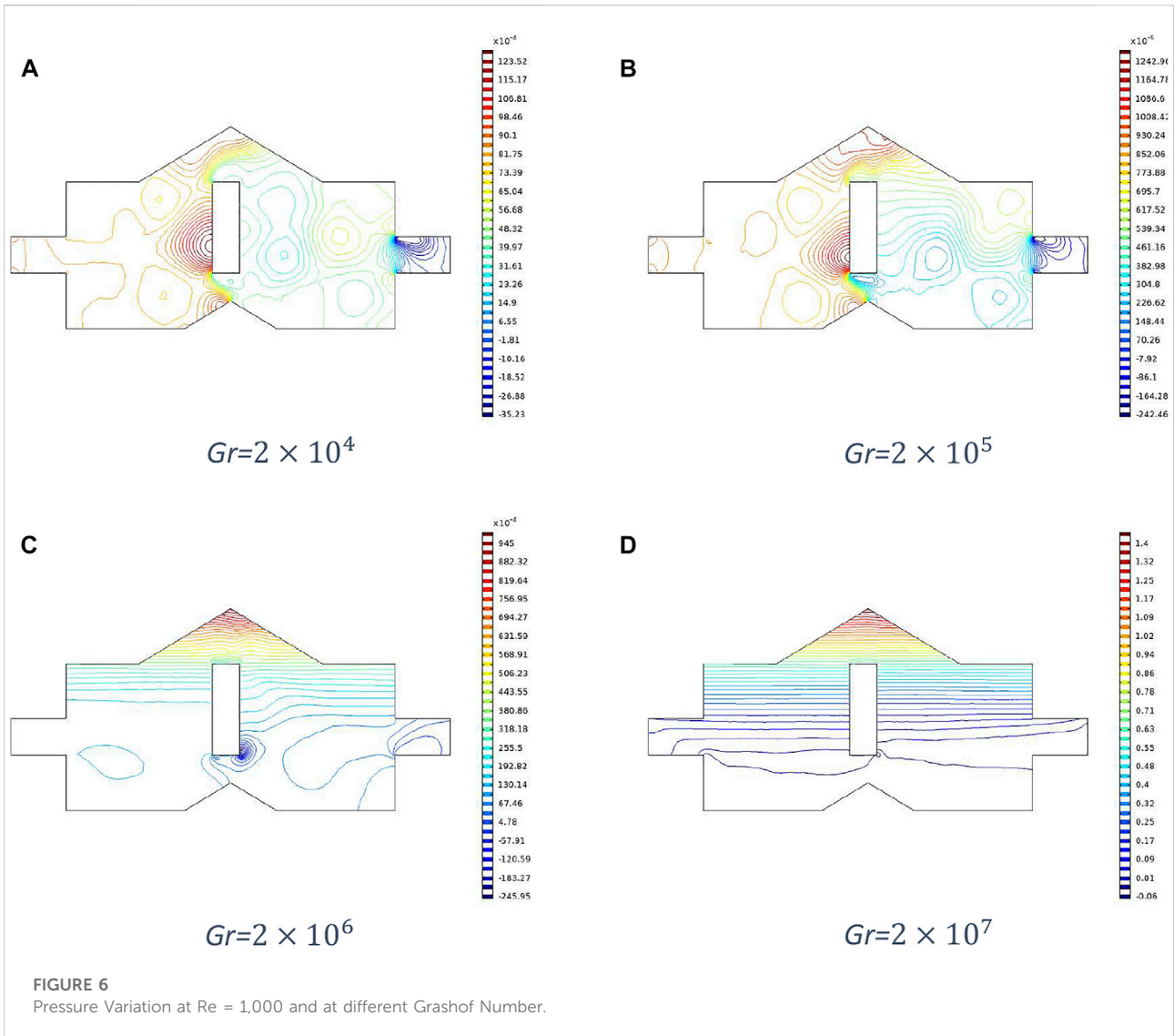
and boundary conditions as the following:

For  $\hat{t} > 0$ :

$$U = 1, \quad V = 0 \quad (\text{At inlet}), \tag{17}$$

$$\theta = 1 \quad (\text{for the walls of metal rod}) \tag{18}$$





$$\frac{\partial \theta}{\partial \hat{n}} = 0 \quad (\text{for walls of the cavity}), \quad (19)$$

where

$$Gr = \frac{g\beta L^3 \Delta T}{\nu^2}, \quad Re = \frac{U_0 L}{\nu}, \quad Pr = \frac{\mu c_p}{k}, \quad \nu = \frac{\mu}{\rho}, \quad \Delta T = T_h - T_c.$$

### Solution method

Since analytical methods to solve the system of equations given by Eqs. 11–19 fail for complicated cases, numerical techniques are used to obtain the solution. As discussed before, the Galerkin finite element method is applied to solve the system of Eqs. 11–19. The pressure term is penalized by the

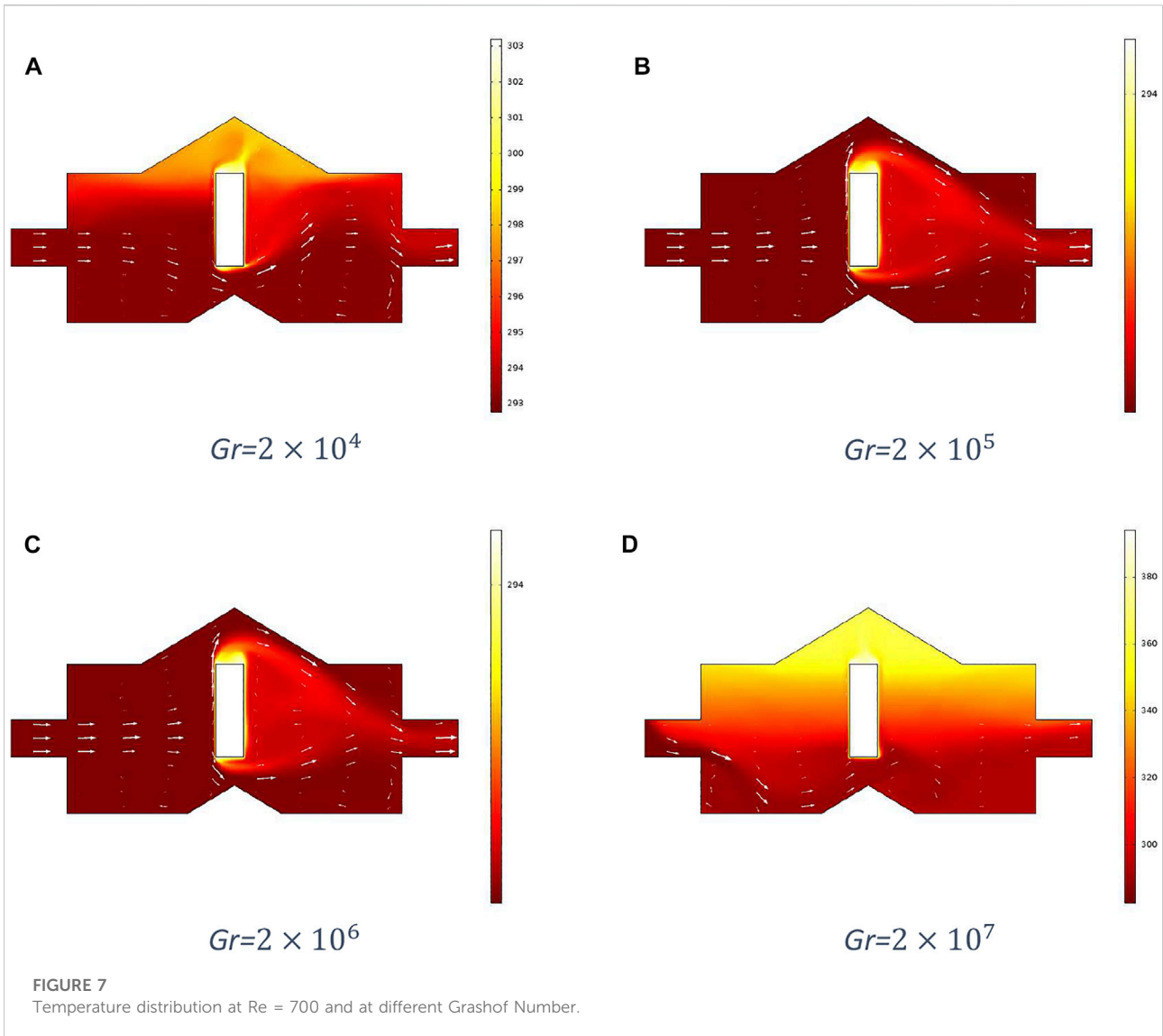
virtue of the penalty parameter  $\gamma$  using the penalty finite element method [57, 58] as follows:

$$P = -\gamma \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right). \quad (20)$$

Using Eq. 20 in Eqs. 12 and 13, we get the following:

$$\frac{\partial U}{\partial \hat{t}} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \gamma \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 V}{\partial X \partial Y} \right) + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \quad (21)$$

$$\begin{aligned} \frac{\partial V}{\partial \hat{t}} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = & \gamma \left( \frac{\partial^2 U}{\partial Y \partial X} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \\ & + \frac{Gr}{Re^2} \theta. \end{aligned} \quad (22)$$



### Weak formulation

Weight functions are used to give the weak formulation of Eqs. 14, 21, and 22. The weak formulations of these equations for a triangular element  $A^e$  of the cavity using  $w_1$ ,  $w_2$ , and  $w_3$  as the weight functions are as follows:

$$\int_{A^e} w_1 \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) dA - \int_{A^e} w_1 \gamma \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 V}{\partial X \partial Y} \right) dA - \int_{A^e} w_1 \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) dA = 0, \quad (23)$$

$$\int_{A^e} w_2 \left( \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) dA - \int_{A^e} w_2 \gamma \left( \frac{\partial^2 U}{\partial Y \partial X} + \frac{\partial^2 V}{\partial Y^2} \right) dA - \int_{A^e} w_2 \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) dA - \frac{Gr}{Re^2} \int_{A^e} w_2 \theta dA = 0, \quad (24)$$

$$\int_{A^e} w_3 \left( \frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) dA - \frac{1}{Re \times Pr} \int_{A^e} w_3 \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) dA = 0, \quad (25)$$

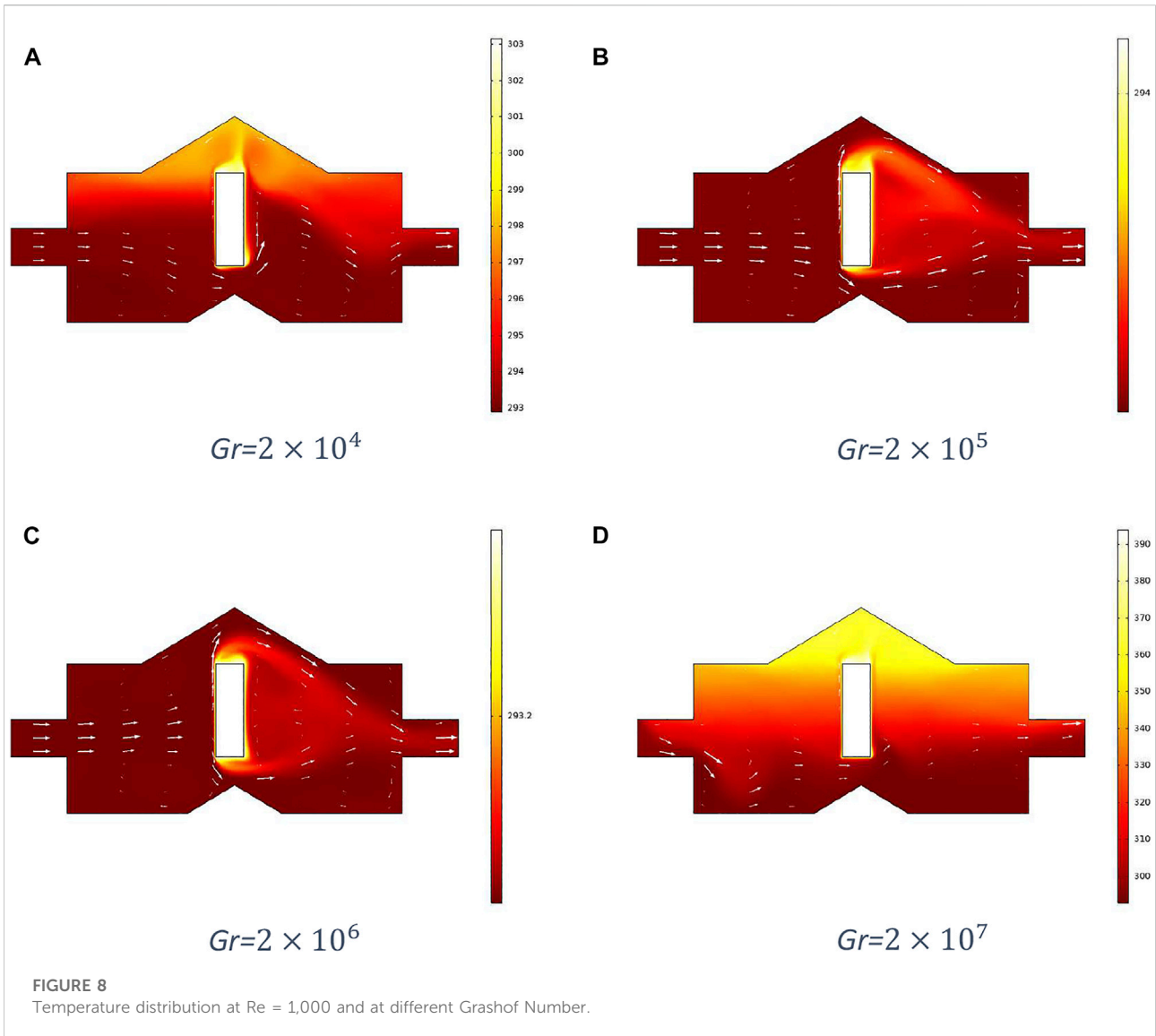


TABLE 2 Computation of Nusselt number for different Grashof numbers and Reynolds numbers.

Reynolds number (Re)	Grashof number (Gr)	Average Nusselt number (Nu)
700	$1 \times 10^4$	0.0516
	$5 \times 10^4$	0.3276
	$1 \times 10^5$	0.6956
	$5 \times 10^5$	2.8664
	$1 \times 10^6$	7.5720
1,000	$1 \times 10^4$	0.0564
	$5 \times 10^4$	0.3204
	$1 \times 10^5$	0.7820
	$5 \times 10^5$	2.9524
	$1 \times 10^6$	6.6796

where the subscript  $A^e$  on the integral is for the triangular discretized elements.

### Finite element method

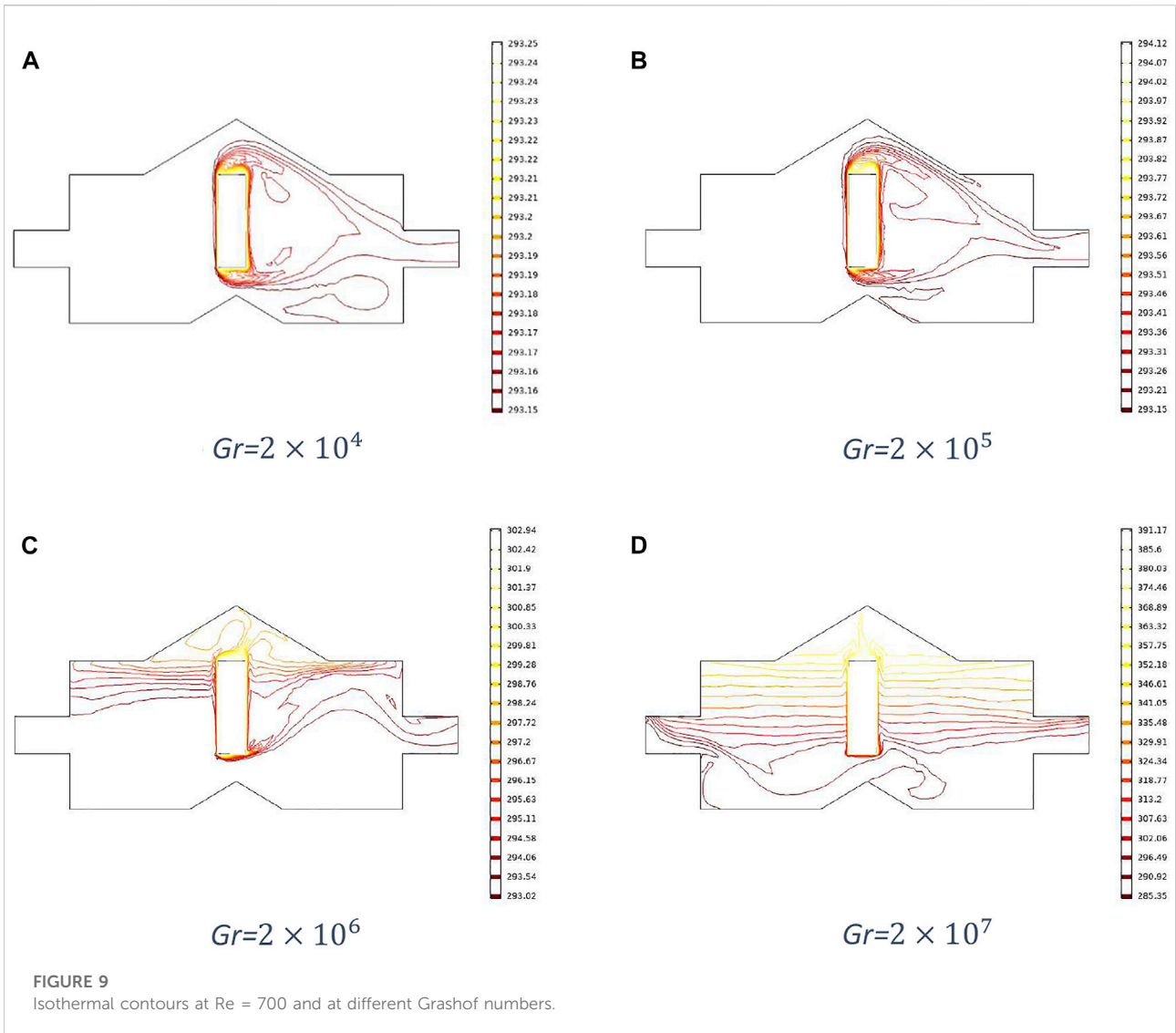
Using FEM, we approximate the functions  $U(X, Y, \hat{t})$ ,  $V(X, Y, \hat{t})$  and  $\theta(X, Y, \hat{t})$  as  $U^e$ ,  $V^e$ , and  $\theta^e$  over the triangular elements  $A^e$ . Thus,

$$U \approx U^e(X, Y, \hat{t}) = \sum_{i=1}^6 U_i^e \phi_i^e(X, Y, \hat{t}), \tag{26}$$

$$V \approx V^e(X, Y, \hat{t}) = \sum_{i=1}^6 V_i^e \phi_i^e(X, Y, \hat{t}), \tag{27}$$

$$\theta \approx \theta^e(X, Y, \hat{t}) = \sum_{i=1}^6 \theta_i^e \phi_i^e(X, Y, \hat{t}), \tag{28}$$

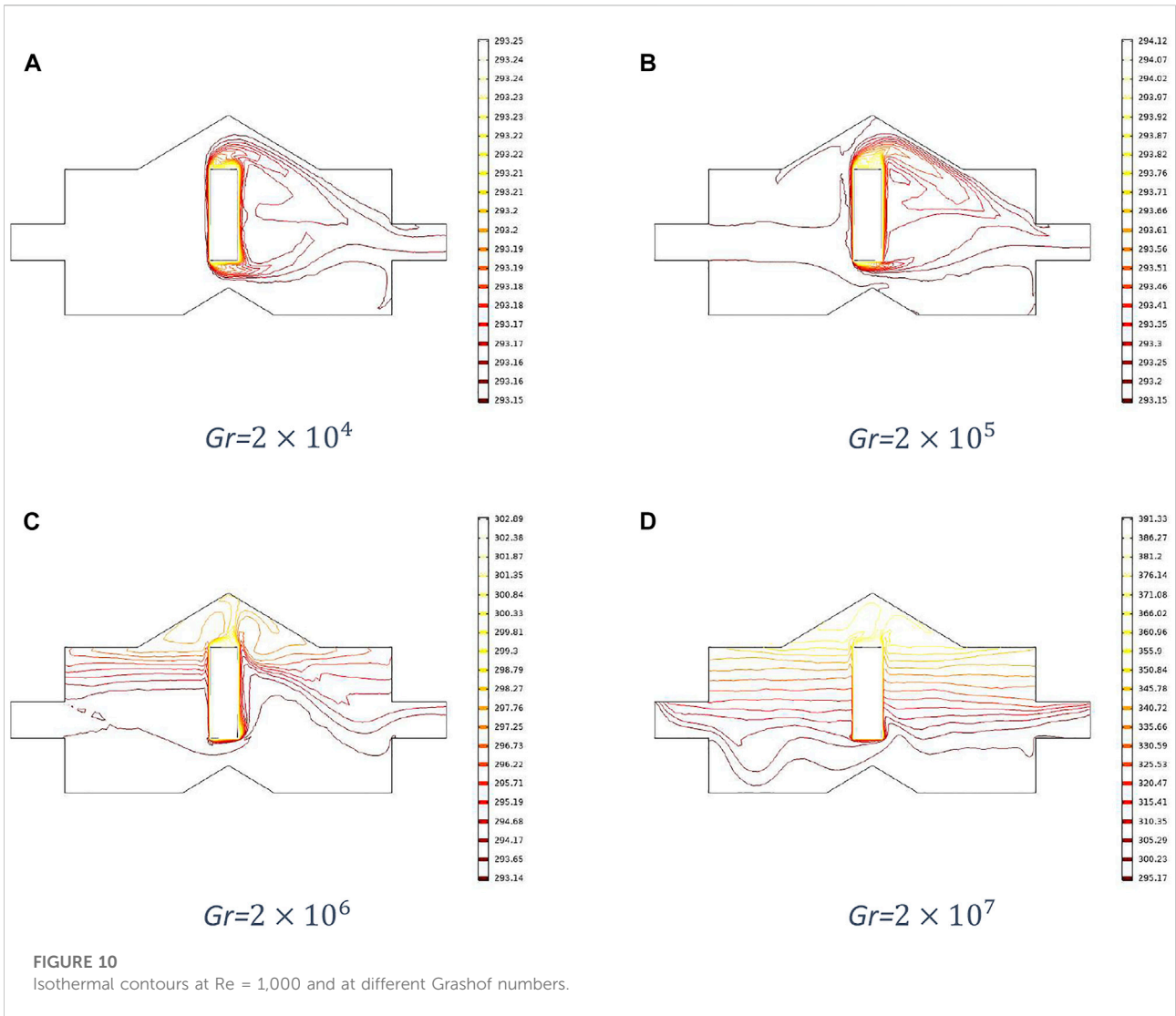




where  $\phi_i^e$  is used as a trial function. Substituting Eqs. 26–28 in Eqs. 23–25 obtains the system of residuals as follows:

$$\begin{aligned}
 R_j^{(1)} = & \sum_{i=1}^6 U_i^e \int_{A^e} \left\{ \frac{\partial \phi_i^e}{\partial t} + \left( \sum_{i=1}^6 U_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial X} \right. \\
 & + \left. \left( \sum_{i=1}^6 V_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial Y} \right\} \phi_j^e dA - \gamma \left\{ \sum_{i=1}^6 U_i^e \int_{A^e} \frac{\partial \phi_j^e}{\partial X} \frac{\partial \phi_i^e}{\partial X} \right. \\
 & + \left. \sum_{i=1}^6 V_i^e \int_{A^e} \frac{\partial \phi_j^e}{\partial X} \frac{\partial \phi_i^e}{\partial Y} \right\} dA - \frac{1}{Re} \sum_{i=1}^6 U_i^e \int_{A^e} \left\{ \frac{\partial \phi_j^e}{\partial X} \frac{\partial \phi_i^e}{\partial X} \right. \\
 & \left. + \frac{\partial \phi_j^e}{\partial Y} \frac{\partial \phi_i^e}{\partial Y} \right\} dA, \tag{29}
 \end{aligned}$$

$$\begin{aligned}
 R_j^{(2)} = & \sum_{i=1}^6 V_i^e \int_{A^e} \left\{ \frac{\partial \phi_i^e}{\partial t} + \left( \sum_{i=1}^6 U_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial X} \right. \\
 & + \left. \left( \sum_{i=1}^6 V_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial Y} \right\} \phi_j^e dA - \gamma \left\{ \sum_{i=1}^6 V_i^e \int_{A^e} \frac{\partial \phi_j^e}{\partial Y} \frac{\partial \phi_i^e}{\partial X} \right. \\
 & + \left. \sum_{i=1}^6 V_i^e \int_{A^e} \frac{\partial \phi_j^e}{\partial Y} \frac{\partial \phi_i^e}{\partial Y} \right\} dA - \frac{1}{Re} \sum_{i=1}^6 V_i^e \int_{A^e} \left\{ \frac{\partial \phi_j^e}{\partial X} \frac{\partial \phi_i^e}{\partial X} \right. \\
 & + \left. \frac{\partial \phi_j^e}{\partial Y} \frac{\partial \phi_i^e}{\partial Y} \right\} dA - \frac{Gr}{Re^2} \\
 & \int_{A^e} \sum_{i=1}^6 \theta_i^e \phi_i^e dA \tag{30}
 \end{aligned}$$



$$R_j^{(3)} = \sum_{i=1}^6 \theta_i^e \int_{A^e} \left\{ \frac{\partial \phi_i^e}{\partial t} + \left( \sum_{i=1}^6 U_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial X} + \left( \sum_{i=1}^6 V_i^e \phi_i^e \right) \frac{\partial \phi_i^e}{\partial Y} \right\} \phi_j^e dA - \frac{1}{Re \times Pr} \sum_{i=1}^6 \theta_i^e \int_{A^e} \left\{ \frac{\partial \phi_j^e}{\partial X} \frac{\partial \phi_i^e}{\partial X} + \frac{\partial \phi_j^e}{\partial Y} \frac{\partial \phi_i^e}{\partial Y} \right\} dA. \quad (31)$$

The above integrals are evaluated by numerical integration.

### Newton method

Thus, we get a linear system of equations and is tackled using Newton-Raphson form:

$$J(d^m - d^{m+1}) - R(d^m) = 0. \quad (32)$$

The above system of equations is solved for every iteration. Here  $d$  represents the iterative index,  $R(d^m)$  the vector of residual, and  $J(d^m)$  the Jacobian matrix. The Jacobian matrix  $J(d^m)$  includes

the partial derivatives of the family of residuals in respect of  $U$ ,  $V$ , and  $\theta$ . Divided differences are used to evaluate the Jacobian.

## Computation of the Nusselt number

The Nusselt number measures the rate of heat transfer from the heated rod. It is calculated as follows:

$$Nu = -\frac{\partial \theta}{\partial n}, \quad (33)$$

where  $n$  is normal to the plane.

At the vertical wall of the rod, it is defined as follows:

$$Nu = -\sum_{i=1}^6 \theta_i^e \frac{\partial \phi_i^e}{\partial X}. \quad (34)$$

And at the horizontal wall, it is defined as follows:

$$Nu = -\sum_{i=1}^6 \theta_i^e \frac{\partial \phi_i^e}{\partial Y}, \quad (35)$$

The expressions that calculate the average Nusselt number over vertical and horizontal sides of the rod will be given by the following:

$$\overline{Nu} = \frac{1}{L} \int_0^L Nu \, dY, \quad \overline{Nu} = \frac{1}{L} \int_0^L Nu \, dX, \quad (36)$$

respectively.

## Meshing

A triangular mesh of 3,930 elements was used to study the problem numerically as shown in the [Figure 2](#).

## Algorithm validation and grid independent test

The grid independent test is necessary to ensure the accuracy of the results. For this, the Nusselt number is calculated for a different number of triangular mesh elements. It is noted that the percentage error in the Nusselt number for mesh elements 3,001 is about 0.01% as compared with the refined mesh of 3,930 triangular elements. Therefore, the refined mesh of 3,930 triangular elements is used to explore the present problem. [Table 1](#) shows different values of the Nusselt number for different mesh elements at  $Re = 700$  and  $Gr = 5 \times 10^3$ .

## Results and discussion

In this portion, the flow and heat transfer through the cavity formed due to the fault have been shown graphically for various parameters using COMSOL Multiphysics. In [Figure 3](#), the Reynolds number is kept constant at 700 and the Grashof number is varied, and in [Figure 4](#), the Reynolds number is kept fixed at 1,000 and the Grashof number is varied. It can be seen that the flow pattern is varied slightly for greater Grashof numbers, but velocity increases remarkably for  $Gr \geq 2 \times 10^6$  (this can be seen from the attached color legend in [Figures 3](#) and [4](#)) which shows that velocity of the fluid increases with the increased Grashof number. On the other hand, from [Figures 5](#) and [6](#), it can be observed that pressure is less in the surrounding of the lower wall than that of the upper wall, showing that the velocity is higher near the lower wall. This can also be analyzed with reference to the buoyancy force effect.

Moreover, from [Figures 7–10](#), significant variation can be observed in the convection dominant region. [Figures 7](#) and [8](#) represent temperature distributions and isotherms, respectively at different Grashof numbers and fixed Reynolds number 700, and [Figures 9](#) and [10](#) show temperature distributions and isotherms, respectively for different Grashof numbers and fixed Reynolds number 1,000. The temperature distribution becomes more uniform, and isotherms spread more within the cavity for  $Gr \geq 2 \times 10^6$ . The physical meaning is that the temperature variation between the metal piece and surrounding fluid increases with the Grashof number. The heat transfer rate is evaluated in the form of the average Nusselt number for various Grashof numbers and Reynolds numbers, which is displayed in [Table 2](#).

## Conclusion

In this paper, heat transfer has been investigated in laminar flow due to natural convection through the cavity formed by faults. The finite element method discretizes the prevailing equations. The discretized equations are dealt with through COMSOL Multiphysics. The computed results are displayed in the Discussion section. From the simulation of the flow, it is analyzed that the velocity of the fluid increases as the Grashof number increases, and it is greater below the metal piece due to the effect of the buoyancy force. Moreover, the Nusselt number also increases with the increased Grashof number. It is also concluded that the tensional fault zones formed by the energy accumulation of radioactive material disintegration cause the cavities which are responsible for the transportation of energy in the form of heat. There are many applications of the analysis of

heat transfer and fluid flow characteristics of hydrothermal fluid through such cavities.

## Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

## Author contributions

All authors have contributed to solve and write the paper under the supervision of SN.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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## Nomenclature

$u, v$  Velocity components ( $\text{ms}^{-1}$ )

$T$  Temperature (K)

$p$  Pressure (Pa)

$c_p$  Specific heat at constant pressure ( $\text{kJ kg}^{-1}\text{K}^{-1}$ )

$\mu$  Dynamic viscosity of fluid ( $\text{N s m}^{-2}$ )

$\nu$  Kinematic viscosity of fluid ( $\text{m}^2 \text{s}^{-1}$ )

$k$  Thermal conductivity of fluid ( $\text{W m}^{-1} \text{K}^{-1}$ )

$\beta$  Coefficient of thermal expansion ( $\text{K}^{-1}$ )

$\rho$  Density of fluid ( $\text{kg m}^{-3}$ )

$g$  Acceleration due to gravity ( $\text{m s}^{-2}$ )

$T_h$  Temperature of metal piece (K)

$T_c$  Temperature of fluid (K)

$\rho_0$  Density of fluid at  $T_c$  ( $\text{kg m}^{-3}$ )

$U_0$  Characteristic velocity ( $\text{m s}^{-1}$ )

$L$  Characteristic length (m)

$U, V$  Dimensionless velocity components (—)

$\hat{t}$  Dimensionless time (—)

$P$  Dimensionless pressure (—)

$\theta$  Dimensionless temperature (—)

$Re$  Reynolds number (—)

$Pr$  Prandtl number (—)

$Gr$  Grashof number (—)

$Nu$  Nusselt number (—)