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On the role of chirality flips for the muon magnetic moment and its relation to the muon mass

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The muon mass and the anomalous magnetic moment a_μ are quantities which require chirality flips, i.e., transitions between left- and right-handed muons. Muon chirality flips are connected to electroweak symmetry breaking and Yukawa couplings. Scenarios for physics beyond the Standard Model motivated by the quest to understand electroweak symmetry breaking and/or the origin of flavour often introduce new sources of chirality flips; they hence provide potentially large contributions to a_μ, and the current a_μ measurement provides relevant constraints on such scenarios. This connection between a_μ, chirality flips, and the muon mass generation mechanism is important and underlies much of the current research on a_μ. The present article provides a brief pedagogical introduction to this role of chirality flips and an overview of general relationships. The general statements are illustrated with several concrete models involving e.g., leptoquarks and supersymmetry.

KEYWORDS

muon (g - 2), quantum field theory, electroweak symmetry breaking, leptoquark, supersymmetry, beyond the standard model

1 Introduction

The anomalous magnetic moment of the muon, $a_\mu = (g-2)_\mu/2$, provides a unique constraint on physics beyond the Standard Model (BSM). The recent run-1 measurement at Fermilab has sharpened the deviation from the corresponding Standard Model prediction. Using the world average taking into account the Fermilab measurement [1] and the Theory Initiative prediction [2],[1] this deviation amounts to

$$\Delta a_\mu^{2021} = (25.1 \pm 5.9) \times 10^{-10}. \tag{1}$$

The observable a_μ not only motivates the existence of BSM physics but also constrains BSM physics complementarily to other constraints from the collider, intensity and cosmic frontiers.

a_μ is an observable with four distinctive properties:

1 This prediction is based on original Refs. [16–41]. Progress on lattice gauge theory evaluations of the hadronic vacuum polarization contributions is under scrutiny but not yet taken into account in Ref. [2].

- loop-induced,
- flavour-conserving,
- CP-conserving,
- chirality-flipping.

Among these properties, the last one is arguably the least obvious and least intuitive. But the role of chirality flips is extremely important, in particular for BSM phenomenology of a_μ. The present article aims to provide a pedagogical introduction to the role of chirality flips and to elucidate how different mechanisms for chirality flips impact BSM a_μ phenomenology.

In a nutshell, the notion and the importance of chirality may be explained as follows. In a relativistic theory, massive and massless particles are fundamentally different. Massive particles can be put into the rest frame, where the spin can point into any direction. Massless particles always move at the speed of light, and their spin degree of freedom reduces to helicity, the spin in the direction of velocity. For spin 1/2 fermions, the massless limit is characterized by an additional symmetry—chiral symmetry.[2]

The massless limit of the muon would thus lead to an additional, chiral symmetry. The measurement principle of the magnetic moment anomaly a_μ, however, directly uses the continuous spin precession relative to the direction of velocity. This makes clear that non-zero a_μ is related to a non-zero muon mass, and thus to a breaking of the corresponding chiral symmetry. Accordingly, the two operators for the muon mass and a_μ involve *chirality flips*, i.e., products of left- and right-handed spinors such as $\overline{\psi_L}\psi_R$ and $\overline{\psi_L}\sigma^{\mu\nu}\psi_R$.

This connection is of high interest. In the Standard Model the muon mass is explained in complicated way: there is a Higgs field, subject to a wine-bottle shaped potential, which acquires a vacuum expectation value and breaking electroweak gauge invariance; the muon couples to the Higgs field *via* Yukawa interactions and thereby receives its rest mass. Many open questions are related to the origin of the Higgs field, its potential as well as the origins and the hierarchical family-structure of the Yukawa couplings.

In the quest to answer such questions many BSM scenarios have been proposed which modify the Higgs and/or Yukawa sector of the Standard Model. Through the connection *via* chirality flips, such scenarios can often lead to enhanced contributions to a_μ and thus provide promising explanations of the deviation (1). Conversely the determination of the value (1) leads to constraints on such scenarios and thus helps learning more about the origin of muon mass generation.

Section 2 of the present article presents the background of these connections and discusses the role of chirality flips for a_μ and for the muon mass in general terms. Section 3 draws general, model-independent conclusions. Section 4 provides an illustration in terms of generic one-loop results and Section 5 discusses three concrete BSM scenarios which represent three very different ways how chirality flipping mechanisms can appear and how the corresponding contributions to a_μ and the muon mass behave. As the present article aims to present theoretical background knowledge it will not present an overview of the vast phenomenological literature. As an example of a broad up-to-date phenomenological study with detailed literature survey we mention Ref. [3].

2 Throughout this text, the notion of chiral symmetry is more general and not the same as the notion of chiral symmetry in low-energy QCD related to the light meson masses. We will later refer to muon-specific chiral symmetry to highlight the distinction.

2 Relationships between $g-2$, chirality flips, and the muon mass generation mechanism

2.1 Derivation of the relationships

There is a deep connection between the anomalous magnetic dipole moment and the rest mass of a fermion. The connection follows from the relationships of both observables to chirality flips. To illustrate the connection we begin with the quantum field theory operators describing a generic mass term for a Dirac fermion ψ and the operator for the anomalous magnetic dipole moment $a = (g-2)/2$,

$$\mathcal{L}_m = -m\,\bar{\psi}\psi, \tag{2a}$$

$$\mathcal{L}_{\text{dip}} = -a\frac{Qe}{4m}\,\bar{\psi}\sigma^{\mu\nu}\psi\,F_{\mu\nu}. \tag{2b}$$

Here $F^{\mu\nu}$ is the electromagnetic field strength tensor, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and our sign conventions for charges and gauge couplings are defined by the QED gauge covariant derivative $D^\mu = \partial^\mu + iQeA^\mu$, where $Q = -1$ for an electron or muon.

Any Dirac fermion field can be decomposed into its left- and right-handed (or: left- and right-chiral) parts as

$$\psi = \psi_L + \psi_R \qquad \psi_{L,R} = P_{L,R}\psi \equiv \frac{1}{2}(1 \mp \gamma_5)\psi. \tag{3}$$

The left- and right-handed parts are eigenstates of chirality, i.e., of the γ_5 matrix with eigenvalues ∓ 1, respectively. It is useful to record the relations for barred fields,

$$\overline{\psi_L} = \bar{\psi}P_R \qquad \overline{\psi_R} = \bar{\psi}P_L. \tag{4}$$

In a free massless quantum field theory, chirality is related to helicity, the spin in the direction of the spatial momentum: ψ_L describes massless fermions with helicity −1/2 and corresponding antifermions with helicity +1/2; ψ_R has the opposite property—hence the terminology left-/right-handed. In a theory for massive Dirac fermions, however, ψ_L or ψ_R alone do not describe energy eigenstates, and generally there are no simultaneous eigenstates of chirality (i.e., of γ_5) and of the Dirac Hamiltonian.[3]

3 There are simultaneous eigenstates of energy/momentum and helicity, but for massive fermions, helicity and chirality are distinct concepts.

Chiral fermion fields are useful even in case of massive fermions because of their Lorentz transformation properties. ψ_L and ψ_R transform differently; they do not mix under Lorentz transformations, and Lorentz invariant Lagrangians can be expressed completely in terms of chiral fermion fields. For this reason it is possible to assign e.g., independent gauge transformations to left- and right-handed fermion fields and to construct a gauge theory with different gauge interactions of left- and right-handed fields—the electroweak Standard Model is an example.

Now we can rewrite the two Lagrangian terms (2) in terms of chiral fermion fields,

$$\mathcal{L}_m = -m\ (\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L), \quad (5a)$$

$$\mathcal{L}_{\rm dip} = -a\frac{Qe}{4m}\ (\overline{\psi_L}\sigma^{\mu\nu}\psi_R + \overline{\psi_R}\sigma^{\mu\nu}\psi_L)\, F_{\mu\nu}. \quad (5b)$$

Both the fermion mass term and the $(g-2)$-term connect left- and right-handed fields, i.e., both *need a chirality flip.*

The need for chirality flips has very important general consequences, which we now explain. We first focus on the muon mass; the consequences for $(g-2)$ will be similar.

In QED or QCD, left- and right-handed fermions have the same gauge transformations and Dirac mass terms such as Eqs 2a, 5a are gauge invariant. However in a theory with electroweak interactions and associated $SU(2)_L\times U(1)_Y$ gauge invariance, no gauge invariant Dirac mass terms for leptons or quarks are possible. Specifically the left-handed muon μ_L is part of an $SU(2)_L$ doublet $L = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$ with $U(1)_Y$ hypercharge $-1/2$ while the right-handed muon μ_R is an $SU(2)_L$ singlet with hypercharge -1. The only way to generate a muon mass in a theory with electroweak gauge invariance (SM or beyond) is to have spontaneous electroweak symmetry breaking (EWSB) and to couple the muon in a gauge invariant way to the corresponding vacuum expectation value. In the SM, EWSB is realized *via* the vacuum expectation value of the Higgs doublet $\langle\Phi\rangle = \begin{pmatrix} 0 \\ v^{\rm SM}/\sqrt{2} \end{pmatrix}$, and the gauge invariant coupling is realized *via* the Yukawa interaction

$$\mathcal{L}_m = -y_\mu \bar{L}\Phi\mu_R + h.c. \quad \rightarrow \quad -\frac{y_\mu v^{\rm SM}}{\sqrt{2}}\overline{\mu_L}\mu_R + h.c.. \quad (6)$$

Thus a tree-level muon mass $m_\mu = \frac{y_\mu v^{\rm SM}}{\sqrt{2}}$ is generated. While this formula is specific to the SM and to tree level, an analogous more general conclusion is true in any extension of the SM which has electroweak gauge invariance. Any such theory must have some (set of) expectation values (VEVs) $v^{\rm (B)SM}$ responsible for EWSB (these may be expectation values of fundamental or of composite fields). The physical muon mass m_μ will always be generated from (tree-level and/or loop-induced) couplings to these VEVs, and thus there will inevitably be the proportionality

$$m_\mu \propto v^{\rm (B)SM}. \quad (7)$$

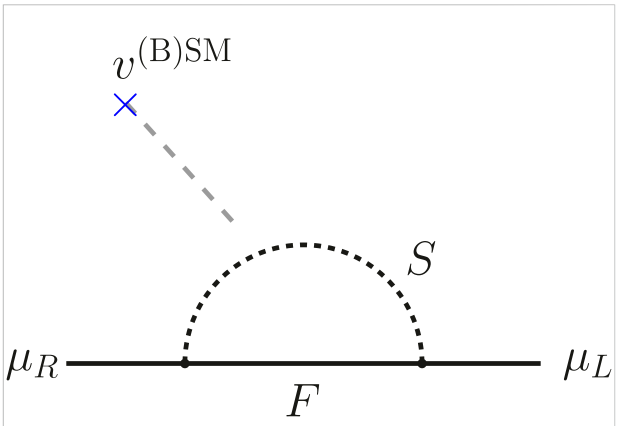

FIGURE 1
One-loop Feynman diagram illustrating Eqs 10, 11. Each contribution to the muon mass and to a_μ must involve one factor of some SM or BSM VEV breaking electroweak symmetry, illustrated by $v^{\rm (B)SM}$, and a chirality flip between μ_L and μ_R. In order to obtain a contribution to a_μ, a photon needs to couple to any of the charged internal lines. *S* and *F* denote generic scalar and fermionic particles.

Next we can connect the general relation (7) to chirality. It is useful to define a muon-specific chiral symmetry, under which the left- and right-handed muons transform with opposite phases:

$$\mu_R \rightarrow e^{i\alpha}\mu_R \qquad L \rightarrow e^{-i\alpha}L. \quad (8)$$

Under this chiral symmetry transformation, the muon mass term in the form (2a) or (5a) is not invariant. Hence in a theory which is invariant under the chiral symmetry (8), the muon mass must be zero—not only at tree level but exactly. Conversely, a non-zero muon mass requires a breaking of the chiral symmetry. In the SM Lagrangian, there is precisely one source of breaking of this chiral symmetry, namely the Yukawa interaction (6). The muon Yukawa coupling y_μ thus acts as the breaking parameter of the chiral symmetry. Combining this discussion with Eq. 7 we obtain that

$$m_\mu \propto y_\mu v^{\rm SM} \quad (9)$$

in the SM not only at tree level but at all orders. Any contribution to the physical muon mass m_μ must involve one factor of $v^{\rm SM}$ and one factor of y_μ (and possibly other factors).

Again an analogous conclusion is true in any extension of the SM which contains electroweak gauge invariance. Any such theory must contain some (set of) parameter(s) which break the chiral symmetry (8). We call them collectively $y_\mu^{\rm (B)SM}$. Any contribution to the physical muon mass in any such theory will then involve the factors

$$\Delta m_\mu \propto \left[y_\mu^{\rm (B)SM} v^{\rm (B)SM}\right] \times (\text{other couplings}). \quad (10)$$

This expresses that any contribution to m_μ must be proportional to the parameter(s) responsible for chiral symmetry breaking and to the VEVs responsible for EWSB; Figure 1 shows an illustrative Feynman diagram. The "other couplings" may be any couplings of the theory in question which appear in appropriate Feynman diagrams and do not have to be related to EWSB or chiral symmetry breaking. Apart from the proportionality factors given here, the full results for m_μ may contain numerical prefactors or loop functions of dimensionless quantities.

The chiral symmetry breaking parameters $y_\mu^{\mathrm{(B)SM}}$ in Eq. 10 are often equal or similar to the SM Yukawa coupling y_μ, but they don't have to be. They may be generalized Yukawa couplings to the SM-like Higgs doublet, Yukawa couplings to other Higgs-like fields, or even different types of couplings which nevertheless contribute to the breaking of the chiral symmetry.

Exactly the same kind of discussion applies to $(g-2)$ in view of the analogous structure of Eqs 5a, 5b. The $(g-2)$ Lagrangian (5b) breaks electroweak gauge invariance as well as the chiral symmetry associated with the respective fermion in the same way as the mass term (5a). Any non-zero contribution to $(g-2)$ therefore must involve the same kind of factors as any contribution to the mass. Specializing to the muon, we can write the generic relationship

$$\Delta a_\mu \propto m_\mu \times \left[y_\mu^{\mathrm{(B)SM}} v^{\mathrm{(B)SM}} \right] \times \frac{(\text{other couplings})}{M^2_{\text{typical}}} \tag{11}$$

for any contribution to a_μ in any model. The factors in the square bracket reflect the present discussion; they are required for any non-zero contribution to the coefficient in the Lagrangian (5b). Up to numerical factors the coefficient is given by the ratio a_μ/m_μ; hence solving for a_μ produces the explicit factor m_μ on the r.h.s. The denominator M^2_{typical} represents a typical mass scale of the theory and must appear for dimensional reasons. The "other couplings" and further, not explicitly written factors are as in Eq. 10.

2.2 Application to the standard model

Equation 11 provides crucial insight into the structure of possible BSM contributions to a_μ and allows to draw many general conclusions. We will explore such conclusions in the subsequent sections. Here we briefly remark that the relation also applies to all sectors of the SM itself.

In all pure QED and the hadronic contributions of the SM the factors in the square bracket of (11) can simply be replaced by the muon mass $[\ldots] \to m_\mu$. In QED Feynman diagrams which contain only photon and muon lines ("mass-independent contributions") the only typical mass scale is m_μ itself, hence (11) in this case simply reduces to the known structure [2].

$$a_\mu^{\text{QED,mass-indep.}} = \sum_{n\geq 1} A_1^{(2n)} \left(\frac{\alpha}{\pi}\right)^n \tag{12}$$

with the finestructure constant $\alpha = e^2/4\pi$ and where the coefficients $A_1^{(2n)}$ are pure numbers, including the famous result $A_1^{(2)} = 1/2$.

Among the QED mass-dependent contributions are diagrams with τ-lepton loops. For those diagrams, the "typical" mass scale is the τ-lepton mass m_τ, and e.g., the leading contribution from the τ-lepton is

$$a_\mu^{\text{QED},\tau,\text{leading}} = \frac{\alpha^2}{45\pi^2}\frac{m_\mu^2}{m_\tau^2}, \tag{13}$$

again in line with the generic result (11).

The hadronic SM contributions also follow the generic pattern (11), with the square bracket simply replaced by m_μ. As an example we mention here the analytical result for the leading logarithmic hadronic light-by-light contribution obtained in an effective-field-theory approach in Ref. [4]:

$$a_\mu^{\text{Hlbl,Knecht et al. (2002)}} = \left(\frac{\alpha}{\pi}\right)^3 \ln^2\left(\frac{m_\mu}{\mu_0}\right)\mathcal{C}, \quad \mathcal{C} = 3\left(\frac{N_C}{12\pi}\right)^2\left(\frac{m_\mu}{F_\pi}\right)^2. \tag{14}$$

Here the relevant mass scale is the pion decay constant F_π, explaining the appearance of the ratio $(m_\mu/F_\pi)^2$.

For the weak SM contributions to a_μ, the discussion of gauge invariance and chiral symmetry breaking is obviously far more important. Nevertheless, in the generic pattern (11) the interesting factors in the square bracket can only correspond to the single Higgs VEV and the single muon Yukawa coupling in the SM. For this reason ultimately the square bracket essentially reduces to the muon mass m_μ even in the electroweak SM. Correspondingly, the SM one-loop contributions from the weak interactions, i.e., from W- and Z-boson Feynman diagrams can be written as

$$a_\mu^{\text{EW}(1)} \propto \frac{\alpha}{4\pi s_W^2}\frac{m_\mu^2}{M_{W,Z}^2} \tag{15}$$

where s_W is the sine of the weak mixing angle and $M_{W,Z}$ are the W- and Z-boson masses. This formula is valid up to numerical factors of order unity, and it is in line with (11) for $M_{\text{typical}} = M_{W,Z}$.

It is also useful to note that in this way the generic pattern (11) allows to correctly predict the order of magnitude of the weak contributions, which is around 10×10^{-10} according to the estimate (15). The actual value is $a_\mu^{\text{EW}} = 15.36(10) \times 10^{-10}$ [2].

3 General conclusions for physics beyond the SM

The analysis of chirality flips, electroweak gauge invariance and muon-chiral symmetry has led to the generic patterns (10) and (11) for any contribution to m_μ and a_μ in the SM or any BSM extension. Here we draw several general conclusions from this analysis.

First, the relationship between a_μ and m_μ means that a_μ can provide a window to the muon mass generation mechanism: According to the SM the muon mass is generated by EWSB generating a Higgs VEV and the Yukawa interaction which couples the muon to the Higgs VEV. But many open questions remain. Importantly, understanding the muon mass generation mechanism does not only involve understanding EWSB and the origin of the Higgs potential and the Higgs VEV but also understanding the origin and the structure of Yukawa couplings.

Many BSM scenarios extend or modify the Higgs sector and/or the Yukawa sector. Technically, the muon mass generation mechanism is reflected by the factors in square brackets in Eqs 10, 11, and many BSM scenarios substantially modify these factors. This implies on the one hand that BSM scenarios with modified Higgs/Yukawa sectors can lead to strong enhancements in these factors and thereby provide potentially large contributions to a_μ and promising explanations of the current a_μ deviation. On the other hand, the enhancement mechanisms can differ significantly between various BSM scenarios. The measurement of a_μ thus provides a sensitive probe of BSM scenarios with modified muon mass generation mechanism.

As a second conclusion, we may introduce a universal, model-independent expression for contributions to m_μ and to a_μ in some given BSM scenario as (see also Refs. [5, 6]),

$$\frac{\Delta m_\mu^{\mathrm{BSM}}}{m_\mu} = \mathcal{O}(C_{\mathrm{BSM}}), \tag{16a}$$

$$\Delta a_\mu^{\mathrm{BSM}} = C_{\mathrm{BSM}} \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}, \tag{16b}$$

where M_{BSM} is the relevant mass scale. The dimensionless coefficient C_{BSM} introduced here summarizes the interesting chirality-flipping factors in square brackets in Eq. 11, the "other couplings" and the not explicitly written factors. It also includes by definition a factor $1/m_\mu$ such that C_{BSM} corresponds to the relative muon mass correction and such that the explicit factor m_μ^2 appears in the expression for Δa_μ. In writing (16) we used that the proportionality factors appearing in Eqs 10, 11 are equal up to potential $\mathcal{O}(1)$ differences.

Equation 16 highlight that any BSM scenario contributing to a_μ will simultaneously contribute to m_μ, and the contributions are related. The complicated origin of the muon mass, the necessity for EWSB and chiral symmetry breaking are all encapsulated in the factor C_{BSM}. This factor is very model-dependent and its value reflects the interesting dynamical details of the model. But the relation (16) is model independent and holds at any loop order.

A third conclusion is that one may impose a criterion that the BSM corrections to the muon mass do not introduce fine-tuning, i.e., do not exceed the actual muon mass. In models where this criterion is satisfied, C_{BSM} can be at most of order unity and a generic upper limit[4],

$$\Delta a_\mu^{\mathrm{BSM}} \lesssim \mathcal{O}(1) \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}, \tag{17}$$

is obtained [5, 6]. In this wide class of models, imposing this criterion then implies an order-of-magnitude upper limit on the mass scale for which the current observed value Δa_μ can be accommodated. This upper limit is approximately 2 TeV,[5] which is obviously of high interest in view of complementary searches for BSM particles at LHC. If the fine-tuning criterion applies, particles responsible for the a_μ deviation should be in reach of the LHC—even though it is not guaranteed that they can be discovered in view of background and their interactions with LHC detectors.

4 Generic one-loop results

Here we provide explicit one-loop results for contributions to a_μ and m_μ which illustrate the general relationships explained in the previous sections. To be specific we consider the case of one-loop diagrams as in Figure 1, with one neutral scalar particle S and one charged fermion F which couple to the muon *via* the interaction

$$\mathcal{L}_{\mathrm{int}} = S^\dagger \bar{F}(c_L P_L + c_R P_R)\mu + h.c.. \tag{18}$$

We note that if S and F are interaction eigenstates with definite gauge quantum numbers at most one of the two couplings c_L and c_R can be non-zero. If S and/or F are linear combinations of states with different quantum numbers, both c_L and c_R can be non-zero

4 We note that Eq. 16 does not imply the naive scaling $\Delta a_e^{\mathrm{BSM}}: \Delta a_\mu^{\mathrm{BSM}}: \Delta a_\tau^{\mathrm{BSM}} \approx m_e^2: m_\mu^2: m_\tau^2$ with the lepton generation since the coefficient C_{BSM} does not have to be generation-independent. As mentioned, the definition of C_{BSM} contains an explicit factor $1/m_\mu$; the later example of a leptoquark model will provide further illustration. Still, the prefactor m_l^2/M_{BSM}^2 in a_l implies that the muon magnetic moment is more sensitive to BSM physics than the electron magnetic moment and that typical models which explain e.g., the BNL deviation for a_μ give negligible contributions to a_e. For detailed discussions and examples for deviations from naive scaling in models with leptoquarks, two Higgs doublets or supersymmetry we refer to Refs. [42, 43].

5 The relation (16) and the resulting (17) assume that BSM contributions to a_μ and to m_μ arise at the same loop order. This is generally true, but an exception can arise in models which allow BSM contributions to m_μ already at tree level. An example is provided by models vector-like leptons which mix at tree level with muons, see Refs. [44–46] and the brief discussion in Ref. [3]. There, also tree-level BSM contributions to the muon mass exist, and the ratio between Δa_μ and $\Delta m_\mu^{\mathrm{tree}}$ does not scale as $1/M_{\mathrm{BSM}}^2$ as above but as $1/(16\pi^2 v^2)$. This might seem to allow arbitrarily high masses without fine-tuning, circumventing the upper mass limit of around 2 TeV. However, even using only tree-level effects in the muon mass, these references also find upper mass limits from perturbativity and constraints on the Higgs–muon coupling.

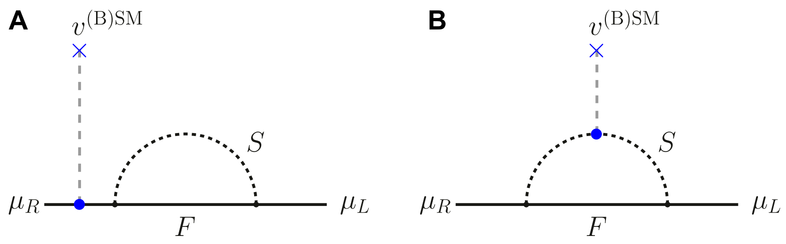

FIGURE 2
Concrete instances of the generic diagram of Figure 1. In the **(A)** diagram, the muon chirality is flipped at the external muon line *via* a coupling to an EWSB VEV. In the **(B)** diagram, the muon chirality is flipped *via* the loop, and the EWSB VEV couples to the scalar particle in the loop.

(e.g., F could be a SM quark or lepton, or F and S could be SUSY charginos and sfermions).

The one-loop contributions of S and F with these interactions to the muon mass and to a_μ read

$$\Delta m_\mu = \frac{1}{16\pi^2}\left\{ \frac{m_\mu}{2}\left[|c_L|^2 + |c_R|^2\right]B_1 - m_F \,\mathrm{Re}\left[c_L c_R{}^*\right]B_0\right\}, \tag{19a}$$

$$\Delta a_\mu = \frac{m_\mu}{16\pi^2}\left\{\frac{m_\mu}{12m_S^2}\left[|c_L|^2 + |c_R|^2\right]F_1^C + \frac{2m_F}{3m_S^2}\mathrm{Re}\left[c_L c_R{}^*\right]F_2^C\right\}. \tag{19b}$$

The results are evaluated in the limit $m_\mu \ll m_{S,F}$. The loop functions $B_{0,1} \equiv B_{0,1}\,(0, m_F, m_S)$ are standard Passarino-Veltman functions; the loop functions $F_i^C \equiv F_i^C\,(m_F^2/m_S^2)$ are given e.g., in Ref. [3]. In the limit $m_S = m_F$ the loop functions reduce to $B_0 = -2B_1 = 1/\bar{\epsilon} + \ln(\mu^2/m_S^2)$ and $F_i^C = 1$, where μ is the renormalization scale and where $1/\bar{\epsilon}$ is the dimensional regularization parameter which is set to zero in the $\overline{\mathrm{MS}}$-renormalization scheme. Hence in general all appearing loop functions have values of $\mathcal{O}(1)$.

The two results (19) indeed have an analogous structure. Each contribution to a_μ has a counterpart contribution to m_μ; the contributions differ in the relative factor m_μ^2/m_S^2 and in the $\mathcal{O}(1)$ coefficients and loop functions. Hence each term reflects the general relationship (16), which holds in all cases. But the interesting details and the dynamics of the BSM scenario is summarized in the quantity C_{BSM}. We can now go deeper and relate each term to the discussion of chirality flips and EWSB and to the generic patterns (10.11).

We first focus on the $|c_{L,R}|^2$-terms, illustrated in Figure 2A. For the $|c_{L,R}|^2$-terms we would define

$$C_{\mathrm{BSM}} \sim \frac{|c_{L,R}|^2}{192\pi^2}. \tag{20}$$

Here Δm_μ is proportional to an explicit factor of m_μ itself, and Δa_μ is proportional to an explicit factor m_μ^2. These terms have a very simple behaviour with respect to chirality flips. The $|c_{L,R}|^2$ structure means that the S–F–loop does not change the muon chirality. Hence the muon chirality must be flipped at the external muon line (in the diagrammatic computation this corresponds to an application of the Dirac equation $p\!\!/u(p) = m_\mu u(p)$ of the external muon spinor). The chirality flip at the external muon line may also be interpreted as a coupling of the muon line to the Higgs background field, as illustrated in the Feynman diagram.

Hence BSM contributions behaving like this neither involve new sources of chirality flips nor new sources of EWSB. The factors in square brackets in Eqs 10, 11 originate purely from the SM and amount simply to m_μ. Accordingly, such BSM scenarios also do not provide significant enhancement mechanisms.

In contrast, the $c_L c_R{}^*$-terms in Eq. 19 behave more interestingly. Here we would define

$$C_{\mathrm{BSM}} \sim \frac{\mathrm{Re}\left[c_L c_R{}^*\right]}{24\pi^2}\,\frac{m_F}{m_\mu}. \tag{21}$$

This case exhibits the explicit factor $1/m_\mu$ mentioned in the context of the definition of C_{BSM}.[6] These contributions are illustrated in the Feynman diagram of Figure 2B. At one vertex, a right-handed muon couples to the loop particles S and F, *via* the coupling c_R. At the other vertex a left-handed muon couples to the loop, *via* the coupling c_L. In the computation of the diagram, the fermion mass m_F also arises *via* the propagator of the fermion F in the loop, explaining the total combination of factors $c_L c_R^* m_F$. An important point is that this loop diagram effectively breaks electroweak gauge invariance and it breaks the muon-chiral symmetry (8). As mentioned before, the product $c_L c_R^*$ can only be non-zero if F and/or S are no gauge eigenstates. Now we see in more detail that the combination

$$[\ldots] \to c_L c_R^* m_F \tag{22}$$

corresponds to the square brackets in Eqs 10, 11, i.e., to the factors related to EWSB and muon-chiral symmetry breaking. It may be that the fermion mass m_F arises from a VEV, in which case $m_F \propto v^{(B)SM}$, where the VEV could arise either from the SM

6 See also the related discussion in footnote 4.

Higgs or from some BSM Higgs field. It may also be that the scalar field is a mixture of fields of different quantum numbers such that the couplings $c_{L,R}$ involve mixing matrix elements which effectively are $\propto v^{(B)SM}$. In all cases, a non-zero product $c_L c_R^* m_F$ implies that the muon-chiral symmetry (8) is broken, no matter how the fields S, F might be assigned to transform under that symmetry.

In general, therefore, the $c_L c_R^*$-terms may be strongly enhanced—the factor $c_L c_R^* m_F$ may be much larger than $|c_{L,R}|^2 m_\mu$. This is the chiral enhancement due to new sources of chirality flips and possibly new sources of EWSB. For this reason BSM scenarios with such contributions can provide particularly promising explanations of the current a_μ value, and the precise measurement of a_μ provides stringent constraints on the parameter spaces of such scenarios.

One may wonder about the fact that these terms involve one power of m_μ less. In some models (e.g., in certain leptoquark models) the apparent behaviour of Eq. 19 is real; the factors $c_L c_R^* m_F$ are indeed independent of m_μ. The corresponding terms then provide additive contributions to m_μ which are independent of m_μ itself; the contributions to a_μ are only proportional to m_μ instead of m_μ^2. The interpretation is that such models involve new parameters breaking muon-chiral symmetry, and those parameters are unrelated to the muon Yukawa coupling. In such models the chiral enhancement of a_μ contributions can be particularly large, but at the same time the discussion of fine-tuning in the muon mass becomes particularly relevant. We will encounter an example of this in the discussion of the leptoquark model in Section 5.2.

In some models, however, the behaviour of the $c_L c_R^* m_F$-terms is more involved (e.g., in supersymmetric models). Here, even though there are additional new sources of muon chirality flips beyond the muon Yukawa coupling, all chirality flips can be traced to one common origin. Hence the new sources of chirality flips can be fundamentally related to the original muon Yukawa coupling, and effectively we can write

$$c_L c_R^* m_F \propto m_\mu \tag{23}$$

i.e., the relevant enhancement factors are actually proportional to the muon mass. In terms of the generic relations (10.11), the square brackets are proportional to the muon mass. There can still be important enhancements in the proportionality factors such as $\tan\beta$ in supersymmetric models. This abstract discussion will be made concrete in the context of explicit examples in the following section.

5 Examples of concrete BSM scenarios

Here we discuss the role of chirality flips in three concrete BSM scenarios. The scenarios are also phenomenologically interesting in their own right, and they are discussed extensively in the literature. Here we use them to illustrate the general discussion of the previous sections and the range of possibilities.

The first example has a very simple behaviour and no chiral enhancement; the second example is a specific kind of leptoquarks which leads to strong chirality enhancement proportional to the top-quark mass; the third example is supersymmetry, where the chiral enhancement is given by as $\tan\beta$, the ratio of Higgs VEVs.

5.1 Simple 2-field model without chiral enhancement

We begin with a very straightforward extension of the SM by two new fields, one fermion F and one scalar S, which together couple to the muon as in Section 4. However now we assume in addition that this setup already forms a complete gauge invariant extension of the SM, i.e., both F and S are gauge eigenstates with definite gauge quantum numbers, and the fermion mass m_F is a gauge invariant Dirac mass term.

To be specific we choose F to be a Dirac fermion $SU(2)_L$ doublet with hypercharge −1/2 and S to be an $SU(2)_L$ singlet with hypercharge zero. We also assign F and S to be odd under a Z_2 symmetry. In this way, the scalar S constitutes a neutral dark matter candidate, while the fermion doublet F contains one neutral and one charged component. The only Z_2 and gauge invariant interaction with the muon is possible *via* a Lagrangian

$$\mathcal{L}_{\text{int}} = \lambda_L S^\dagger \bar{F} P_L L + h.c.. \tag{24}$$

As mentioned in Section 4 such models where F and S are gauge eigenstates allow either only couplings to the left-handed or to the right-handed muon. In this case, there is only a coupling to the left-handed muon doublet L with a coupling constant λ_L. In addition the model involves gauge invariant mass terms m_F and m_S for the two fields.

Because there is only a left-handed coupling λ_L this model—and such models in general—cannot lead to new sources of chirality flips. In terms of the discussion of Section 4 the chirality can only be flipped at the external muon line, and the Feynman diagrams behave as illustrated in Figure 2B.

The explicit one-loop contributions of the two new particles to the muon mass and to a_μ simply read

$$\Delta m_\mu = \frac{1}{16\pi^2}\left\{\quad \frac{m_\mu}{2}\ |\lambda_L|^2 B_1\right\}, \tag{25a}$$

$$\Delta a_\mu = \frac{m_\mu}{16\pi^2}\left\{\frac{m_\mu}{12 m_S^2}|\lambda_L|^2 F_1^C\right\}. \tag{25b}$$

Similarly to the generic case we can define a quantity C_{BSM} as

$$C_{\text{BSM}} = \frac{|\lambda_L|^2}{192\pi^2}. \tag{26}$$

It is simply given by a typical loop factor and by a squared coupling. No special enhancement or suppression mechanism exists. For values of couplings around $\lambda_L \sim 1$, it amounts to around one per-mille.

With this quantity we can parametrize the contributions to the muon mass and to a_μ as

$$\frac{\Delta m_\mu}{m_\mu} = C_{\mathrm{BSM}} \times 6B_1, \tag{27}$$

$$\Delta a_\mu = C_{\mathrm{BSM}} \times \frac{m_\mu^2}{m_S^2} F_1^C. \tag{28}$$

This highlights that the relative contribution to the muon mass is very simply given by a few per-mille, a typical one-loop magnitude if no enhancements are present. Similarly, plugging in numbers, a_μ is numerically given by

$$\Delta a_\mu \sim 25 \times 10^{-10} |\lambda_L|^2 \left(\frac{100\,\mathrm{GeV}}{m_S}\right)^2. \tag{29}$$

Hence the scenario could explain the current a_μ deviation for rather large coupling values $|\lambda_L| \gtrsim 1$ and small BSM masses around 100 . . . 200 GeV.

Models such as this have been discussed in detail in the literature, in particular in Refs. [3, 7–9]. Such models can indeed accommodate the current a_μ value in a viable way. However, Ref. [3] has shown that no such gauge invariant two-field extension is able to explain a_μ simultaneously with the dark matter relic density while evading constraints from dark matter and LHC searches. Along similar lines, Ref. [10] has shown that if one also requires an explanation of B-physics measurements, BSM scenarios with at least four new fields are required.

5.2 Leptoquark model with chiral enhancement

Next we consider an example of a leptoquark model, a model with the so-called leptoquark S_1. It involves only one new field and is in this sense even simpler than the model of the previous subsection, but the leptoquark S_1 allows more complicated interactions. The field S_1 is defined as a colour anti-triplet, and $SU(2)_L$ singlet, with hypercharge 1/3. With this assignment, two different interactions with quarks and leptons are possible in a gauge invariant way.

The relevant part of the interaction Lagrangian reads

$$\mathcal{L}_{S_1} = -\left(\lambda_L Q \cdot L S_1 + \lambda_R t_R \mu_R S_1 + h.c.\right), \tag{30}$$

where $Q \cdot L$ denotes the SU(2) invariant product of the left-handed quark doublet and left-handed lepton doublet. Generation indices are suppressed but we consider only quarks of the third generation (i.e., the top/bottom quark doublet) and leptons of the second generation. Accordingly, $t_R \mu_R$ is the product of the right-handed top-quark and muon singlets (we assume here 2-spinor notation for the fermion fields).

Hence the coupling λ_L governs the interaction between S_1 and left-handed muon and left-handed top quark (and muon neutrino and bottom quark); the coupling λ_R governs the interaction between S_1 and the right-handed muon and right-handed top quark. The structure of the relevant Feynman diagrams contributing to the muon mass and to a_μ is shown in Figure 3B.

The fact that both left-handed and right-handed couplings exist indicates that new sources of muon chirality flips are possible in this scenario. It is instructive to connect the discussion to the notion of the muon-specific chiral symmetry introduced in Eq. 8. We might ask: is the additional Lagrangian $\mathcal{L}_{S_1}$ invariant under this chiral symmetry? The most general ansatz of the chiral transformations of the relevant fields, extending (8), is

$$\mu_R \to e^{i\alpha} \mu_R \qquad L \to e^{-i\alpha} L \tag{31a}$$

$$t_R \to e^{i n_{t_R} \alpha} Q \qquad Q \to e^{i n_Q \alpha} Q \tag{31b}$$

$$S_1 \to e^{i n_{S_1} \alpha} S_1. \tag{31c}$$

In order for the λ_L-term to be invariant we need to choose $n_{S_1} = 1 - n_Q$. In order for the λ_R-term to be invariant we need to choose $n_{S_1} = -1 - n_{t_R}$. This is compatible only if $n_Q - n_{t_R} = 2$. If this is the case, the top–Higgs Yukawa interaction governed by the top Yukawa coupling y_t is not invariant under the chiral symmetry.

Overall, we learn that if the product of the three couplings $\lambda_L \lambda_R y_t$ is non-zero, there is no way to define the chiral symmetry such that all Lagrangian terms are invariant. Hence the muon-specific chiral symmetry in this scenario is not only broken by the muon Yukawa coupling y_μ, but also by the product of these three couplings, i.e., there are two sources of breaking

$$y_\mu, \qquad \lambda_L \lambda_R y_t, \tag{32}$$

which are independent of each other.

In the explicit one-loop contributions to the muon mass and to a_μ, the chirality-flip enhanced contributions dominate for realistic parameter choices. Focusing only on them, we can write

$$\Delta m_\mu = \frac{1}{16\pi^2} \left\{ -m_t \, \mathrm{Re}\left[\lambda_L \lambda_R^*\right] B_0 \right\}, \tag{33a}$$

$$\Delta a_\mu = \frac{m_\mu}{16\pi^2} \left\{ \frac{2m_t}{3m_{S_1}^2} \mathrm{Re}\left[\lambda_L \lambda_R^*\right] F_2^C \right\}. \tag{33b}$$

Feynman diagrammatically (see Figure 3A), these contributions correspond to a left-handed muon coupling *via* λ_L to a left-handed top-quark, the top-quark changing its chirality *via* the term m_t or equivalently *via* the product $y_t v^{\mathrm{SM}}$, and the right-handed top-quark coupling to a right-handed muon *via* λ_R. The appearing factors $\lambda_L \lambda_R m_t$ reflect the discussion of chirality flips and EWSB in Section 2.1.

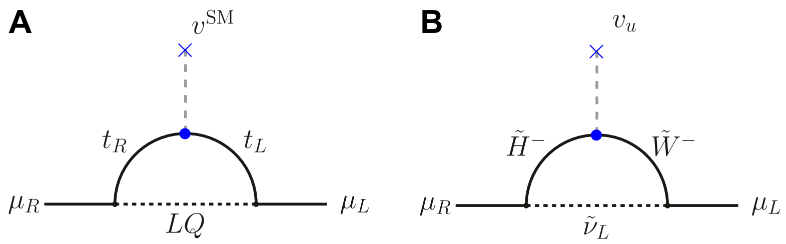

FIGURE 3
Concrete instances of the generic diagrams Figure 1 for a leptoquark model **(A)** and supersymmetry **(B)**. The diagrams correspond to the results (35.36) and (43.44), respectively.

Expressing the top-quark mass as $m_t = y_t v^{\rm SM}/\sqrt{2}$, the essential factors can be written as the product

$$\lambda_L \lambda_R y_t v^{\rm SM}, \tag{34}$$

and this product precisely corresponds to the factors in square brackets of Eqs 10, 11. They involve the new source of muon-specific chiral symmetry breaking, $\lambda_L\lambda_R y_t$, and the SM Higgs VEV $v^{\rm SM}$, which is the only source of EWSB in this model.

As mentioned in Section 2.1, the factors in the SM corresponding to (34) are simply $y_\mu v^{\rm SM}$. Hence the leptoquark model provides an example where the chirality flip is governed by a different, *a priori* unrelated set of factors. Thanks to the large top-Yukawa coupling and the potentially large couplings $\lambda_{L,R}$ this provides a very strong enhancement.

We can also connect to the discussion of the parametrization (16) and define the quantity $C_{\rm BSM}$ as

$$C_{\rm BSM} = \frac{{\rm Re}[\lambda_L \lambda_R^*]}{24\pi^2} \frac{m_t}{m_\mu} \tag{35}$$

such that

$$\frac{\Delta m_\mu}{m_\mu} = -C_{\rm BSM} \times \frac{3B_0}{2}, \tag{36a}$$

$$\Delta a_\mu = C_{\rm BSM} \times \frac{m_\mu^2}{m_{S_1}^2} F_2^C. \tag{36b}$$

Here the quantity $C_{\rm BSM}$ involves the mass ratio m_t/m_μ because of the new source of chirality flips which is unrelated to the muon mass. Plugging in numbers we obtain

$$C_{\rm BSM} \approx 7{\rm Re}[\lambda_L \lambda_R^*], \tag{37}$$

which highlights that very large contributions are possible. The relative corrections to the muon mass can easily be larger than 100%, and the contributions to a_μ can easily reach 25×10^{-10} even if the leptoquark mass m_{S_1} is in the several-TeV region. Accordingly, leptoquark models with such chiral enhancements are among the most promising and most discussed potential explanations of the current deviation (1). In particular, more complicated leptoquark models are promising to explain a_μ simultaneously with other deviations from the SM in B-physics, see e.g., Refs. [11, 12].

Since the muon mass is pushed up by the top-quark mass, we provide here further discussion of the contributions to the muon mass. The definition of $C_{\rm BSM}$ and writing Eq. 36 hides the fact that the absolute correction to the muon mass is actually independent of the muon mass itself. This fact is clearly exhibited by the original Eq. 33 and by the discussion of origins of chirality flips, Eq. 32. Schematically, the structure of the tree-level and leading one-loop contributions to the muon mass in this model read

$$m_\mu = \frac{y_\mu v^{\rm SM}}{\sqrt{2}} + \frac{\lambda_L \lambda_R y_t v^{\rm SM}}{16\pi^2 \sqrt{2}} \times (\text{loop functions}). \tag{38}$$

This way of writing emphasizes the additive structure and the fact that the one-loop correction is independent of the tree-level term. In view of the large value of the top-Yukawa coupling the one-loop term can easily dominate.

It also raises a question related to fine-tuning and naturalness. If the one-loop correction is significantly larger than the physical muon mass (say, in the $\overline{\rm MS}$-scheme), it must be cancelled precisely by a corresponding tree-level contribution. The larger the one-loop correction becomes, the more fine-tuned the tree-level term has to be and the less natural the model appears. In phenomenological discussions it is therefore motivated to impose an upper limit on the level of fine-tuning one is willing to accept. This will lead to upper limits on the couplings and correspondingly to an upper limit on the leptoquark mass for which the current a_μ deviation can be explained. This upper mass limit will be in the ball-park of 2 TeV, corresponding to the discussion around Eq. 17. For an example we refer to Ref. [3].

5.3 Minimal supersymmetric standard model with tan β enhancement

Third we consider the case of the Minimal Supersymmetric Standard Model (MSSM). The MSSM is well known as one of the most studied extensions of the SM and one of the most promising explanations of the current a_μ deviation. Here we will not discuss its phenomenology and not review the extensive literature. Instead we focus on the leading MSSM one-loop contributions to a_μ and discuss them from the perspective of chirality flips, in analogy to the previous examples.

The leading MSSM contributions arise from one-loop diagrams with virtual charginos χ_i^- and a sneutrino $\tilde{\nu}$. The couplings depend on the chargino index $i \in \{1, 2\}$ and read

$$c_L = -g_2 V_{i1}^* \qquad c_R = y_\mu U_{i2}, \tag{39}$$

where g_2 is the $SU(2)_L$ gauge coupling and V_{i1} and U_{i2} are mixing matrix elements corresponding to the gaugino-like and Higgsino-like components of the chargino i. With these definitions, the generic one-loop formulas (19) apply. Figure 3B shows a Feynman diagram in mass-insertion approximation, where the interaction eigenstate gaugino and Higgsino mix *via* a coupling to a Higgs VEV.

In this MSSM case, the generic formulas are deceptive. The $c_L c_R^*$-terms are proportional to the large chargino mass $m_{\chi_i^-}$, so one might expect a relative enhancement factor like

$$\frac{m_{\chi_i^-}}{m_\mu}, \text{or } \frac{y_\mu m_{\chi_i^-}}{m_\mu} \propto \frac{m_{\chi_i^-}}{v}, \tag{40}$$

depending on whether one takes into account that the coupling c_R is proportional to the muon Yukawa coupling. The first option is similar to the enhancement by m_t/m_μ in case of leptoquarks; the second option still seems to allow very large contributions for very high chargino masses. Both expectations are incorrect. The techical reason is the chargino mixing, and the correct behaviour can be obtained by combining the values of the mixing matrices with the behaviour of the loop functions as a function of the mass eigenvalues, see e.g., [13, 14].

The true behaviour of these chargino–sneutrino contributions can be better understood by analyzing the underlying chirality flips and EWSB, as discussed in Section 2.1. In the MSSM, one may extend the muon-specific chiral symmetry (8) to supermultiplets. In this way, the chiral symmetry is broken precisely only by the muon Yukawa coupling y_μ, like in the SM. All other terms in the MSSM Lagrangian are invariant under this chiral symmetry.[7] Therefore, there are no new sources of muon-specific chiral symmetry breaking in the MSSM.

However, there is a new source of EWSB. The MSSM contains two Higgs doublets, with two different VEVs v_u and v_d. Their ratio is defined as $\tan\beta = v_u/v_d$, and the case of large $\tan\beta$ is of particular relevance for a_μ. The tree-level muon mass is given by the Yukawa coupling to the small VEV, $y_\mu v_d/\sqrt{2}$; hence the value of the Yukawa coupling y_μ is bigger than in the SM. In the one-loop diagrams like Figure 3B, the charginos can couple to the large VEV v_u. This coupling fundamentally originates from the supersymmetrized gauge interaction between gaugino, Higgsino, and Higgs, and thus contributes a factor $g_2 v_u$.

Therefore, the true behaviour of these MSSM contributions can be brought into the form of Eqs 10, 11 as follows. The factors in square brackets become

$$y_\mu v_u, \tag{41}$$

a product of the single chiral symmetry breaking parameter and the large VEV; the "other couplings" amount to g_2^2 from the coupling c_L and from the gaugino–Higgsino–Higgs-VEV coupling.[8] The relative enhancement compared to the tree-level muon mass $y_\mu v_d$ is thus given by the famous enhancement by

$$\tan\beta. \tag{42}$$

Similarly, up to $\mathcal{O}(1)$ factors we can write

$$\frac{\Delta m_\mu}{m_\mu} \sim C_{\text{BSM}}, \tag{43a}$$

$$\Delta a_\mu \sim C_{\text{BSM}} \frac{m_\mu^2}{m_{\text{SUSY}}^2} \tag{43b}$$

with

$$C_{\text{BSM}} \sim \frac{g_2^2}{16\pi^2} \tan\beta. \tag{44}$$

This version of the formulas makes manifest several important properties. It exhibits the well-known $\tan\beta$-enhancement, compared to a typical one-loop prefactor $g_2^2/16\pi^2$, which is around one per-mille. For values such as $\tan\beta = \mathcal{O}(50)$, the current a_μ deviation can be explained if the relevant SUSY masses are of the order 500 GeV, a mass region which is constrained but not excluded by the LHC. The formulas also show that despite the enhancement, there is no issue of potential fine-tuning in the muon mass, in contrast to the case of leptoquarks.

7 This is true if we follow the customary treatment of trilinear soft SUSY-breaking terms and write the corresponding parameters as products $y_f A_f$ of the appropriate Yukawa coupling times the so-called A-parameters.

8 In addition to the discussion presented here, it is possible to analyze the consequences of further symmetries related to MSSM parameters: the $\tan\beta$-enhanced chirality flips actually break a Peccei-Quinn symmetry and an R-symmetry, which implies that the $\tan\beta$-enhanced terms must also be proportional to products of the MSSM Higgsino mass and gaugino mass parameters μM_1 or μM_2; hence actually the "other couplings" and C_{BSM} also need to contain ratios such as $\mu M_i/M_{\text{SUSY}}^2$, where $i = 1, 2$ and where M_{SUSY} is a generic mass scale of the relevant particles. These additional factors are typically of $\mathcal{O}(1)$ but they can lead to enhancements in special regions of parameter space, see e.g., [3, 14].

6 Summary

This article has focused on the role of chirality flips for a_μ and the resulting connection to the muon mass. Both a_μ and the muon mass correspond to chirality-flipping operators, and non-vanishing contributions require breaking of electroweak symmetry and of a muon-specific chiral symmetry.[9]

These basic connections are visible in the generic relations (10.11) which expose the role of the chirality flips and EWSB for m_μ and a_μ. Any contribution in any model (and at any loop order) must involve some SM or BSM VEV and some chirality-flipping coupling (Yukawa-like coupling or generalization). Hence BSM scenarios with new sources of EWSM and/or new flavour structures, new Yukawa-like couplings can provide large chirality flip enhancements, and accordingly to large contributions to a_μ and promising explanations of the deviation (1). Conversely, the a_μ measurement helps to identify promising parameter regions in concrete BSM scenarios[10].

The generic relations (10.11) can be simplified to Eq. 16 by introducing the abbreviation C_{BSM} which encapsulates the model-dependent details. The abbreviated equations highlight the $1/M^2_{\mathrm{typical}}$ dependence of contributions to a_μ and the correlation to large contributions to m_μ. As an example, 100% corrections to m_μ generally correlate to an explanation of the a_μ deviation (1) at a mass scale $M_{\mathrm{typical}} \sim 2$ TeV, see Eq. 17.

The explicit examples of Sections 4 and 5 illustrate models without chirality flip enhancements, as well as different kinds of models with chirality flip enhancements. The first example is a simple 2-field model without chiral enhancement, and correspondingly large contributions to a_μ are only possible for rather light BSM masses and large couplings. The second example is a specific kind of leptoquarks which leads to new sources of chirality flips and strong chiral enhancement. The enhancement is proportional to m_t and formally unrelated to the muon mass itself; hence this scenario provides a potentially large additive contribution to m_μ (and potential fine-tuning) and a contribution to a_μ which is linear in the muon mass and which leads to potential explanations of (1) for multi-TeV scale leptoquark masses. The third example is the MSSM, where the chiral enhancement is related to the muon mass. The actual chirality flipping mechanism exemplifies Eq. 23 and the enhancement scales as $\tan\beta$, the ratio of Higgs VEVs. These examples are representative of important trends in current phenomenological literature, and we refer to Ref. [3] for a survey including an overview of the literature.

The discussion of chirality flips and the resulting interpretation of a_μ as a window to the muon mass generation mechanism raises the question: which other key observables allow to test models with large potential contributions to a_μ and with new sources of chirality flips? One such observable is clearly the H–μ–μ coupling between the Higgs boson and muons. Like the muon mass and a_μ, this coupling is directly affected by new sources of EWSB and chirality flips, and its role and connection to a_μ is explored in a general setting in Ref. [15]. Similarly, a_μ is strongly correlated with charged lepton flavour violating processes such as $\mu \to e\gamma$ which are calculated from the similar Feynman diagrams with a chirality flip. Such lepton flavour-violating processs are governed by flavour violating coupling constants. Therefore in the BSM scenarios with significant contributions to a_μ, large lepton flavour violation is possible and may be seen in future experiments; conversely current experimental limits constrain values of corresponding lepton flavour-violating parameters.

9 The need for chirality flips also distinguishes a_μ from many other precision observables. In particular, electroweak precision observables such as the mass of the W boson (predicted *via* its relationship to the muon decay constant G_μ, the Z boson mass, α, and model-specific parameters) are chirality conserving. For this reason BSM contributions to a_μ and the W boson mass tend to be only weakly correlated. Accordingly, the recent CDF measurement of the W boson mass [47] can be explained in a variety of BSM scenarios simultaneously with Eq. 1, by appropriately choosing parameters in different model sectors.

10 We also refer to the workshop webpage http://pheno.csic.es/g-2Days21/

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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