frontiers | Frontiers in Physics

PERSPECTIVE
published: 01 June 2022
doi: 10.3389/fphy.2022.920238

On Gravitational Fields in Superconductors

*Giorgio Papini**

Physics Department and Prairie Particle Physics Institute, University of Regina, Regina, SK, Canada

DeWitt's theory on superconductors in gravitational fields is re-derived in the low velocity limit and linear gravity from the Klein-Gordon equation for an ensemble of charged spinless bosons. The solution has a phase singularity that gives rise to boson condensation and enables the description of type-II superconductors in the presence of gravity.

Keywords: gravitatation, superconductivity, boson condensation, topological singularities, klein—gordon equation

OPEN ACCESS

Edited by:
Antonio Gallerati, Politecnico di Torino, Italy

Reviewed by:
Gaetano Scarpetta, University of Salerno, Italy

***Correspondence:**
Giorgio Papini
giorgio.papini@gmail.com

Specialty section:
This article was submitted to Interdisciplinary Physics, a section of the journal Frontiers in Physics

Received: *14 April 2022*
Accepted: *02 May 2022*
Published: *01 June 2022*

Citation:
Papini G (2022) On Gravitational Fields in Superconductors. Front. Phys. 10:920238. doi: 10.3389/fphy.2022.920238

1 INTRODUCTION

In the middle of the 1960s, the Institute of Field Physics of the University of North Carolina at Chapel Hill was an exciting place for people interested in the interaction of gravity with quantum systems. Bryce and Cecile DeWitt were the body and soul of the activities of the Institute that they had founded. I landed at the Institute in the fall of 1964. Robert Forward of the Hughes Research Laboratories, at times a visitor there, and I shared a problem: Could a small general relativistic effect produced by a rotating mass in a terrestrial laboratory, be made measurable by integrating over time the action that gravity would have on a supercurrent? Would the current, once started, flow indefinitely, because of its lack of viscosity? Bryce was intrigued and found a solution to the problem [1] using the BCS theory [2]. His approach applied mainly to stationary gravitational fields, a limitation in view of the intended applications. His findings could be extended to non-inertial gravitational fields and agreed well [3], in the appropriate limit, with the experimental data on the London moment [4] of rotating superconductors that were being obtained at the time [5, 6]. The action of gravity on the electron-phonon interaction was assumed small, and it was indeed small, but this was shown only later [7]. Several other applications have since been conceived and are collected in [9].

In hindsight, I would solve the problem of superconductors as follows. Taking into account the condensation of electrons into bosons, the gas of spinless bosons, flowing against a background of positive charge represented by the lattice, can be described by the covariant Klein-Gordon equation

$$[(\nabla_\alpha - qA_\alpha)(\nabla^\alpha - qA^\alpha) + m^2]\Phi(x) = 0. \quad (1)$$

In (1), ∇_α is the covariant derivative. Notations and units ($\hbar = c = k_B = 1$) are as in [10]. Given the typical value of the electron velocities $\sim 10^{-5}$, the choice of the KG equation might seem excessive. It has, however the merit that it only requires the equivalence principle. To first order in the metric deviation $\gamma_{\alpha\beta} = g_{\alpha\beta} - \eta_{\alpha\beta}$, **Eq. 1** has the exact solution [8–10].

$$\Phi(x) = \exp(-i\chi(x))\phi_0, \quad (2)$$

where $\phi_0(x)$ is a plane wave solution of the free Klein–Gordon equation, $\chi(x)$ is the gravitational Berry's phase [8, 9, 11],

$$\partial_\beta\chi = qA_\beta + \Phi_{G,\beta} \equiv \widetilde{A}_\beta, \quad (3)$$

and

$$\Phi_G(x) = -\frac{1}{2}\int_P^x dz^\lambda \left[\gamma_{\alpha\lambda,\beta}(z) - \gamma_{\beta\lambda,\alpha}(z)\right](x^\alpha - z^\alpha)k^\beta + \frac{1}{2}\int_P^x dz^\lambda \gamma_{\alpha\lambda}(z)k^\alpha, \quad (4)$$

is a two-point function [12, 13]. The point P, omitted for simplicity in the following, refers to some fixed base event and k_α is the wave vector of ϕ_0. It is also useful to re-write (4) as

$$\Phi_G \equiv \int^x dz^\lambda \partial_{z_\lambda}\Phi_g(z,x). \quad (5)$$

Extensions of (2) to any order in $\gamma_{\mu\nu}$, and to other wave equations can be found in [9, 10, 14–19]. Differentiating (4) with respect to x, we obtain

$$\Phi_{G,\alpha} = -\frac{1}{2}\int^x dz^\lambda\left(\gamma_{\alpha\lambda,\beta} - \gamma_{\beta\lambda,\alpha}\right)k^\beta + \frac{1}{2}\gamma_{\beta\alpha}k^\beta. \quad (6)$$

Using the Lanczos-DeDonder condition $\gamma^\nu_{\alpha,\nu} - 1/2\gamma^\beta_{\beta,\alpha} = 0$, we can also prove the useful relations $\Phi_{G,\alpha\beta} = k_\sigma\Gamma^\sigma_{\alpha\beta}, \eta^{\alpha\beta}\Phi_{g,\alpha\beta} = 0, \tilde{A}^\alpha_{,\alpha} = 0$. It is then easy to show that **(1)** can be re-written in Minkowski space-time by replacing A_α with $\tilde{A}_\alpha$. Because the momentum of the superelectrons is

$$\tilde{P}_\mu = k_\mu - \tilde{A}_\mu, \quad (7)$$

we also find to first order in $\gamma_{\alpha\beta}$

$$g^{\alpha\beta}\tilde{P}_\alpha\tilde{P}_\beta = g^{\alpha\beta}P_\alpha P_\beta. \quad (8)$$

Then, using the "Lorentz gauge" $\tilde{A}^\alpha{}_{,\alpha} = 0$, we arrive at the result

$$\left(-\partial_t^2 - 2iq\tilde{A}_0\partial_t + q^2\tilde{A}_0{}^2\right)\psi = \left(-\partial_i\partial^i + 2iq\tilde{A}_i\partial^i + q^2\tilde{A}_i\tilde{A}^i + m^2\right)\psi, \quad (9)$$

where the correction term $m^2\gamma_{\alpha\beta}u^\alpha u^\beta$, small for typical electron velocities, has been dropped. Subtracting m by means of the transformation $\psi \to \tilde{\psi}(x)\exp(-imt)$ and assuming that $q\tilde{A}_0\tilde{\psi} < m\tilde{\psi}$ and that $\partial_t\tilde{\psi}$ and $\partial_t^2\tilde{\psi}$ can be neglected, we arrive at the result

$$-\frac{1}{i}\partial_t\tilde{\psi} = \frac{1}{2m}\left(\frac{1}{i}\partial_i - qA_i\right)\times\left(\frac{1}{i}\partial^i - qA^i\right)\tilde{\psi} + qA_0\tilde{\psi} \quad (10)$$

which coincides with the usual Schroedinger equation obtained by DeWitt. The term containing $\tilde{A}_i\tilde{A}^i$ in (10) has been discussed in [3, 4]. An additional interesting proposal concerning nonlinear terms has been recently made by Ummarino and Gallerati [20].

The physical content of (10) is well-known. The Schiff-Barnhill [21] field $-\partial_i\tilde{A}_0 = 0$ gives an electric field inside the superconductor where in zero-gravity there is none. Moreover, in order that the wave-function be single-valued, the condition $\oint \tilde{A}_j dz^j = 2n\pi$, where n is an integer, must be satisfied for any loop linking a multiply connected region of the superconductor. It is this condition that is used in most applications aimed at detecting small gravitational effects. It also accounts for the London moment in the case of rotation.

2 TOPOLOGICAL SINGULARITY

Despite the weak field approximation, solution (2) of (1), together with the definitions 3) and 4) offers interesting insights into the symmetry and symmetry violations of the vacuum of the system of spinless bosons considered. **Eq. 2** represents, in fact, a space-time transformation of the vacuum that renders the ground state degenerate. This symmetry violation can lead to condensation phenomena. By straight differentiation with respect to z, we can, in fact, find that [10].

$$\frac{\partial^2\Phi_g(z)}{\partial z^\tau\partial z^\sigma} - \frac{\partial^2\Phi_g(z)}{\partial z^\sigma\partial z^\tau} = R_{\alpha\beta\sigma\tau}(x^\alpha - z^\alpha)k^\beta \equiv [\partial z_\tau, \partial z_\sigma]\Phi_g(z) \quad (11)$$

where

$$R_{\mu\nu\sigma\tau} = \frac{1}{2}\left(\gamma_{\mu\sigma,\nu\tau} + \gamma_{\nu\tau,\mu\sigma} - \gamma_{\mu\tau,\nu\sigma} - \gamma_{\nu\sigma,\mu\tau}\right), \quad (12)$$

is the linearized Riemann tensor. **Eq. 11** indicates the presence of a singularity. Phase singularities are known to give rise to strings of magnetic flux in magnetism, vortices in optics and in charged and neutral superfluids, and lines of silence in acoustics [22]. The connection with condensation follows from

$$g^{\alpha\beta}\tilde{P}_\alpha\tilde{P}_\beta = g^{\alpha\beta}(k_\alpha + qA_\alpha)\left(k_\beta + qA_\beta\right) \simeq m^2\left(1 + 2\gamma^{\alpha\beta}u_\alpha u_\beta\right) \equiv H, \quad (13)$$

and from applying the Ising model in one, or two dimensions [10] to the energy function H. Condensation takes the form of oscillations of particle momenta about the average direction of P_μ. When quantized, the oscillations are quasiparticles similar to spin waves with dispersion relations that can be found by applying textbook procedures [23].

3 SUMMARY AND CONCLUSION

From the Klein-Gordon equation for a gas of charged, spinless bosons flowing against the positively charged background represented by the metal lattice we have re-obtained the Schroedinger equation used by DeWitt to describe the superelectrons in a gravitational field.

It is useful, at this point, to summarize the approximations made. The gravitational field has been treated to linear order in the metric deviation. The derivation also applies to slowly time-dependent gravitational fields provided $\partial_0\tilde{\psi}$ and $\partial_0^2\tilde{\psi}$ remain small and $q\tilde{A}_0\tilde{\psi} < m\tilde{\psi}$. Terms non-linear in the electromagnetic potentials give small corrections to any electric fields that may occur within a superconductor and have also been neglected, though London's non-linear hydrodynamical correction [4] has been measured [24]. Gravitational corrections to the electron-phonon interaction are small and have also been neglected. All results reported by

DeWitt have been re-obtained. Gravity is present in the wave-function of a quantum system as the classical external field $\gamma_{\alpha\beta}$. **Eq. 1** is, however, fully covariant and gauge invariant, hence applicable to time-dependent gravitational fields.

Condensation in the ensemble of identical particles takes the form of alignment in the direction of an average momentum with oscillations that satisfy a dispersion relation and have a Planck distribution [10]. Using Stokes' theorem, **Eqs 2** and **4** can be re-written in the form

$$\Phi = \phi_0 + \frac{i}{4}\int_S ds^{\alpha\beta} R_{\mu\nu\beta\alpha}\left[J^{\mu\nu}, \phi_0\right], \tag{14}$$

where $J_{\mu\nu}$ is the angular momentum operator. Thus ϕ_0, transported along z in a closed path bounded by a surface S loses its rotational symmetry and Φ leads to condensation. We can then define a correlation length (per unit of spacing) $\xi \sim \exp\left(\frac{m\gamma_\alpha^\alpha}{4T}\right)$, and a susceptibility $\frac{d\Gamma}{d\gamma} \simeq \frac{m}{4T}$ which is the gravitational counterpart of Curie's law. Both ensure that there is vanishing alignment when $T \to \infty$ and strong alignment when $T \to 0$. Numerical examples involving astrophysical objects [10] show that condensation effects can be substantial even when gravity is weak.

The singularity present in Φ_g vanishes when $x_\alpha - z_\alpha = 0$. It is a line of normal phase surrounded by superelectrons, as in type-II superconductors which are characterized by vortical lines threading the superconductor. The extension of the results to neutral superfluids is obvious. If the model is two-dimensional, there is a critical temperature T_c below which the state of the system is represented by bound vortices that become unbound when $T > T_c$.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

The (single) author was responsible of conceptualization, methodology and writing.

REFERENCES

1. DeWitt BS. Superconductors and Gravitational Drag. *Phys Rev Lett* (1966) 16: 1092. doi:10.1103/physrevlett.16.1092
2. Bardeen J, Cooper LN, Schriffer JR. Microscopic Theory of Superconductivity. *Phys Rev* (1957) 108:1175. doi:10.1103/physrev.108.1175
3. Papini G. London Moment of Rotating Superconductors and Lense-Thirring fields of General Relativity. *Nuovo C* (1966) 45:66. doi:10.1007/bf02710584
4. London F. *Superfluids*, Vol. 1. New York: Dover Publications (1961).
5. Hildebrandt AF, Saffren MM, King CA, Jr, Hendricks JB, Rorschach HE, Bol M, et al. In: Proc. 9th Int. Conf. on Low-Temp. Phys. Pt. A 459; ibidem Pt. A, 466; ibidem Pt.A, 471.
6. Zimmerman JE, Mercereau JE. Compton Wavelength of Superconductive Electrons. *Phys Rev Lett* (1965) 14:887. doi:10.1103/physrevlett.14.887
7. Leung M-C, Papini G, Rystephanick RG. Gravity-Induced Electric Fields in Metals. *Can.J Phys* (1971) 49:2754. doi:10.1139/p71-334
8. Cai YQ, Papini G. Applying Berry's Phase to Problems Involving Weak Gravitational and Inertial Fields. Class. *Quan Grav.* (1990) 7:269. doi:10.1088/0264-9381/7/2/021
9. Lambiase G, Papini G. *The Interaction of Spin with Gravity in Particle Physics*. Heidelberg, Germany: Springer Nature (2021).
10. Papini G. Some Classical and Quantum Aspects of Gravitoelectromagnetism. *Entropy* (2020) 22:1080. doi:10.3390/e22101089
11. Berry MV. Quantal Phase Factors Accompanying Adiabatic Changes. *Proc Roy Soc A* (1984) 392:45–57. doi:10.1098/rspa.1984.0023
12. Ruse HS. Taylor's Theorem in the Tensor Calculus. *Proc Lond Math. Soc.* (1931) 32:87–92. doi:10.1112/plms/s2-32.1.87
13. Synge JL. *Relativity: The General Theory*. Amsterdam: Netherlands: North-Holland Publishing Company (1966).
14. Lambiase G, Papini G, Punzi R, Scarpetta G. Neutrino Optics and Oscillations in Gravitational fields. *Phys Rev D* (2005) 71:073011. doi:10.1103/physrevd.71.073011
15. Papini G. In: Rizzi G, Ruggiero ML, editors. *Relativity in Rotating Frames*. Dordrecht, Netherlands: Kluwer Academic Publishers (2004).
16. Papini G. Spin–gravity Coupling and Gravity-Induced Quantum Phases. *Gen Rel Gravit* (2008) 40:1117. doi:10.1007/s10714-007-0595-z
17. Papini G. Spin-2 Particles in Gravitational fields. *Phys Rev D* (2007) 75:044022. doi:10.1103/physrevd.75.044022
18. Papini G, Scarpetta G, Feoli A, Lambiase G. Optics of Spin-1 Particles from Gravity-Induced Phases. *Int J Mod Phys D* (2009) 18:485. doi:10.1142/s0218271809014595
19. Papini G. Classical and Quantum Aspects of Particle Propagation in External Gravitational fields. *Int J Mod Phys D* (2017) 26:1750137. doi:10.1142/s0218271817501371
20. Ummarino GA, Gallerati A. Superconductors in a weak Static Gravitational Field, 77, 1; Exploiting Weak Field Gravity-Maxwell Symmetry in Superconductive Fluctuations Regime 7723:193; 2019,11,1341; Possible Alterations of Local Gravitational Field inside a Superconductor. 202123, 193. *EPJC; Symmetry; Entropy* (2017) 7723:193.
21. Schiff LI, Barnhill MV. Gravitation-Induced Electric Field Near a Metal. *Phys Rev* (1966) 151:1067. doi:10.1103/physrev.151.1067
22. Berry M. Singularities in Waves and Rays. In: Balian R, editor. *Les Houches, Session V, 1980-Physique des Défauts/Physics of Defects*. Amsterdam, Netherlands: North-Holland Publishing Company (1981).
23. Kittel C. *Introduction to Solid State Physics*. 7th ed.. New York, NY, USA: John Wiley & Sons (1996).
24. Bok J, Klein J. Electric Fields in Superconductors. *Phys Rev Lett* (1968) 20:660. doi:10.1103/physrevlett.20.660

Conflict of Interest: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's Note: All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

Copyright © 2022 Papini. This is an open-access article distributed under the terms of the Creative Commons Attribution License (CC BY). The use, distribution or reproduction in other forums is permitted, provided the original author(s) and the copyright owner(s) are credited and that the original publication in this journal is cited, in accordance with accepted academic practice. No use, distribution or reproduction is permitted which does not comply with these terms.