



# Scattering Cancellation Coating Composed of Periodical Subwavelength Structures

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We propose to cancel the scattered wave using two layers of periodical subwavelength structures by coating the scattering object with the designed composite material. As a demonstration, it is possible to obtain such scattering cancellation effect for a fiber optical nanoprobe, and fabricate the coating layers by simply etching cylinders or doping elements in silicon/SOI wafer. The required quasistatic parameters of the coating material are homogeneous and isotropic. The simulation results show that the 632.8 nm TE polarized light travels through the coated fiber optical nanoprobe with the wavefront undisturbed, and serve as evidence of the effectiveness of the designed invisibility coating. The proposed scattering cancellation scheme could be useful in non-invasive probing applications at visible and near-infrared wavelengths.

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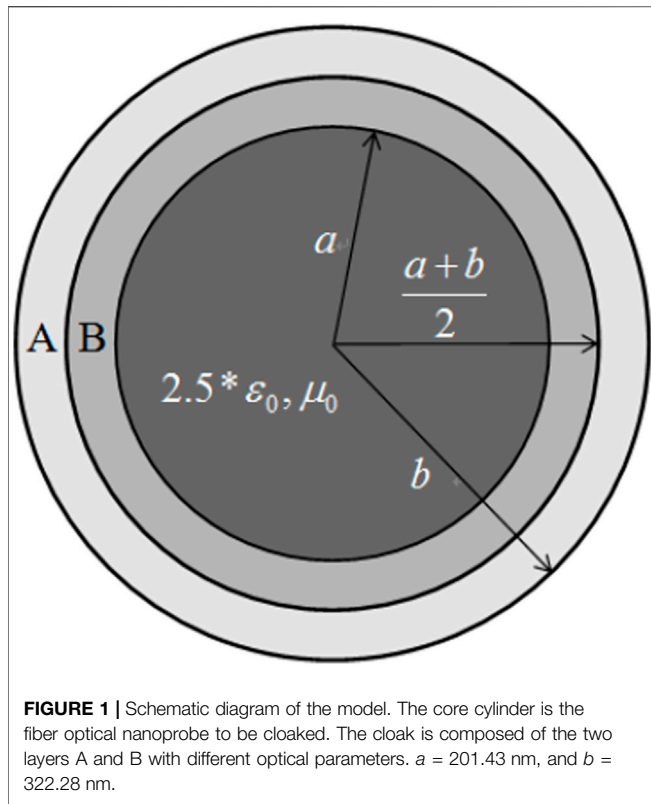
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## INTRODUCTION

Invisibility cloak has drawn lots of attention in recent years [1–11]. Invisibility effects are usually generated with absorbing screens for eliminating the backscattering or reflection from objects. Recently, due to the possibility of cancellation or reduction of the scattered wave from the scattering object, and the potential significance in non-invasive probing/sensing applications [12–14], invisibility cloaking based on metamaterials become a hot topic.

For designing the invisibility devices, the parameter distribution is usually obtained with the technique of coordinate transformation [5, 6] or scattering cancellation [15–17]. The transformation technique often requires singular dielectric parameters or anisotropic parameters [2–6], that are difficult to implement in practice. Although the recent development of metamaterial make the nontrivial material with uncommon parameters possible to fabricate in real world [18–22], and some invisibility designs based on coordinate transformation, such as carpet cloak or illusion cloak, have been demonstrated in laboratories [9–11], the ideal omnidirectional invisibility cloak with broad working frequency bandwidth still challenges the current fabrication technologies, especially in optical frequencies [8].

To overcome the difficulty in the invisibility design with simple parameter distributions, transparent coatings based on scattering cancellation have been proposed in the field of electromagnetics, optics and acoustics [16, 23], showing how this technique bring invisibility cloak into reality by coating the scattering object with layers of metamaterial. Although this technique is initially proposed to generate the effect of optical invisibility [13], the experimental realizations are demonstrated in the field of electromagnetics [13, 24, 25] and acoustics [14, 17]. The difficulty of achieving plasmonic material in optical frequencies impedes the development of optical invisibility coatings. To our best knowledge, a feasible implementing scheme to achieve



omnidirectional and broadband invisibility with low loss materials remains challenging and still needs to be put forward.

In this paper, we present the design of the invisibility coating composed of two layers of metamaterial, to address the abovementioned problems in the existing devices. The scattering cross section of a concentric cylindrical system are derived and written as a function of the component material parameter. The scattering cross section is minimized with optimized parameter distribution, i.e., the materials parameters for the coating layers are determined by searching for the minimal value of the derived scattering cross section of the modeled system, chosen as the objective function of the optimization algorithm. The resulting values of the constituent parameters are different from the previous plasmonic-type coatings with much larger permittivity than air, and can be feasibly implemented by etching in silicon wafer and SOI wafer. The resulting device could be low loss compared with the devices composed of metallic material or negative-index metamaterials.

We consider an simplified fiber optical nanoprobe, i.e., an infinitely long cylindrical object of radius  $a$ , with isotropic and homogeneous parameter distribution of permittivity  $2.5 \epsilon_0$  and permeability  $\mu_0$ , where  $\epsilon_0$  and  $\mu_0$  are permittivity and permeability of air. As shown in **Figure 1**, the nanoprobe is wrapped with the bilayer concentric cylindrical coating which forms the invisibility cloak to be further designed, and illuminated by a TE polarized optical plane wave. Both the layers A and B have the same thickness of  $(b - a)/2$ . In the following, we use the scattering cancellation method to obtain the

optical parameters of the layers, which will further be implemented with metamaterials.

As the incident wave is TE polarization mode with the electric field vector perpendicular to the cross-section of the concentric cylinder, then the electric field can be mathematically regarded as a scalar field in the plane perpendicular to the vector. Firstly, we consider a two-boundary model, i.e., the cylinder only coated by one layer with the thickness of  $h$ . The wave field in the background medium, in the coating layer and in the fiber can be respectively expanded with Bessel or Hankel series as

$$E_0 = E_{0,inc} + E_{0,scr} = \sum_n \alpha_{0,n} J_n(k_0 r) e^{in\theta} + \sum_n R_{1,n} \alpha_{0,n} H_n^{(1)}(k_0 r) e^{in\theta}, \tag{1}$$

$$E_1 = E_{1,sta} + E_{1,f} = \sum_n S_{1,n} \alpha_{0,n} J_n(k_1 r) e^{in\theta} + \sum_n F_{1,n} \alpha_{0,n} H_n^{(1)}(k_1 r) e^{in\theta}, \tag{2}$$

$$E_2 = E_{2,trs} = \sum_n T_{1,n} \alpha_{0,n} J_n(k_0 r) e^{in\theta}, \tag{3}$$

where  $E_{0,inc}$ ,  $E_{0,scr}$  is the electric field of the incident wave and scattering wave filed in the background medium,  $E_{1,sta}$ ,  $E_{1,f}$  is the standing field and outgoing cylindrical wave in the coating layer, and  $E_{2,trs}$  is the field in the region inside the hidden nanoprobe.

The boundary condition equations at the interfaces of the layers are:

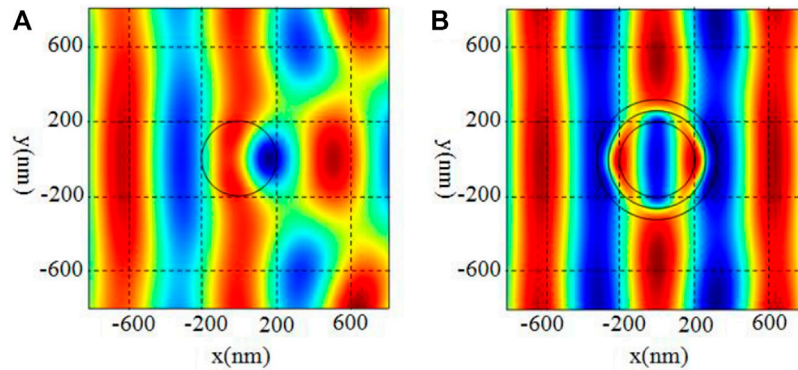
$$\begin{cases} E_{inc}|_{r=a,a+h} + E_{scr}|_{r=a,a+h} = E_{tra}|_{r=a,a+h} \\ \vec{n} \cdot \frac{1}{\mu_0} (\nabla E_{inc}|_{r=a,a+h}) \vec{n} \cdot \frac{1}{\mu_0} (\nabla E_{scr}|_{r=a,a+h}) = \vec{n} \cdot \frac{1}{\mu_1} (\nabla E_{tra}|_{r=a,a+h}) \end{cases} \tag{4}$$

Substituting **Eqs. 1–3** into **Eq. 4**, the coefficients  $R_{1,n}$  of the single-layer structure that give the scattering field of the system can be obtained. Our model, however, is made up of concentric cylinders with more coating layers. For the goal of calculating the wave field scattered by this more complicated system, the layer B can be regarded as an effective boundary, and the coefficients  $R_{1,n}$ ,  $T_{1,n}$  can be regarded as the reflection and transmission coefficients at this effective boundary, then the bilayer model become an effective single-layer structure and the corresponding scattering problem can be calculated using **Eqs 1–4**. This process turns the calculation of the multilayer scattering problem into a recursive progress, starting from the innermost layer, to the outermost layer. When the scattering problem of the whole system are solved, we obtain the total scattering coefficients  $R_{j,n}$  and define the scattering cross section of the cylinder with  $j$  coating layers in the far field as

$$\sigma = \frac{2}{\pi k_0} \int_0^{2\pi} \left| \sum_n R_{j,n} \exp\left(in\left(\theta - \frac{\pi}{2}\right)\right) \right|^2 d\theta, \tag{5}$$

The smaller the scattering cross section is, the better stealth effect one gets.

Based on the previous equations, the scattering cross section of the system shown in **Figure 1** can be regarded as a function of the material parameters of layers A and B. By taking the scattering cross section as the objective function of



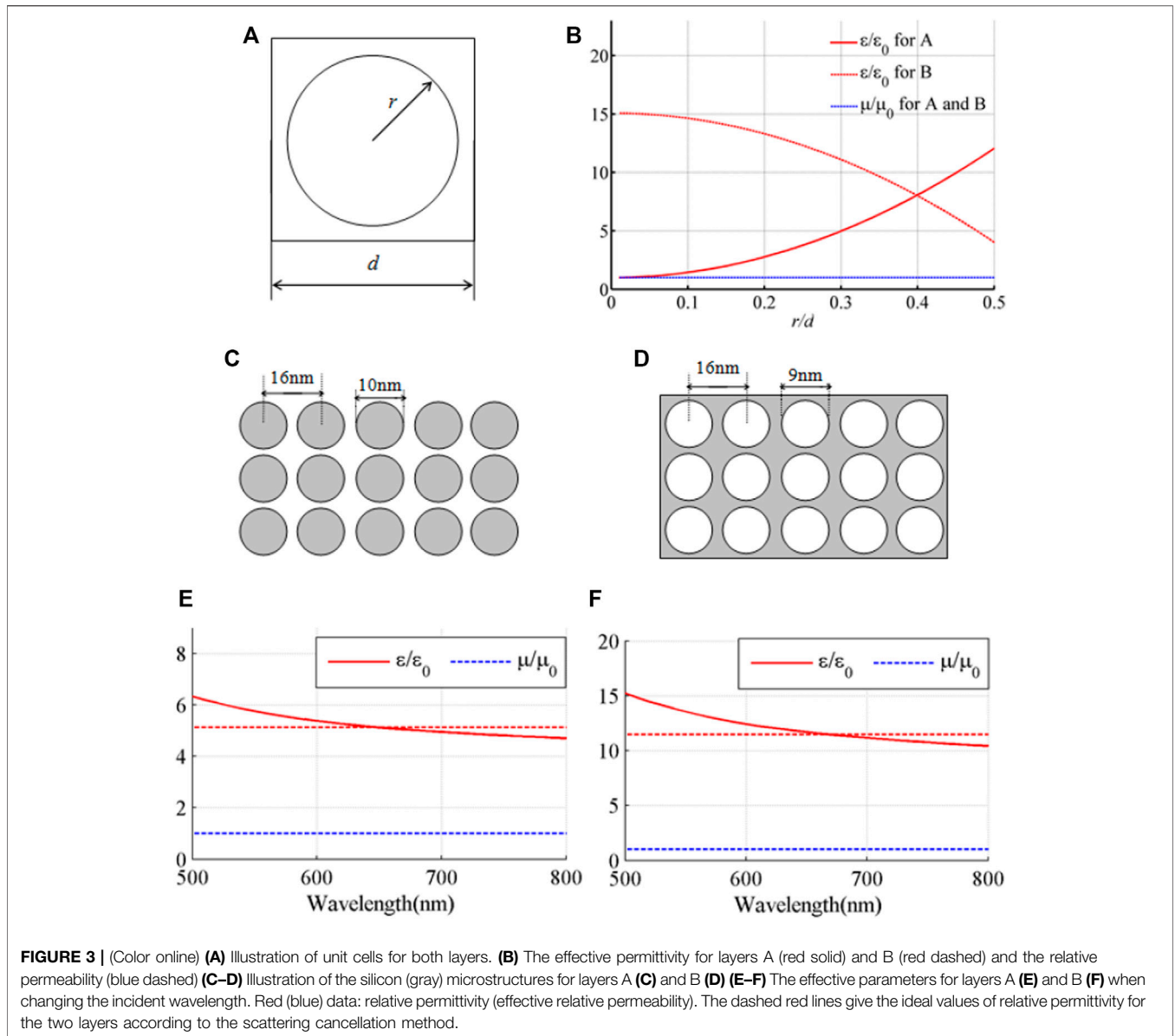
**FIGURE 2** | (Color online) The electric field excited by the 632.8 nm plane optical wave **(A)** without and **(B)** with the cloak.

the standard genetic algorithm, and searching in the realizable parameter space for the best stealth effect, the optimum material parameters of the coating layers for constructing the scattering-cancellation type cloak are found. The optimized constitutive material parameters that can make the nanoprobe undetectable to the detecting signals are  $5.1\epsilon_0, \mu_0$  and  $11.5\epsilon_0, \mu_0$  for layers A and B respectively, which are different with the plasmonic cloak, where effective parameters of the coating layers are far smaller than those in air [15]. To demonstrate the performance of this cloak, the electric fields excited by the plane optical wave with wavelength of 632.8 nm for the nanoprobe without and with the cloak are calculated using the above recursive method by taking  $S_{j,n}$  and  $F_{j,n}$  in Eq 1 into account, and shown in **Figures 2A,B** respectively. For the case without the cloak, the back scattering and shadow area can be clearly observed in **Figure 2A**. But in **Figure 2B** it can be seen that, under the protecting of the coating layers, the scattering from the nanoprobe and the corresponding shadow area are both cancelled, with the field for plane wave in free space recovered. By applying this cloak, the scattering cross section is reduced to 4.2 percent of the case without cloak.

In order to realize the effective parameters, the subwavelength structures designed by the effective medium theory (EMT) are periodically arranged as shown in **Figure 3**. As the required parameters are isotropic, the inserted structures are chosen as cylinders and arranged in square lattice with lattice constant of  $d$ . The radius of the embedded cylinder is  $r$ , which should be small enough to meet the requirement of EMT [26]. The important configurations that eventually determine the effective features of the composite material are the parameters of the component material and the ratio of the cylinder radius to the lattice constant. For constructing layer A with a relative permittivity 5.1, silicon cylinders are chosen and properly arranged in air. And layer B with permittivity  $11.5\epsilon_0$  is achieved by drilling cylindrical holes on silicon wafer. The illustration of unit cells for both layers is shown in **Figure 3A**, and the effective parameters can be precisely modulated by tuning the filling ratio of silicon. The effective parameters can

be retrieved by studying the reflection and transmission properties when plane wave incident to the unit cells. According to this well-established retrieving method [26], the effective parameters when changing the filling ratio are calculated and shown in **Figure 3B**. As expected, for layer A, with the increase of  $r/d$ , the relative permittivity increases monotonously, while for layer B decreases. For both cases, permeability remains the same. According to **Figure 3B**, the required effective parameters for layer A and B can be achieved at  $r/d = 0.31$  and  $r/d = 0.29$  respectively.

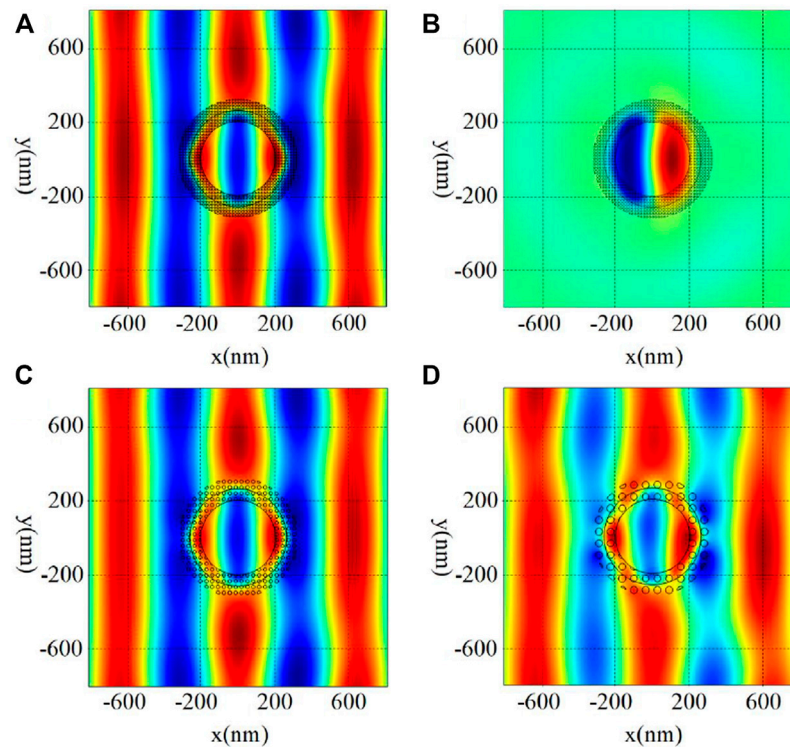
Once the ratio of the embedded cylindrical radius to the size of the unit is determined, the size of the unit cells could be designed by considering the geometry of our cloaking layers. In order to ensure the validation of effective medium approximation, the dimension of the unit cell should be small enough, but smaller nanoscale structures are more difficult to fabricate. After balancing the invisibility effect and the difficulty in practical fabrication, we choose the dimension of the unit cell as 16 nm, which is more than 20 times smaller than the wavelength and sufficiently ensure the validation of effective medium approximation. The silicon (gray) microstructures for layers A and B are shown in **Figures 3C,D**. For checking the broadband performance of the unit cell, we have studied its effective parameters between the range of 500–800 nm. The relative effective permittivity and permeability for layers A and B is given in **Figures 3E,F** by red (solid), and blue (dashed). By comparing the results with the ideal parameters (red dashed), it can be seen that in the long wavelength range from 600 to 800 nm, all the effective parameters agree well with the ideal values, which promise the good broadband performance of the unit cell and the resulting cloak. For all the cases, the relative permeability is approximately around one. As the dimension of the unit cell is enough small, all the obtained effective parameters approximately remains constant near 632.8 nm, suggesting the subwavelength structures are effective in broadband, while the value of permittivity begins to increase gradually at around 530 nm due to the dispersion effect of the lattices and the silicon material. This dispersion effect is caused by the multiple scattering among the cylinders as the wavelength



become smaller and the requirement of EMT is gradually deviated. Better invisibility effects, broader bandwidth and lower loss can be achieved with smaller unit structures, but the fabrication of such nanostructures would be difficult. All the conclusions agree well with the EMT.

In order to demonstrated the effectiveness of the design, the electric field when the 632.8 nm optical wave incidents to the nanoprobe coated by the two metamaterial layers is calculated and shown in **Figure 4A**. It can be observed that the plane wave field pattern is well recovered as if the wave is traveling in free space, and the field is nearly the same with the results given in **Figure 2B**. The scattering field from the nanoprobe is suppressed under the protecting of the coating layers as shown in **Figure 4B**, and loss is hardly observed as the structures are deep subwavelength and multiple scattering among them is very weak. For showing the robustness of

the design, we also give the performances of the coatings composed of unit cells with different sizes in **Figures 4C,D** respectively. The stealth effects of the corresponding cloaks when the cell dimension is 32 and 63 nm are also acceptable, but the coating with the 16 nm unit size generate the best invisibility effect. The results confirm the above conclusions that smaller unit cells will generate better stealth effects, as requirement of the effective medium theory is better satisfied when the unit structures are deeper subwavelength, but further decreasing the dimensions may lead to more stringent requirements for the manufacturing process. The integral is done over the intensity of scattering field in the area surrounding layer A. The performance of the device is quantitatively verified by calculating the scattering cross section when changing the incident wavelength from 500–800 nm. The scattering cross section is generally



**FIGURE 4** | (Color online) **(A,C–D)** The electric field excited by the 632.8 nm plane optical wave for the cloak constructed by the subwavelength structures with cell dimension of 16 nm **(A)**, 32 nm **(C)** and 63 nm **(D)**. **(B)** The scattering field from the nanoprobe which is under the protecting of the coating layers constructed by the subwavelength structures with cell dimension of 16 nm.

decreased by the proposed coating layers to less than 0.2 for most of the wavelength from 600–680 nm, except at the wavelength of 653 nm, where additional scattering is generated by a whispering gallery mode.

In conclusion, we proposed the scattering cancellation type invisibility coating for realizing non-invasive probing with a fiber optical nanoprobe. The coating layers can be feasibly constructed by etching in silicon wafer and SOI wafer. The effective parameters of the composite material are precisely modulated in order to meet the requirements of the scattering cancellation. Stealth effects are obtained at the wavelength of around 532 nm with the scattering cross section decreased to less than 0.2. The results show that for the given configuration, the structures are still effective even when the wavelength is 600 nm, and the scattering cross section remains a small value in the whole investigated frequency range. The ability of guiding optical waves around objects like the nanoprobe makes this design potentially useful for versatile wave manipulation applications.

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## DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

## AUTHOR CONTRIBUTIONS

WK: Methodology, writing, reviewing and editing. CT: Programming, methodology, writing, reviewing and editing. YC: Methodology, reviewing and editing.

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