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A New Four-Dimensional Chaotic System and its Circuit Implementation

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A new four-dimensional chaotic system is designed in the paper. The equilibrium point and stability of the chaotic system are analyzed, and the dynamical behaviors of the system under different parameters are analyzed by using Lyapunov exponents, Bfurcation diagram, SE and C0 complexity algorithms. The special phenomenon of the coexistence of attractors is also found. Finally, the implementation of circuit of the new system is carried out using digital signal processing (DSP) technology, and the results are consistent with the numerical simulation results, which prove the validity of the theoretical analysis. Through analysis and simulation of the system, it can be found that it has relatively rich dynamic characteristics and can be applied in areas such as confidential communication and image encryption.

Keywords: chaotic system, hidden attractor, coexistence of attractors, SE and C0 complexity, DSP implementation

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1 INTRODUCTION

In 1963, Lorenz proposed the famous Lorenz chaotic system [1], which has complex dynamical behavior although it is only three-dimensional. As an important branch of nonlinear systems, chaotic systems have good prospects for applications in secure communications, image encryption and neural networks because of their unique properties [2–24]. As the research progresses, chaotic systems have been expanded to the fractional-order domain [25–33], not only in the integer-order direction [34–40]. In both integer-order and fractional-order systems, some of them have the special phenomenon of hidden attractors.

The Sil'nikov criterion considers that a chaotic system requires at least one unstable equilibrium point [41]. However, the Sil'nikov criterion is a sufficient and unnecessary condition for the emergence of chaotic phenomena. A class of systems discovered in recent years may have only stable equilibrium points, no equilibrium points or infinitely many equilibrium points, the chaotic state is still apparent, with the attractor basin not meeting the adjacent domain of the equilibrium point, such fields of attraction are called hidden attractors [42–44].

For classical systems of chaos, such as Lorenz [1], Chen [45] and Lü [46] systems, the equilibrium points of these systems are unstable and therefore generate self-excited attractors. However, if the equilibrium point of the system is unstable, then hidden attractors may be created. Since the 21st century, more studies has been done on hidden attractors. In 2010, Vagaitsev et al. [47] discovered hidden attractors in generalized Chua's circuits firstly, and since then there has been a boom in the study of hidden attractors. In 2012, G.A. Leonov et al. [48] studied hidden attractors in smooth Chua's systems and proposed an algorithm for hidden attractors localization. In 2014, Lao et al. [49] introduced a cost function in chaotic systems with hidden attractors and used it for parameter estimation in chaotic circuit systems. In 2015, Chen et al. [50] discovered the coexistence of hidden attractors by improving the classical Chua's circuit using memristors. In 2017, M. Borah et al. [51] proposed a fractional-order chaotic system and it does not have an equilibrium point, which is a system of chaos with hidden attractors. In 2019, Cang et al. [52] discovered the coexistence of hidden attractors under different parameters in a class of Lorenz-like systems. In the same year, Zhang et al. [53] introduced a multiscroll hyperchaotic system with hidden attractors based on the Jerk system. In 2020, Deng

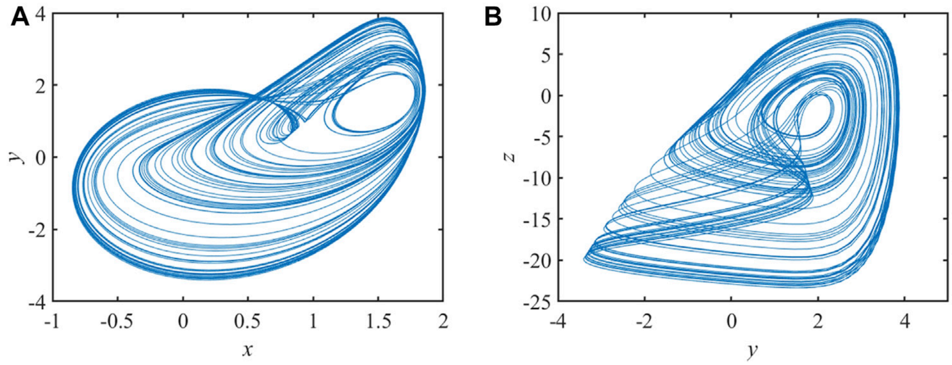

FIGURE 1 | Hidden chaotic attractor of system (1), **(A)** *x*–*y* plane **(B)** *y*–*z* plane.

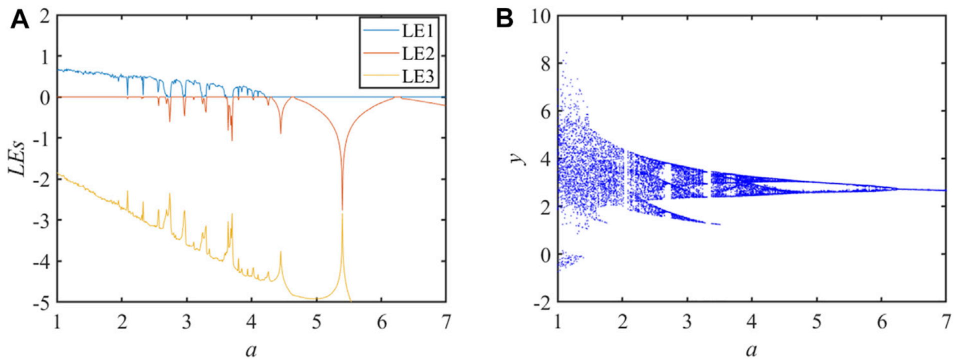

FIGURE 2 | Dynamical analysis with parameter *a*, **(A)** LEs, **(B)** BD.

TABLE 1 | The state and LEs under the change of parameter *a*.

Range	LEs	State	Range	LEs	State
1–2.69	+0 - -	Chaos	3.72–3.79	+0 - -	Chaos
2.70–2.75	0 - - -	Period	3.80–3.81	0 - - -	Period
2.76–2.95	+0 - -	Chaos	3.82–4.00	+0 - -	Chaos
2.96–2.97	0 - - -	Period	4.01–4.03	0 - - -	Period
2.98–3.23	+0 - -	Chaos	4.04–4.09	+0 - -	Chaos
3.24–3.30	0 - - -	Period	4.10–4.11	0 - - -	Period
3.31–3.59	+0 - -	Chaos	4.12–4.20	+0 - -	Chaos
3.60–3.71	0 - - -	Period	4.21–7.00	0 - - -	Period

et al. [54] proposed a multi-wing system of hidden attractors with only a single stable equilibrium point. In addition, Mou et al. [55] presented a chaotic circuit based on memristor–memcapacitor, which is a circuit system with an abundance of hidden attractors.

In addition, the coexistence of attractors is an interesting phenomenon that occurs in some chaotic systems where the parameters remain the same and the system enters different orbits and thus forms different attractors under different initial conditions due to the properties of the chaotic system itself. In dynamical systems, the presence of this phenomenon can lead to very complex behavior of the system. In the article, a novel four-dimensional chaotic system is arised from a modification of Wei system [54]. It is found that the system has stable equilibrium points and can be classified as a chaotic system with hidden attractors, while the system also has complex dynamical phenomena, including the coexistence of periodic windows and attractors. The stability, interchangeability, and agility of system can be significantly improved by using software implementation, and therefore the effectiveness of the system is verified by execution onto a DSP platform.

The remainder of this article is presented below. In **Section 2**, the mathematical model of the system is presented and the type of equilibrium points and their stability are analyzed. In **Section 3**, the dynamical behavior of the 4-dimensional system is analyzed, including Lyapunov exponent spectrum (LEs) and bifurcation diagram (BD), coexistence of attractors, SE and C0 complexity. In **Section 4**, a circuit realization of the 4-dimensional chaotic system is performed through a DSP platform. Finally, conclusions are given in **Section 5**.

2 MATHEMATICAL MODELS

2.1 Analysis of Equilibrium Points

A new chaotic system is created by inserting extra variables w and a constant k into the three-dimensional system of chaos, as follows:

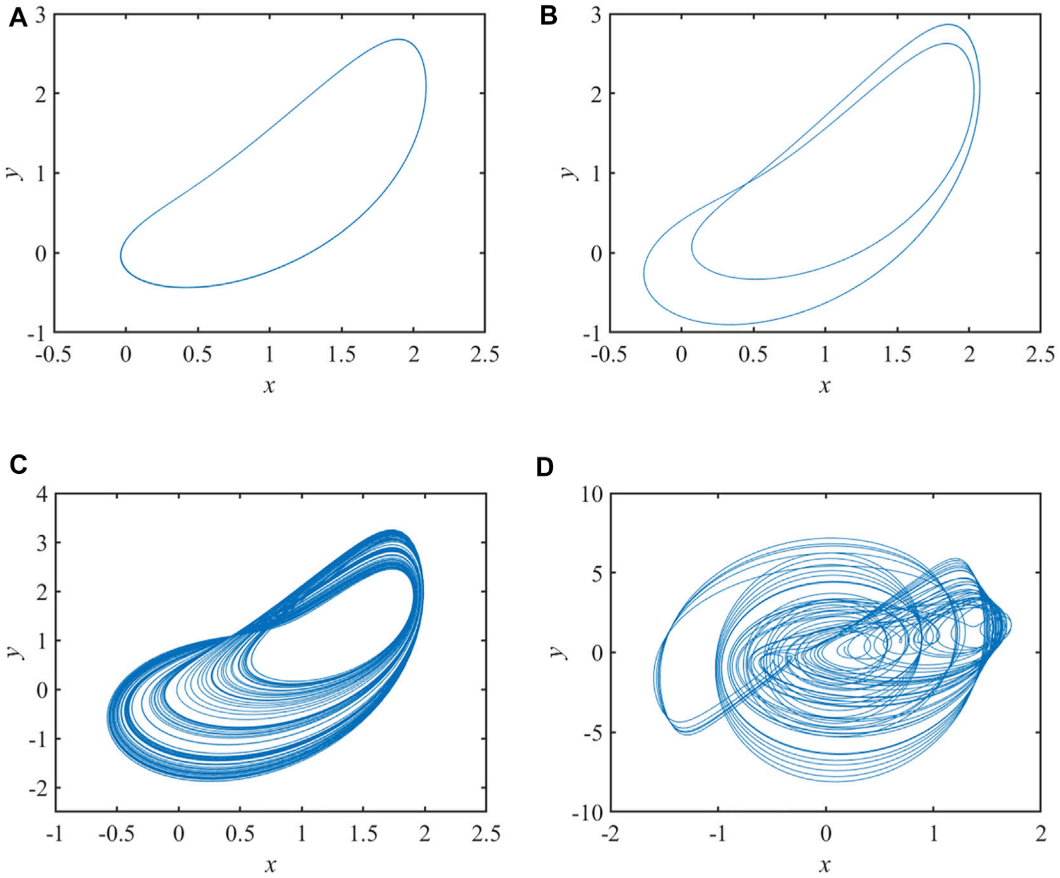

FIGURE 3 | Phase diagram of the system, **(A)** a = 6.8, **(B)** a = 5.8, **(C)** a = 4, **(D)** a = 1.2.

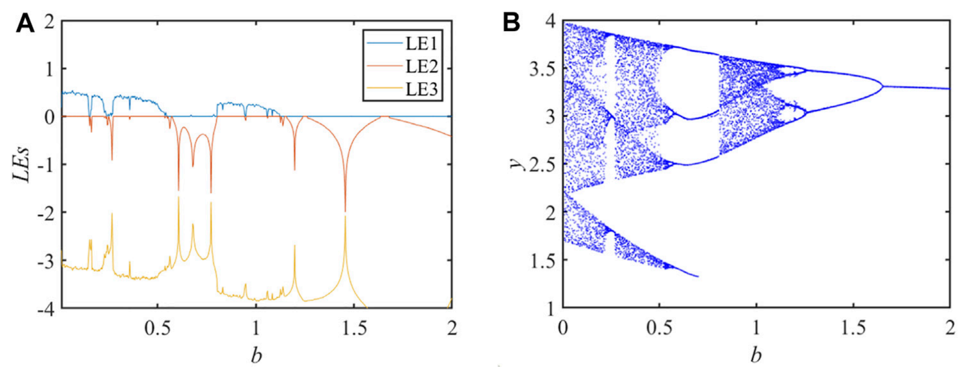

FIGURE 4 | Dynamical analysis with parameter b, **(A)** LEs, **(B)** BD.

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = -by + xz + k \\ \dot{z} = d - e^{xy} \\ \dot{w} = czw \end{cases}. \tag{1}$$

The dispersion of the system is obtained from the system dynamical equations and can be expressed as

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w}. \tag{2}$$

Setting a = 2.6, b = 0.2, c = 5, d = 17, k = 3, and the initial conditions as (1, −3, 0.1, 7). And then

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -2.3 < 0, \tag{3}$$

TABLE 2 | The state and LEs under the change of parameter *b*.

Range	LEs	State	Range	LEs	State
0.010–0.142	+0 - -	Chaos	0.786–0.798	0 - - -	Period
0.143–0.156	0 - - -	Period	0.799–0.939	+0 - -	Chaos
0.157–0.234	+0 - -	Chaos	0.940–0.945	0 - - -	Period
0.235–0.241	0 - - -	Period	0.946–1.052	+0 - -	Chaos
0.242–0.546	+0 - -	Chaos	1.053–1.057	0 - - -	Period
0.547–0.662	0 - - -	Period	1.058–1.078	+0 - -	Chaos
0.663–0.665	+0 - -	Chaos	1.079–1.081	0 - - -	Period
0.666–0.777	0 - - -	Period	1.082–1.119	+0 - -	Chaos
0.778–0.785	+0 - -	Chaos	1.120–2.000	0 - - -	Period

Eq. 2 smaller than zero, it is proved that the system is dissipative and therefore may have chaotic attractors.

Setting $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$, it means that

$$\begin{cases} a(y-x) = 0 \\ -by + xz + k = 0. \\ d - e^{xy} = 0 \\ czw = 0 \end{cases} \tag{4}$$

It can be obtained that when $d > 1$, the system has two equilibrium points E_1 $(\sqrt{\ln d}, \sqrt{\ln d}, \frac{k}{\sqrt{\ln d}} + b, 0)$ and E_2 $(\sqrt{\ln d}, \sqrt{\ln d}, \frac{k}{\sqrt{\ln d}} + b, 0)$.

Let the set of equilibrium points be P. The Jacobian matrix of system 3) at the set of equilibrium points p is

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ z & -b & x & 0 \\ -ye^{xy} & -xe^{xy} & 0 & 0 \\ 0 & 0 & cw & cz \end{bmatrix}, \tag{5}$$

and then the characteristic equation for point set

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \tag{6}$$

where $a_1 = 3.3$, $a_2 = 2.2298$, $a_3 = 0.9327$, and $a_4 = 0.2589$. The eigenvalues can be calculated from the characteristic equation as $\lambda_1 = -2.5545$, $\lambda_2 = -0.5$, $\lambda_3 = -0.1227\text{-}0.4332i$, $\lambda_4 = -0.1227 + 0.4332i$, λ_1 and λ_2 are negative real roots, and λ_3 and λ_4 are a pair of complex conjugate eigenvalues, so these two equilibrium points are saddle-focus equilibrium points, which are important for chaotic systems. According to the Routh-Hurwitz criterion, the real part of all roots is negative and satisfied with

$$\Delta_1 = a_1 > 0, \tag{7}$$

$$\Delta_2 = a_1a_2 - a_0a_3 > 0, \tag{8}$$

$$\Delta_3 = a_1a_2a_3 - a_1^2a_4 - a_0a_3^3 > 0, \tag{9}$$

$$\Delta_4 = a_4\Delta_3 > 0, \tag{10}$$

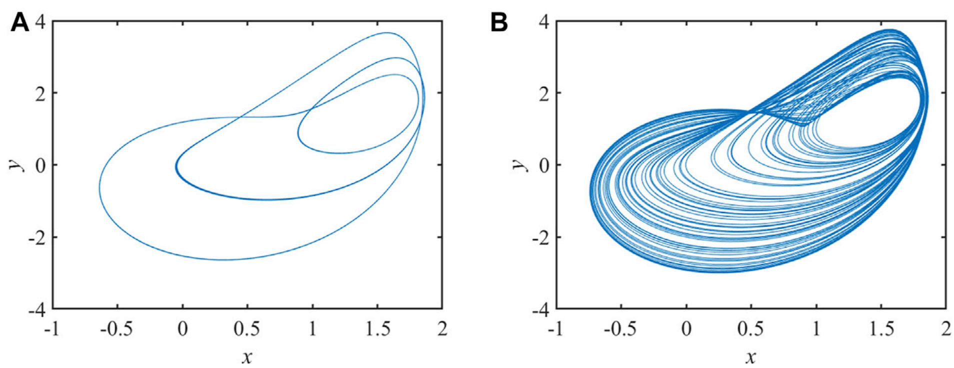

FIGURE 5 | Phase diagram of the system, **(A)** *b* = 0.70, **(B)** *b* = 0.46.

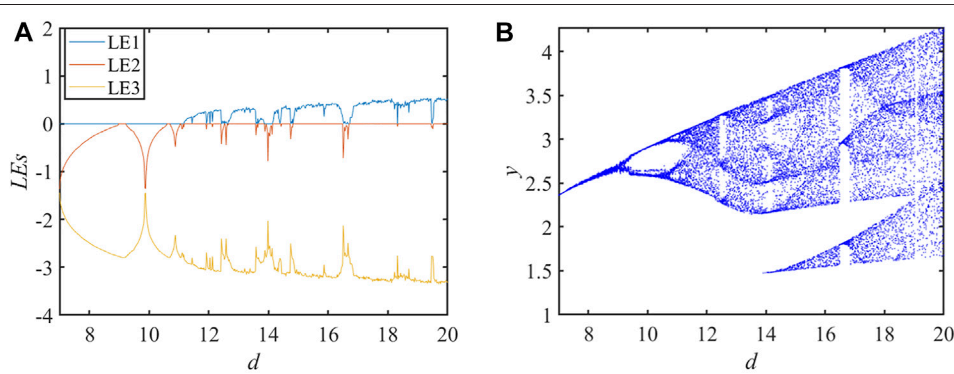

FIGURE 6 | Dynamical analysis with parameter *d*, **(A)** LEs, **(B)** BD.

TABLE 3 | The state and Les under the change of parameter *d*.

Range	LEs	State	Range	LEs	State
0–11.13	0 - - -	Period	13.65–13.84	+0 - -	Chaos
11.14–11.43	+0 - -	Chaos	13.85–14.12	0 - - -	Period
11.44–11.45	0 - - -	Period	14.13–14.37	+0 - -	Chaos
11.46–11.91	+0 - -	Chaos	14.38–14.39	0 - - -	Period
11.92–11.93	0 - - -	Period	14.40–14.73	+0 - -	Chaos
11.94–12.01	+0 - -	Chaos	14.74–14.82	0 - - -	Period
12.02–12.03	0 - - -	Period	14.83–16.48	+0 - -	Chaos
12.04–12.39	+0 - -	Chaos	16.49–16.71	0 - - -	Period
12.40–12.42	0 - - -	Period	16.72–18.30	+0 - -	Chaos
12.43–12.62	+0 - -	Chaos	18.31–18.32	0 - - -	Period
12.63–12.64	0 - - -	Period	18.33–19.45	+0 - -	Chaos
12.65–13.55	+0 - -	Chaos	19.46–19.47	0 - - -	Period
13.56–13.64	0 - - -	Period	19.48–20.00	+0 - -	Chaos

Thus, both equilibria E_1 and E_2 are asymptotically stable.

3 DYNAMICAL BEHAVIORS

3.1 Chaotic Attractor

In system (1), let $a = 2.6$, $b = 0.2$, $c = 5$, $d = 17$, $k = 3$, and the initial values are (1, −3, −0.1, 7). The phase diagrams of chaotic attractors in the system in different phase planes are shown in **Figure 1**.

3.2 Lyapunov Exponents and Bifurcation Diagram of the System

In the system, which is a four-dimensional chaotic system, the BD is analyzed in combination with the LEs to obtain the states with different parameters. In the following, parameters *a*, *b* and *d* will be set as variables with initial values of (1, −3, −0.1, 7) and step size $h = 0.01$. Only parameters *a*, *b* and *d* will be changed, the rest will remain unchanged and the state of the system will be observed.

With the parameters $a\in$ [1, 7], let $b = 0.2$, $c = 5$, $d = 17$, $k = 3$, and initial values of (1, -3, −0.1, 7), the LEs and BD of the system are shown in **Figure 2**.

The LEs can clearly see the change of the system state when the parameter *a* changes, and the BD can be obtained from the multiplicative period bifurcation into chaos, and it is known after analysis that the BD corresponds exactly to the LEs. The system states are shown in **Table 1**, which gives a clear view of the range of parameters of the system in different states.

According to the data in **Table 1**, the cyclical and chaotic states of the system under the transformation of parameter *a* are shown in **Figure 3**.

Taking the parameter $b\in[0.01, 2]$, let $a = 2.6$, $c = 5$, $d = 17$, $k = 3$, and the initial values are (1, −3, −0.1, 7), the LEs and the

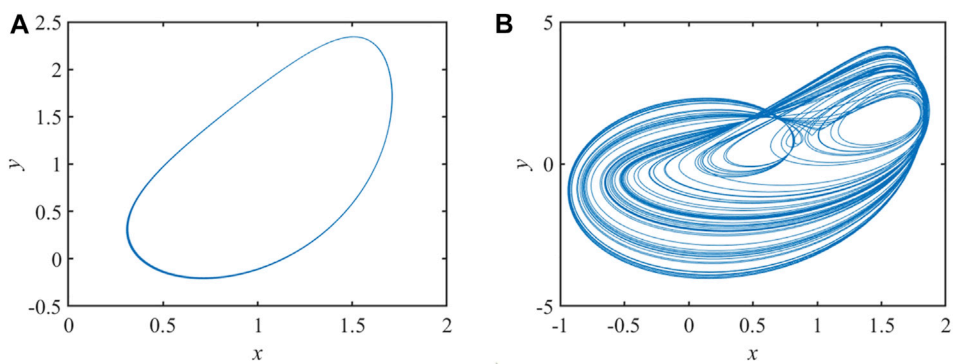

FIGURE 7 | Phase diagram of the system, **(A)** $d = 7.0$, **(B)** $d = 19.0$.

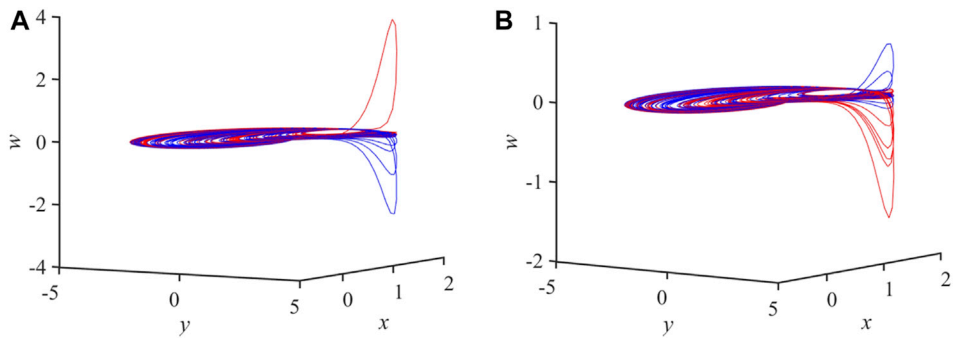

FIGURE 8 | The phase diagram of coexistence attractor, **(A)** initial values are (1, −3, −0.4, 7) and (−0.1, 0.4, −0.1,7), **(B)** initial values are (−2, 3.6, −0.1, 7) and (1, −3, −0.1, 7).

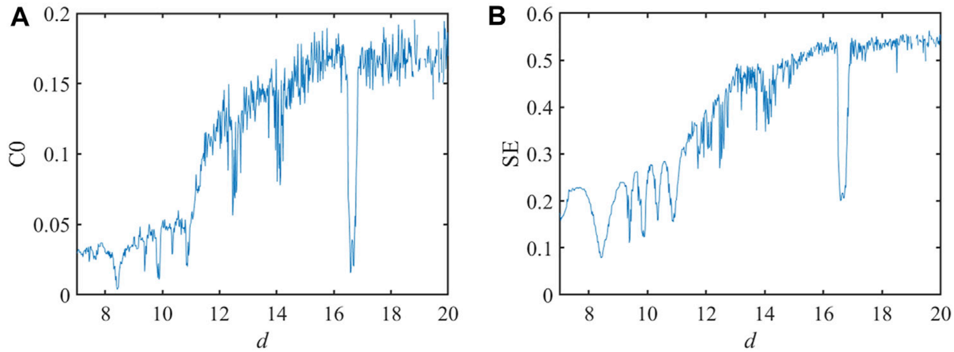

FIGURE 9 | Structural complexity curve with parameter d, **(A)** C0 Complexity, **(B)** SE Complexity.

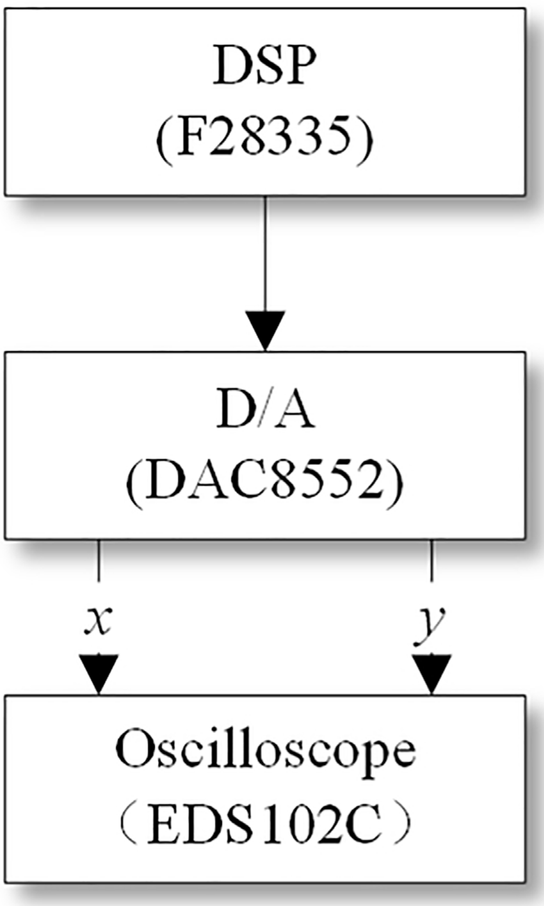

FIGURE 10 | The flow chart.

BD under this condition are displayed at **Figure 4**. As the parameter b varies, cyclical and chaotic regions are evident in the system.

The state table of the system as the parameters b change is shown in **Table 2**. It is evident that the system undergoes another period-doubling bifurcation after going through a period-doubling bifurcation into chaos. The analysis shows that the BD corresponds exactly to the LEs.

The state of the system with parameter b transformed is displayed at **Figure 5**.

Taking the parameter $d \in [7, 20]$, setting $a = 2.6$, $b = 0.2$, $c = 5$, $k = 3$, and initial values of (1, −3, −0.1, 7), LEs and BD for this system are displayed at **Figure 6**. The system exhibits complex dynamics of chaos and alternating periods when the parameter d is varied.

The state table of the system when the state varies with the parameter d is shown in **Table 3**. It can be seen that the chaotic range of the system is relatively large at this time, while there is a clear periodic window.

When the parameter d is varied, the system produces the periodic and chaotic state displayed at **Figure 7**.

3.3 Coexistence of Attractors

An interesting phenomenon that exists in chaotic systems, namely the coexistence of attractors. When the parameters of the system are constant and the initial values are varied, the trajectory of the system can gradually vary to a different state of motion. Setting $a = 2.6$, $b = 0.2$, $c = 5$, $d = 17$, $k = 3$. When the initial values are set to (1, −3, −0.4, 7) and (−0.1, 0.4, −0.1, 7), the coexistence of attractors of the system is displayed at **Figure 8A**, and when the initial values are set to (−2, 3.6, −0.1, 7) and (1, −3, −0.1, 7), the coexistence of attractors of the system is displayed at **Figure 8B**.

3.4 Complexity Analysis

Another important aspect of the study of chaotic systems is the study of their complexity. The degree of closeness of a chaotic sequence to a random sequence, as shown using the corresponding algorithm, is the complexity of a chaotic system. The higher the value of complexity, the closer the system is to a random sequence and the safer it is [56, 57]. In this paper, the SE algorithm and the C0 algorithm are used to analyse the complexity of the structure.

The SE (spectral entropy) algorithm performs a Fourier transform on the sequence and then obtains the spectral entropy value by combining the energy density in the frequency domain with the Shannon entropy. The C0

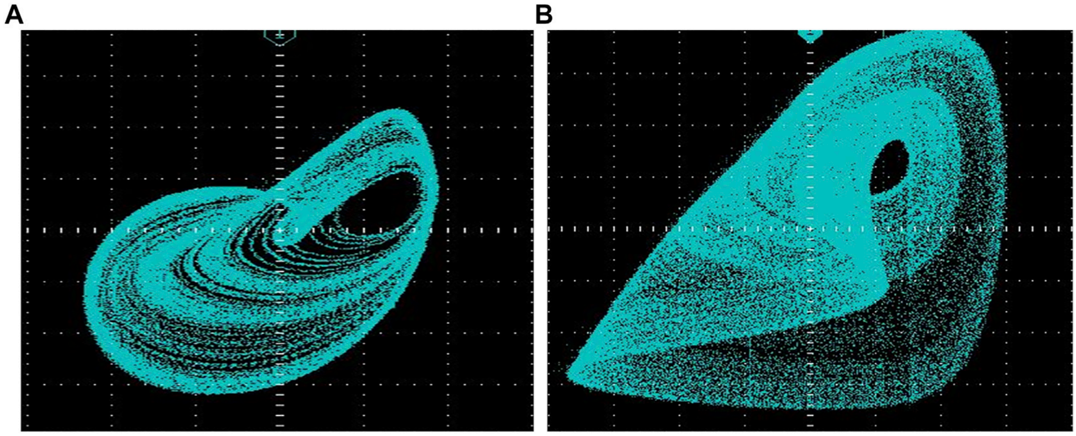

FIGURE 11 | The phase diagram of DSP simulation, **(A)** *x–y* plane **(B)** *y–z* plane.

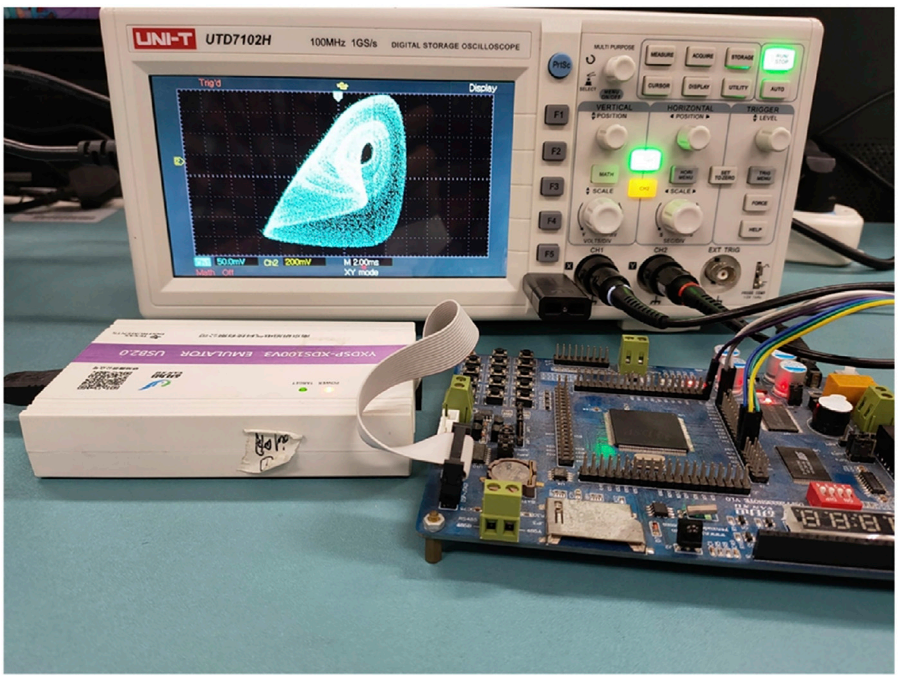

FIGURE 12 | DSP simulation physical picture.

algorithm, on the other hand, involves decomposing the sequence to produce two parts of regularity and irregularity, and then measuring the proportion of the irregular part to obtain the result.

In the section, we will analyse the complexity of the system under the variation of parameter d. When taking parameters $a = 2.6$, $b = 0.2$, $c = 5$, $k = 3$, and $d \in [7, 20]$, the results obtained are shown in **Figure 9**.

As can be seen from **Figure 9**, the SE algorithm and the C0 algorithm are highly synchronized, and the system is in the limit ring state when $d < 11.30$, in the chaotic state when $d \in [11.30, 12.43]$, in the limit ring state when $d \in [12.44, 12.60]$, in the chaotic state when $d \in [12.61, 14.01]$, in the limit ring state when $d \in [14.02, 14.17]$, in the chaotic state when $d \in [14.18, 16.48]$, in the limit ring state when $d \in [16.49, 16.82]$, and when $d > 16.83$, the system enters the chaotic state again. By analyzing the SE complexity diagram and the C0 complexity diagram, it can be found that the complexity is at a low point when the system is in the periodic state and increases significantly when the system enters the chaotic state, and the presented results are consistent with the LEs and the BD displayed at **Figure 6**.

4 DSP IMPLEMENTATION

Since chaotic systems are susceptible to external perturbations when implemented using analog circuits, the relevant characteristic conditions in the actual circuit are more difficult to control accurately, so they can be implemented using DSP. The DSP chip F28335, with its fast computing speed, high accuracy and low environmental impact, was chosen as the empirical platform to validate the novel system of chaos. As software can only receive digital signals, a continuous chaotic system must be discretized in order to handle the system on a DSP platform firstly. Therefore, we discretize the continuous system of chaos and transform it into a discrete chaotic sequence by the approach of fourth-order Runge-Kutta, and then write into the DSP chip using a programming language. Here a stack operation is devised to make sure the data is not corrupted as much as possible. On the output side of the DSP, a D/A converter (DAC8552) is used to convert the digital sequence into an analogue sequence which is then sent to an oscilloscope (EDS102C) so that the oscilloscope can better capture the image. The overall workflow is shown in **Figure 10**.

The parameters are given as $a = 2.6$, $b = 0.2$, $c = 5$, $d = 17$, $k = 3$, and the initial conditions as (1, −3, −0.1,7), the images are obtained as shown in **Figure 11**. By comparing with **Figure 1**, it is clear that the results obtained using DSP are consistent with the simulations carried out in MATLAB.

The f28335 chip, D/A converter and oscilloscope used for the system DSP simulation are shown in **Figure 12**.

5 CONCLUSION

The article proposes a new four-dimensional chaotic system and investigated its dynamic properties. In the study of this chaotic system, different chaotic attractors are found. By analysing the LEs, the BD and the complexity of this system, it can be understood that the system exhibits a dramatic degree of complexity in its dynamic properties as the parameters changed of the system. By numerical simulations, the peculiar phenomenon of the coexistence of chaotic attractors was observed by us. The results show that the chaotic system has very complex dynamic properties. Finally, the circuit was built and tested on a DSP platform, and a comparison of the test results with the numerical simulation results shows a high degree of consistency. The article provides a reference for the study of chaotic systems and circuit experiments, and has good prospects for applications in information encryption and secure communication.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

XW built the system model, analysed the dynamics, performed the circuit implementation and wrote the manuscript. YF and YC supervised the work and revised the manuscript. Both authors read and approved the final manuscript.

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