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Asymmetric Quantum Steering Generated by Triple-Photon Down-Conversion Process With Injected Signals

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Asymmetric quantum steering generated by the triple-photon down-conversion process in an injected signal optical cavity is investigated. The triple-photon down-conversion process can be realized in an optical superlattice by quasi-phase-matching technology. Asymmetric quantum steering can be obtained in this triple-photon down-conversion process. The direction of asymmetric quantum steering can be controlled by adjusting the parameters of the nonlinear process. The generation of asymmetric quantum steering in the present scheme has potential applications in quantum secret sharing and quantum networks.

Keywords: quantum steering, EPR steering, asymmetric quantum steering, cascaded nonlinear, quasi-phase-matching

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1 INTRODUCTION

As an extension of the Einstein–Podolsky–Rosen (EPR) paradox [2], Schrödinger found the phenomenon of quantum steering in 1935 [1]. Quantum steering is a sufficient condition for quantum entanglement and a necessary condition for Bell nonlocality [3]. Wiseman *et al.* [4] gave a mathematically operable definition for quantum steering in 2007. In addition, they raised the question of whether there is asymmetric quantum steering, that is, A can steer B, but conversely B cannot steer A. According to Reid's quantum steering criteria [5], this question was quickly answered both theoretically [6, 7] and experimentally [8]. The research shows that asymmetric quantum steering is a universal property, which does not depend on Gaussian measurement [9]. Intracavity second harmonic generation [10] and atomic Bose–Hubbard chain [11, 12] can produce asymmetric quantum steering based on continuous variables. The asymmetric quantum steering in four-mode cluster states was measured experimentally [13]. An optical parametric oscillator (OPO) is used in many quantum optical experiments [14]. For example, it can produce three-color entanglement [15, 16]. By using the two down-converted optical fields of a nondegenerate OPO, Ou *et al.* proposed that EPR steering was experimental feasible [17]. For both degenerate [18] and nondegenerate [19] cases, the nonlinear conversion efficiency can be improved by injecting signals into the low-frequency mode. He and Reid confirmed the existence of N-partite EPR steering and developed the concept of genuine N-partite EPR steering, and put forward the criteria for genuine multipartite EPR steering [20]. A scheme is proposed for experimental generation of a highly versatile and flexible repository of multipartite steering using an optical frequency comb and ultrafast pulse shaping [21]. Collective multipartite EPR steering can be generated by cascaded four-wave mixing of rubidium atoms [22]. The research on multipartite quantum steering has attracted much attention [23–28]. Olsen [29]

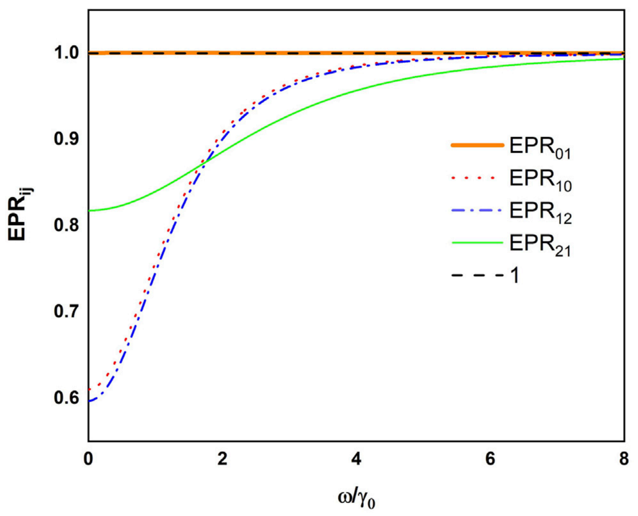

FIGURE 1 | Values of EPR$_{jk}$ and EPR$_{kj}$ versus the normalized Fourier analysis frequency ω/γ_0 with γ_0=0.01, γ_1=2γ_0, γ_2=4γ_0, γ_3=3γ_0, κ =0.01, ϵ_0=0.45, ϵ_1=0.1, and ϵ_2=0.2.

used a nondegenerate parametric oscillator with an injected signal to show how the directionality and extent of the steering can be readily controlled for output modes. Wang and Li analyzed theoretically and experimentally bichromatic entanglement between the signal and the idle [30]. Kalaga and Leoński analyzed the relations between entanglement and steering for a two-mode mixed state [31] and three qubit system [32], respectively. Cao and Guo [33] not only provided the mathematical basis and characterization for Bell delocalization and EPR steering but also derived a sufficient condition to judge whether the state can be steered. Recently, the hybrid ferrimagnet–light system of two macroscopic magnons has made a huge breakthrough. Zheng *et al.* [34] found that entanglement can be significantly enhanced and strong two-way asymmetric quantum steering appears between two magnons.

Rojas González *et al.* [35] gave the first theoretical demonstration of continuous-variable triple-photon state quantum entanglement. They also found that quantum entanglement among the three modes disappeared in the case of spontaneous parametric triple-photon generation. However, the genuine triple-photon entanglement can be obtained in the case of injection signal [35]. Agustí *et al.* [36] showed that the state generated by a three-mode spontaneous parametric down-conversion (SPDC) was the non-Gaussian state, and the states that were generated by superconducting-circuit implementation of the three-mode SPDC had tripartite entanglement based on the criteria built from three-mode correlation functions. However, the quantum steering correlation, especially the asymmetric quantum steering correlation, in the triple-photon down-conversion process has not been studied. In this article, the asymmetric quantum steering generated by the triple-photon down-conversion process with two injected signals is investigated. We demonstrate that the asymmetric quantum steering can be generated by this nonlinear process and also show how to control the asymmetry of quantum steering by adjusting the intensity of the injected signals. Similar to quantum key distribution, in quantum secret sharing [37–39], the confidentiality of shared information does not depend on computational assumptions, but on the uncertainty and non-cloning of quantum mechanics. Xiang *et al.* [23] designed a protocol based on EPR steering and extended the protocol to three-user scenarios to distribute richer steerability properties including one-to-multimode steering and collective steering that can be used for one-sided device-independent quantum secret sharing. We think that the present scheme of the generation of asymmetric quantum steering has potential applications in quantum secret sharing and quantum networks.

2 THEORY

The system consists of a nondegenerate OPO that is driven by an external coherent pump field with the frequency of ω_0. Triple-photon SPDC can be achieved in the optical cavity by using the quasi-phase-matching (QPM) technology [40]. The frequencies of the three parametric optical fields are ω_1, ω_2, and ω_3, respectively, which satisfies the energy conservation relationship $\omega_0 = \omega_1 + \omega_2 + \omega_3$. The phase mismatch in this nonlinear process is compensated by the reciprocal lattice vector provided by the optical superlattice. The interaction Hamiltonian for the triple-photon SPDC can be written as

$$\mathcal{H}_I = i\hbar\kappa\hat{a}_0\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3^\dagger + h.c., \tag{1}$$

where κ represents the effective nonlinearity of optical superlattice that can be taken as real [41]. $\hat{a}_i\,(i = 0, 1, 2, 3)$ is the bosonic annihilation operator of the cavity mode with the frequency ω_i. The Hamiltonian of the external input fields is

$$\mathcal{H}_{ext} = i\hbar\left(\epsilon_0\hat{a}_0^\dagger + \epsilon_1\hat{a}_1^\dagger + \epsilon_2\hat{a}_2^\dagger\right) + h.c., \tag{2}$$

where ϵ_0 is the amplitude of pump. ϵ_1 and ϵ_2 represent the injected signal fields, and they are also considered to be real. If $\epsilon_1 = \epsilon_2 = 0$, it is the case of spontaneous down-conversion. Because there is no quantum correlation among the output optical fields in the spontaneous down-conversion case [35], in this work, we will study the asymmetric quantum steering characteristics among the output optical fields with two injected signals. We assume that all the optical fields are resonant in the cavity. Following the description of Lindblad superoperator, the losses of the optical fields in the cavity can be written as

$$\mathcal{L}\hat{\rho} = \gamma_i\left(2\hat{a}_i\hat{\rho}\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\hat{\rho} - \hat{\rho}\hat{a}_i^\dagger\hat{a}_i\right), \tag{3}$$

where $\gamma_i\,(i = 0, 1, 2, 3)$ represents the cavity loss of the optical field with the frequency ω_i. $\hat{\rho}$ is the system density matrix. The master equation of this system can be expressed as

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\mathcal{H}_I + \mathcal{H}_{pump}, \hat{\rho}\right] + \sum\mathcal{L}\hat{\rho}. \tag{4}$$

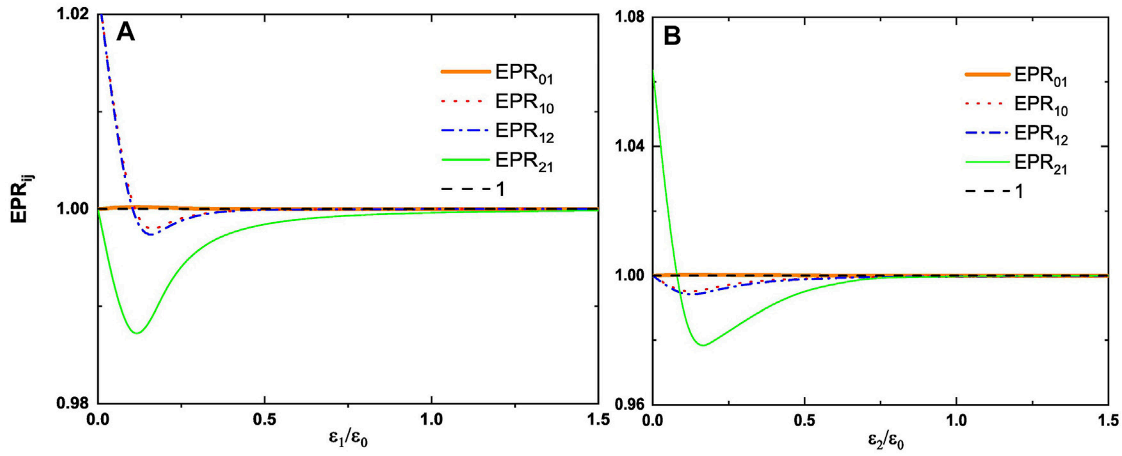

FIGURE 2 | Values of EPR$_{jk}$ and EPR$_{kj}$ versus **(A)** ϵ_1/ϵ_0 and **(B)** ϵ_2/ϵ_0, respectively.

One can obtain the Fokker–Planck equation in the positive-P representation for studying the characteristics of quantum steering [42, 43]. The third-order derivatives can be reasonably negligible. Then, the Fokker–Planck equation can be given as

$$\begin{aligned}\frac{dP}{dt} = \Big\{&-(\epsilon_0-\gamma_0\alpha_0-\kappa\alpha_1\alpha_2\alpha_3)\frac{\partial}{\partial\alpha_0}-(\epsilon_0^*-\gamma_0^\dagger\alpha_0^\dagger-\kappa\alpha_1^\dagger\alpha_2^\dagger\alpha_3^\dagger)\frac{\partial}{\partial\alpha_0^\dagger}\Big\}\\ &-(\epsilon_1-\gamma_1\alpha_1+\kappa\alpha_0\alpha_2^\dagger\alpha_3^\dagger)\frac{\partial}{\partial\alpha_1}-(\epsilon_1^*-\gamma_1\alpha_1^\dagger+\kappa\alpha_0^\dagger\alpha_2\alpha_3)\frac{\partial}{\partial\alpha_1^\dagger}\\ &-(\epsilon_2-\gamma_2\alpha_2+\kappa\alpha_0\alpha_2^\dagger\alpha_3^\dagger)\frac{\partial}{\partial\alpha_2}-(\epsilon_2^*-\gamma_2\alpha_2^\dagger+\kappa\alpha_0^\dagger\alpha_1\alpha_3)\frac{\partial}{\partial\alpha_2^\dagger}\\ &-(-\gamma_3\alpha_3+\kappa\alpha_0\alpha_1^\dagger\alpha_2^\dagger)\frac{\partial}{\partial\alpha_3}-(-\gamma_3\alpha_3^\dagger+\kappa\alpha_0^\dagger\alpha_1\alpha_2)\frac{\partial}{\partial\alpha_3^\dagger}\\ &+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}(2\kappa\alpha_0\alpha_3^\dagger)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1^\dagger\partial\alpha_2^\dagger}(2\kappa\alpha_0^\dagger\alpha_3)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1\partial\alpha_3}(2\kappa\alpha_0\alpha_2^\dagger)\\ &+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1^\dagger\partial\alpha_3^\dagger}(2\kappa\alpha_0^\dagger\alpha_2)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_2\partial\alpha_3}(2\kappa\alpha_0\alpha_1^\dagger)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1^\dagger\partial\alpha_3^\dagger}(2\kappa\alpha_0^\dagger\alpha_1).\end{aligned} \tag{5}$$

Following the normal processing, the equations of motion of the cavity modes can be obtained as

$$\begin{aligned}\frac{d\alpha_1}{dt} &= \epsilon_1-\gamma_1\alpha_1+\kappa\alpha_0\alpha_2^\dagger\alpha_3^\dagger+\sqrt{2\kappa\alpha_0\alpha_3^\dagger}\eta_1+\sqrt{2\kappa\alpha_0\alpha_2^\dagger}\eta_2,\\ \frac{d\alpha_1^\dagger}{dt} &= \epsilon_1^*-\gamma_1\alpha_1^\dagger+\kappa\alpha_0^\dagger\alpha_2\alpha_\dagger+\sqrt{2\kappa\alpha_0^\dagger\alpha_3}\eta_1^\dagger+\sqrt{2\kappa\alpha_0^\dagger\alpha_2}\eta_2^\dagger,\\ \frac{d\alpha_2}{dt} &= \epsilon_2-\gamma_2\alpha_2+\kappa\alpha_0\alpha_2^\dagger\alpha_3^\dagger+\sqrt{2\kappa\alpha_0\alpha_3^\dagger}\eta_1+\sqrt{2\kappa\alpha_0\alpha_1^\dagger}\eta_3,\\ \frac{d\alpha_2^\dagger}{dt} &= \epsilon_2^*-\gamma_2\alpha_2^\dagger+\kappa\alpha_0^\dagger\alpha_1\alpha_3+\sqrt{2\kappa\alpha_0^\dagger\alpha_3}\eta_1^\dagger+\sqrt{2\kappa\alpha_0^\dagger\alpha_1}\eta_3^\dagger,\\ \frac{d\alpha_3}{dt} &= -\gamma_3\alpha_3+\kappa\alpha_0\alpha_1^\dagger\alpha_2^\dagger+\sqrt{2\kappa\alpha_0\alpha_2^\dagger}\eta_2+\sqrt{2\kappa\alpha_0\alpha_1^\dagger}\eta_3,\\ \frac{d\alpha_3^\dagger}{dt} &= -\gamma_3\alpha_3^\dagger+\kappa\alpha_0^\dagger\alpha_1\alpha_2+\sqrt{2\kappa\alpha_0^\dagger\alpha_2}\eta_2^\dagger+\sqrt{2\kappa\alpha_0^\dagger\alpha_1}\eta_3^\dagger,\\ \frac{d\alpha_0}{dt} &= \epsilon_0-\gamma_0\alpha_0-\kappa\alpha_1\alpha_2\alpha_3, \frac{d\alpha_0^\dagger}{dt} = \epsilon_0^*-\gamma_0^\dagger\alpha_0^\dagger-\kappa\alpha_1^\dagger\alpha_2^\dagger\alpha_3^\dagger.\end{aligned} \tag{6}$$

The aforementioned eight coupled stochastic differential equations can be solved by the linearization method. One can expand the positive-P variables into their steady-state expectation values plus delta-correlated Gaussian fluctuation terms as $\alpha_i = A_i + \delta\alpha_i$ ($i = 0, 1, 2, 3$) with $\delta\alpha_i \ll A_i$. A_i is the steady-state solution of the cavity mode $\hat{a}_i$, which can be obtained when the noise terms are ignored by setting $d\alpha_i/dt = 0$ in **Eq. 6**. Complex variable α_i corresponds to the normally ordered expectation value of the operator $\hat{a}_i$. In this case, **Eq. 6** can be linearized as

$$\begin{aligned}\frac{d}{dt}\delta\alpha_1 &= -\gamma_1\delta\alpha_1+\kappa A_2'A_3'\delta\alpha_0+\kappa A_0A_3'\delta\alpha_2^\dagger+\kappa A_0A_2'\delta\alpha_3^\dagger+\sqrt{2\kappa A_0A_3'}\eta_1+\sqrt{2\kappa A_0A_2'}\eta_2,\\ \frac{d}{dt}\delta\alpha_1^\dagger &= -\gamma_1\delta\alpha_1^\dagger+\kappa A_2A_3\delta\alpha_0^\dagger+\kappa A_0'A_3\delta\alpha_2+\kappa A_0'A_2\delta\alpha_3+\sqrt{2\kappa A_0'A_3}\eta_1^\dagger+\sqrt{2\kappa A_0'A_2}\eta_2^\dagger,\\ \frac{d}{dt}\delta\alpha_2 &= -\gamma_2\delta\alpha_2+\kappa A_1'A_3'\delta\alpha_0+\kappa A_0A_3'\delta\alpha_1^\dagger+\kappa A_0A_1'\delta\alpha_3^\dagger+\sqrt{2\kappa A_0A_3'}\eta_1+\sqrt{2\kappa A_0A_1'}\eta_3,\\ \frac{d}{dt}\delta\alpha_2^\dagger &= -\gamma_2\delta\alpha_2^\dagger+\kappa A_1A_3\delta\alpha_0^\dagger+\kappa A_0'A_3\delta\alpha_1+\kappa A_0'A_1\delta\alpha_3+\sqrt{2\kappa A_0'A_3}\eta_1^\dagger+\sqrt{2\kappa A_0'A_1}\eta_3^\dagger,\\ \frac{d}{dt}\delta\alpha_3 &= -\gamma_3\delta\alpha_3+\kappa A_1'A_2'\delta\alpha_0+\kappa A_0A_2'\delta\alpha_1^\dagger+\kappa A_0A_1'\delta\alpha_2^\dagger+\sqrt{2\kappa A_0A_2'}\eta_2+\sqrt{2\kappa A_0A_1'}\eta_3,\\ \frac{d}{dt}\delta\alpha_3^\dagger &= -\gamma_3\delta\alpha_3^\dagger+\kappa A_1A_2\delta\alpha_0^\dagger+\kappa A_0'A_2\delta\alpha_1+\kappa A_0'A_1\delta\alpha_2+\sqrt{2\kappa A_0'A_3}\eta_2^\dagger+\sqrt{2\kappa A_0'A_1}\eta_3^\dagger,\\ \frac{d}{dt}\delta\alpha_0 &= -\gamma_0\delta\alpha_0-\kappa A_1A_2\delta\alpha_3-\kappa A_1A_3\delta\alpha_2-\kappa A_2A_3\delta\alpha_1,\\ \frac{d}{dt}\delta\alpha_0^\dagger &= -\gamma_0\delta\alpha_0^\dagger-\kappa A_1'A_2'\delta\alpha_3^\dagger-\kappa A_1'A_3'\delta\alpha_2^\dagger-\kappa A_2'A_3'\delta\alpha_1^\dagger.\end{aligned} \tag{7}$$

The resulting equations can be written for the vector of fluctuation terms as

$$d\delta\tilde{\alpha} = -\mathbf{A}\delta\tilde{\alpha}dt + \mathbf{B}dW, \tag{8}$$

where **A** is the drift matrix, **B** is the noise term that contains the steady-state solutions, and dW is a vector of Wiener increments [43]. The drift matrix **A** is obtained as

$$\mathbf{A} = \begin{pmatrix} \gamma_0 & 0 & \kappa A_2A_3 & 0 & \kappa A_1A_3 & -0 & \kappa A_1A_2 & 0\\ 0 & \gamma_0 & 0 & \kappa A_2'A_3' & 0 & \kappa A_1'A_3' & 0 & \kappa A_1'A_2'\\ -\kappa A_2'A_3' & 0 & \gamma_1 & 0 & 0 & -\kappa A_0A_3' & 0 & -\kappa A_0A_2'\\ 0 & -\kappa A_2A_3 & 0 & \gamma_1 & -\kappa A_0A_3' & 0 & -\kappa A_0'A_2 & 0\\ -\kappa A_1'A_3' & 0 & 0 & -\kappa A_0A_3' & \gamma_2 & 0 & 0 & -\kappa A_0A_1'\\ 0 & -\kappa A_1A_3 & -\kappa A_0A_3' & 0 & 0 & \gamma_2 & -\kappa A_0'A_1 & -0\\ -\kappa A_1'A_2' & 0 & 0 & -\kappa A_0A_2' & 0 & -\kappa A_0A_1' & \gamma_3 & 0\\ 0 & -\kappa A_1A_2 & -\kappa A_0'A_2 & 0 & -\kappa A_0'A_1 & 0 & 0 & \gamma_3 \end{pmatrix}. \tag{9}$$

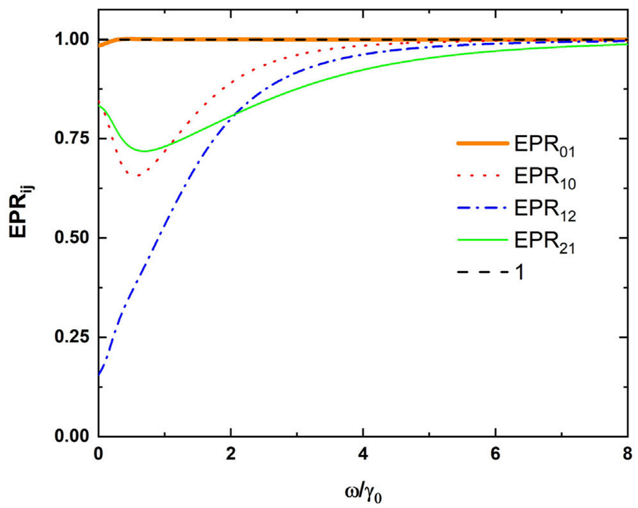

FIGURE 3 | Values of EPR_{jk} and EPR_{kj} versus the Fourier analysis frequency ω with γ_0=0.1, γ_1=2γ_0, γ_2=4γ_0, γ_3=0.3γ_0, κ =0.01, ϵ_0=0.45, ϵ_1=0.1, and ϵ_2=0.2.

Equation 8 can be solved via the Fourier transform. Then, one can obtain the intracavity spectra as

$$\mathbf{S}(\omega) = (\mathbf{A} + i\omega\mathbf{I})^{-1}\mathbf{B}\mathbf{B}^{\mathrm{T}}(\mathbf{A}^{\mathrm{T}} - i\omega\mathbf{I})^{-1}, \tag{10}$$

where ω is the Fourier analysis frequency and $\mathbf{I}$ is the identity matrix. According to the standard input–output relationship [44], the output spectra can be calculated through **Eq. 10**. The quadrature amplitude and phase can be defined as $\hat{X}_j = \hat{a}_j + \hat{a}_j^{\dagger}$ and $\hat{Y}_j = -i(\hat{a}_j - \hat{a}_j^{\dagger})$. We calculate $S^q(\omega) = QSQ^T$, where Q is the block diagonal **8 × 8** matrix. Then, the output spectral variances and covariances for the cavity modes i and j can be obtained from $S^q(\omega)$. EPR steering can be demonstrated based on the Reid criterion, and the inferred variances are written as [5]

$$V_{\inf}(\hat{X}_{ij}) = V(\hat{X}_i) - \frac{[V(\hat{X}_i, \hat{X}_i)]^2}{V(\hat{X}_j)},$$
$$V_{\inf}(\hat{Y}_{ij}) = V(\hat{Y}_i) - \frac{[V(\hat{Y}_i, \hat{Y}_i)]^2}{V(\hat{Y}_j)}, \tag{11}$$

where $V(\hat{X}, \hat{Y}) = \langle \hat{X}\hat{Y}\rangle - \langle \hat{X}\rangle\langle \hat{Y}\rangle$ and $V_{\inf}(\hat{X}_{ij})$ denotes the variance of $\hat{X}_i$ as inferred by measurement made of $\hat{X}_j$. If the product of these two inferred variances is less than 1, one can say that mode i can be steered by the measurement of mode j, and EPR steering is demonstrated for the two cavity modes.

EPR_{jk} is the product of the $\hat{X}_{jk}$ and $\hat{Y}_{jk}$ inferred variance. However, EPR_{jk} is not always equal to EPR_{kj}. If one of EPR_{jk} and EPR_{kj} is more than or equal to 1 and the other is less than 1, there is asymmetric quantum steering between cavity modes k and j.

3 RESULTS

Because asymmetric quantum steering between the other optical fields are not obvious enough, we only investigate the asymmetric quantum steering characteristics between the optical fields $\hat{a}_0$ and $\hat{a}_1$ and between the optical fields $\hat{a}_1$ and $\hat{a}_2$, respectively. In the following, we choose the pairs (ω_0, ω_1) and (ω_1, ω_2) to analyze the influences of the normalized analysis frequency ω/γ_0 and the injected signal amplitudes ϵ_1 and ϵ_2 on the asymmetric quantum steering.

Figure 1 depicts the values of EPR_{jk} and EPR_{kj} versus the normalized analysis frequency ω/γ_0. One can see that $\mathrm{EPR}_{01} = 1$, while $\mathrm{EPR}_{10} < 1$ in the whole range of the normalized analysis frequency, which shows that the output fields $\hat{a}_0$ and $\hat{a}_1$ exhibit asymmetric quantum steering. That is, the optical field $\hat{a}_0$ can steer the optical field $\hat{a}_1$, but $\hat{a}_1$ cannot steer $\hat{a}_0$. However, both EPR_{12} and EPR_{21} are less than 1, which shows that the output

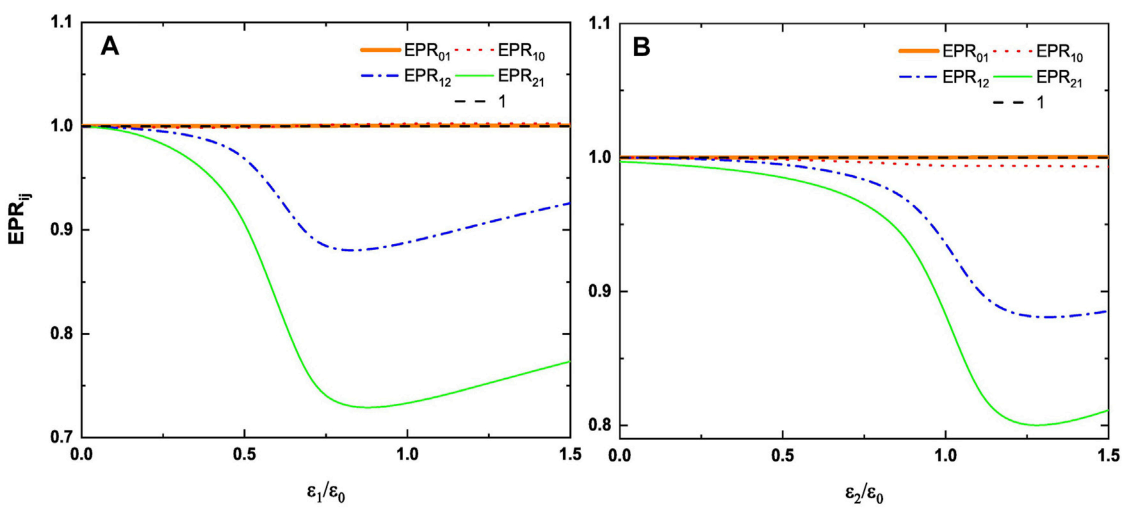

FIGURE 4 | Values of EPR_{jk} and EPR_{kj} versus **(A)** ϵ_1/ϵ_0 and **(B)** ϵ_2/ϵ_0 with γ_0=0.1.

fields $\hat{a}_1$ and $\hat{a}_2$ do not exhibit asymmetric quantum steering. The optical fields $\hat{a}_1$ and $\hat{a}_2$ can be steered with each other.

Figure 2 plots EPR_{jk} and EPR_{kj} versus 1) ϵ_1/ϵ_0 and 2) ϵ_2/ϵ_0 with $\omega = 8\gamma_0$, $\gamma_0 = 0.01$, $\gamma_1 = 0.02$, $\gamma_2 = 0.04$, $\gamma_3 = 0.03$, and $\kappa = 0.01$. We found that by changing the amplitude of the injected signal $\epsilon_1$or ϵ_2, one can control whether there is asymmetric quantum steering between the output modes. As shown in **Figure 2A**, EPR_{21} is less than 1 and EPR_{12} is more than 1 when $\epsilon_1/\epsilon_0 < 0.13$, which shows that the output fields $\hat{a}_1$ and $\hat{a}_2$ exhibit asymmetric quantum steering. When $\epsilon_1/\epsilon_0 > 0.13$, EPR_{10} is less than 1 and EPR_{01} is more than 1, which shows that the output fields $\hat{a}_0$ and $\hat{a}_1$ have asymmetric quantum steering in this range. Different from the case in **Figure 2A**, **Figure 2B** shows that the output fields $\hat{a}_1$ and $\hat{a}_2$ exhibit asymmetric quantum steering when ϵ_2 is small. However, the optical fields $\hat{a}_0$ and $\hat{a}_1$ have asymmetric quantum steering in the whole range of ϵ_2. This shows that the injected signal will affect the asymmetric quantum steering characteristics among the output optical fields. The influence of different cavity loss rates on quantum steering is also worth studying. Therefore, we choose a different set of γ_i to recalculate the quantum steering characteristics among the output optical fields.

Figure 3 shows the quantum steering of the output field $(\hat{a}_0, \hat{a}_1)$ and $(\hat{a}_1, \hat{a}_2)$ versus the normalized analysis frequency ω/γ_0 with a new set of γ_i. Asymmetric quantum steering does not exist between $\hat{a}_0$ and $\hat{a}_1$ only when ω is extremely small. Apart from that, $\hat{a}_0$ and $\hat{a}_1$ have asymmetric quantum steering. $\hat{a}_1$ and $\hat{a}_2$ have symmetric quantum steering in the whole range of ω which is similar to the case in **Figure 1**. However, the influences of ϵ_1 and ϵ_2 on the asymmetric quantum steering are different from the case in **Figure 2**. **Figure 4A** depicts the values of EPR_{jk} and EPR_{kj} versus ϵ_1/ϵ_0 with $\omega = 8\gamma_0$, $\gamma_0 = 0.1$, $\gamma_1 = 0.2$, $\gamma_2 = 0.4$, $\gamma_3 = 0.03$, and $\kappa = 0.01$. One can see that $\hat{a}_0$ and $\hat{a}_1$ show asymmetric quantum steering when about $\epsilon_1/\epsilon_0 < 0.7$, and $\hat{a}_1$ and $\hat{a}_2$ have symmetric steering in the whole range. In **Figure 4B**, the quantum steering between $\hat{a}_0$ and $\hat{a}_1$ is always asymmetric, but the quantum steering between $\hat{a}_1$ and $\hat{a}_2$ is always symmetric in the whole range, which is different from the case in **Figure 2**. This may be due to the increase of the cavity loss rates, which affects the quantum properties of the output optical fields. Moreover, the injected signal of the optical field $\hat{a}_1$ has a greater influence on the quantum steering than the injected signal of the optical field $\hat{a}_2$. The asymmetry of quantum steering can be controlled by adjusting the intensities of the injected signals and the cavity loss rates.

4 CONCLUSION

Asymmetric quantum steering produced by the triple-photon down-conversion process with two injected signals is investigated. Asymmetric quantum steering can be obtained in some parameter regimes between optical fields $\hat{a}_0$ and $\hat{a}_1$ or between $\hat{a}_1$ and $\hat{a}_2$. Both the loss rates of the cavity modes and the intensities of the injected signals have the influences on the asymmetric quantum steering among the output optical fields. Our scheme provides a new idea for generating asymmetric quantum steering, which has potential applications in quantum secret sharing and continuous variable teleportation.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material; further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

TC wrote the manuscript. YY designed and directed the study. CX, KP, and AC contributed to the discussion and edited the manuscript.

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