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Novel Global Asymptotic Stability and Dissipativity Criteria of BAM Neural Networks With Delays

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In this article, issues of both stability and dissipativity for a type of bidirectional associative memory (BAM) neural systems with time delays are investigated. By using generalized Halanay inequalities and constructing appropriate Lyapunov functionals, some novelty criteria are obtained for the asymptotic stability for BAM neural systems with time delays. Also, without assuming boundedness and differentiability for activation functions, some new sufficient conditions for proving the dissipativity are established by making use of matrix theory and inner product properties. The received conclusions extend and improve some previously known works on these problems for general BAM neural systems. In the end, numerical simulation examples are made to show the availability of the theoretical conclusions.

Keywords: BAM neural network, global asymptotic stability, dissipativity, inner product, generalized Halanay inequalities, matrix theory

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1 INTRODUCTION

The BAM neural network model, proposed by Kosko in [1], consists of neurons in two layers, the x-layer and the y-layer. The neurons of the same layer are sufficiently interconnected to the neurons arranged in the other layer, but neurons do not interconnect among the same layer. A useful feature of BAM is its ability to invoke stored pattern pairs in the case of noise. For detailed memory structure and examples of the BAM neural network, please refer to [2]. In recent years, BAM neural systems have received significant attention due to their wide applications in a lot of fields such as pattern recognition, image processing, signal processing, associative memories, optimization problems, and other engineering areas [3–6].

In general, due to the limited switching speed and signal propagation speed of neuron amplifiers, the implementation of a neural network will inevitably have a time delay. We also know that using a delayed version of the neural network is very important to solve some motion-related optimization problems. However, research shows that time delay may lead to divergence, oscillation, and instability, which may be bad for BAM neural systems [7, 8]. Therefore, these applications of the BAM neural systems with delays greatly rely on the dynamical behavior of the neural systems. For these reasons, it is necessary to study the dynamical behavior of the neural systems with delays, and it has been widely studied by a great number of researchers [9, 10].

In the design and analysis of neural networks, stability analysis is a very important and essential link. As small as a specific control system or as large as a social system, financial system, and ecosystem, it is always carried out under various accidental or continuous disturbances. After bearing this kind of interference, it is very important whether the system can keep running or working without losing control or swinging. For neural networks, because the output of the network is a function of time, for a given input, the response of the network may converge to a stable output, oscillate, increase infinitely, or follow a chaotic mode. Therefore, if a neural network system wants to play a role in engineering, it must be stable.

The notion of global dissipativity proposed in the 1970s is a common notion in dynamical systems, and it is applied in the fields of chaos and synchronization theory, stability theory, and robust control and system norm estimation [11–14]. Hence, it is a special and interesting problem to study the dissipativity of dynamical networks. Up to now, the dissipativity for several classes of simple neural networks with delays has begun to attract initial interest in investigation, and some sufficient conditions have been received [15–17]. Yet, to our knowledge, only a few articles have not been used for Lyapunov–Krasovskii functionals or Lyapunov functionals [18–22]. In this study, a few dissipativity conclusions have been received for BAM neural networks with varying delays *via* inner product properties and matrix theory, which are different from the neural systems' model investigated in [23, 24].

Inspired by the previous discussion, the global asymptotic stability and dissipativity of BAM neural systems with time delays are investigated. Some new criteria to ensure the dissipation and stability of the BAM neural system are received. Compared with the previous results, our main results are more general and less conservative. The innovations of the study are at least the following aspects.

1) The BAM neural network model studied in this article has a time-varying delay.
2) In our article, the nonlinear activation functions we assumed are not differentiable and bound.
3) In this article, the sufficient conditions for the dissipativity of BAM neural networks with time-varying delay are obtained by using only the inner product property and matrix theory.
4) Moreover, the global attraction sets, namely, positive invariant sets, are obtained.

The structure of the article is organized in the following. The model description and some preliminary knowledge with some necessary definitions and lemmas are given in **Section 2**. In **Section 3**, by constructing Lyapunov functionals, we discussed the global asymptotic stability for the equilibrium point of delayed BAM neural systems. Some sufficient criteria are obtained and discussed to guarantee the global dissipativity by using inner product properties in **Section 4**. Two examples and their simulation conclusions are provided in **Section 5**. In the end, some results are reached in **Section 6**.

2 PRELIMINARIES

Notations: In this article, let R^n be a Euclidean space with the inner product $<x,y> = y^T x$ and the norm $\|x\|_2 = \sqrt{<x,x>}$, where $x = (x_1, x_2, \ldots, x_n)^T$ *and* $y = (y_1, y_2, \ldots, y_n)^T \in R^n$. The matrix norm is $\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}$ for $A \in R^{n\times n}$, where $\lambda_{\max}(A^T A)$ denotes the maximum eigenvalue of $A^T A$. $\lambda_{\min}(A)$ denotes the minimum eigenvalue of A. $A > 0$ denotes that matrix A is symmetric positive definite. E is a unit matrix.

In this article, the model of delayed BAM neural networks is investigated.

$$\begin{cases} \dot{x}(t) = -Ax(t) + Cf(y(t)) + \tilde{C}f(y(t-\tau)) + I, \\ \dot{y}(t) = -By(t) + Dg(x(t)) + \tilde{D}g(x(t-\sigma)) + J, \end{cases} \tag{1}$$

for $t > 0$, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ represents neuron in the first layer at time t, and $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ represents neuron in the second layer at time t; $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ and $B = \mathrm{diag}(b_1, b_2, \ldots, b_n)$, in which $a_i > 0$ and $b_j > 0\, (i, j \in \mathcal{I} = \{1, 2, \ldots, n\})$ denote passive decay rates, respectively; $C = (c_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$, $\tilde{C} = (\tilde{c}_{ij})_{n\times n}$, *and* $\tilde{D} = (\tilde{d}_{ji})_{n\times n}$ are synaptic connection strengths; $f(y(t)) = (f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t)))^T$ and $g(x(t)) = (g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t)))^T$ denote nonlinear activation functions; $I = (I_1, I_2, \ldots, I_n)^T$, $J = (J_1, J_2, \ldots, J_n)^T$ represents the external inputs to the neurons; $\tau = (\tau_1, \tau_2, \ldots, \tau_n)^T$, $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T$ which are required for axonal transmission and neural processing of signals are time delays.

In this study, we considered the following continuous activation functions:

(H₁): $\forall x,\ y \in R,\ x \neq y,\ i, j \in \mathcal{I}$, activation functions $f_j(\cdot)$ and $g_i(\cdot)$ satisfy $f_j(0) = g_i(0) = 0$, and there exist constants $l_j, m_i > 0$ such that

$$0 \le \frac{f_j(x) - f_j(y)}{x - y} \le l_j, \qquad 0 \le \frac{g_i(x) - g_i(y)}{x - y} \le m_i.$$

Remark 1: The hypothesis of activation function H_1 in this study has been widely used in some references. In particular, when discussing the stability, synchronization, and dissipation of neural networks, H_1 is a common assumption. In the study, the activation function is Lipschitz continuous, so it is monotonously increasing. But it may not be differentiable or bounded. However, in [8, 13], the activation function should not only satisfy the hypothesis H_1 of this study but also satisfy the boundedness. In [15], the derivative of the activation function also satisfies boundedness. In this study, the activation function only needs to satisfy the hypothesis H_1. Compared with [8, 13, 15], the assumption of excitation function in this study is more general.

The initial condition of the system (1) is considered as

$$\begin{cases} x(s) = \varphi(s),\ s \in [-\alpha + t_0, t_0], \\ y(s) = \psi(s),\ s \in [-\alpha + t_0, t_0], \end{cases}$$

where $\bar{\tau} = \max_{j\in\mathcal{I}}\{\tau_j\}$, $\bar{\sigma} = \max_{i\in\mathcal{I}}\{\sigma_i\}$, $\alpha = \max\{\bar{\tau}, \bar{\sigma}\}$, and $\varphi(s), \psi(s) \in C[(-\alpha + t_0, t_0), R^n]$.

Definition 1: [25]. The neural system (1) is globally dissipative if there exists a compact set $S \subseteq R^{2n}$, such that $\forall z_0 \in S$, $\exists\ T(z_0) > 0$, when $t \ge t_0 + T(z_0)$, $z(t, t_0, z_0) \subseteq S$, in which $z(t, t_0, z_0)$ represents the solution for (1) from initial time t_0 and initial state z_0. A set S is said to be forward invariant if $\forall z_0 \in S$ indicates $z(t, t_0, z_0) \subseteq S$ for $t \ge t_0$.

Definition 2: [26]. The point $(x^{\star T}, y^{\star T})^T$ with $x^\star = (x_1^\star, x_2^\star, \ldots, x_n^\star)^T$ and $y^\star = (y_1^\star, y_2^\star, \ldots, y_n^\star)^T$ is the equilibrium of system (1) if

$$\begin{cases} -Ax^\star + Cf(y^\star) + \tilde{C}f(y^\star) + I = 0, \\ -By^\star + Dg(x^\star) + \tilde{D}g(x^\star) + J = 0. \end{cases}$$

Lemma 1: [27]. For every positive $k > 0$ and every a, $b \in R^n$,

$$2a^T b \le k a^T X a + k^{-1} b^T X^{-1} b$$

holds, in which $X > 0$.

Lemma 2: (Generalized Halanay inequalities) [28]. If $V(t) \ge 0$, $t \in (-\infty, +\infty)$ and

$$D^+ V(t) \le \gamma(t) + \xi(t) V(t) + \eta(t) \sup_{t-\tau(t) \le s \le t} V(s), \ t \ge t_0,$$

for $t \in [t_0, +\infty)$, in which $\gamma(t) \ge 0$, $\eta(t) \ge 0$, and $\xi(t) \le 0$ are continuous functions and $\tau(t) \ge 0$, and there exists $\alpha > 0$ such that

$$\xi(t) + \eta(t) \le -\alpha, \ for \ t \ge t_0.$$

Then,

$$V(t) \le \frac{\gamma^\star}{\alpha} + \left(\sup_{-\infty \le s \le t_0} V(s) - \frac{\gamma^\star}{\alpha} \right) \cdot e^{-\mu^\star (t - t_0)},$$

where
$\gamma^\star = \sup_{t_0 \le t \le \infty} \gamma(t)$, $\mu^\star = \inf_{t \ge t_0} \{\mu(t) \colon \mu(t) + \xi(t) + \eta(t) e^{\mu(t)\tau(t)} = 0\}$, and the upper-right Dini derivative $D^+ y(t) = \overline{\lim_{h \to 0^+}} \frac{y(t+h) - y(t)}{h}$.

3 GLOBAL ASYMPTOTIC STABILITY FOR BAM NEURAL NETWORKS

First of all, under condition (H_1), neural system (1) always at least has an equilibrium point. In the following, the asymptotic stability of the equilibrium point will be proved. For simplicity, we transformed the equilibrium point of system (1) to the origin. We assumed that $z^\star = (x_1^\star, x_2^\star, \ldots, x_n^\star, y_1^\star, y_2^\star, \ldots, y_n^\star)^T$ is an equilibrium of neural system (1). By the transformation $u_i(\cdot) = x_i(\cdot) - x_i^*$, $w_j(\cdot) = y_j(\cdot) - y_j^\star$, one can transform system (1) into the system as follows:

$$\begin{cases} \dot{u}(t) = -Au(t) + C\tilde{f}(w(t)) + \tilde{C}\tilde{f}(w(t-\tau)), \\ \dot{w}(t) = -Bw(t) + D\tilde{g}(u(t)) + \tilde{D}\tilde{g}(u(t-\sigma)), \end{cases} \tag{2}$$

where $\tilde{f}(w(t)) = (\tilde{f}_1(w_1(t)), \tilde{f}_2(w_2(t)), \ldots, \tilde{f}_n(w_n(t)))^T$, $\tilde{g}(u(t)) = (\tilde{g}_1(u_1(t)), \tilde{g}_2(u_2(t)), \ldots, \tilde{g}_n(u_n(t)))^T$, in which $\tilde{f}_j(w_j(t)) = f_j(w_j(t) + y_j^\star) - f_j(y_j^\star)$, and $\tilde{g}_i(u_i(t)) = g_i(u_i(t) + x_i^*) - g_i(x_i^*)$. Functions $f_j(\cdot)$, $g_i(\cdot)$ satisfy the condition (H_1); hence, $\tilde{f}_j(\cdot)$, $\tilde{g}_i(\cdot)$ satisfy

$$\begin{cases} \tilde{f}_j^2(w_j(\cdot)) \le l_j w_j(\cdot) \tilde{f}_j(w_j(\cdot)), \\ \tilde{f}_j^2(w_j(\cdot)) \le l_j^2 w_j^2(\cdot), \ \tilde{f}_j(0) = 0, \end{cases} \tag{3}$$

$$\begin{cases} \tilde{g}_i^2(u_i(\cdot)) \le m_i u_i(\cdot) \tilde{g}_i(u_i(\cdot)), \\ \tilde{g}_i^2(u_i(\cdot)) \le m_i^2 u_i^2(\cdot), \ \tilde{g}_i(0) = 0. \end{cases} \tag{4}$$

Remark 2: It is easy to verify that systems (1) and (2) have the same stability. Therefore, to prove the stability of the equilibrium point $z^\star$ of the system (1), it is sufficient to prove the stability of the trivial solution of the system (2).

Theorem 1: Under condition (H_1), if there exist positive definite diagonal matrices $P = \{p_i\} \in R^{n \times n}$, $N = \{n_i\} \in R^{n \times n}$ and constants ς_1, ς_2, β_1, $\beta_2 > 0$ such that

$$-2PA + \varsigma_1^{-1} P\tilde{C}N^{-1}\tilde{C}^T P + \beta_1^{-1} PCC^T P + \beta_2 M^2 + \varsigma_2 PM^2 < 0,$$
$$-2NB + \varsigma_2^{-1} N\tilde{D}P^{-1}\tilde{D}^T N + \beta_2^{-1} NDD^T N + \beta_1 L^2 + \varsigma_1 NL^2 < 0,$$

where $M = \text{diag}\{m_1, \ldots, m_n\}$, $L = \text{diag}\{l_1, \ldots, l_n\}$; then the zero solution of neural system (2) is a unique equilibrium point and is globally asymptotically stable. Proof. Now, we chose Lyapunov functional.

$$V(u(t), w(t)) = \sum_{i=1}^n p_i u_i^2(t) + \varsigma_1 \sum_{j=1}^n \int_{t-\tau_j}^t n_j \tilde{f}_j^2(w_j(s)) ds$$
$$+ \sum_{j=1}^n n_j w_j^2(t) + \varsigma_2 \sum_{i=1}^n \int_{t-\sigma_i}^t p_i \tilde{g}_i^2(u_i(s)) ds.$$

Then,

$$\begin{aligned} \dot{V}(u(t), w(t)) &= 2\sum_{i=1}^n p_i u_i(t) \dot{u}_i(t) + \varsigma_1 \sum_{j=1}^n n_j \left[\tilde{f}_j^2(w_j(t)) - \tilde{f}_j^2(w_j(t - \tau_j)) \right] \\ &\quad + 2\sum_{j=1}^n n_j w_j(t) \dot{w}_j(t) + \varsigma_2 \sum_{i=1}^n p_i [\tilde{g}_i^2(u_i(t)) - \tilde{g}_i^2(u_i(t - \sigma_i))] \\ &= 2u^T(t) P \dot{u}(t) + \varsigma_1 \tilde{f}^T(w(t)) N \tilde{f}(w(t)) - \varsigma_1 \tilde{f}^T(w(t-\tau)) N \\ &\quad \times \tilde{f}(w(t-\tau)) + 2w^T(t) N \dot{w}(t) + \varsigma_2 \tilde{g}^T(u(t)) P \tilde{g}(u(t)) \\ &\quad - \varsigma_2 \tilde{g}^T(u(t-\sigma)) P \tilde{g}(u(t-\sigma)) \\ &= 2u^T(t) P\left(-Au(t) + C\tilde{f}(w(t)) + \tilde{C}\tilde{f}(w(t-\tau))\right) + \varsigma_1 \tilde{f}^T(w(t)) N \\ &\quad \times \tilde{f}(w(t)) - \varsigma_1 \tilde{f}^T(w(t-\tau)) N \tilde{f}(w(t-\tau)) + 2w^T(t) N (-Bw(t) \\ &\quad + D\tilde{g}(u(t)) + \tilde{D}\tilde{g}(u(t-\sigma))\big) + \varsigma_2 \tilde{g}^T(u(t)) P \tilde{g}(u(t)) \\ &\quad - \varsigma_2 \tilde{g}^T(u(t-\sigma)) P \tilde{g}(u(t-\sigma)). \end{aligned} \tag{5}$$

By Lemma 1, we obtained

$$-\varsigma_1 \tilde{f}^T(w(t-\tau)) N \tilde{f}(w(t-\tau))$$
$$+ 2u^T(t) P\tilde{C}\tilde{f}(w(t-\tau)) \le \varsigma_1^{-1} u^T(t) P\tilde{C}N^{-1}\tilde{C}^T P u(t), \tag{6}$$

$$-\varsigma_2 \tilde{g}^T(u(t-\sigma)) P \tilde{g}(u(t-\sigma))$$
$$+ 2w^T(t) N\tilde{D}\tilde{g}(u(t-\sigma)) \le \varsigma_2^{-1} w^T(t) N\tilde{D}P^{-1}\tilde{D}^T N w(t). \tag{7}$$

From **Eqs 6**, **7**, then

$$\begin{aligned} \dot{V}(u(t), w(t)) &\le -2u^T(t) PAu(t) + 2u^T(t) PC\tilde{f}(w(t)) + \varsigma_1^{-1} u^T(t) P\tilde{C}N^{-1}\tilde{C}^T P u(t) \\ &\quad + \varsigma_1 \tilde{f}^T(w(t)) N \tilde{f}(w(t)) - 2w^T(t) NBw(t) + 2w^T(t) ND\tilde{g}(u(t)) \\ &\quad + \varsigma_2^{-1} w^T(t) N\tilde{D}P^{-1}\tilde{D}^T N w(t) + \varsigma_2 \tilde{g}^T(u(t)) P \tilde{g}(u(t)) \\ &\le -2u^T(t) PAu(t) + \beta_1^{-1} u^T(t) PCC^T P^T u(t) + \beta_1 \tilde{f}^T(w(t)) \tilde{f}(w(t)) \\ &\quad + \varsigma_1^{-1} u^T(t) P\tilde{C}N^{-1}\tilde{C}^T P u(t) + \varsigma_1 w^T(t) NL^2 w(t) - 2w^T(t) NBw(t) \\ &\quad + \beta_2^{-1} w^T(t) NDD^T N^T w(t) + \beta_2 \tilde{g}^T(u(t)) \tilde{g}(u(t)) \\ &\quad + \varsigma_2^{-1} w^T(t) N\tilde{D}P^{-1}\tilde{D}^T N w(t) + \varsigma_2 u^T(t) PM^2 u(t) \\ &= u^T(t)\left(-2PA + \varsigma_1^{-1} P\tilde{C}N^{-1}\tilde{C}^T P + \beta_1^{-1} PCC^T P + \beta_2 M^2 + \varsigma_2 PM^2\right) u(t) \\ &\quad + w^T(t)\Big(-2NB + \varsigma_2^{-1} N\tilde{D}P^{-1}\tilde{D}^T N + \beta_2^{-1} NDD^T N + \beta_1 L^2 \\ &\quad + \varsigma_1 NL^2\Big) w(t) \\ &< 0, \ \forall u(t) \ne 0, \ w(t) \ne 0. \end{aligned} \tag{8}$$

This implies that the origin solution of system (2) is asymptotically stable. So the equilibrium point of system (1) is asymptotically stable.

Corollary 1: Under condition (H_1), suppose $L = M = E$, $\varsigma_1 = \varsigma_2 = \beta_1 = \beta_2 = 1$, if there exist positive definite diagonal matrices $P = \{p_i\} \in R^{n\times n}$, $N = \{n_i\} \in R^{n\times n}$ such that

$$-2PA + P\tilde{C}N^{-1}\tilde{C}^T P + PCC^T P + E + P < 0,$$
$$-2NB + N\tilde{D}P^{-1}\tilde{D}^T N + NDD^T N + E + N < 0,$$

then, the origin solution of network (2) is a unique equilibrium point, and it is globally asymptotically stable.

4 GLOBAL DISSIPATIVITY FOR BAM NEURAL NETWORKS

In this part, the global dissipativity for the BAM neural system (1) is considered.

Theorem 2: Under assumption (H_1), suppose $z(t) = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_n(t))^T$ is a solution of system (1) and

$$\xi(t) + \eta(t) \leq -\alpha < 0,$$

then for any given $\varepsilon > 0$, there exists T such that for all $t \geq T$

$$\|z(t)\|_2 \leq \sqrt{\frac{\gamma^2}{\alpha} + \varepsilon}.$$

So, network (1) is dissipative, and the closed ball $E = E\left(0, \sqrt{\frac{\gamma^2}{\alpha} + \varepsilon}\right)$ is an absorbing set, where $\gamma = \delta_3\|I\|_2^2 + \rho_3\|J\|_2^2$, $\xi(t) = \max\{-2\lambda_{\min}(A) + \delta_1^{-1} + \delta_2^{-1} + \rho_1 m^2\|D\|_2^2 + \delta_3^{-1},\ -2\lambda_{\min}(B) + \rho_1^{-1} + \rho_2^{-1} + \delta_1 l^2\|C\|_2^2 + \rho_3^{-1}\}$, $\eta(t) = \max\{\delta_2 l^2\|\tilde{C}\|_2^2, \rho_2 m^2\|\tilde{D}\|_2^2\}$, δ_1, δ_2, $\delta_3, \rho_1, \rho_2, \rho_3 > 0$, $l = \max_{j\in\mathcal{I}}\{l_j\}, and\ m = \max_{i\in\mathcal{I}}\{m_i\}$. Proof. The Lyapunov functional should be considered:

$$V(t) = \|x(t)\|_2^2 + \|y(t)\|_2^2. \tag{9}$$

Then,

$$\begin{aligned}\dot{V}(t) &= 2 < x(t), \dot{x}(t) > + 2 < y(t), \dot{y}(t) > \\ &= 2 < x(t), -Ax(t) > + 2 < x(t), Cf(y(t)) > + 2 < x(t), \tilde{C}f(y(t-\tau)) > \\ &\quad +2 < x(t), I > + 2 < y(t), -By(t) > + 2 < y(t), Dg(x(t)) > \\ &\quad +2 < y(t), \tilde{D}g(x(t-\sigma)) > + 2 < y(t), J > \\ &\leq -2\lambda_{\min}(A)\|x(t)\|_2^2 + 2f^T(y(t))C^T x(t) + 2f^T(y(t-\tau))\tilde{C}^T x(t) + 2I^T x(t) \\ &\quad -2\lambda_{\min}(B)\|y(t)\|_2^2 + 2g^T(x(t))D^T y(t) + 2g^T(x(t-\sigma))\tilde{D}^T y(t) + 2J^T y(t).\end{aligned} \tag{10}$$

By $< x, y > = y^T x$, (H_1), and Lemma 1, there exists σ_1, σ_2, σ_3, ρ_1, ρ_2, $\rho_3 > 0$ such that

$$\begin{aligned}2f^T(y(t))C^T x(t) &\leq \delta_1 f^T(y(t))C^T Cf(y(t)) + \delta_1^{-1}x^T(t)x(t) \\ &\leq \delta_1\lambda_{\max}(C^T C)\|f(y(t))\|_2^2 + \delta_1^{-1}\|x(t)\|_2^2 \\ &\leq \delta_1\lambda_{\max}(C^T C)l^2\|y(t)\|_2^2 + \delta_1^{-1}\|x(t)\|_2^2 \\ &\leq \delta_1 l^2\|C\|_2^2\|y(t)\|_2^2 + \delta_1^{-1}\|x(t)\|_2^2,\end{aligned} \tag{11}$$

$$\begin{aligned}2f^T(y(t-\tau))\tilde{C}^T x(t) &\leq \delta_2 f^T(y(t-\tau))\tilde{C}^T\tilde{C}f(y(t-\tau)) + \delta_2^{-1}x^T(t)x(t) \\ &\leq \delta_2\lambda_{\max}\left(\tilde{C}^T\tilde{C}\right)\|f(y(t-\tau))\|_2^2 + \delta_2^{-1}\|x(t)\|_2^2 \\ &\leq \delta_2\lambda_{\max}\left(\tilde{C}^T\tilde{C}\right)l^2\|y(t-\tau)\|_2^2 + \delta_2^{-1}\|x(t)\|_2^2 \\ &\leq \delta_2 l^2\|\tilde{C}\|_2^2\|y(t-\tau)\|_2^2 + \delta_2^{-1}\|x(t)\|_2^2,\end{aligned} \tag{12}$$

$$\begin{aligned}I^T x(t) &\leq \delta_3 I^T I + \delta_3^{-1}x^T(t)x(t) \\ &\leq \delta_3\|I\|_2^2 + \delta_3^{-1}\|x(t)\|_2^2.\end{aligned} \tag{13}$$

Similar to **Eqs 11–13**, then

$$2g^T(x(t))D^T y(t) \leq \rho_1 m^2\|D\|_2^2\|x(t)\|_2^2 + \rho_1^{-1}\|y(t)\|_2^2, \tag{14}$$

$$2g^T(x(t-\sigma))\tilde{D}^T y(t) \leq \rho_2 m^2\|\tilde{D}\|_2^2\|x(t-\sigma)\|_2^2 + \rho_2^{-1}\|y(t)\|_2^2, \tag{15}$$

$$J^T y(t) \leq \rho_3\|J\|_2^2 + \rho_3^{-1}\|y(t)\|_2^2. \tag{16}$$

By using **Eqs 11–16** in **Eq. 10**, it is easy to obtain

$$\begin{aligned}\dot{V}(t) &\leq \left(-2\lambda_{\min}(A) + \delta_1^{-1} + \delta_2^{-1} + \rho_1 m^2\|D\|_2^2 + \delta_3^{-1}\right)\|x(t)\|_2^2 \\ &\quad +\left(-2\lambda_{\min}(B) + \rho_1^{-1} + \rho_2^{-1} + \delta_1 l^2\|C\|_2^2 + \rho_3^{-1}\right)\|y(t)\|_2^2 \\ &\quad +\delta_2 l^2\|\tilde{C}\|_2^2\|y(t-\tau)\|_2^2 + \rho_2 m^2\|\tilde{D}\|_2^2\|x(t-\sigma)\|_2^2 \\ &\quad +\delta_3\|I\|_2^2 + \rho_3\|J\|_2^2 \\ &\leq \gamma + \xi(t)\left(\|x\|_2^2 + \|y\|_2^2\right) + \eta(t)\left(\|x(t-\sigma)\|_2^2 + \|y(t-\tau)\|_2^2\right) \\ &\leq \gamma + \xi(t)V(t) + \eta(t)\sup_{t-\max\{\bar{\tau},\bar{\sigma}\}\leq s\leq t} V(s).\end{aligned} \tag{17}$$

Then, by Lemma 2, we obtain

$$\|z(t)\|_2^2 \leq \|x(t)\|_2^2 + \|y(t)\|_2^2 = V(t) \leq \frac{\gamma^*}{\alpha} + \left(\sup_{-\infty\leq s\leq 0} V(s) - \frac{\gamma^*}{\alpha}\right)e^{-\mu^* t},$$

where $\mu^* = \inf_{t\geq 0}\{\mu(t)\colon \mu(t) + \xi(t) + \eta(t)e^{\mu(t)\max\{\bar{\tau},\bar{\sigma}\}} = 0\}$.

So, for the given sufficient small $\varepsilon > 0$, there exists $T \geq 0$ such that

$$\|z(t)\|_2 \leq \sqrt{\frac{\gamma^*}{\alpha} + \varepsilon}, \quad \forall t \geq T,$$

where $\varepsilon > 0$ is sufficiently small. □

Corollary 2: If taking $\delta_1, \delta_2, \delta_3, \rho_1, \rho_2, \rho_3 = 1$, under assumptions (H_1), suppose that $z(t) = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_n(t))^T$ is a solution of network (1) and

$$\xi(t) + \eta(t) \leq -\alpha < 0,$$

then network (1) is dissipative, and the closed ball $E = E\left(0, \sqrt{\frac{\gamma^2}{\alpha} + \varepsilon}\right)$ is an absorbing set for any $\varepsilon > 0$, where $\gamma = \|I\|_2^2 + \|J\|_2^2$, $\xi(t) = \max\{-2\lambda_{\min}(A) + m^2\|D\|_2^2 + 3, -2\lambda_{\min}(B) + l^2\|C\|_2^2 + 3\}$, $\eta(t) = \max\{l^2\|\tilde{C}\|_2^2, m^2\|\tilde{D}\|_2^2\}$.

Corollary 3: Under assumptions (H_1), suppose that $z(t) = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_n(t))^T$ is a solution of network (1), if

$$\xi(t) + \eta(t) \leq -\alpha < 0$$

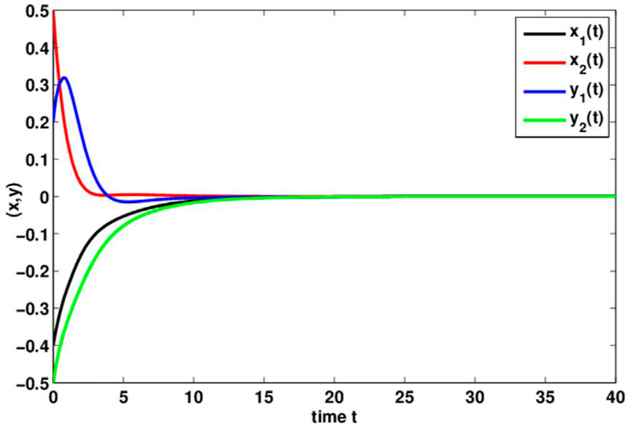

FIGURE 1 | Trajectories of system (18) for $[x(0), y(0)]^T = (-0.4, 0.5, 0.2, -0.5)^T$.

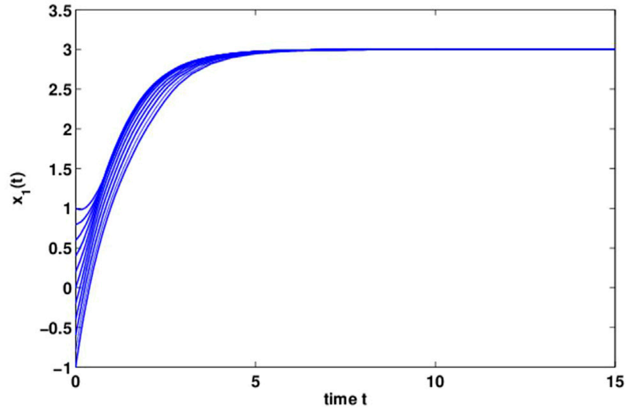

FIGURE 2 | Time response of the state variable $x_1(t)$ with different initial values.

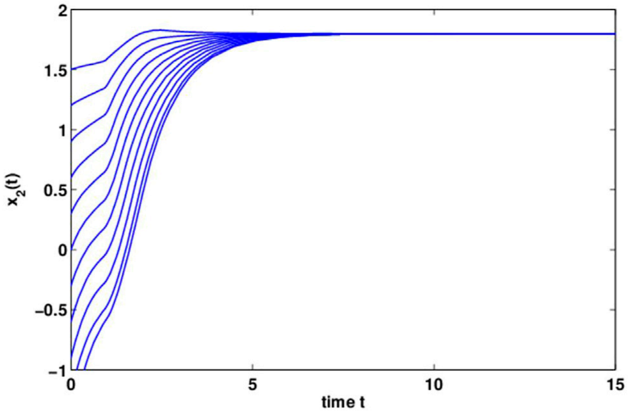

FIGURE 3 | Time response of the state variable $x_2(t)$ with different initial values.

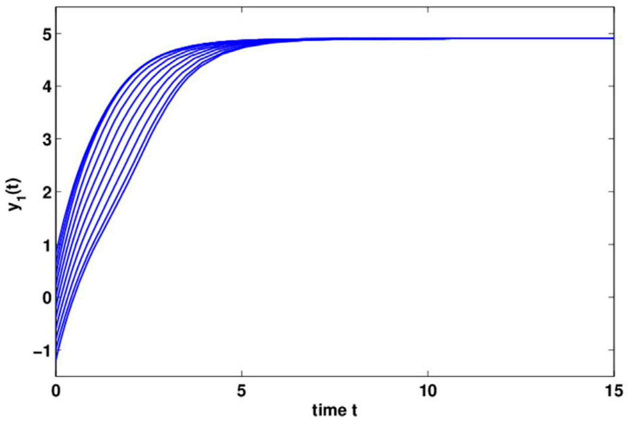

FIGURE 4 | Time response of the state variable $y_1(t)$ with different initial values.

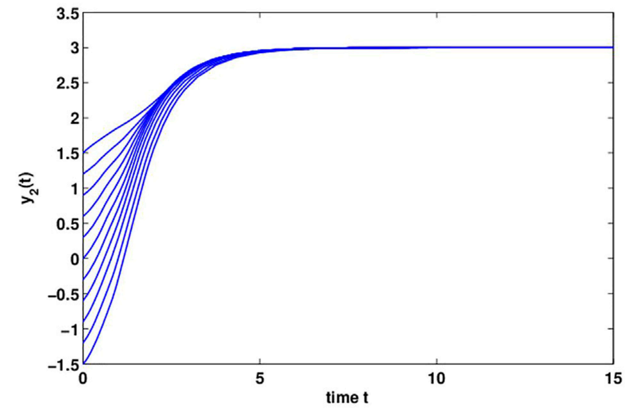

FIGURE 5 | Time response of the state variable $y_2(t)$ with different initial values.

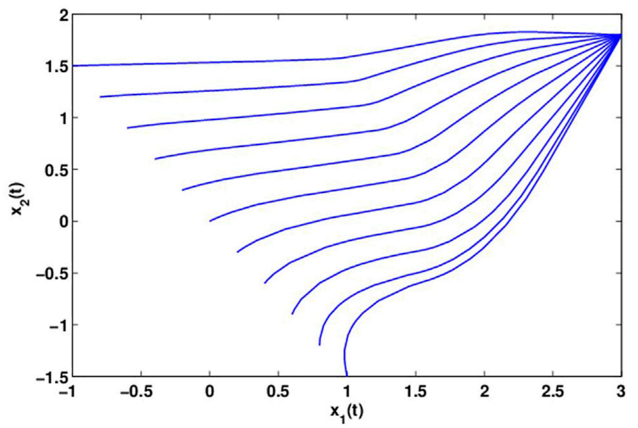

FIGURE 6 | Time response of the state variable $x_1(t)$ and $x_2(t)$ with different initial values.

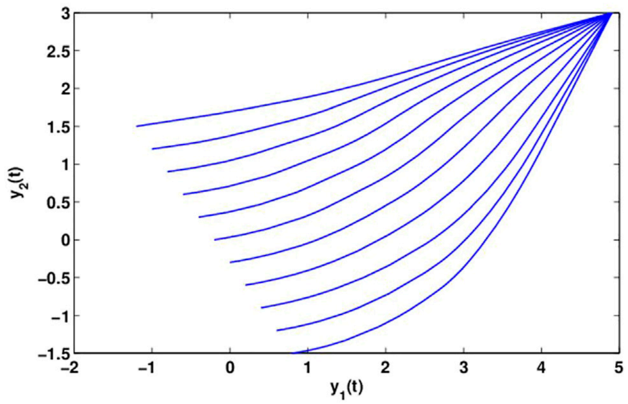

FIGURE 7 | Time response of the state variable $y_1(t)$ and $y_2(t)$ with different initial values.

and

$$\overline{\lim_{t\geq 0}}\{\delta_3\|I\|_2^2+\rho_3\|J\|_2^2\}=0,$$

then system (1) is globally stable, where $\gamma=\delta_3\|I\|_2^2+\rho_3\|J\|_2^2$, $\xi(t)=\max\{-2\lambda_{\min}(A)+\delta_1^{-1}+\delta_2^{-1}+\rho_1 m^2\|D\|_2^2+\delta_3^{-1},-2\lambda_{\min}(B)+\rho_1^{-1}+\rho_2^{-1}+\delta_1 l^2\|C\|_2^2+\rho_3^{-1}\}$, $\eta(t)=\max\{\delta_2 l^2\|\tilde{C}\|_2^2,\rho_2 m^2\|\tilde{D}\|_2^2\}$, δ_1, δ_2, δ_3, ρ_1, ρ_2, $\rho_3>0$.

Remark 3: In the existing articles, a lot of researchers studied the qualitative behaviors of neural systems *via* the Lyapunov function with linear matrix inequality techniques [26, 29, 30]. However, in this article, some new sufficient criteria of dissipativity of BAM neural networks with time delays are given by only using the property of matrix theory and inner product.

5 NUMERICAL SIMULATIONS

In the part, two examples are presented to show the effectiveness.

Example 1. Investigation of the delayed BAM neural network model.

$$\begin{cases}\dot{x}(t)=-Ax(t)+Cf(y(t))+\tilde{C}f(y(t-\tau))+I,\\ \dot{y}(t)=-By(t)+Dg(x(t))+\tilde{D}g(x(t-\sigma))+J,\end{cases}\tag{18}$$

in which $x(t)=(x_1(t),x_2(t))^T$ and $y(t)=(y_1(t),y_2(t))^T$. Let $\tau_1=1$, $\tau_1=0.9$, $\sigma_1=0.8$, $\sigma_2=0.7$, $A=B=E$, $I=J=0$ and

$$C=\begin{pmatrix}0&0.2\\-0.2&0.1\end{pmatrix},\ \tilde{C}=\begin{pmatrix}0.1&0.2\\0&-0.1\end{pmatrix},\ D=\begin{pmatrix}0.2&1\\1&0.4\end{pmatrix},\ \tilde{D}=\begin{pmatrix}0.2&0.5\\0&-0.1\end{pmatrix}.$$

Choose $f_j(y_j)=(|y_j+1|+|y_j-1|)/2$, $g_j(x_j)=(|x_j+1|+|x_j-1|)/2$, $j=1,2$, $l_1=l_2=m_1=m_2=L=M=\beta_1=\beta_2=\varsigma_1=\varsigma_2=1$.

By computing, we can get

$$-2PA+\varsigma_1^{-1}P\tilde{C}N^{-1}\tilde{C}^TP+\beta_1^{-1}PCC^TP+\beta_2M^2+\varsigma_2PM^2<0,$$

$$-2NB+\varsigma_2^{-1}N\tilde{D}P^{-1}\tilde{D}^TN+\beta_2^{-1}NDD^TN+\beta_1L^2+\varsigma_1NL^2<0.$$

So, from Theorem 1, network (18) has a unique equilibrium, and it is globally asymptotically stable. By MATLAB, a unique equilibrium of network (18) $(0,0,0,0)^T$ is given, and the simulation results are given in **Figure 1**.

Example 2. The BAM neural model with delays is considered as (**Eq. 18**), where $x(t)=(x_1(t),x_2(t))^T$, $y(t)=(y_1(t),y_2(t))^T$ and $x(0)=(-1,1.5)^T$, $y(0)=(0.8,-1.5)^T$. Let $\tau_1=0.9$, $\tau_2=0.9$, $\sigma_1=0.8$, $\sigma_2=0.8$, $A=B=E$, $I=(1,0.5)^T$, $J=(2.5,0.5)^T$ and

$$C=\begin{pmatrix}1&0.2\\-0.2&0.1\end{pmatrix},\ \tilde{C}=\begin{pmatrix}0.1&0.2\\1&-0.1\end{pmatrix},\ D=\begin{pmatrix}0.2&1\\1&0.4\end{pmatrix},\ \tilde{D}=\begin{pmatrix}0.2&0.5\\1&-0.1\end{pmatrix}.$$

Choose $f_j(y_j)=(|y_j+1|+|y_j-1|)/2$, $g_j(x_j)=(|x_j+1|+|x_j-1|)/2$, $j=1,2$ and $l_1=l_2=m_1=m_2=l=m=\delta_1=\delta_2=\delta_3=\rho_1=\rho_2=\rho_3=1$.

By computing, we can get $\gamma=7.75$, $\xi(t)=2.703$, $\eta(t)=1.04$. Let $\alpha=4$, $\varepsilon=0.98$, it follows from Theorem 2 and is observed that system (18) is global dissipativity. **Figures 2**, **3** reflect the behaviors for the states $x_1(t)$ and $x_2(t)$ with different initial conditions. **Figures 4**, **5** show the phase plane behaviors of $y_1(t)$ and $y_2(t)$ with different initial conditions. **Figures 6**, **7** demonstrate the behaviors of the time domain for the states $x_1(t)$, $x_2(t)$ and $y_1(t)$, $y_2(t)$ with different initial conditions. System (18) is globally dissipative from the numerical simulations.

Remark 4 : In the numerical simulation part of [13], the author only gives the simulation diagram of the BAM neural network model with one node. This article presents the simulation diagram of the BAM neural network model with two nodes. Moreover, in [13], the values of $\sigma(t)$ and $\tau(t)$ are all 1, while the values of $\sigma_1(t)$, $\sigma_2(t)$, $\tau_1(t)$, and $\tau_2(t)$ in this study are different. Therefore, in numerical simulation, this study is more general in the value of the model and time delay. In addition, the unique equilibrium point $(0,0,0,0)^T$ of the system (18) is obtained by MATLAB. **Figure 1** shows an image which is globally asymptotically stable of the system (18) under initial conditions $(x_1(t), x_2(t), y_1(t), y_2(t))^T = (-0.4, 0.5, 0.2, -0.5)^T$. **Figures 2–5** show the state diagram of $x_1(t)$, $x_2(t)$, $y_1(t)$, and $y_2(t)$ under different initial conditions with respect to time t. **Figures 6**, **7** show the state diagrams of $x_1(t)$, $x_2(t)$ and $y_1(t)$, $y_2(t)$ under different initial conditions with respect to time t. The previous figures given in this study can more intuitively reflect the stability and dissipation of the BAM neural network model.

6 CONCLUSION

In this study, by using matrix theory, inner product properties, generalized Halanay inequalities, and constructing appropriate Lyapunov functionals, novel sufficient criteria of the global asymptotic stability of the system and the global dissipativity of the equilibrium point have been derived for a type of BAM neural systems with delays. The given results might have an impact on investigating the instability, the existence of periodic solutions, and the stability of BAM neural networks. A comparison between the results and the correspondingly previous works implies that the derived criteria are less conservative and more general through numerical simulations.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

ML established the mathematical model, theoretical analysis, and wrote the original draft; HJ provided modeling ideas and analysis methods; CH checked the correctness of theoretical results; BL and ZL performed the simulation experiments. All authors read and approved the final manuscript.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.