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# A possible trigger for a wave-function collapse

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The measurement problem is usually analysed with the quantum decoherence or wave-function collapse theory. Both theories seem to be equivalent with respect to experiments. The trigger/reason for a wave-function collapse is not clearly defined (in the corresponding theory) and the present work is an attempt to close this gap. The present thought experiment describes a measurement to determine the localization of an electron inside a box. The present work assumes that the determination of information (about the localization of the electron) induces a wave-function collapse. It seems possible to identify a clearly defined trigger for a wave-function collapse in the described model system.

## KEYWORDS

quantum measurement, wave-function collapse, localization, quantum information, quantum physics

## Introduction

A wave-function can consist of a superposition of several eigenstates and can be reduced to a single eigenstate by an interaction with (for example) a measuring instrument. This process of reduction is usually called wave-function collapse. The wave-function collapse can be regarded as the connecting element between quantum and classical world [1], [2].

The idea of the wave-function collapse dates back to the days, when quantum mechanics was developed. The discussions between Heisenberg and Bohr led to the development of the Copenhagen interpretation of quantum mechanics, in which the wave-function collapse plays an important role [3]. According to the Copenhagen interpretation, the wave-function collapse has a non-deterministic character [4], [5]. Other theories, such as quantum decoherence, try to explain this phenomenon by an apparent wave-function collapse, in which the classical appearance of a system (after a measurement) is the consequence of a loss of quantum coherence [6], [7], [8]. Nevertheless, quantum decoherence does not solve the measurement problem [9].

## Discussion of the thought experiment

Now imagine an electron in a (one dimensional) box, which is moving backwards and forwards by reflection at the impenetrable barriers of the box. Barrier one is located at  $x =$

0 and barrier two at  $x = L$ , then the following wave-functions are possible solutions for the Schrodinger equation of this model system:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(kx) \tag{1}$$

with  $k = \frac{n\pi}{L}$ .

Now imagine an electron in the ground-state  $\psi_1(x)$ . This wave-function can be expressed by a superposition of the electron to be in the left or to be in the right part of the box:

$$\psi_1(x) = \psi_L(x) + \psi_R(x) \tag{2}$$

$\psi_L(x)$  and  $\psi_R(x)$  describes the electron to be in the left and to be in the right part of the box, respectively:

$$0 \leq x \leq L/2: \quad \psi_L(x) = \psi_1(x) \quad \psi_R(x) = 0 \tag{3}$$

$$L/2 < x \leq L: \quad \psi_L(x) = 0 \quad \psi_R(x) = \psi_1(x) \tag{4}$$

$\psi_L(x)$  and  $\psi_R(x)$  can be described by a series expansion with the eigenfunctions  $\psi_n(x)$ :

$$\psi_L = 1/2 \cdot \psi_1 - \sum_{n=2}^{\infty} \frac{2n \cdot \cos(n\pi/2)}{\pi(-1+n^2)} \psi_n \tag{5}$$

$$\psi_R = 1/2 \cdot \psi_1 + \sum_{n=2}^{\infty} \frac{2n \cdot \cos(n\pi/2)}{\pi(-1+n^2)} \psi_n \tag{6}$$

Any function for the probability density of the electron in the box can be described by a series expansion with the eigenfunctions. Consider, for example, the following wave-function  $\psi_E$ :

$$\psi_E = \sum_n c_n \psi_n \tag{7}$$

The coefficients of this series expansion can be determined with the following integral:

$$c_n = \int \psi_n^* \psi_E dx \tag{8}$$

The coefficients in formula 5 and 6 were determined in such a way, that formula 2 is normalized correctly.

By the superposition of the  $\psi_L(x)$ - and  $\psi_R(x)$ -wave-function, the two terms with the summation are cancelled out and the result is the  $\psi_1(x)$ -wave-function.

$\psi_1(x)$  has the lowest possible energy for the electron in the box and therefore the following energy (or more precisely, the energy expectation value) is larger:

$$E' = \frac{\int \psi_L^* \hat{H} \psi_L dx}{\int \psi_L^* \psi_L dx} = \frac{\int \psi_R^* \hat{H} \psi_R dx}{\int \psi_R^* \psi_R dx} \tag{9}$$

$E'$  is larger than the energy of the  $\psi_1(x)$ -wave-function due to the additional terms in formula 5 and 6, respectively.

Now imagine that the electron is in the  $\psi_1$ -state and therefore its probability is the same to be in the left or right part of

the box. Then a measuring instrument is activated, which determines if the electron is localized in the left or in the right part of the box (but with no higher resolution). If the electron is in the left part of the box, then the instrument is in the  $\varphi_L$  state and the instrument- $\varphi_R$  state corresponds to the electron to be in the right part of the box. Consequently the wave-function of the system is:

$$\psi = \varphi_L \psi_L + \varphi_R \psi_R \tag{10}$$

Due to the entanglement of the instrument with the electron, the summation terms in  $\psi_L$  and  $\psi_R$  (see formulas 5 and 6) do not cancel each other out. The energy of the electron can be calculated with the following formula:

$$E = \frac{\iint \psi^* \hat{H} \psi dx_1 dx_2}{\iint \psi^* \psi dx_1 dx_2} \tag{11}$$

$dx_1$  and  $dx_2$  integrate over the wave-function of the instrument and the electron, respectively. Consequently formula 11 is:

$$E = \frac{\int \varphi_L^* \varphi_L dx_1 \int \psi_L^* \hat{H} \psi_L dx_2 + \int \varphi_L^* \varphi_R dx_1 \int \psi_L^* \hat{H} \psi_R dx_2 + \int \varphi_R^* \varphi_L dx_1 \int \psi_R^* \hat{H} \psi_L dx_2 + \int \varphi_R^* \varphi_R dx_1 \int \psi_R^* \hat{H} \psi_R dx_2}{\int \varphi_L^* \varphi_L dx_1 \int \psi_L^* \psi_L dx_2 + \int \varphi_L^* \varphi_R dx_1 \int \psi_L^* \psi_R dx_2 + \int \varphi_R^* \varphi_L dx_1 \int \psi_R^* \psi_L dx_2 + \int \varphi_R^* \varphi_R dx_1 \int \psi_R^* \psi_R dx_2} \tag{12}$$

The detection of the electron to be in the left or to be in right part of the box corresponds to two distinguishable and orthogonal states of the instrument and therefore the following formulas can be assumed to be correct:

$$\int \varphi_L^* \varphi_R dx_1 = \int \varphi_R^* \varphi_L dx_1 = 0 \tag{13}$$

$$\int \varphi_L^* \varphi_L dx_1 = \int \varphi_R^* \varphi_R dx_1 = 1 \tag{14}$$

With the help of formulas 13 and 14 it is possible to simplify formula 12:

$$E = \frac{\int \psi_L^* \hat{H} \psi_L dx_2 + \int \psi_R^* \hat{H} \psi_R dx_2}{\int \psi_L^* \psi_L dx_2 + \int \psi_R^* \psi_R dx_2} = E' \tag{15}$$

According to formula 15 the entanglement of the measuring instrument with the electron increases its energy from the ground-state energy to  $E'$ . It is important to note that the reason for this energy increase is the determination of information about the localization of the electron by the measuring instrument and is not the consequence of a perturbation (such as a potential) in the box. The present work assumes that the determination of information about a quantum system induces a wave-function collapse. According to the wave-function collapse theory, this energy increase can be interpreted as the consequence of a wave-function collapse with a localization of the electron either in the left or right part of the box. The energy  $E'$  corresponds exactly to an electron to be in a pure  $\psi_L$ - or to be in a pure  $\psi_R$ -state (or, more generally, it corresponds exactly to the total energy expectation value of the two possible states, which can be realized by the wave-function collapse). The reason for the energy increase in formula 12 is the

term  $\int \varphi_L^* \varphi_R dx_1 = 0$  and this term is the consequence of the information determination by the measuring instrument. Before the determination of information by the measuring instrument, the wave-function of the system is:

$$\psi' = \varphi_N \psi_L + \varphi_N \psi_R \quad (16)$$

$\varphi_N$  is the wave-function of the measuring instrument before the measurement process. According to this wave-function  $\psi'$  the electron has the ground-state energy, this can be shown by the application of the Hamilton-operator on this wave-function. Therefore no localization of the electron to be in the left or right part of the box has taken place. After the measurement the wave-function of the system is:

$$\psi = \varphi_L \psi_L + \varphi_R \psi_R \quad (17)$$

Now the energy of the electron is increased and this increase can be interpreted as the consequence of a wave-function collapse with a localization of the electron in the left or right part of the box. Therefore the appearance of the  $\int \varphi_L^* \varphi_R dx_1 = 0$ -term in the energy-calculation formula (due to the entanglement of the electron with the measurement instrument) can be regarded as the trigger/reason for this wave-function collapse.

Or analyzed from a different perspective: The activation of the measuring instrument causes a wave-function collapse with a localization of the electron in the left or right part of the box. Due to the decreased localization area of the electron, its uncertainty in momentum is increased compared to the electron state before the measurement. In the described model system with a one-dimensional box, an increase of the uncertainty in momentum is only possible by an increase of the energy expectation value of the electron. Therefore the mechanism for the energy increase by the appearance of the  $\int \varphi_L^* \varphi_R dx_1 = 0$ -term in the energy calculation formula can be identified as the trigger for the wave-function collapse. Or in other words: As soon as the energy of the electron is increased (by the described mechanism) and corresponds to the energy of a more localized electron, then the electron is more localized and the wave-function collapse has taken place.

The present analysis is also applicable for the system, in which the two measuring instrument states  $\varphi_L$  and  $\varphi_R$  corresponds to two localization areas of different size. Then appropriate functions for the  $\psi_L$ - and  $\psi_R$ -states are to be used. The energy expectation value of the wave-function of the system corresponds then exactly to the total energy expectation value of the two states, which can be realized by the wave-function collapse.

It is also not necessary that the initial state of the electron (before the measurement) is the  $\psi_1$ -wave-function. The electron can be in any  $\psi_n$ -state - it is then only necessary to define corresponding  $\psi_L$ - and  $\psi_R$ -wave-functions (for example in accordance with the instructions below):

$$0 \leq x \leq L/2: \quad \psi_L(x) = \psi_n(x) \quad \psi_R(x) = 0 \quad (18)$$

$$L/2 < x \leq L: \quad \psi_L(x) = 0 \quad \psi_R(x) = \psi_n(x) \quad (19)$$

Due to the entanglement of the measuring instrument with the  $\psi_L$ - and  $\psi_R$ -wave-functions (see formula 10), the energy of the electron is increased and corresponds exactly to the energy expectation value of the electron to be in the left or right part of the box. Therefore the described theoretical approach is generally applicable to describe the localization of the electron (by a measurement) in the described box.

## Conclusion

The present discussion of the thought experiment shows that the entanglement of the electron with the measuring instrument increases the energy of the electron. This energy increase can be interpreted as the consequence of a wave-function collapse. The reason for the energy increase is the appearance of the  $\int \varphi_L^* \varphi_R dx_1 = 0$ -term in the energy-calculation formula and can therefore be regarded as the trigger/reason for the wave-function collapse. It seems possible that also a corresponding entanglement with microscopic elements (in the described model system) can increase the energy of the electron, as long as a  $\int \varphi_L^* \varphi_R dx_1 = 0$ -term appears in the energy calculation formula.

## Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

## Author contributions

The author confirms being the sole contributor of this work and has approved it for publication.

## Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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