



Effect of Medium on Fundamental Interactions in Gravity and Condensed Matter

Alexander Zhuk^{1,2,3} and Valerii Shulga^{1,4*}

¹International Center of Future Science, Jilin University, Changchun City, China, ²Astronomical Observatory, Odessa I.I. Mechnikov National University, Odessa, Ukraine, ³Center for Advance Systems Understanding (CASUS), Görlitz, Germany, ⁴Institute of Radio Astronomy, National Academy of Sciences of Ukraine, Kharkiv, Ukraine

Recently, it was shown that the gravitational field undergoes exponential cutoff at large cosmological scales due to the presence of background matter. In this article, we demonstrate that there is a close mathematical analogy between this effect and the behavior of the magnetic field induced by a solenoid placed in a superconductor.

Keywords: cosmology, scalar perturbations, gravitational potential, magnetic field, superconductor

1 INTRODUCTION

It seems quite natural that the presence of the medium influences the propagation of fundamental interactions. The simplest example is the Debye screening of the electric field of an individual particle in a plasma by particles of opposite sign. Here, the potential produced by an external point charge has the form of the Yukawa potential (but not the Coulomb one) with the Debye screening length (*see, e.g.* [1]). A similar screening mechanism of the electron charge due to vacuum polarization takes place in quantum electrodynamics (*see, e.g.* [2]). The Anderson-Higgs mechanism is another example of the influence of the medium on fundamental interactions, which are carried by gauge fields. In this case, after symmetry breaking, the Higgs vacuum field acts as a medium [3–5]. As a result of interaction with this medium, the initially massless gauge fields gain mass [6]. It is also known that medium in the form of the superconductor affects the electromagnetic interaction. For example, external magnetic field undergoes the exponential cutoff inside the superconductor due to the Meissner effect (*see, e.g.* [7]).

The examples above did not concern the gravitational interaction between massive bodies. It is known that in a vacuum in the weak field limit the gravitational potential satisfies the Poisson equation and has the form of Newton's potential [8]. From a naive point of view, since all masses have the same sign and are attracted to each other, one should hardly expect a screening of the gravitational interaction, as, for example, for electric charges in a plasma. However, it was demonstrated recently [9–11] that medium in the case of gravity also plays important role. It was shown that, due to the interaction of the gravitational potential with background matter, there is an exponential cutoff of the gravitational interaction at large cosmological scales. In **section 2** we reproduce this result. For many, this result turned out to be rather unexpected. Therefore, in this paper, in **section 3**, we present a close mathematical analogue of this phenomenon by the example of the magnetic field induced by a solenoid placed in a superconductor.

2 SCREENING OF THE GRAVITATIONAL INTERACTION IN COSMOLOGY

We consider the Universe containing the cosmological constant Λ and filled with discrete point-like gravitating sources (galaxies and the group of galaxies) with comoving mass density

OPEN ACCESS

Edited by:

Antonio Gallerati,
Politecnico di Torino, Italy

Reviewed by:

Douglas Alexander Singleton,
California State University, Fresno,
United States
Yurii Aleshchenko,
The Russian Academy of Sciences
(RAS), Russia

*Correspondence:

Valerii Shulga
shulga@rian.kharkov.ua

Specialty section:

This article was submitted to
Cosmology,
a section of the journal
Frontiers in Physics

Received: 14 February 2022

Accepted: 02 May 2022

Published: 24 May 2022

Citation:

Zhuk A and Shulga V (2022) Effect of
Medium on Fundamental Interactions
in Gravity and Condensed Matter.
Front. Phys. 10:875757.
doi: 10.3389/fphy.2022.875757

$$\rho = \sum_n \rho_n = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n), \tag{1}$$

Where $\mathbf{r} = (x^1, x^2, x^3)$ is comoving distance. This is our medium. Such matter has a dust-like equation of state and the average energy density $\bar{\epsilon} = \bar{\rho}c^2/a^3$ where comoving averaged mass density $\bar{\rho} = \text{const}$, c is the speed of light and a is the conformal factor. The corresponding background metric is described by Friedmann-Lemaître-Robertson-Walker (FLRW) one.

The discrete inhomogeneities perturb the FLRW metric [12, 13]:

$$ds^2 = a^2 \left[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\delta_{\alpha\beta} dx^\alpha dx^\beta \right], \tag{2}$$

Where we restrict ourselves to scalar perturbations in conformal Newtonian gauge. Scalar function $\Phi(\eta, \mathbf{r})$ is the gravitational potential created at the point with the radius-vector \mathbf{r} by all gravitating masses in the Universe [8]. The perturbed Einstein equations are [12, 13]:

$$\Delta\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = \frac{1}{2}\kappa a^2 \delta\epsilon, \tag{3}$$

$$\Phi' + \mathcal{H}\Phi = -\frac{1}{2}\kappa a^2 \bar{v}, \tag{4}$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' - \mathcal{H}^2)\Phi = 0, \tag{5}$$

Where $\Delta \equiv \delta^{\alpha\beta}\partial_\alpha\partial_\beta$ is the Laplace operator, the prime denotes the conformal time η derivative, $\mathcal{H} \equiv (da/d\eta)/a = (a/c)H$ and $H \equiv (da/dt)/a$ is the Hubble parameter, $v(\eta, \mathbf{r})$ is the peculiar velocity potential and $\kappa \equiv 8\pi G_N/c^4$, where G_N is the gravitational constant. The energy density fluctuation reads [14, 15]:

$$\delta\epsilon = \frac{c^2}{a^3} \delta\rho + \frac{3\bar{\rho}c^2}{a^3} \Phi, \tag{6}$$

Where $\delta\rho(\eta, \mathbf{r}) \equiv \rho - \bar{\rho}$ is the fluctuation of the mass density (1) around its constant average value $\bar{\rho}$.

Equation 4 demonstrates that the peculiar velocities affect the gravitational potential. If we neglect this influence (i.e. $\Phi' + \mathcal{H}\Phi = 0$), then **Eq. 3** takes the form

$$\Delta\Phi - \frac{a^2}{\lambda^2}\Phi = \frac{\kappa c^2}{2a} \delta\rho, \tag{7}$$

Where the screening length

$$\lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}}. \tag{8}$$

With the help of the transformation (to remove the $\bar{\rho}$ contribution on the RHS of **Eq. 7**)

$$\phi = c^2 a \Phi - \frac{4\pi G_N \bar{\rho}}{a^2} \lambda^2 = c^2 a \Phi - \frac{c^2 a}{3} \tag{9}$$

Equation 7 is reduced to

$$\Delta\phi - \frac{a^2}{\lambda^2}\phi = 4\pi G_N \rho. \tag{10}$$

For the mass density (1), we can easily solve this Helmholtz equation, and applying transformation (9) obtain:

$$\Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a} \sum_n \frac{m_n}{|\mathbf{r} - \mathbf{r}_n|} \exp\left(-\frac{a|\mathbf{r} - \mathbf{r}_n|}{\lambda}\right). \tag{11}$$

It is worth noting that the physical distance is $R = ar$. The term $1/3$ (which is due to $\bar{\rho}$ in $\delta\rho$) plays an important role since only with this term the averaged over all volume value of the gravitational potential $\bar{\Phi}$ is equal to zero as it should be for fluctuations [9].

In **Eq. 11**, we neglect the peculiar velocities of the inhomogeneities. However, they also play an important role [16, 17] and must be taken into account. For the considered model, as was shown in [16], it is sufficient in (7, 9–11) to replace λ with λ_{eff} and additionally in (11): $1/3 \rightarrow 1/3(\lambda_{\text{eff}}/\lambda)^2$ where

$$\lambda_{\text{eff}} = \sqrt{\frac{c^2 a^2 H}{3} \int \frac{da}{a^3 H^3}}. \tag{12}$$

To get this result, we should take into consideration **Eq. 5**. This screening length (as well as λ) depends on time. For example, for the standard Λ CDM model at present time $\lambda_{\text{eff}} = 2.57$ Gpc [16].

Therefore, the gravitational potential Φ satisfies the Helmholtz equation, not the Poisson equation. This is due to the interaction of the gravitational potential with the medium. We can see it directly from **Eq. 6** where the term $\sim \bar{\rho}\Phi$ describe this interaction. Due to the peculiar velocity, **Eq. 3** also acquires an additional term proportional to Φ [16]. If the medium is absent that corresponds to the limit $\bar{\rho} \rightarrow 0, v \rightarrow 0$, then the screening lengths λ and λ_{eff} tends to infinity, and the Yukawa potentials in (11) are reduced to the Newton's ones without screening of the gravitational interaction.

3 SOLENOID IN A SUPERCONDUCTOR. SCREENING OF THE INDUCED MAGNETIC FIELD

In this section, in order to present the mathematical analog of the screening effect described above, we render some of equations of the paper [18] in a form suitable for our purpose. Following this paper, we consider a thin solenoid placed in a superconductor. Thin means that the diameter of the solenoid is much smaller than the magnetic field penetration length λ_m . It is well known that the magnetic field of the solenoid \mathbf{B}_{sol} is absent from the outside it, but the vector potential \mathbf{A}_{sol} is not equal to zero. The interaction of this potential with the superconducting medium induces a current \mathbf{J}_{sc} , which, in turn, leads to the appearance of an induced magnetic field \mathbf{B}_{sc} . Thus, the Maxwell equation has the form¹

$$\text{curl}\mathbf{B}_{\text{tot}} = \text{curl}(\mathbf{B}_{\text{sc}} + \mathbf{B}_{\text{sol}}) = \mathbf{J}_{\text{sc}} + \mathbf{J}_{\text{sol}}. \tag{13}$$

Since outside the solenoid $\mathbf{B}_{\text{sol}}, \mathbf{J}_{\text{sol}} = 0$, we get

$$\text{curl}\mathbf{B}_{\text{sc}} = \mathbf{J}_{\text{sc}}, \tag{14}$$

Where in the London limit the superconducting current density is [7, 18]

¹In this section, we use the system of units adopted in book [7].

$$\mathbf{J}_{sc} = -\frac{1}{\lambda_m^2} \left(\frac{1}{q} \nabla \theta + \mathbf{A}_{tot} \right). \tag{15}$$

Here, $\mathbf{A}_{tot} = \mathbf{A}_{sc} + \mathbf{A}_{sol}$, θ is the phase of the order parameter and the magnetic field penetration length

$$\lambda_m = \frac{1}{q\sqrt{n_s}}, \tag{16}$$

Where n_s is the superfluid density, parameter q defines the superconducting flux quanta (see, e.g., Eq. 19 below) and in the real superconductor $q = 2e/\hbar c$ [7]. The absence of a superconducting medium corresponds to the limit $n_s \rightarrow 0 \Rightarrow \lambda_m \rightarrow \infty$. Expression (16) is an analogue of cosmological formula (8) (and, accordingly, formula (12)). In Eq. 15 the term $\lambda_m^{-2} \mathbf{A}_{sol} \sim n_s \mathbf{A}_{sol}$ describes the interaction of the solenoid magnetic field with the superconducting medium just as the term $\sim \bar{\rho} \Phi$ on the RHS of Eq. 6 describes the interactions of the gravitational potential with the cosmological medium.

Now, applying curl operation to both sides of (15), we obtain

$$\mathbf{B}_{sc} - \lambda_m^2 \Delta \mathbf{B}_{sc} = 0, \tag{17}$$

Where we took into account that outside of the solenoid $\text{curl} \nabla \theta = 0$ and $\mathbf{B}_{sol} = 0$. Δ is the Laplace operator in flat space. To solve this equation, we need to define the boundary conditions. Let the solenoid be extended along the z -axis. Obviously, due to the cylindrical symmetry the induced magnetic field inside the superconductor is also parallel to the z -axis: $\mathbf{B}_{sc}(r) = B_{sc}(r) \hat{z}$, where \hat{z} is the unit vector along z -axis. In cylindrical coordinates, \mathbf{r} is the radius-vector in the xy -plane (it is worth noting that in the previous section \mathbf{r} denotes the comoving three-dimensional radius-vector). At distances $r \gg \lambda_m$, the superconducting current goes to zero: $\mathbf{J}_{sc} \rightarrow 0$. Therefore, at this distances Eq. 15 reads

$$\mathbf{A}_{tot} = -\frac{1}{q} \nabla \theta. \tag{18}$$

Integrating both sides of this equation over an area inside the contour $r = \text{const}$, and performing the Stokes area-to-contour transformation for the RHS, we find

$$\Phi_{tot} = -\frac{2\pi}{q} N \equiv -\Phi_0 N, \quad N = 0, 1, 2, \dots, \tag{19}$$

Where $\Phi_{tot} = \Phi_{sc} + \Phi_{sol}$ is the total magnetic flux consisting of the sum of the magnetic fluxes of the induced magnetic field and the magnetic field inside the solenoid. Φ_0 is the superconducting flux quanta. Therefore,

$$\Phi_{sc} = \Phi_{tot} - \Phi_{sol}. \tag{20}$$

This is our boundary condition. We can include it directly into Eq. 17:

$$B_{sc} - \lambda_m^2 \Delta B_{sc} = \Phi_{sc} \delta(\mathbf{r}), \tag{21}$$

Where we took into account 2D cylindrical symmetry of the problem and, consequently, Δ is a radial Laplace operator.

Obviously, integrating this equation over an area inside the contour $r = \text{const}$ we arrive at identity. Equation 21 is the Helmholtz one (similar to Eq. 10), and has the decreasing solution

$$B_{sc} = \frac{\Phi_{sc}}{2\pi\lambda_m^2} K_0 \left(\frac{r}{\lambda_m} \right), \tag{22}$$

Where K_0 is the modified Bessel function. The induced magnetic field behaves asymptotically as follows:

$$B_{sc}(r \rightarrow 0) \sim -\ln(r), \quad B_{sc}(r \rightarrow \infty) \sim \frac{1}{\sqrt{r}} \exp(-r/\lambda_m) \tag{23}$$

This behavior reflects the cylindrical symmetry of the model. For example, Yukawa's potential has been transformed: $(1/r) \exp(-r/\lambda_m) \rightarrow (1/\sqrt{r}) \exp(-r/\lambda_m)$. As expected, the screening length coincides with the magnetic field penetration length λ_m . Formula (23) is 2D analog of Eq. 11.

4 CONCLUSION

In this paper, we have touched upon the problem of the influence of the medium on fundamental interactions. First, on the basis of articles [9–11], we showed that as a result of the interaction of the gravitational field with the cosmological medium, the gravitational potential is subject to exponential screening on large cosmological scales. Then, following the model considered in paper [18], we have traced a close analogy between the interaction of the gravitational field with the cosmological medium and the interaction of the magnetic field of a solenoid with a superconducting medium. As a result of this interaction, the induced magnetic field in the superconductor undergoes exponential screening at distances exceeding the magnetic field penetration length.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

AZ: Conceptualization, Investigation, Writing. VS: Formal analysis, Editing.

ACKNOWLEDGMENTS

The authors are grateful to Boris Svistunov for fruitful discussions and valuable comments.

REFERENCES

1. Brydges DC, Martin PA. Coulomb Systems at Low Density. *J Stat Phys* (1999) 96:1163–330. arXiv:cond-mat/9904122 [cond-mat.stat-mech]. doi:10.1023/A:1004600603161
2. Merches I, Tatomir D, Lupu RE. *Basics of Quantum Electrodynamics*. NY, USA: Taylor & Francis (2019).
3. Anderson PW. Plasmons, Gauge Invariance, and Mass. *Phys Rev* (1963) 130(1):439–42. doi:10.1103/physrev.130.439
4. Englert F, Brout R. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys Rev Lett* (1964) 13:321–3. doi:10.1103/PhysRevLett.13.321
5. Higgs PW. Broken Symmetries and the Masses of Gauge Bosons. *Phys Rev Lett* (1964) 13:508–9. doi:10.1103/PhysRevLett.13.508
6. Linde AD. *Particle Physics and Inflationary Cosmology (Contemporary Concepts in Physics Series)*. NY, USA: Taylor & Francis (1990).
7. Svistunov B, Babaev E, Prokofev N. *Superfluid States of Matter*. NY, USA: Taylor & Francis (2015).
8. Landau LD, Lifshitz EM. *The Classical Theory of Fields (Course of Theoretical Physics Series, V.2)*. Oxford, UK: Pergamon Press (2000).
9. Eingorn M. First-order Cosmological Perturbations Engendered by Point-like Masses. *Astrophys J* (2016) 825:84. arXiv:1509.03835 [gr-qc]. doi:10.3847/0004-637X/825/2/84
10. Eingorn M, Kiefer C, Zhuk A. Scalar and Vector Perturbations in a Universe with Discrete and Continuous Matter Sources. *J Cosmol Astropart Phys* (2016) 2016:032. arXiv:1607.03394 [gr-qc]. doi:10.1088/1475-7516/2016/09/032
11. Eingorn M, Kiefer C, Zhuk A. Cosmic Screening of the Gravitational Interaction. *Int J Mod Phys D* (2017) 26:1743012. arXiv:1711.01759 [gr-qc]. doi:10.1142/S021827181743012X
12. Mukhanov V, Feldman HA, Brandenberger RH. Theory of Cosmological Perturbations. *Phys Rept* (1992) 215:203–333. doi:10.1016/0370-1573(92)90044-Z
13. Gorbunov DS, Rubakov VA. *Introduction to the Theory of the Early Universe: Cosmological Perturbations and Inflationary Theory*. Singapore: World Scientific (2011).
14. Eingorn M, Zhuk A. Hubble Flows and Gravitational Potentials in Observable Universe. *J Cosmol Astropart Phys* (2012) 2012:026. arXiv:1205.2384 [astro-ph.CO]. doi:10.1088/1475-7516/2012/09/026
15. Eingorn M, Zhuk A. Remarks on Mechanical Approach to Observable Universe. *J Cosmol Astropart Phys* (2014) 2014:024. arXiv:1309.4924 [astro-ph.CO]. doi:10.1088/1475-7516/2014/05/024
16. Canay E, Eingorn M. Duel of Cosmological Screening Lengths. *Phys Dark Universe* (2020) 29:100565. arXiv:2002.00437 [gr-qc]. doi:10.1016/j.dark.2020.100565
17. Canay E, Eingorn M, McLaughlin A, II, Arapoğlu AS, Zhuk A. Effect of peculiar Velocities of Inhomogeneities on the Shape of Gravitational Potential in Spatially Curved Universe (2022). arXiv:2201.07561 [gr-qc].
18. Mangel I, Kapon I, Blau N, Golubkov K, Gavish N, Keren A. Stiffnessometer: A Magnetic-field-free Superconducting Stiffness Meter and its Application. *Phys Rev B* (2020) 102:024502. arXiv:1705.00624 [cond-mat.supr-con]. doi:10.1103/PhysRevB.102.024502

Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's Note: All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

Copyright © 2022 Zhuk and Shulga. This is an open-access article distributed under the terms of the Creative Commons Attribution License (CC BY). The use, distribution or reproduction in other forums is permitted, provided the original author(s) and the copyright owner(s) are credited and that the original publication in this journal is cited, in accordance with accepted academic practice. No use, distribution or reproduction is permitted which does not comply with these terms.