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# Scalar Field and Particle Dynamics in Conformal Frame

*Arbab I. Arbab*[1,2]*, *Razan S. Albughylil*[1] and *Alnadhief H. A. Alfedeel*[2,3]

[1]Department of Physics, College of Science, Qassim University, Buraidah, Saudi Arabia, [2]Department of Physics, Faculty of Science, University of Khartoum, Khartoum, Sudan, [3]Department of Mathematics and Statistics, College of Sciences, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh, Saudi Arabia

The dynamics of the scalar field and particle in a conformal frame are considered. The conformal Klein-Gordon equation describing the scalar field is transformed into the quantum Telegraph equation in Minkowski space-time. The conformal factor acts like a background field having a perfect energy-momentum tensor. The scalar field decays exponentially with time during inflation allowing the conformal field to induce space energy. The conformal field grows with time at the expense of decreasing the energy density of the real scalar field. Einstein's tensor embodies an energy-momentum tensor representing the background contribution reflecting the matter aspect of the gravitational field. The energy density arising from the conformal field is negative. The background energy associated with Einstein's curvature tensor gives rise to massive gravitons that act like a cosmological constant. In an expanding Universe, this particular case yields a background energy proportional to the square of the scalar field mass giving rise to the massive graviton. Because of the background fluid, which is intrinsically coupled to space curvature, the particle's motion is found to exhibit a drag force and therefore moves in a curved path even no matter around exists. It is found that breaking the conformal invariance gives rise to the mass generation of gravitons.

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***Edited by:***
*Peter Dunsby,*
*University of Cape Town, South Africa*

***Reviewed by:***
*Kazuharu Bamba,*
*Fukushima University, Japan*
*Ignazio Licata,*
*Institute for Scientific Methodology (ISEM), Italy*

***Correspondence:**
*Arbab I. Arbab*
*aiarbab@uofk.edu*

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## INTRODUCTION

The general theory of gravitation describes the gravitational effects as manifestations of the curvature of space-time. The concept of force may not be needed in Einstein's formalism. The motion of an object in a gravitational field can not be distinguished from that one due to a non-inertial motion. While Newton used scalars and vectors to formulate his theory of gravitation, Einstein, on the other hand, employed tensors to envisage the effect of curvature on the motion of a particle that has interesting mathematical properties. The particle path in curved space is defined by a geodesic equation that incorporates a quantity embodying the gravitational field. The above equivalence between curvature and gravity is a crucial factor on which Einstein built his theory [1].

Interestingly, the general theory of relativity (GTR) reproduces and generalizes Newton's theory of gravitation. The mathematical properties of the tensors in which the GTR was built gave rise to new and additional mathematical quantities that are found to bear physical meanings. The overwhelmingly confirmed predictions of the GTR qualify it to be the appropriate theory of gravitation. Besides the GTR, a rival theory of gravitation was introduced by Brans-Dicke (BD) in 1961 where the gravitational interaction is partaken by a scalar field [2].

In this formalism, the gravitational constant is considered to be a function of this scalar field. This assumption dictates that Newton's constant had to vary with space-time. Besides the gravitational constant, the BD theory encompasses an additional coupling that remained un-determined by the theory. To yield the same prediction for the solar system constraints as that of the GTR, the Cassini mission limited the coupling constant to a value of $\omega > 10^4$ that makes the two paradigms indistinguishable for the present epoch [3]. They however differ from other cosmic eras.

Owing to Noether's theorem, the conserved physical quantities emerge from the invariance of the equation of motion (or the Lagrangian) under a given transformation. A new transformation called the conformal transformation that was known to preserve the angle between two curves is believed to be appropriate to fundamental physical theories. The electromagnetic and quantum theories are found to be invariant under the conformal transformation, but the GTR is not. This poses an impasse to GTR. Hence, the need for a conformal theory of gravitation is raised. This time the BD is found to be invariant under the conformal transformation. Further attempts to quantize gravity following the same prescription as that made for the electromagnetic theory were doomed to failure.

Weyl formulated a conformal theory of gravitation by introducing the Weyl tensor which is unlike the Riemann and Ricci tensor and the scalar is conformally invariant. The Weyl tensor represents the curvature of the space where the Ricci tensor vanishes. The conformal factor, transforming a given metric tensor into a conformal one, is a function of space and time. It is found that this factor acts as representing a background field coupled to gravity. It induces an energy-momentum tensor in a gravitational theory that can be attributed to the background (the material aspect of the space).

A disturbance in the gravitational field will give rise to a gravitational wave that travels at the speed of light in a vacuum. Thus, if this is mediated by a graviton it must be massless. A possibility that gravity is mediated by massive graviton were entertained by Freund *et al.* [4]. In this theory, a finite-range gravitation theory is proposed which introduces the idea that the graviton could be massive. He then found that the effect of such a proposal modified the Einstein equations by introducing a cosmological-like term proportional to the square of the graviton mass ($-m^2$) in the field equations. Recall that the mass term in the electromagnetic Lagrangian is of the form $m^2 A_\mu A^\mu$, which in GTR would become $m^2 g^{\mu\nu} g_{\mu\nu} = 4m^2$, which is a constant. For such a reason, the mass term can be included in the cosmological constant in the Einstein-Hilbert action.

This is reminiscent of the massive electrodynamics proposed by Proca that was known to break the gauge invariance of electromagnetism. Proca theory could be applicable inside a superconductor where the electromagnetic field becomes a short-range interaction. An electrodynamics that respects the gauge invariance in conjunction with the introduction of the photon mass is proposed in [5].

We would like here to explore a gravitational scalar-tensor theory and see how the scalar field proceeds in conformal coordinates. The scalar field partakes the gravitational interactions with the graviton. The wave equation of the scalar field is found to be described by a new matter-wave equation proposed by Arbab [6, 7]. This is a quantum Telegraph equation whose classical analog governs the propagation of the electric signals in transmission lines. Hence, a conformal scalar gravitational theory can be compared with a massive gravitational theory. We in a sense deal with a quantization of the gravitational scalar-tensor theory. We found the conformal factor increases exponentially with time. The exponent term involves a mass that is related to the mass of the graviton.

An exponential conformal factor is connected, in the Robertson-Walker Universe, to an inflationary period in the history of the Universe. In this era, the inflationary period is driven by a cosmological constant that was initially introduced by Einstein in his static Universe to hold it from collapse, if gravity is the only existing force. Today the cosmological constant is believed to account for the background energy contained in the Universe. This cosmological constant is found to be related to the conformal factor. Thus to resolve the dark energy problem, one is inclined to assume the actual space we live in is a conformal Riemannian space, however.

The above particular quantization associated with the conformal gravity could bring a lot of clues to the quantization of a gravitational theory. Furthermore, we anticipate that only a conformal gravity can be quantized, and not any other gravitational theory. This could be the reason why up to now no single quantized gravitational theory existed with a large consensus among physicists.

We would like in this work to study the evolution of the scalar-tensor gravity and study the conformal aspect of the theory. In particular, we would like to explore the scalar wave equation, governed by the Klein-Gordon equation, and see how it evolves in a conformal coordinate. Note that in a conformal representation the background field extracted from the Einstein curvature acts as if it has an energy-momentum tensor. This energy-momentum tensor depends on the functional form of the conformal factor.

We would like to find the conformal factor by comparing the Klein-Gordon equation, describing the scalar field in the conformal coordinate with a recently proposed quantum Telegraph equation [6, 7]. We then calculate the Ricci tensor and scalar and see how the Universe with these new quantities evolves. In particular, we want to see the effect of the mass of the scalar field on the evolution of the Einstein field equations without solving them. A conformal theory is found to be equivalent to introducing a cosmological constant that is also equivalent to a massive graviton field theory. The appearance of the mass term in the system is like the Higgs mechanics that generates the particle's mass that were once assumed massless [8]. Because a background fluid could be inherently coupled to curvature, that background field is reflected in the particle's motion which tends to be a drag-like type. The presence of this background field influences the particle motion mimicking that one moving in a curved path. The existence of a background fluid permeating the whole space will therefore justify why all objects experience inertia. It can be seen as tantamount to the effect of space on the motion of gravitating bodies.

## CONFORMAL GRAVITATION

Einstein's field equations describing the gravitational field can be expressed as

$$G_{\mu\nu} = \kappa T_{\mu\nu}\,, \qquad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R\,, \qquad \kappa = \frac{8\pi G}{c^4}, \tag{1}$$

where $T_{\mu\nu}$ is the energy momentum tensor of the matter representing the gravitational source. The above Einstein equations are not invariant under the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu}. \tag{2}$$

This is because the Einstein tensor is not conformally invariant. The Ricci and tensor and scalar transform in $D$—dimensions as [9–11].

$$\tilde{R}_{\mu\nu} = R_{\mu\nu} + \Omega^{-2}\left[2(D-2)\,\Omega_{,\mu}\Omega_{,\nu} - (D-3)\,\Omega_{,\lambda}\Omega^{,\lambda} g_{\mu\nu}\right] - \Omega^{-1}\left[(D-2)\Omega_{;\mu\nu} + g_{\mu\nu}\Box\Omega\,\right], \tag{3}$$

$$\tilde{R} = \Omega^{-2}\left[R - 2(D-1)\frac{\Box\Omega}{\Omega} - (D-1)(D-4)\,g^{\mu\nu}\frac{\Omega_{,\mu}\Omega_{,\nu}}{\Omega^2}\right], \tag{4}$$

and the Einstein's tensor transforms as

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} + \frac{D-2}{2\Omega^2}\left[4\Omega_{,\mu}\Omega_{,\nu} + (D-5)\Omega_{,\lambda}\Omega^{,\lambda} g_{\mu\nu}\right] - \frac{D-2}{\Omega}\left[\Omega_{;\mu\nu} - g_{\mu\nu}\Box\Omega\,\right]. \tag{5}$$

Assume the energy momentum tensor of the matter present to be an ideal fluid, *i.e.,*

$$T_m^{\mu\nu} = \left(\rho + p\,c^{-2}\right) u_\mu u_\nu - p\, g_{\mu\nu}, \tag{6}$$

whose energy and momentum conservation equation transforms, under the conformal transformation, as

$$\tilde{T}^{\mu\nu}_{m;\nu} = -\frac{\Omega^{,\mu}}{\Omega}\, T_m, \tag{7}$$

which is conformally conserved when $T_m = 0$. This case is usually valid for radiation fluid.

Now **Eq. 5** can be rewritten in the form

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} + T^{\Omega}_{\mu\nu}, \tag{8}$$

where

$$T^{\Omega}_{\mu\nu} = \frac{D-2}{2\Omega^2}\left[\,4\Omega_{,\mu}\Omega_{,\nu} + (D-5)\,\Omega_{,\lambda}\Omega^{,\lambda} g_{\mu\nu}\,\right] - \frac{D-2}{\Omega}\left[\,\Omega_{;\mu\nu} - g_{\mu\nu}\Box\Omega\,\right], \tag{9}$$

is an expression of the energy momentum tensor that represents the background contribution, and acts as if the space is full of a scalar field $\Omega$ having the above energy momentum tensor.

One can express the above tensor in an ideal fluid form to deduce the form of the pressure and energy density, *viz.,*

$$\mathcal{T}_{\mu\nu} = \kappa^{-1}c^{-2}\left(-\frac{4\Omega_{,\mu}\Omega_{,\nu}}{\Omega^2} + 2\,\frac{\Omega_{;\mu\nu}}{\Omega}\right) - \kappa^{-1}c^{-2}\left[-\frac{\Omega_{,\lambda}\Omega^{,\lambda}}{\Omega^2} + \frac{2\Box\Omega}{\Omega}\right] g_{\mu\nu}, \tag{10}$$

where $D = 4$. This energy-momentum tensor represents the matter aspect (content) of the gravitational field. Hence, comparing **Eq. 10** with **Eq. 6** indicates that the background pressure is given by

$$\mathcal{P} = \kappa^{-1}c^{-2}\left[-\frac{\Omega_{,\lambda}\Omega^{,\lambda}}{\Omega^2} + \frac{2\Box\Omega}{\Omega}\right]. \tag{11}$$

One can now associate an angular momentum with the background field defined by

$$J^{\mu\nu\,0} = x^\mu\,\mathcal{T}^{\nu\,0} - x^\nu\,\mathcal{T}^{\mu\,0}. \tag{12}$$

The orbital angular momentum corresponds to the $J^{ij0}$ components

$$J^{ij\,0} = x^i\,\mathcal{T}^{j\,0} - x^j\,\mathcal{T}^{i\,0}. \tag{13}$$

Let us now consider the wave equation of the scalar field $\phi$, and see how it transforms under the conformal transformation. Let us assume the scalar field transform under the conformal transformation as

$$\tilde{\phi} = \Omega^{\xi}\,\phi, \tag{14}$$

where $\xi$ is some real number (weight).

The D'alembertian of the scalar field $\phi$ is found to transform as [9, 10].

$$\tilde{\Box}\tilde{\phi} = \Omega^{\xi-2}\left[\Box\phi + (2\xi + D - 2)\,\frac{\Omega_{,\mu}}{\Omega}\phi_{,\mu} + \left(\xi\frac{\Box\Omega}{\Omega} + \xi(\xi + D - 3)\frac{\Omega^{,\mu}\,\Omega_{,\mu}}{\Omega^2}\right)\phi\right], \tag{15}$$

where

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \tag{16}$$

The conformal Klein-Gordon equation will be

$$\tilde{\Box}\tilde{\phi} + \left(\frac{\tilde{m}c}{\hbar}\right)^2\tilde{\phi} = 0, \tag{17}$$

where $\tilde{m}$ is the conformal mass. A conformal covariance and invariant formulation of scalar wave equations was studied in [12]. It is proposed in this work that the conformal factor is proportional to the field mass. Another study of such models was initiated by a treatment of interacting conformal scalar and gravitational fields [13].

The massless wave equation in conformal frame, $\tilde{\Box}\tilde{\phi} = 0$, becomes

$$\Box\phi + \frac{2}{c^2}\frac{\dot{\Omega}}{\Omega}\frac{\partial\phi}{\partial t} = 0\,, \qquad \xi = 0\,, \qquad \Omega = \Omega(t). \quad (17a)$$

However, for $\xi \neq 0$, we will have a Telegraph equation with a distortion given by

$$\Box\phi + \frac{2(\xi+1)}{c^2}\frac{\dot{\Omega}}{\Omega}\frac{\partial\phi}{\partial t} + \left[\xi\frac{\Box\Omega}{\Omega} + \frac{\xi(\xi+1)}{c^2}\frac{\dot{\Omega}^2}{\Omega^2}\right]\phi = 0. \quad (17b)$$

The Telegraph equation is the equation that governs the propagation of electric signals in transmission lines. Thus, the presence of the conformal factor $\Omega$ (background) induces a dissipation in the wave equation that is similar to the effect of a conductor on the propagation of the electromagnetic wave inside it. However, if $\xi = -1$, and $\Omega \propto e^{imc^2t/\hbar}$, then **Eq.17b** yields the Klein-Gordon equation.

In a recent work, it shown that the quantum Telegraph equation describing the matter wave (de Broglie wave) is found to yield the Schrodinger, Klein-Gordon and Dirac equations [6, 7]. It is expressed as

$$\Box\phi + \frac{2m}{\hbar}\frac{\partial\phi}{\partial t} + \left(\frac{mc}{\hbar}\right)^2\phi = 0, \quad (18)$$

where $\hbar$ is the reduced Planck's constant. **Eq. 18** is a distortionless Telegraph equation that preserves the wave entity. The solution of the above equation is a traveling wave to left and right with speed of light, $c$, but with time-decaying amplitude. It is of the form

$$\phi = \phi_0 e^{-mc^2t/\hbar}\, e^{-i\left(\pm c\,k\,t - \vec{k}\cdot\vec{r}\right)}, \quad (19)$$

where $\phi_0$ is a constant. The above quantum equation can be seen as a resistive (dissipative) Klein-Gordon equation. It represents a propagation of a massive scalar field in a medium (background).

Applying **Eq. 15** in **Eq. 17** yields

$$\Box\phi + 2\frac{\dot{\Omega}}{c^2\Omega}\frac{\partial\phi}{\partial t} + \left(\frac{mc}{\hbar}\right)^2\phi = 0\,, \qquad \tilde{m} = \Omega^{-1}m\,, \qquad D = 4, \quad (20)$$

if $\Omega = \Omega(t)$. Moreover, it seems that as if the scalar wave travels in a resistive medium. Or, one may envisage space as a medium. It is also similar to the equation of the propagation of the electric field in a conducting medium [14]. In the latter case the coefficient, $2\frac{\dot{\Omega}}{c^2\Omega} = \mu_0\sigma$, impling that the background medium bears an electric conductivity given by

$$\sigma = 2\,\varepsilon_0\frac{\dot{\Omega}}{\Omega}. \quad (21)$$

Therefore, when light propagates in a Minkowski space-time it experiences a dissipation while no dissipation occurs in the conformal frame.

## CONFORMAL BACKGROUND FIELD

The invariance of a gravitational theory under the conformal transformation should bring new insight to the theory. Owing to Noether's theorem, an invariance of a given theory under a given symmetry should result in a conserved physical quantity. Thus, a question is posed what is the benefit of a theory being invariant under conformal transformation? Motivated by **Eqs 8–10**, one can hypothesize that the conformal factor, $\Omega$, is not a mere function but is linked to a field representing the background medium. This field can be connected with some scalar fields. Comparing **Eq. 18** with **Eq. 20** yields

$$\Omega = \Omega_0\, e^{mc^2t/\hbar}, \quad (22)$$

where $\Omega_0$ is a constant, and $m$ is the mass of the conformal field. This holds in an expanding homogenous Universe. It is interesting to see that one can treat the conformal factor as a field (conformal field). The conformal field is the field due to the background medium outlined above. But since the background field emanated from the curvature term, we may say that the scalar field is coupled to curvature *via* the background field, as evident from **Eq. 20**.

Thus, a scalar field satisfying the Klein-Gordon will be equivalent to the quantum Telegraph equation in the conformal coordinate. **Eq. 18** is similar to the wave equation of an electric field propagating in a conducting medium [14]. Because the wave is coupled to the wire, it experiences a decay in its amplitude.

**Eq. 19** indicates that $\Omega \propto |\phi|^{-1}$. A time-dependent conformal factor makes the metric behave like an expanding one. In the Robertson-Walker metric, the scale factor of an expanding Universe, $a$, is proportional to the conformal factor $\Omega$ so that

$$\frac{\dot{\Omega}}{\Omega} = \frac{\dot{a}}{a} = H, \quad (23)$$

is Hubble's parameter. Recall that during inflation the Hubble's parameter is constant. Hence, $H = \frac{mc^2}{\hbar}$, where $m$ is the mass of the inflaton (scalar) field. This implies that the Universe was scaled by a factor depending on the scalar mass and time (homogeneous Universe). The mass of this scalar field can be calculated from the knowledge of Hubble's parameter. Notice that here the scalar field, $\phi$, mediates the gravitational interactions which act like a massive propagator (massive graviton). Consequently, the gravitational interactions become short-range.

Interesting studies related to the interaction of scalar field in curved space are considered [15].

Owing to **Eqs 10**, **11**, and the fact that $\mathcal{T}_{\mu\nu}$ is that of a perfect fluid, then the background pressure is

$$\mathcal{P} = \kappa^{-1}c^{-2}\left(\frac{\dot{\Omega}}{\Omega}\right)^2 = \kappa^{-1}c^{-2}H^2, \quad (24)$$

and its energy density is

$$\rho_\Omega = -3\kappa^{-1}c^{-2}\left(\frac{\dot{\Omega}}{\Omega}\right)^2 = -3\kappa^{-1}c^{-2}H^2. \quad (25)$$

Notice that a negative energy density gives rise to inflation. During the inflationary period the scalar field and its energy density ($\phi^2$) decay exponentially with time, as evident from **Eq. 19**. The energy density of the background field $\Omega$ is $\mathcal{T}_{00}$ that yields

$$\rho_{\Omega}c^2 = -3\kappa^{-1}\left(\frac{\dot{\Omega}}{\Omega}\right)^2 = -3\kappa^{-1}\left(\frac{mc^2}{\hbar}\right)^2 = -\frac{3m^2c^6}{8\pi G\hbar^2}, \qquad (26)$$

upon using **Eq. 22**, where $m$ is the mass of the inflaton (scalar) field. This particular energy density corresponds to a Universe filled with a cosmological constant ($\Lambda$). Notice that the scalar field, $\phi$, decreases with time and so does its energy density, while the conformal field, $\Omega$, increases with time. Therefore, since $\Omega$ is the background field, the background energy increases with time while the ordinary scalar field decays with time. This will ultimately drive our Universe into a background-dominated era.

From **Eqs 3**, **4**, one finds

$$\tilde{R}_{00} = R_{00} + 2c^{-2}\left(\frac{\dot{\Omega}}{\Omega}\right)^2, \qquad \tilde{R}_{00} = R_{00} + 2\left(\frac{mc}{\hbar}\right)^2, \qquad (27)$$

and

$$\tilde{R} = \Omega^{-2}\left(R - 6\left(\frac{mc}{\hbar}\right)^2\right), \qquad \tilde{R} = \Omega^{-2}\left(R - 6\frac{H^2}{c^2}\right). \qquad (28)$$

**Equations 27** and **28** imply that the background alone can curve the space. Here the curvature scalar $R$ is that one due to the presence of matter in space. The background curvature is equal to $R_\Omega = -6\left(\frac{mc}{\hbar}\right)^2$. This term can be considered as a quantum correction that rarely appears at a classical level.

Recall that the background energy per oscillator is non-zero and is equal to $E_0 = \frac{1}{2}\hbar\omega$. It is now apparent that the presence of the background energy is equivalent to including a mass term in the Einstein equations. This, in other words, is a new mechanism of giving the graviton a mass. It remarkable that our results agree with that of [16] where they attribute the mass $m$ to the graviton mass. Our results are obtained without solving Einstein's equations, without assuming any form of the energy-momentum tensor of the scalar field. The effect of the background conformal field on the matter source, in a scalar-tensor theory, can be studied in future endeavors.

One can now express the conformal Einstein's tensor in **Eq. 5**, for $D = 4$, in the form

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} + \Lambda\, g_{\mu\nu} + 2\Lambda\, \delta^0_\mu\, \delta^0_\nu\,, \qquad \Lambda = \frac{m^2c^2}{\hbar^2}. \qquad (8a)$$

The second term in Einstein's tensor mimics the cosmological constant which in the present context reflects the effect of the background that can be extracted from Einstein's curvature. It represents a perturbation in the space-time curvature which has now a quantum signature. Quantum effects can break conformal invariance too [17].

The last term in **Eq.8a** can be seen as a representation of the energy-momentum tensor due to the presence of a background field that signifies the interaction of the gravitational field with the quantum scalar field. Such a field also emerges when an electromagnetic field is coupled to a scalar field that led to the Higgs field permeating the whole space. The remaining scalar (Higgs) field would have an energy-momentum tensor of a form expressed in **Eq.8a**. The remaining field filling the space can be thought of like that assigned to ether. Here the ether is more physical than the hypothetical one early introduced to help the electromagnetic field propagate in space. Thus, the notion of a quantum ether appears when a quantum field is coupled to the gravitational field. Similarly, a classical ether will result from the coupling of the gravitational field with a classical scalar field.

Remarkably, the background energy is not exhibited in Einstein's ordinary equations that overlooked the material contribution of space so that the conformal transformation highlights the material aspect of space that is attached to the space-time curvature. It is evident from **Eq.8a** that the generation of mass breaks the time-translation symmetry of Einstein's equations.

Like the degeneracy in a quantum system which is sometimes lifted when a perturbation is applied to a system, we see here that the conformal transformation of Einstein's curvature separated (extracted) the background contribution from the curvature of space-time. This contribution is of paramount importance. We see that the background field couples to massless scalar field giving it a mass and to the massive field inducing a dissipative effect in its evolution. It is thus very interesting that breaking the conformal transformation (when time-symmetry is broken) leads to a massive field, *i.e.,* a mechanism to give the massless field a mass. Recall that in quantum field theory paradigms (spontaneous symmetry breaking), breaking a local gauge invariance leads to the emergence of a massive field too.

Breaking of symmetry could lead to tangible cosmological consequences since it adds appreciable energy content to the Universe when the massive field responsible for breaking the symmetry acquires mass. It is argued that the axion field which results from its interaction of photons can impart its energy as a kind of dark energy (matter) that helps accelerate the Universe expansion [18]. Moreover, the presence of a background field could also provide a resistive force that alters the orbital motion of stars inside galaxies modifying it to a non-Keplerian behavior. That is because the velocity of stars in elliptical galaxies exhibits an unexpectedly flat pattern. This latter behavior is resolved upon imposing dark matter contribution [19]. Scalar field dark matter models are considered owing to the recent accurate cosmological and astrophysical observations [20].

## PARTICLE DYNAMICS IN CONFORMAL FRAME

The motion of a particle in the curved space is described by the geodesic equation

$$\frac{dv^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda}\, v^\nu\, v^\lambda = 0, \qquad (29)$$

where $v^\mu$ is the particle's velocity and $\Gamma^\mu_{\nu\lambda}$ are the Christoffel symbol. The conformal transformation of the Christoffel symbols is given by

$$\tilde{\Gamma}^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} + \Omega^{-1}\left[\Omega_{,\nu}\,\delta^\mu_\lambda + \Omega_{,\lambda}\,\delta^\mu_\nu - \Omega_{,\delta}\, g^{\mu\delta}\, g_{\nu\lambda}\right], \qquad (30)$$

so that the equation of the geodesic in the conformal frame will become

$$\frac{d\tilde{v}^\mu}{d\tilde{\tau}} + \tilde{\Gamma}^\mu_{\nu\lambda}\,\tilde{v}^\nu\,\tilde{v}^\lambda = \frac{dv^\mu}{ds} + \Gamma^\mu_{\nu\lambda}\,v^\nu\,v^\lambda + \left[2\partial_\nu(\ln\Omega)\,v^\nu\,v^\mu - c^2(\partial_\delta\ln\Omega)\,g^{\mu\delta}\right]. \quad (31)$$

If the particle in the conformal frame is described by the geodesic equation (when the right-hand side of **Eq. 31** vanishes), then in the Minkowski space-time, one has

$$\frac{dv^\mu}{d\tau} + 2\gamma\,\frac{\dot{\Omega}}{\Omega}\,v^\mu = 0, \quad (32)$$

if $\Omega = \Omega(t)$, where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ is the Lorentz factor, and the last term in **Eq. 31** vanishes. The additional acceleration in the left-hand side of **Eq. 32** is due to the conformal field that is coupled to the particle's motion. It is like a viscous (drag) term that arises when a particle travels in a medium. This is so since the conformal field represents the field of the background fluid. Upon employing **Eq. 22** this viscous-like term would become

$$a^\mu_\Omega = -2\gamma\left(\frac{mc^2}{\hbar}\right)v^\mu, \quad (33)$$

which depends on the background field mass, $m$. Therefore, as long as $m \neq 0$, all particles travel in a curved space, no matter if other sources of matter exist, experience a drag force. This could justify why all objects resist motion or exhibit inertia. Therefore, the influence of the background on the motion of quantum particles would be prominent when subject to motion in a narrow space that could lead to prominent phenomena (e.g., Casimir effect). A self-force on a charged particle in an external scalar field is recently considered and found to lead to interesting phenomena [21].

Let us now consider the particle dynamics in conformal frame. To this end, we define the force acting on the particle in a conformal frame as follows

$$\tilde{f}^\mu = \frac{d\tilde{p}^\mu}{d\tilde{\tau}}, \quad (34)$$

where

$$d\tilde{\tau} = \Omega\,d\tau\,, \qquad \tilde{m}_p = \Omega^n m_p, \quad (35)$$

where $m_p$ is the particle's mass. Applying **Eq. 35** in **Eq. 34** yields the particle's acceleration as

$$\tilde{a}^\mu = \Omega^{n-2}\left[\frac{dv^\mu}{d\tau} + (n-1)\frac{\Omega_{,\nu}}{\Omega}\,v^\mu\,v^\nu\right]. \quad (36)$$

Thus, because of the second term in the right-hand side of **Eq. 36**, a free particle in a conformal frame receives an extra acceleration when compared with that one in the Minkowski space-time. It is given by

$$a^\mu_\Omega = (n-1)\frac{\Omega_{,\nu}}{\Omega}\,v^\mu\,v^\nu. \quad (37)$$

Therefore, if $\Omega = \Omega(t)$, then the above acceleration becomes

$$a^\mu_\Omega = (n-1)\,\gamma\,\frac{\dot{\Omega}}{\Omega}\,v^\mu, \quad (38)$$

which upon using **Eq. 22** yields

$$a^\mu_\Omega = (n-1)\,\gamma\,\frac{mc^2}{\hbar}\,v^\mu. \quad (39)$$

If we compare **Eq. 39** with **Eq. 33**, we deduce that $n = 3$ so that the conformal particle mass, $\tilde{m}_p = \Omega^3 m_p$. Because of the appearance of $\hbar$, we call this acceleration a quantum-conformal acceleration.

It is pertinent to mention that the inappropriate choice of a frame of reference (*e.g.*, rotating frame) has induced a fictitious force, *i.e.,* the Coriolis force.

## CONCLUDING REMARKS

We have found that a scalar field satisfying the Klein-Gordon equation reduces to a scalar field satisfying the quantum Telegraph equation in the conformal representation. The conformal factor acts like a scalar field having a negative energy density giving rise to inflation. The energy-momentum tensor of the background field has a perfect fluid form. The background energy density is found to be proportional to the square mass ($-m^2$) of the scalar (inflaton) field. This agrees with a theory of finite-range gravitation proposed by Freund et al. It is equivalent to introducing a cosmological constant in the GTR. The conformal Ricci scalar involves the mass of the scalar field partaking in the gravitational interactions. A massive scalar field coupled to a background field endows space with non-zero curvature. The conformal field ($\Omega$), representing the background, is coupled to the scalar field by the relation, $\Omega \propto |\phi|^{-1}$. In this scenario, the massive scalar field mediates the gravitational interactions which make them short-range. Therefore, the scalar field acts like the graviton. One can assume that the real Universe has a space structure that is different from that one due to Riemann. In such a space the Universe is filled with a background field that is necessary for the gravitational interaction to propagate which mimics the aether that was advocated to allow the electromagnetic wave to travel in space. We have shown that the background fluid (field) is coupled to the particle motion as well. This field couples to a massless scalar field giving it a mass and to the massive field inducing a dissipative effect in its evolution.

## DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

## AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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