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Neutrinos in a Minimal 3-3-1 Model

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In this work, we present a general review of neutrino physics in the minimal 331 model. New gauge and scalar interactions are present, with violation of both flavor and lepton numbers. Including mixing angles and possible CP-violating phases, 15 new parameters arise in vector and scalar neutrino interactions. We also bring to light a discussion at the different neutrino bases that naturally appear in most beyond the Standard Model physics and, in particular, in the minimal 331 model.

Keywords: neutrinos, gauge models, 331 models, CP violation, flavor violation

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1 INTRODUCTION

Although proposed in 1930 by Pauli in his famous letter to the nuclear physics community [1], neutrinos are still a mystery today. CP violation [2–5], the origin of neutrino masses [5, 6], and, potentially, even dark matter and cosmology [7, 8] are examples of problems related to neutrino physics still to be solved. Yet, the mere fact of having non-zero masses already puts these elusive particles in the spotlight for new physics beyond the Standard Model (BSM).

To address some of these points, and many others not related to neutrino physics, many extensions of the usual $SU(2) \times U(1)$ Electroweak Standard Model (SM) have been studied, both modifying its gauge symmetry structure and particle content. One of the simplest extensions which modify the gauge structure is the so-called 331 models, where the electroweak gauge group enlarged to a $SU(3) \times U(1)$ symmetry.

These models were originally proposed as a natural explanation for the number of fermion generations observed in nature. In the SM, the triangle gauge anomalies are canceled out individually in each fermion generation. Hence, the number of families is theoretically unrestricted. The same conclusion holds in many extensions of the SM, as well. On the other hand, with an appropriate choice of representations, the 331 models can be made anomaly-free only when the number of generations is a multiple of three [9]. Moreover, asymptotic freedom constrains this number to be exactly three.

Many other interesting, and phenomenologically rich, characteristics arise in the 331 models as, for example, an automatic Peccei-Quinn symmetry [10], the natural presence of Zee and Zee-Babu loop mechanisms to generate neutrino masses [11], and the presence of a new neutral gauge boson together with flavor-changing neutral currents (FCNC) at tree-level in the quark sector [12, 13]. As a simple extension of the SM, these models can also be seen as an intermediate step towards grand unification theories, which arise at large energy scales [14].

Several phenomenological consequences of the beyond standard physics in this class of models have been studied by several authors which include meson decays and oscillations with new sources of CP violation phases [15, 16]; new particles and new interactions beyond those predicted in the Standard Model in collider experiments, e.g., [17], where it was found that some collider anomalies of excess of WW events could be solved by the vector bosons present in the theory; possible solution to the muon $g - 2$ anomaly [18, 19] by means of a neutral scalar boson and a VEV in the region of 7.2 TeV to 12.2 TeV; as well as stability analysis of the scalar potential [20] and dark matter constraints [18, 21–23]. Some have also proposed possible experimental tests for non-standard neutrino interactions arising in this class of models [24].

As we can see, this model has been getting a lot of attention lately, with promising prospects, and, yet, there is much to explore.

Throughout this work, we introduce the minimal version of these models, called the Minimal 331 Model (m331), with an emphasis on its leptonic sector, introducing a parameter counting in this sector and focusing on neutrino physics.

2 THE MINIMAL MODEL

The 331 models consist of a chiral theory based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$, with the electric charge operator given by

$$Q = T_3 + \beta T_8 + X1, \tag{1}$$

where T_3 and T_8 are the $SU(3)_L$ generators, and X is the hypercharge. In the fundamental representation, we have $T_3 = \lambda_3/2 = \mathrm{Diag}\,(1/2, -1/2, 0)$ and $T_8 = \lambda_8/2 = (1/\sqrt{3})\mathrm{Diag}\,(1/2, 1/2, -1)$. The parameter β determines the particle content of the model and, hence, the possible versions of 331 models. The most interesting choices are $\beta = \pm\sqrt{3}$ and $\beta = \pm 1/\sqrt{3}$, each with unique features and phenomenology.

For $\beta = -\sqrt{3}$, we can either work with only the currently known lepton degrees of freedom or introduce new exotic charged leptons. The former corresponds to the minimal version (m331) [25–27], while the latter is known in the literature as the 331 Model with Heavy Leptons [28]. With $\beta = -1/\sqrt{3}$, new neutral fields (or right-handed neutrinos) must be introduced in the lepton sector, leading to a fermionic candidate for dark matter [29–32]. Finally, for $\beta = \sqrt{3}$ or $1/\sqrt{3}$, similar versions of the previous models arise.

Henceforth, we focus our attention only on the m331, with $\beta = -\sqrt{3}$ in **Eq. 1**. No new leptonic fields will be introduced and, particularly, no right-handed neutrino is needed. As we will see, even in this minimalistic framework, many modifications arise in the neutrino phenomenology, such as new CP-violating parameters, and flavor and lepton number violating interactions. Such modifications will introduce difficulties in defining a flavor basis as is usually done in the description of neutrino oscillations.

In the m331, the leptonic fields transform under the gauge group as

$$\Psi_{aL} = \left(\nu'_{aL}, l'_{aL}, (l'_{aR})^c\right)^T \sim (1, 3, 0), \tag{2}$$

where $a = 1, 2, 3$ is the generation index and the primes indicate that these fields are gauge symmetry states. The charge conjugation operation is defined as usual[1] $\psi^c = C\bar{\psi}^T$.

In the quark sector, one of the generations, it does not matter which, transform in a different representation of the gauge group

$$Q_{1L} = \left(u'_{1L}, d'_{1L}, J'_{1L}\right)^T \sim (3, 3, 2/3), \tag{3}$$

$$Q_{iL} = \left(d'_{iL}, -u'_{iL}, J'_{iL}\right)^T \sim (3, 3^*, -1/3), \tag{4}$$

where $i = 2, 3$ and the J'_{aL} ($a = 1, 2, 3$) are exotic quarks with electric charge 5/3 (J_1) and −4/3 ($J_{2,3}$).

Such choice is responsible for enforcing the anomaly cancellation only within (a multiple of) three families, as commented in the Introduction. Furthermore, because one of the families transforms differently, FCNC will occur in this model at the tree level, both in gauge and scalar interactions.

In this sector, we also have right-handed (RH) singlets

$$u'_{aR} \sim (3, 1, 2/3), \quad d'_{aR} \sim (3, 1, -1/3), \tag{5}$$

$$J'_{1R} \sim (3, 1, 5/3), \quad J'_{iR} \sim (3, 1, -4/3), \tag{6}$$

where, again, $i = 2, 3$.

To generate mass for all gauge bosons, except the photon, we need to introduce three scalar triplets in the Higgs sector of this model, given by

$$\eta = \left(\eta^0, \eta_1^-, \eta_2^+\right)^T \sim (1, 3, 0), \tag{7}$$

$$\rho = \left(\rho^+, \rho^0, \rho^{++}\right)^T \sim (1, 3, 1), \tag{8}$$

$$\chi = \left(\chi^-, \chi^{--}, \chi^0\right)^T \sim (1, 3, -1). \tag{9}$$

Assuming all VEVs are non-zero, these three scalar fields are sufficient to generate the intended spontaneous symmetry breaking (SSB) pattern

$$SU(3)_C \times SU(3)_L \times U(1)_X \xrightarrow{\chi} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\eta,\rho} SU(3)_C \times U(1)_Q,$$

where $U(1)_Q$ represents the electromagnetic gauge symmetry, with the generator Q given in **Eq. 1** (with $\beta = -\sqrt{3}$).

However, as shown in [27], we cannot generate a non-zero mass for all the charged leptons with only the above triplets. To remedy this situation, a scalar field that transforms as a sextet under SU(3) is introduced by

$$S = \begin{pmatrix} \sigma_1^0 & \sigma_1^- & \sigma_2^+ \\ \sigma_1^- & 2s_1^{--} & \sigma_2^0 \\ \sigma_2^+ & \sigma_2^0 & 2s_2^{++} \end{pmatrix} \sim (1, 6, 0). \tag{10}$$

3 YUKAWA INTERACTIONS AND LEPTON MASSES

For the leptons, the relevant Yukawa interactions are given by

$$-2\mathcal{L}_{\text{yuk}} = G^\eta_{ab}\bar{\Psi}_{aL}\Psi^c_{bL}\eta^* + G^S_{ab}\bar{\Psi}_{aL}S\Psi^c_{bL} + \text{H.c.}, \tag{11}$$

where, by Fermi statistics, G^η (respectively G^S) must be an anti-symmetric (symmetric) matrix.

Explicitly, these interactions correspond to

$$\begin{aligned} -\mathcal{L}_{\text{yuk}} = &\left[\overline{l'_L}\cdot G^\eta \cdot l'_R\right]\eta^{0*} + \left[\overline{\nu'_L}\cdot G^\eta \cdot l'_R\right]\eta_1^+ \\ &-\left[\overline{l'_L}\cdot G^\eta \cdot (\nu'_L)^c\right]\eta_2^- + \frac{1}{2}\left[\overline{\nu'_L}\cdot G^S \cdot (\nu'_L)^c\right]\sigma_1^0 \\ &+\left[\overline{l'_L}\cdot G^S \cdot (\nu'_L)^c\right]\sigma_1^- + \left[\overline{\nu'_L}\cdot G^S \cdot l'_R\right]\sigma_2^+ \\ &+\left[\overline{l'_L}\cdot G^S \cdot l'_R\right]\sigma_2^0 + \left[\overline{l'_L}\cdot G^S \cdot (l'_L)^c\right]s_1^{--} \\ &+\left[\overline{(l'_R)^c}\cdot G^S \cdot l'_R\right]s_2^{++} + \text{H.c.,} \end{aligned} \tag{12}$$

where we omitted the generation indices for convenience.

After SSB, the (non-diagonal) charged lepton mass matrix is given by

$$M^l = \frac{1}{\sqrt{2}}\left(v_\eta G^\eta + v_{\sigma_2} G^S\right), \tag{13}$$

where $\langle \eta^0 \rangle = v_\eta/\sqrt{2}$ and $\langle \sigma_2^0 \rangle = v_{\sigma_2}/\sqrt{2}$.

Now, we can see the importance of the sextet in **Eq. 10** to the lepton masses. Without it, M^l would be an anti-symmetric 3×3 matrix. However, a known result from linear algebra states that the eigenvalues of an anti-symmetric matrix always come in pairs. Hence, as noted in [27], the lightest lepton (the electron) would have zero mass. Adding the sextet makes M^l a general 3×3 matrix, and all charged leptons can obtain a non-zero mass.

As is well known, a general complex matrix can be diagonalized by a bi-unitary transformation, see e.g. [33]. Hence

$$\hat{M}_l = V_L^\dagger \cdot M^l \cdot V_R = \mathrm{Diag}\left(m_e, m_\mu, m_\tau\right), \tag{14}$$

and we define the physical fields for the charged leptons by

$$l_L = V_L^\dagger \cdot l_L', \text{ and } l_R = V_R^\dagger \cdot l_R'. \tag{15}$$

From **Eq. 12**, if $v_{\sigma_1^0} \neq 0$, the neutrinos obtain Majorana masses, given by

$$M^\nu = \frac{v_{\sigma_1}}{\sqrt{2}} G^S, \tag{16}$$

with $\langle \sigma_1^0 \rangle = v_{\sigma_1}/\sqrt{2}$.

Notice that, from **Eqs 13, 16**, the neutrinos and charged lepton masses are related by

$$M^\nu = \left(\frac{v_{\sigma_1}}{v_{\sigma_2}}\right)\left[M^l - \frac{v_\eta}{\sqrt{2}} G^\eta\right]. \tag{17}$$

Since the neutrino mass matrix is symmetric, following the Takagi decomposition [33], we can diagonalize M^ν with only one unitary transformation V_N by

$$\hat{M}^\nu = V_N^\dagger \cdot M^\nu \cdot V_N^\star = \mathrm{Diag}\left(m_1, m_2, m_3\right), \tag{18}$$

and physical fields for the neutrinos are given by

$$\nu_L = V_N^\dagger \cdot \nu_L'. \tag{19}$$

With definitions (15) and (19), we can rewrite the Yukawa interactions with the triplet η as

$$\begin{aligned} -\mathcal{L}_{\text{yuk}}^\eta = & \left[\overline{l_L} \cdot F_{LR}^\eta \cdot l_R\right]\eta^{0*} + \left[\overline{\nu_L} \cdot F_{NR}^\eta \cdot l_R\right]\eta_1^+ \\ & -\left[\overline{l_L} \cdot F_{LN}^\eta \cdot (\nu_L)^c\right]\eta_2^- + \text{H.c.}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} F_{LR}^\eta &= V_L^\dagger \cdot G^\eta \cdot V_R, \\ F_{NR}^\eta &= V_N^\dagger \cdot G^\eta \cdot V_R, \\ F_{LN}^\eta &= V_L^\dagger \cdot G^\eta \cdot V_N^\star. \end{aligned} \tag{21}$$

The interactions with the sextet, in the physical bases for the leptons, are given by

$$\begin{aligned} -\mathcal{L}_{\text{yuk}} = & \frac{1}{2}\left[\overline{\nu_L} \cdot F_{NN}^S \cdot (\nu_L)^c\right]\sigma_1^0 + \left[\overline{l_L} \cdot F_{LN}^S \cdot (\nu_L)^c\right]\sigma_1^- \\ & + \left[\overline{\nu_L} \cdot F_{NR}^S \cdot l_R\right]\sigma_2^+ + \left[\overline{l_L} \cdot F_{LR}^S \cdot l_R\right]\sigma_2^0 \\ & + \left[\overline{l_L} \cdot F_{LL}^S \cdot (l_L)^c\right]s_1^{--} + \left[\overline{(l_R)^c} \cdot F_{RR}^S \cdot l_R\right]s_2^{++} + \text{H.c.}, \end{aligned} \tag{22}$$

with

$$\begin{aligned} F_{NN}^S &= V_N^\dagger \cdot G^S \cdot V_N^\star, & F_{LN}^S &= V_L^\dagger \cdot G^S \cdot V_N^\star, \\ F_{NR}^S &= V_N^\dagger \cdot G^S \cdot V_R, & F_{LR}^S &= V_L^\dagger \cdot G^S \cdot V_R, \\ F_{LL}^S &= V_L^\dagger \cdot G^S \cdot V_L^\star, & F_{RR}^S &= V_R^T \cdot G^S \cdot V_R. \end{aligned} \tag{23}$$

We end this section noting that, although all the lepton fields are in their respective physical basis, the scalar fields in **Eqs 20, 22** are still gauge symmetry states.

4 GAUGE INTERACTIONS

In all 331 models, after SSB, we obtain eight massive gauge bosons, being four singly-charged, $W^\pm$, $V^\pm$, two doubly-charged $U^{\pm\pm}$, and two neutral ones Z and Z'. Finally, because of the residual gauge symmetry of Quantum Electrodynamics, we will have a massless vector boson A (the photon). As the notation suggests, the $W^\pm$ and Z bosons correspond to the known SM vector particles.

In particular, in the m331, the lepton charged current (CC) V-A interactions are given by

$$\begin{aligned} -\mathcal{L}_{\text{cc}} = & \frac{g}{\sqrt{2}}\left[\overline{l_L'}\gamma^\mu \nu_L'\right]W_\mu^- + \frac{g}{\sqrt{2}}\left[\overline{\nu_L'}\gamma^\mu (l_R')^c\right]V_\mu^- \\ & + \frac{g}{\sqrt{2}}\left[\overline{l_L'}\gamma^\mu (l_R')^c\right]U^{--} + \text{H.c.}, \end{aligned} \tag{24}$$

where g is the $SU(3)_L$ coupling constant and, again, generation indices are omitted.

In the physical basis for the leptons, we have

$$\begin{aligned} -\mathcal{L}_{\text{cc}} = & \frac{g}{\sqrt{2}}\left[\overline{l_L}\mu^\mu \cdot U_{LN} \cdot \nu_L\right]W_\mu^- \\ & + \frac{g}{\sqrt{2}}\left[\overline{\nu_L}\gamma^\mu \cdot U_{NR} \cdot (l_R)^c\right]V_\mu^- \\ & + \frac{g}{\sqrt{2}}\left[\overline{l_L}\gamma^\mu \cdot U_{LR} \cdot (l_R)^c\right]U^{--} + \text{H.c.}, \end{aligned} \tag{25}$$

where we have the unitary matrices

$$\begin{aligned} U_{LN} &= V_L^\dagger \cdot V_N, \\ U_{NR} &= V_N^\dagger \cdot V_R^\star, \\ U_{LR} &= V_L^\dagger \cdot V_R^\star. \end{aligned} \tag{26}$$

In the Standard Model, is customary to parametrize the unitary matrix coupling leptons and the W-boson, U_{LN}, as

$$U_{LN} = P_l^\dagger \cdot U_{\text{PMNS}} \cdot P_\nu, \tag{27}$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, containing three Euler-like angles and a complex CP-violating phase (also called the Dirac phase). The matrices P_l and P_ν are defined as

$$\begin{aligned} P_l &= \mathrm{Diag}\left(e^{if_1}, e^{if_2}, e^{if_3}\right), \\ P_\nu &= \mathrm{Diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, 1\right), \end{aligned} \tag{28}$$

with f_a, α_1, α_2 (a = 1, 2, 3) being real parameters.

Another common practice is to absorb the P_l phases in the definition of the physical charged lepton fields, which corresponds to redefining **Eq. 15** as

$$l_L = \left(P_l \cdot V_L^\dagger\right) \cdot l_L', \quad l_R = \left(P_l \cdot V_R^\dagger\right) \cdot l_R'. \tag{29}$$

Notice that the above combination still diagonalizes the mass matrix for the charged leptons in **Eq. 13**. Since the neutrinos are Majorana fields, we cannot absorb the matrix P_ν in their physical states.

With this rephasing, **Eq. 25** can finally be written as

$$\begin{aligned}-\mathcal{L}_{cc} = &\frac{g}{\sqrt{2}}\left[\overline{l_L}\gamma^\mu \cdot U_{\rm PMNS}^{\rm Maj} \cdot \nu_L\right]W_\mu^- \\ &+\frac{g}{\sqrt{2}}\left[\overline{\nu_L}\gamma^\mu \cdot U_V \cdot (l_R)^c\right]V_\mu^- \\ &+\frac{g}{\sqrt{2}}\left[\overline{l_L}\gamma^\mu \cdot U_U \cdot (l_R)^c\right]U^{--} + \text{H.c.},\end{aligned} \tag{30}$$

with

$$U_{\rm PMNS}^{\rm Maj} = U_{\rm PMNS} \cdot P_\nu \tag{31}$$
$$U_V = U_{NR} \cdot P_l, \tag{32}$$
$$U_U = P_l \cdot U_{LR} \cdot P_l. \tag{33}$$

5 INTERLUDE: NEUTRINO BASES—GAUGE VS. MASS VS. FLAVOR

In the last two sections, we made a distinction between two types of fermion fields: the gauge symmetry eigenstates (primed fields) and the physical (or mass) eigenstates (unprimed). These fields are related by unitary transformations, as in **Eqs 15, 19**. This distinction arises due to the presence of spontaneous symmetry breaking in our theory. Before the SSB takes place, all particles are massless, including the fermions. Hence, the states can only be labeled by their definite gauge transformations. After the SSB happens, the fermion fields develop a new quantum number, their masses, and the physical states are determined by diagonalizing the Yukawa couplings, as in **Eqs 14, 18**.

In the literature, an yet third basis is used for neutrinos: the *flavor basis*. Within the SM interactions, in the unitary gauge, the only term where the V_L, V_R, V_N matrices explicitly appear in the Lagrangian is in the W-boson interaction as the combination

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{l_L}\gamma^\mu \cdot U_{\rm PMNS}^{\rm Maj} \cdot \nu_L\right]W_\mu^- + \text{H.c.} \tag{34}$$

All other SM interactions are invariant under unitary transformations of the lepton fields. Hence, at the Lagrangian level, we can define a neutrino flavor eigenstate by

$$\nu_L^{\mathfrak{f}} = U_{\rm PMNS}^{\rm Maj} \cdot \nu_L = \left(\nu_e, \nu_\mu, \nu_\tau\right)^T. \tag{35}$$

In the flavor basis, all SM interactions are diagonal, as with the original gauge symmetry eigenstates. Therefore, three new global symmetries arise, corresponding to the conservation of the electron-, muon- and tau-lepton numbers, L_e, L_μ and L_τ.

However, caution should be taken when working at the quantum (field) level since we cannot change basis at our will. To be physically equivalent, not only the Lagrangian must be invariant, but we also must preserve the canonical (anti-) commutation algebra. Following [34], it can be shown that flavor and mass neutrinos cannot be simultaneously quantized. Let $a_a(\mathbf{p}, h)$, $b_a(\mathbf{p}, h)$ be creation operators for the massive fields, with $\mathbf{p}$ being the momentum and $h = \pm 1$ the helicity of each mode, satisfying the usual anti-commutation relation

$$\{a_a(\mathbf{p}, h; t),\, a_b^\dagger(\mathbf{p}', h'; t)\} = (2\pi)^3 \delta_{ab}\delta_{hh'}\delta(\mathbf{p} - \mathbf{p}'), \tag{36}$$
$$\left\{b_a(\mathbf{p}, h; t),\, b_b^\dagger(\mathbf{p}', h'; t)\right\} = (2\pi)^3 \delta_{ab}\delta_{hh'}\delta(\mathbf{p} - \mathbf{p}'), \tag{37}$$

and all other anti-commutators vanish.

Then, as a direct consequence of **Eq. 35**, the creation operators $A_a(\mathbf{p}, h)$, $B_a(\mathbf{p}, h)$ for the flavor eigenstates satisfy

$$\begin{aligned}\{A_a(\mathbf{p}, h, t),\, A_b^\dagger(\mathbf{p}', h', t)\} = &(2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')\delta_{hh'} \\ &\times \sum_c U_{ac}^{\rm PMNS}\left(\frac{E_c - h|\mathbf{p}|}{2E_c}\right)\left(U_{cb}^{\rm PMNS}\right)^\dagger,\end{aligned} \tag{38}$$

$$\begin{aligned}\{B_a(\mathbf{p}, h, t) B_b^\dagger(\mathbf{p}', h', t)\} = &(2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')\delta_{hh'} \\ &\times \sum_c U_{bc}^{\rm PMSN}\left(\frac{E_c + h|\mathbf{p}|}{2E_c}\right)\left(U_{ca}^{\rm PMSN}\right)^\dagger,\end{aligned} \tag{39}$$

and

$$\begin{aligned}\{A_a(\mathbf{p}, h) B_b^\dagger(\mathbf{p}', h')\} = &(2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')\delta_{hh'} \\ &\times \sum_c U_{ac}^{\rm PMSN} e^{2i\beta_c} U_{bc}^{\rm PMSN}\, \frac{m_c}{2E_c}.\end{aligned} \tag{40}$$

Notice that, in **Eqs 38, 39**, the anti-commutators are not diagonal in the flavor indices, and the one in **Eq. 40** does not vanish (due to the presence of the neutrino masses)! Therefore, for flavor states, one cannot define a Fock space properly. However, there are certain limits when an approximate Fock space can be defined, for example, when the neutrinos are ultra-relativistic, which is a reasonable assumption in many cases. For a detailed and comprehensive discussion on this subject, see Ref. [34] and references therein.

The above discussion on the quantization of different bases is not exclusive to 331 models. All gauge theories with multiple neutrino bases, the SM included, suffer from the same problem. As we saw, we must be careful in using the proper neutrino basis when dealing with non-relativistic neutrinos.

Another point of caution concerning neutrino oscillation is non-standard interactions at neutrino production and detection. Many BSM models, including the 331 models, have flavor-changing processes, which can be misinterpreted when using flavor states. For example, the V-boson interaction in **Eq. 30** can give a new contribution to muon decay with explicit flavor violation as $\mu^- \to \overline{\nu}_a^{\mathfrak{f}} \nu_b^{\mathfrak{f}} e^-$, where a, b = e, μ, τ.

6 NEW PHASES AND PARAMETERS

At the end of **Section 4**, after absorbing the charged lepton phase matrix P_l, we saw that the W-boson interaction in **Eq. 30** depends on six free parameters: the four parameters in $U_{\rm PMNS}$ plus two complex phases in P_ν (also called Majorana phases). However, unlike what happened in the SM, in the m331, these absorbed phases will resurface in other CC interactions. Furthermore, once we have chosen to write the W-boson interaction as **Eq. 31**, we cannot rephase the charged leptons anymore. Hence, the new interactions will introduce a reasonable number of free parameters in our model, together with new sources of CP violation. Nevertheless, hopefully, these new parameters may turn out to be helpful in some contexts of neutrino physics.

Starting with the gauge interaction, similarly to **Eq. 27**, we can decompose the matrices U_{NR} and U_{LR} as

$$U_{NR} = P(\beta)^\dagger \cdot U^{(2)} \cdot P(\phi), \tag{41}$$

$$U_{LR} = P(\delta)^\dagger \cdot U^{(3)} \cdot P(\omega), \tag{42}$$

where $P(\omega)$ and $P(\phi)$ are diagonal matrices containing three phases each (similarly to P_l), $P(\beta)$ and $P(\delta)$ comprise only two phases (similarly to P_ν). As the PMNS matrix, $U^{(2)}$ and $U^{(3)}$ are unitary matrices having four parameters: three angles and a complex phase.

Hence, we have that

$$U_V = P(\beta)^\dagger \cdot U^{(2)} \cdot [P(\phi) \cdot P_l], \tag{43}$$

$$U_U = \left[P_l \cdot P(\delta)^\dagger\right] \cdot U^{(3)} \cdot [P(\omega) \cdot P_l]. \tag{44}$$

Each product $P_l \cdot P(\delta)^\dagger$, $P(\phi) \cdot P_l$ and $P(\omega) \cdot P_l$ have three free parameters corresponding to the sum of phases. Hence, we found that U_V has nine free parameters (three angles, one intrinsic complex phase and five factorable complex phases) and U_U has ten (three angles, one intrinsic complex phase and six factorable complex phases). Here, the phases absorbed in the standard electroweak interaction by the lepton fields reappear. After redefining the absolute phases by the physical phase difference, we find that one new phase appears in the U-boson interaction. For more details, see [35, 36].

For the scalar sector, we can write the Yukawa couplings in **Eqs 21, 23** as

$$F^\eta_{LR} = \frac{\sqrt{2}}{v_\eta}\hat{M}_l - \frac{v_{\sigma_2}}{v_\eta}U^{\mathrm{Maj}}_{\mathrm{PMNS}} \cdot \hat{M}^\nu \cdot U^*_V, \tag{45}$$

$$F^\eta_{NR} = \frac{\sqrt{2}}{v_\eta}\left(U^{\mathrm{Maj}}_{\mathrm{PMNS}}\right)^\dagger \cdot \hat{M}_l - \frac{v_{\sigma_2}}{v_\eta}\hat{M}^\nu \cdot U^*_V, \tag{46}$$

$$F^\eta_{LN} = \frac{\sqrt{2}}{v_\eta}\hat{M}_l \cdot U^T_V - \frac{v_{\sigma_2}}{v_\eta}U^{\mathrm{Maj}}_{\mathrm{PMNS}} \cdot \hat{M}^\nu, \tag{47}$$

for the η-interactions, and for the S

$$F^S_{NN} = \frac{\sqrt{2}}{v_{\sigma_1}}\hat{M}^\nu, \tag{48}$$

$$F^S_{LN} = \frac{\sqrt{2}}{v_{\sigma_1}}U^{\mathrm{Maj}}_{\mathrm{PMNS}} \cdot \hat{M}^\nu, \tag{49}$$

$$F^S_{NR} = \frac{\sqrt{2}}{v_{\sigma_1}}\hat{M}^\nu \cdot U^*_V, \tag{50}$$

$$F^S_{LR} = \frac{\sqrt{2}}{v_{\sigma_1}}U^{\mathrm{Maj}}_{\mathrm{PMNS}} \cdot \hat{M}_\nu \cdot U^*_V, \tag{51}$$

$$F^S_{LL} = \frac{\sqrt{2}}{v_{\sigma_1}}U^{\mathrm{Maj}}_{\mathrm{PMNS}} \cdot \hat{M}_\nu \cdot \left(U^{\mathrm{Maj}}_{\mathrm{PMNS}}\right)^T, \tag{52}$$

$$F^S_{RR} = \frac{\sqrt{2}}{v_{\sigma_1}}U^\dagger_V \cdot \hat{M}_\nu \cdot U^*_V, \tag{53}$$

where we used the unitarity of V_L, V_R, and V_N, together with the conditions in **Eqs 14, 18**. The matrices $U^{\mathrm{Maj}}_{\mathrm{PMNS}}$ and U_V are defined in **Eqs 31, 32**, respectively.

Therefore, no new parameters arise from the scalar interactions. All Yukawa couplings will be written with the matrices in **Eqs 31, 32** and the lepton masses. Notice that all the interactions with the sextet S depend directly on the neutrino masses, hence, they are considerably suppressed.

7 CONCLUSION

Throughout this paper, an introduction to the 331 model was presented, focusing on its minimal version. Such a model provides a rich phenomenology in non-standard neutrino interactions, both in the vector and the scalar sectors, with lepton number and flavor-changing interactions. As we saw, the Minimal 331 Model contains at least 15 new parameters associated with neutrino physics, plus neutrino masses, which is nine more than the usual SM extended with the PMNS matrix. Many new sources of CP violation are also present. The novel parameters can, in principle, also help to explain some oscillation anomalies. We also address the different neutrino bases that appear in this model, focusing on difficulties for the flavor basis, which is commonly used in neutrino oscillations.

AUTHOR CONTRIBUTIONS

All authors contributed to conception, design of the study and contributed to manuscript revision, read, and approved the submitted version.

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