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Is the Anisotropic Tilt Angle Necessary in the Turbulence Spectrum of Refractive-Index Fluctuations?

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With the deepening of atmospheric turbulence research, scientists have discovered that the anisotropy of turbulence cells cannot be ignored. The anisotropic non-Kolmogorov turbulence model is more in line with the actual situation of atmospheric turbulence. However, the recent experimental results of Korotkova et al. and Beason et al. display that the turbulence cell has an anisotropic tilt angle, that is, the long axis of the turbulence cell may not be horizontal to the ground but has a certain angle with the ground. Therefore, it is urgent to analyze whether the anisotropic tilt angle is necessary in the turbulence spectrum. In this study, we develop the anisotropic non-Kolmogorov turbulence spectra for the horizontal and vertical links in the presence of the anisotropic tilt angle. Then, based on these spectra, the analytical expressions of the scintillation index for the horizontal and vertical links are derived under weak fluctuation condition. The results indicate that the anisotropic tilt angle is necessary in the vertical link but not in the horizontal link.

Keywords: atmospheric optics, atmospheric propagation, anisotropic non-Kolmogorov turbulence, anisotropic tilt angle, free-space optical communications

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1 INTRODUCTION

Although the Kolmogorov turbulence model is the most widely utilized atmospheric turbulence model at present Andrews and Phillips [1], as more and more atmospheric experimental results inconsistent with the Kolmogorov turbulence model are discovered, scientists have found that the anisotropic non-Kolmogorov turbulence model is a more realistic model for atmospheric turbulence Manning [2]; Procaccia and Constantin [3]; Belen'kii et al. [4]; Toselli et al. [5]; Andrews et al. [6]; Kessar et al. [7]; Cui et al. [8]; Wang et al. [9]; Beason et al. [10]; Zhai [11]; Roşu et al. [12]; Xu and Lai [13,14]. Later, in the process of establishing the anisotropic non-Kolmogorov turbulence model and related turbulence theory, in order to simplify the modeling, a hypothesis that the long axis of the turbulence cell should be horizontal to the ground is extensively introduced Andrews et al. [6]; Cui et al. [8]. However, the recent experimental results of Korotkova et al. and Beason et al. have clearly shown that the turbulence cell has an anisotropic tilt angle, that is, the long axis of the turbulence cell may not be horizontal to the ground but has a certain angle with the ground Wang et al. [9]; Beason et al. [10]. Therefore, discussing whether it is necessary to increase the parameter anisotropic tilt angle in the turbulence spectrum has become an urgent and interesting problem to be solved. In this study, considering the anisotropic tilt angle, the anisotropic non-Kolmogorov turbulence spectra for the horizontal and vertical links are developed. Based on these spectra, we derive the analytical expressions of the scintillation index for the horizontal and vertical links under weak fluctuation condition and then, calculations are implemented to evaluate the influence of the anisotropic tilt angle on the scintillation index in the horizontal and vertical links. It is anticipated that this study will benefit the construction of the atmospheric turbulence model and understanding of optical wave transmission in anisotropic non-Kolmogorov links.

2 FORMULATION

As is well-known, for the horizontal link, the power spectrum model of anisotropic non-Kolmogorov turbulence, which considers the uneven distribution of the horizontal and vertical atmosphere, has the following expression Andrews et al. [6].

$$\Phi_n\left(\kappa_x,\kappa_y\right)=\frac{A(\alpha)\tilde{C}_n^2\mu_x\mu_y}{\left(\mu_x^2\kappa_x^2+\mu_y^2\kappa_y^2\right)^{\alpha/2}},\quad 3<\alpha<4,\tag{1}$$

where μ_x and μ_y are the anisotropic factors which can describe the asymmetry of turbulence cells, κ_x and κ_y are the x and y components of the spatial wavenumber vector κ, respectively, $\kappa=\sqrt{\mu_x^2\kappa_x^2+\mu_y^2\kappa_y^2}$, α is the spectral power law, $\tilde{C}_n^2$ is the structure constant for the anisotropic non-Kolmogorov horizontal link which has units of $\mathrm{m}^{3-\alpha}$, and $A(\alpha)=\Gamma(\alpha-1)\cos(\alpha\pi/2)/4\pi^2$ and $\Gamma(x)$ denote the Gamma function. At $\alpha=11/3$ and $\mu_x=\mu_y=1$, **Eq. 1** reduces to the conventional Kolmogorov spectrum. It is to be noted that in the field of laser atmospheric propagation, the turbulence spectrum usually refers to the spatial power spectral density of refractive-index fluctuations.

In order to introduce the anisotropic tilt angle γ into the turbulence spectrum for the horizontal link, we make a rotation with the same angle γ in the κ-space. Hence, the spatial wavenumbers become $\kappa_x=\kappa_{rx}\cos\gamma-\kappa_{ry}\sin\gamma$ and $\kappa_y=\kappa_{rx}\sin\gamma+\kappa_{ry}\cos\gamma$ in place of κ_x and κ_y in **Eq. 1**, respectively. Here, κ_{rx} and κ_{ry} are the newly introduced coordinates after rotating the coordinate system in κ-space through angle γ, and the power spectrum model for the anisotropic non-Kolmogorov horizontal link in the presence of the anisotropic tilt angle γ has the following expression

$$\Phi_n\left(\kappa_{rx},\kappa_{ry}\right)=\frac{A(\alpha)\tilde{C}_n^2\mu_x\mu_y}{\left(m^2\kappa_{rx}^2+n^2\kappa_{ry}^2+g\kappa_{rx}\kappa_{ry}\right)^{\alpha/2}},\quad 3<\alpha<4,\tag{2}$$

where

$$m=\sqrt{\mu_x^2\cos^2\gamma+\mu_y^2\sin^2\gamma},\tag{3}$$

$$n=\sqrt{\mu_x^2\sin^2\gamma+\mu_y^2\cos^2\gamma},\tag{4}$$

$$g=2\sin\gamma\cos\gamma\left(\mu_y^2-\mu_x^2\right).\tag{5}$$

It is to be noted that the definition of the anisotropic tilt angle γ in the horizontal and vertical links is the same. To avoid repetition, the figure for the anisotropy model of the turbulence cell in the horizontal link is not separately given here; please refer to **Figure 2** for the definition of the anisotropic tilt angle γ. As shown in **Figure 2**, the anisotropic tilt angle γ is the angle between the plane XON where the long axes of the ellipsoid are located and the horizontal plane XOY; counterclockwise value is positive, and the value range is 0–180°.

For the weak Kolmogorov horizontal link, the scintillation index of a plane wave is defined by Andrews and Phillips [1].

$$\sigma_I^2=8\pi^2k^2L\int_0^1\int_0^\infty\kappa\Phi_n(\kappa)\left[1-\cos\left(\frac{L\kappa^2\xi}{k}\right)\right]d\kappa d\xi,\tag{6}$$

where L is the link distance, ξ is the normalized path coordinate related to z by $\xi=1-z/L$ and $k=2\pi/\lambda$, and λ is the wavelength. Substituting **Eq. 2** into **Eq. 6** leads to

$$\sigma_I^2=4\pi k^2L\int_0^1\int_0^\infty\int_0^\infty\Phi_n\left(\kappa_{rx},\kappa_{ry}\right)\times\left[1-\cos\left(\frac{L\kappa^2\xi}{k}\right)\right]d\kappa_{rx}d\kappa_{ry}d\xi.\tag{7}$$

We can transform the stretched coordinate system for $\Phi_n(\kappa_{rx},\kappa_{ry})$ into an isotropic coordinate system by use of the following substitutions,

$$\kappa_{rx}=\frac{q_x}{m}=\frac{q\cos\theta}{m},\quad\kappa_{ry}=\frac{q_y}{n}=\frac{q\sin\theta}{n},\quad q=\sqrt{q_x^2+q_y^2},\tag{8}$$

$$\kappa=q\sqrt{\frac{\cos^2\theta}{m^2}+\frac{\sin^2\theta}{n^2}},\quad d\kappa_{rx}d\kappa_{ry}=\frac{dq_xdq_y}{mn}=\frac{qdqd\theta}{mn},\tag{9}$$

$$\Phi_n(q)=\frac{A(\alpha)\tilde{C}_n^2\mu_x\mu_y}{q^\alpha}\left(1+\frac{g\cos\theta\sin\theta}{mn}\right)^{-\alpha/2}.\tag{10}$$

Substituting **Eqs 8–10** into **Eq. 7**, we can obtain

$$\sigma_I^2=4\pi k^2L\int_0^1\int_0^{2\pi}\int_0^\infty A(\alpha)\tilde{C}_n^2q^{1-\alpha}\frac{\mu_x\mu_y}{mn}\left(1+\frac{g\cos\theta\sin\theta}{mn}\right)^{-\frac{\alpha}{2}}\times\left[1-\cos\left(\frac{Lq^2\xi}{k}\left(\frac{\cos^2\theta}{m^2}+\frac{\sin^2\theta}{n^2}\right)\right)\right]dqd\theta d\xi.\tag{11}$$

Then, evaluating this integral, the scintillation index for a plane wave transmitting through the weak anisotropic non-Kolmogorov horizontal link in the presence of the anisotropic tilt angle γ can be obtained

$$\sigma_I^2=-\frac{2}{\alpha}\Gamma\left(1-\frac{\alpha}{2}\right)\Gamma(\alpha-1)\sin\left(\frac{\alpha\pi}{4}\right)\cos\left(\frac{\alpha\pi}{2}\right)\tilde{C}_n^2k^{3-\frac{\alpha}{2}}L^{\frac{\alpha}{2}}\times\frac{\mu_x\mu_y}{mn}\frac{1}{2\pi}\int_0^{2\pi}\left(1+\frac{g\cos\theta\sin\theta}{mn}\right)^{-\frac{\alpha}{2}}\left(\frac{\cos^2\theta}{m^2}+\frac{\sin^2\theta}{n^2}\right)^{\frac{\alpha}{2}-1}d\theta.\tag{12}$$

In **Figure 1**, the influence of the anisotropic tilt angle γ on the scintillation index of a plane wave in the weak anisotropic non-Kolmogorov horizontal link is numerically analyzed. It is to be noted that in the calculation, we have $\lambda=1.55\,\mu\mathrm{m}$, $L=5\,\mathrm{km}$, and $\tilde{C}_n^2=1\times10^{-14}\mathrm{m}^{3-\alpha}$. It can be easily seen from **Figure 1** that for different values of power law α, anisotropic factors μ_x and μ_y and the variation of the anisotropic tilt angle γ has no effect on the scintillation

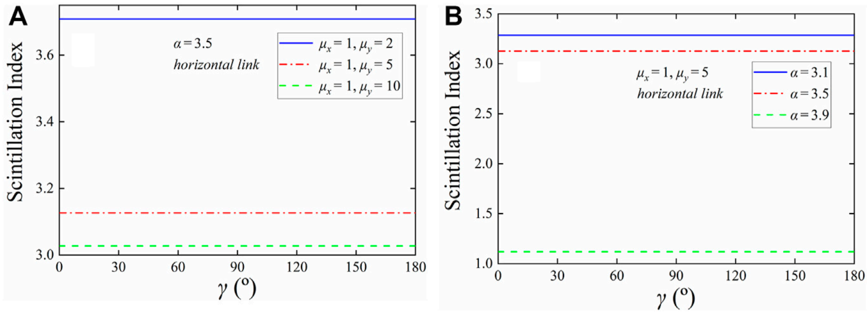

FIGURE 1 | Scintillation index of a plane wave versus the anisotropic tilt angle γ for the horizontal link. **(A)** With various anisotropic factor μ_x and μ_y values; **(B)** with various power law α values.

index for the horizontal link, so the anisotropic tilt angle γ is not necessary in the horizontal link. This result can be physically explained by mentioning the change of curvature of the anisotropic turbulence cells with respect to the isotropic case, and anisotropic turbulence cells will change the focusing properties of the turbulence. When a beam propagates along the long axis of the anisotropic turbulence cells (horizontal link), the radius of curvature for the interface between the beam and anisotropic turbulence cells is small, and the beam will be more deviated from the direction of propagation because these cells act as lenses with a lower radius of curvature Toselli et al. [5]. Correspondingly, while a beam propagates along the short axis of the anisotropic turbulence cells (vertical link), the radius of curvature for the interface between the beam and anisotropic turbulence cells is large, and the beam will be less deviated from the direction of propagation because these cells act as lenses with a higher radius of curvature. Then, based on simple spatial geometry knowledge, we can easily deduce that as the anisotropic tilt angle γ changes, the radius of curvature for the interface between the propagated beam and anisotropic turbulence cells is unchanged in the horizontal link. Hence, the anisotropic tilt angle γ is not necessary in the horizontal link.

Because the anisotropic non-Kolmogorov structure constant for the vertical link is altitude-dependent and the optical wave propagation direction in the horizontal and vertical links is different, that is, the z-axis direction of the corresponding coordinate system is different, so the turbulence spectrum for the vertical link in the presence of the anisotropic tilt angle γ cannot be obtained from the turbulence spectrum for the horizontal link **Eq. 1** by coordinate transformation in κ-space. Then, we will introduce a new method to establish the turbulence spectrum for the vertical link in the presence of the anisotropic tilt angle γ.

First, in order to simplify the modeling, different from the horizontal link, we only consider the existence of an anisotropic factor μ in the vertical link, that is, assuming $OX = ON = \mu s$ and $OM = s$. The anisotropic factor μ denotes the ratio of the horizontal axis to the vertical axis for the turbulence cell. Then, based on simple spatial geometry knowledge, we can easily deduce that when only an anisotropic factor μ is considered, the turbulence cell satisfies the ellipsoid hypothesis. In addition, an ellipsoid model in the presence of the anisotropic tilt angle γ is built to illustrate the actual turbulence cell for the vertical link, as shown in **Figure 2**. Obviously, the individual shape of the turbulence cell is random, not a perfect ellipsoid. However, from the perspective of ensemble average, the ellipsoid hypothesis can be made, which will not lead to a large error between the theoretical model and the actual turbulence.

Second, as displayed in **Figure 2**, the plane XOY is the horizontal plane, and OZ is the plumb line, which denotes the optical wave propagation direction for the vertical link. Then, based on the ellipsoid hypothesis of the turbulence cell, we have $OX = OC = ON = OK = \mu s$ and $OM = OG = s$. In addition, the previous assumption is consistent with many studies on the anisotropic turbulence, in which the length of the horizontal axis for the turbulence cell is usually one to several times the length of the vertical axis Cui et al. [8]; Wang et al. [9]; Beason et al. [10].

Finally, the anisotropic factors of the turbulence cell in the presence of the anisotropic tilt angle γ are evaluated It can be easily seen from **Figure 2** that due to the existence of the anisotropic tilt angle γ, the turbulence cell rotates counterclockwise by angle γ around the x-axis; the y-axis and z-axis intersect the ellipsoid at points Y and Z, respectively. From the symmetry of the ellipse, $OY = OE$ and $OZ = OL$. Because only the vertical link is considered in this study, the optical wave propagation direction is always the z-axis. In addition, the straight-lines XC, YE, and ZL form a rectangular coordinate system and satisfy $OX = OC$, $OY = OE$, and $OZ = OL$. So, we can use the ratio between OX, OY, and OZ to calculate the anisotropic factors of the turbulence cell for the vertical link, that is, $\mu_x = OX/OZ$ and $\mu_y = OY/OZ$. Based on the standard equation of ellipse and simple trigonometric function, we can obtain that $OZ = \mu s\sqrt{(1+\tan^2\gamma)/(\tan^2\gamma+\mu^2)}$ and $OY = \mu s\sqrt{(1+\tan^2\gamma)/(1+\mu^2\tan^2\gamma)}$. Then, the anisotropy model of the turbulence cell for the vertical link in the presence of the anisotropic tilt angle γ is established, and the anisotropic factors μ_x and μ_y can be expressed as

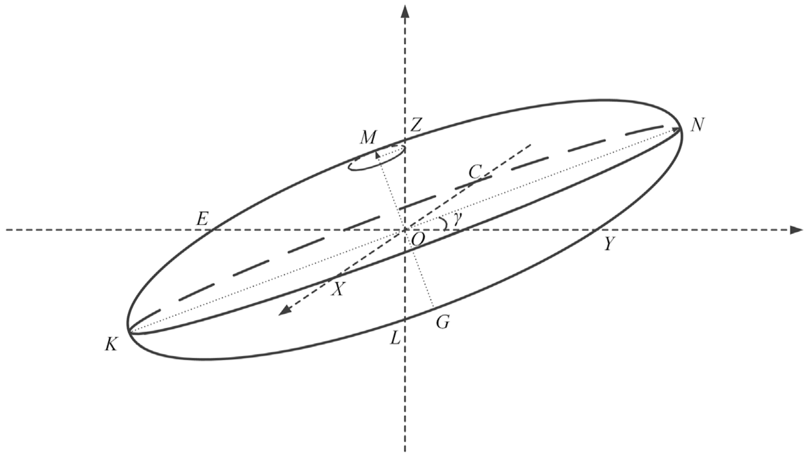

FIGURE 2 | Anisotropy of the turbulence cell for the vertical link in the presence of the anisotropic tilt angle γ.

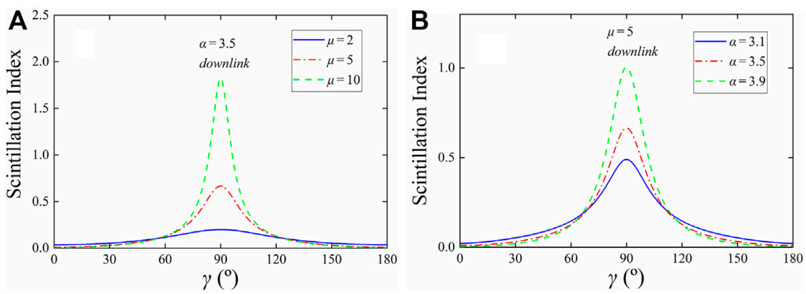

FIGURE 3 | Scintillation index of the downlink versus the anisotropic tilt angle γ. **(A)** With various anisotropic factor μ values; **(B)** with various power law α values.

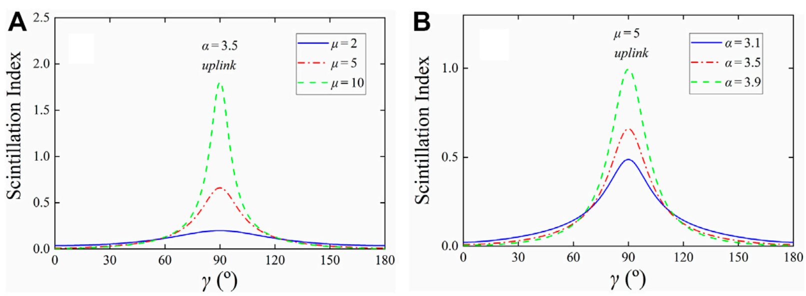

FIGURE 4 | Scintillation index of the uplink versus the anisotropic tilt angle γ. **(A)** With various anisotropic factor μ values; **(B)** with various power law α values.

$$\mu_x = \sqrt{\frac{\mu^2 + \tan^2\gamma}{1 + \tan^2\gamma}},\quad \mu_y = \sqrt{\frac{\mu^2 + \tan^2\gamma}{1 + \mu^2\tan^2\gamma}}. \tag{13}$$

Substituting the anisotropy model of the turbulence cell for the vertical link in the presence of the anisotropic tilt angle γ **Eq. 13** into the extant anisotropic non-Kolmogorov turbulence spectrum model Andrews et al. [6] and considering that the anisotropic non-Kolmogorov structure constant will vary with the altitude, the power spectrum model for the anisotropic non-Kolmogorov vertical link in the presence of the anisotropic tilt angle γ can be obtained

$$\Phi_n\left(\kappa_x, \kappa_y, h\right) = \frac{A(\alpha)\tilde{C}_n^2(h)\mu_x\mu_y}{\left(\mu_x^2\kappa_x^2 + \mu_y^2\kappa_y^2\right)^{\alpha/2}},\quad 3 < \alpha < 4. \tag{14}$$

As displayed in the study by Andrews et al. [15]; Zhai et al. [16], $\tilde{C}_n^2$ for the vertical link is altitude-dependent and provided by

$$\tilde{C}_n^2(h) = \frac{0.033}{A(\alpha)}(k/h)^{(\alpha/2-11/6)} \times \left[0.005\,94\left(\frac{w}{27}\right)^2(10^{-5}h)^{10}\exp\left(-\frac{h}{1000}\right) +2.7\times10^{-16}\exp\left(-\frac{h}{1500}\right)+C_n^2(0)\exp\left(-\frac{h}{100}\right)\right], \quad (15)$$

where w is the rms wind speed with units of m/s, h is the altitude with units of m, and $C_n^2(0)$ is the nominal value of C_n^2 when $h = 0$ with units of $\mathrm{m}^{-2/3}$. It is to be noted that **Eq. 15** is based on non-Kolmogorov turbulence; however we suppose that it is also valid in the presence of anisotropic non-Kolmogorov turbulence, at least approximately.

In the case of a downlink, the scintillation index can be accurately modeled by a plane wave. For an uplink, where the atmospheric turbulence begins just outside the transmitting aperture, a spherical wave can be assumed. Thus, for a plane wave and vertical downlink, the scintillation index under weak fluctuation condition is given by Andrews and Phillips [1].

$$\sigma_{I,downlink}^2 = 8\pi^2k^2\int_{h_0}^{H}\int_0^{\infty}\kappa\Phi_n(\kappa,h) \times\left\{1-\cos\left[\frac{L\kappa^2}{k}\left(\frac{h-h_0}{H-h_0}\right)\right]\right\}d\kappa dh. \quad (16)$$

For a spherical wave and vertical uplink, the scintillation index under weak fluctuation condition is given by Andrews and Phillips [1].

$$\sigma_{I,uplink}^2 = 8\pi^2k^2\int_{h_0}^{H}\int_0^{\infty}\kappa\Phi_n(\kappa,h) \times\left\{1-\cos\left[\frac{L\kappa^2}{k}\frac{(H-h)(h-h_0)}{(H-h_0)^2}\right]\right\}d\kappa dh, \quad (17)$$

where h_0 represents the altitude with units of m for the downlink receiver or the uplink transmitter, L represents the link distance, and $H = h_0 + L$ represents the satellite altitude with units of m. Substituting **Eq. 14** into **Eqs 16**, **17** leads to

$$\sigma_{I,downlink}^2 = 4\pi k^2\int_{h_0}^{H}\int_0^{\infty}\int_0^{\infty}\Phi_n(\kappa_x,\kappa_y,h) \times\left\{1-\cos\left[\frac{L\kappa^2}{k}\left(\frac{h-h_0}{H-h_0}\right)\right]\right\}d\kappa_x d\kappa_y dh, \quad (18)$$

$$\sigma_{I,uplink}^2 = 4\pi k^2\int_{h_0}^{H}\int_0^{\infty}\int_0^{\infty}\Phi_n(\kappa_x,\kappa_y,h) \times\left\{1-\cos\left[\frac{L\kappa^2}{k}\frac{(H-h)(h-h_0)}{(H-h_0)^2}\right]\right\}d\kappa_x d\kappa_y dh. \quad (19)$$

We can transform the stretched coordinate system for $\Phi_n(\kappa_x,\kappa_y,h)$ into an isotropic coordinate system by use of the following substitutions:

$$\kappa_x = \frac{q_x}{\mu_x} = \frac{q\cos\theta}{\mu_x},\quad \kappa_y = \frac{q_y}{\mu_y} = \frac{q\sin\theta}{\mu_y},\quad q=\sqrt{q_x^2+q_y^2}, \quad (20)$$

$$\kappa = q\sqrt{\frac{\cos^2\theta}{\mu_x^2}+\frac{\sin^2\theta}{\mu_y^2}},\quad d\kappa_x d\kappa_y = \frac{dq_x dq_y}{\mu_x\mu_y} = \frac{qdqd\theta}{\mu_x\mu_y}, \quad (21)$$

$$\Phi_n(q,h) = \frac{A(\alpha)\tilde{C}_n^2(h)\mu_x\mu_y}{q^{\alpha}}. \quad (22)$$

Substituting **Eqs 20–22** into **Eqs 18**, **19**, we can obtain

$$\sigma_{I,downlink}^2 = 4\pi k^2\int_{h_0}^{H}\int_0^{2\pi}\int_0^{\infty}A(\alpha)\tilde{C}_n^2(h)q^{1-\alpha} \times\left\{1-\cos\left[\frac{Lq^2}{k}\left(\frac{h-h_0}{H-h_0}\right)\left(\frac{\cos^2\theta}{\mu_x^2}+\frac{\sin^2\theta}{\mu_y^2}\right)\right]\right\}dqd\theta dh, \quad (23)$$

$$\sigma_{I,uplink}^2 = 4\pi k^2\int_{h_0}^{H}\int_0^{2\pi}\int_0^{\infty}A(\alpha)\tilde{C}_n^2(h)q^{1-\alpha} \times\left\{1-\cos\left[\frac{Lq^2}{k}\frac{(H-h)(h-h_0)}{(H-h_0)^2}\left(\frac{\cos^2\theta}{\mu_x^2}+\frac{\sin^2\theta}{\mu_y^2}\right)\right]\right\}dqd\theta dh. \quad (24)$$

Then, evaluating these integrals, the scintillation index for the plane wave vertical downlink and spherical wave vertical uplink under weak anisotropic non-Kolmogorov turbulence in the presence of the anisotropic tilt angle γ can be obtained.

$$\sigma_{I,downlink}^2 = -4\pi^2k^{3-\frac{\alpha}{2}}L^{\frac{\alpha}{2}-1}\Gamma\left(1-\frac{\alpha}{2}\right)\sin\left(\frac{\alpha\pi}{4}\right) \times\int_{h_0}^{H}A(\alpha)\tilde{C}_n^2(h)\left(\frac{h-h_0}{H-h_0}\right)^{\frac{\alpha}{2}-1}dh \times\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\cos^2\theta}{\mu_x^2}+\frac{\sin^2\theta}{\mu_y^2}\right)^{\frac{\alpha}{2}-1}d\theta, \quad (25)$$

$$\sigma_{I,uplink}^2 = -4\pi^2k^{3-\frac{\alpha}{2}}L^{\frac{\alpha}{2}-1}\Gamma\left(1-\frac{\alpha}{2}\right)\sin\left(\frac{\alpha\pi}{4}\right) \times\int_{h_0}^{H}A(\alpha)\tilde{C}_n^2(h)\left[\frac{(H-h)(h-h_0)}{(H-h_0)^2}\right]^{\frac{\alpha}{2}-1}dh \times\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\cos^2\theta}{\mu_x^2}+\frac{\sin^2\theta}{\mu_y^2}\right)^{\frac{\alpha}{2}-1}d\theta. \quad (26)$$

Figure 3; **Figure 4** present the variation of the scintillation index for the downlink and uplink against the function of the anisotropic tilt angle γ under different anisotropic factor μ and power law α values. It is to be noted that in the calculation, we have $\lambda = 1.55\ \mu$m, $L = 1{,}000$ km, $C_n^2(0) = 1\times10^{-14}\mathrm{m}^{-2/3}$, $h_0 = 0$ m, and $w = 30$ m/s. As displayed in **Figure 3**; **Figure 4**, for different values of power law α and anisotropic factor μ, the scintillation index for the downlink and uplink first increase and then decay with the rising of the anisotropic tilt angle γ, and the curves are symmetric about the straight-line $\gamma = 90°$, so the anisotropic tilt angle γ is necessary in the vertical link. As mentioned before, anisotropic turbulence cells will change the focusing properties of the turbulence Toselli et al. [5]. Then, based on simple spatial geometry knowledge and the ellipsoid hypothesis of the turbulence cell, we can easily find that with the increasing of the anisotropic tilt angle γ, the radius of curvature for the interface between the propagated beam and the anisotropic turbulence cells first decays and then increases in the vertical link. In addition, with the change of the radius of curvature, the deviation effect of turbulence on the propagated beam is first strengthened and then weakened. Hence, the anisotropic tilt angle γ is necessary in the vertical link. In addition, the symmetry of the curves about the straight-line $\gamma = 90^{\circ}$ is consistent with the ellipsoid hypothesis of the turbulence cell, so our model is self-consistent.

3 CONCLUSION

In this study, the anisotropic non-Kolmogorov turbulence spectra in the presence of the anisotropic tilt angle γ are developed for the horizontal and vertical links. Then, based on these spectra, the analytical expressions of the scintillation index for the weak anisotropic non-Kolmogorov horizontal and vertical links are derived. The calculation results indicate that for different values of power law α, anisotropic factors μ_x and μ_y and the variation of the anisotropic tilt angle γ has almost no effect on the scintillation index for the horizontal link. In addition, for different values of power law α and anisotropic factor μ, the scintillation index for the downlink and uplink first increase and then decay with the rising of the anisotropic tilt angle γ, and the curves are symmetric about the straight-line $\gamma = 90^{\circ}$. Therefore, the anisotropic tilt angle γ is necessary in the vertical link, but not in the horizontal link. It should be noted that this study will benefit the construction of the atmospheric turbulence model and the understanding of optical wave transmission in anisotropic non-Kolmogorov links.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

CZ proposed the project. CZ conducted the equation derivation, simulation, and image processing. CZ wrote the manuscript.

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