



Stability and Controllability of Various Spatial Solitons in Exciton–Polariton Condensates by a Composite Pumping

Kun Zhang¹, Wen Wen^{2,3}, Ji Lin¹ and Hui-jun Li^{1*}

¹Institute of Nonlinear Physics and Department of Physics, Zhejiang Normal University, Jinhua, China, ²Department of Mathematics and Physics, Hohai University, Changzhou, China, ³College of Science, Hohai University, Nanjing, China

We investigate the stability and controllability of one-dimensional bright and dark solitons, and two-dimensional bright solitons and vortices with the charges $m = 1$ and 2 , respectively, in a nonresonantly incoherent pumped exciton–polariton condensates. A composite pumping, consisting of the constant part and the Bessel-type spatially modulating part, is introduced to balance the gain and loss. We demonstrate that the pumping can not only stabilize all these solitons but also modulate the profiles of these solitons. We also find that all these solitons obtained in this study are different from the ones in the previous studies. Our work may pave a way to modulate these solitons in the nonresonantly pumped exciton–polariton system.

Keywords: exciton–polariton condensate, dark soliton, bright soliton, vortex, Bessel-type pumping

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*Correspondence:

Hui-jun Li
hjli@zjnu.cn

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1 INTRODUCTION

In semiconductor microcavities, exciton–polariton condensates can be formed at a few Kelvin or even at room temperature [1–5]. The strong light–matter interaction [6] observed in the exciton–polariton condensates allows it to be an ideal platform to study quantum nonequilibrium physics and exotic properties of high-orbital condensates [7]. These novel properties are also very important to form various nonlinear phenomena, such as bistability [8, 9], information processing [10–12], pattern formation [13–15], artificial polariton molecules [16], chaos [17], quantum vortices [18–25], and spatial solitons [21, 22, 26–38].

Spatial solitons are formed by balancing the diffraction and nonlinearity. There are rich nonlinear physics and important practical applications [39, 40]. The lattice solitons [41], defect solitons [42], interface kink solitons [43], and surface lattice solitons [44] had been found in the $(1 + 1)$ -dimensional saturated nonlinear Schrödinger equation (SNLSE). And in the coherent atomic media, the $(2 + 1)$ dimensional $[(2 + 1) D]$ SNLSE [45] (or cubic–quintic nonlinear Schrödinger equation [46]) was researched. The bright ground solitons, vortices [47], and double-hump solitons [48] have been found in the $[(2 + 1) D]$ SNLSE, and lattice solitons [41] and discrete solitons [49] have also been obtained in the SNLSE in which there was a periodical modulation potential in the denominator of the saturable nonlinear term. However, except for the bright ground solitons and lattice solitons [41], all others solitons are unstable. In a recent study, a high-dimensional SNLSE including a trapping potential was constructed and various stable nonlinear modes [50] were obtained.

In the nonresonant incoherent exciton–polariton condensations with the homogeneous pumping, dark solitons are unstable, and it would disappear after evolution in a short time for one- [32] and two-dimensional [33] systems. The spatially periodic [22], ring-shaped [15, 23, 35],

and Gaussian-shaped [21, 30, 31] pumping have been proposed to stabilize solitons. However, the balance between the nonlinear gain and the constant loss is not realized, so the stability of solitons is still an open question.

Under the adiabatic approximation of reservoir density, the exciton–polariton condensates can be described by SNLSE with the Kerr nonlinear term and the gain and loss terms, so the formations and stability of nonlinear modes in this system are of particular interest, and the mechanisms are also very complex. When we discuss the nonlinear excitations (such as solitons and vortices) on the basis of homogeneous condensates, it is difficult to realize the balance between the nonlinear gain and the invariable constant loss. Recently, we proposed the composite pumping including the constant part and Gaussian-type spatially modulating part to balance the gain and loss, and generate the stable nonlinear modes [38]. But the roles of constant pumping and spatial modulating pumping had not been elaborated carefully.

In this study, we construct a nonresonant composite pumping, consisting of the constant part and the Bessel-type spatially modulating part, to balance the nonlinear gain and the invariable loss. Then, the Gross–Pitaevskii equations described the dynamics of the exciton–polariton condensates are solved, and the one-dimensional bright and dark solitons and two-dimensional bright solitons and vortices with different charges are obtained. Their stability is proved by the linear stability analysis and evolution, and their controllability is discussed. And we find that the balance or near balance between gain and loss is the necessary condition for the stability of solitons with the nonzero homogeneous background. In addition, we also find that the spatial modulated pumping can be used to modulate the profiles of solitons.

The article is organized as follows. In **Section 2**, the model under study is introduced. In **Section 3**, various soliton solutions, their properties, and their stabilities are discussed. In the last section, the main results are summarized.

2 MODEL

Using the mean-field theory, the dynamics of two-dimensional exciton–polariton condensates are described by using a dissipative Gross–Pitaevskii equation for the polariton field Ψ and the rate equation of the density of the excitonic reservoir n_R :

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + g_C |\Psi|^2 + g_R n_R + i \frac{\hbar}{2} (R n_R - \gamma_C) \right] \Psi, \tag{1}$$

$$\frac{\partial n_R}{\partial t} = P_u(\mathbf{r}) - (\gamma_R + R |\Psi|^2) n_R, \tag{2}$$

where $P_u(\mathbf{r})$ is the nonresonant optical pumping; m^* is the effective polariton mass of the lower polariton branch; γ_C and γ_R are the polariton and exciton loss rates, respectively; R is the condensation rate; g_C represents the strength of the nonlinear interaction between polaritons; and g_R is the interaction strength between polaritons and reservoir excitons. The nonresonant pumping is constructed by $P_u(\mathbf{r}) = P_0 + P_1 J_n(r/w_0)$, which

consists of a cw field P_0 and Bessel-type field, respectively, J_n denotes the n -order Bessel function, $r = x$ for one-dimensional system, and $r = \sqrt{x^2 + y^2}$ for the two-dimensional system.

The dimensionless forms of Eqs 1, 2 can be written as

$$i \frac{\partial u}{\partial s} = -\nabla_{\perp}^2 u - \sigma_1 |u|^2 u - \sigma_2 n u + i(\sigma_3 n - \sigma_4) u, \tag{3}$$

$$\frac{\partial n}{\partial s} = \sigma_5 P(\mathbf{r}) - \sigma_5 (1 + \sigma_6 |u|^2) n, \tag{4}$$

where $s = t/\tau_0$, $(\xi, \eta) = (x/R_x, y/R_y)$, $u = \Psi/\psi_0$, $n = n_R/n_R^0$, $w = w_0/R_x$, $\nabla_{\perp}^2 = \partial^2/\partial \xi^2 + \delta^2 \partial^2/\partial \eta^2$, $\delta = R_x/R_y$, and $P(\mathbf{r}) = \sigma_7 + \sigma_8 J_n(\frac{\sqrt{\xi^2 + \eta^2/\delta^2}}{w})$, with $\tau_0 (\equiv 2m^* R_x^2/\hbar)$, R_x , y , ψ_0^2 , and n_R^0 being, respectively, characteristic time, the width of quantum well layers in x , y directions, the condensate density, and the reservoir density. These coefficients in Eqs 3, 4 are $\sigma_1 = -g_C \psi_0^2 \tau_0/\hbar$, $\sigma_2 = -g_R n_R^0 \tau_0/\hbar$, $\sigma_3 = R n_R^0 \tau_0/2$, $\sigma_4 = \gamma_C \tau_0/2$, $\sigma_5 = \gamma_R \tau_0$, $\sigma_6 = R \psi_0^2/\gamma_R$, $\sigma_7 = \frac{P_0}{n_R^0 \gamma_R}$, and $\sigma_8 = \sigma_7 P_1/P_0$. The characteristic time $\tau_0 = 5.45 \times 10^{-10}$ s can be obtained by the parameters in Ref. [32].

Here, if $R_x = R_y$, this model is (2 + 1) dimensional, that is, the quantum well layers are sandwiched between the two distributed Bragg reflectors [2]. If $R_x \ll R_y$, $\delta \ll 1$, the effect of $\partial^2/\partial \eta^2$ can be neglected, so Eqs 3, 4 are reduced to quasi (1 + 1) dimensional, such as the nanowire system [32].

3 SOLITON SOLUTIONS AND THEIR PROPERTIES

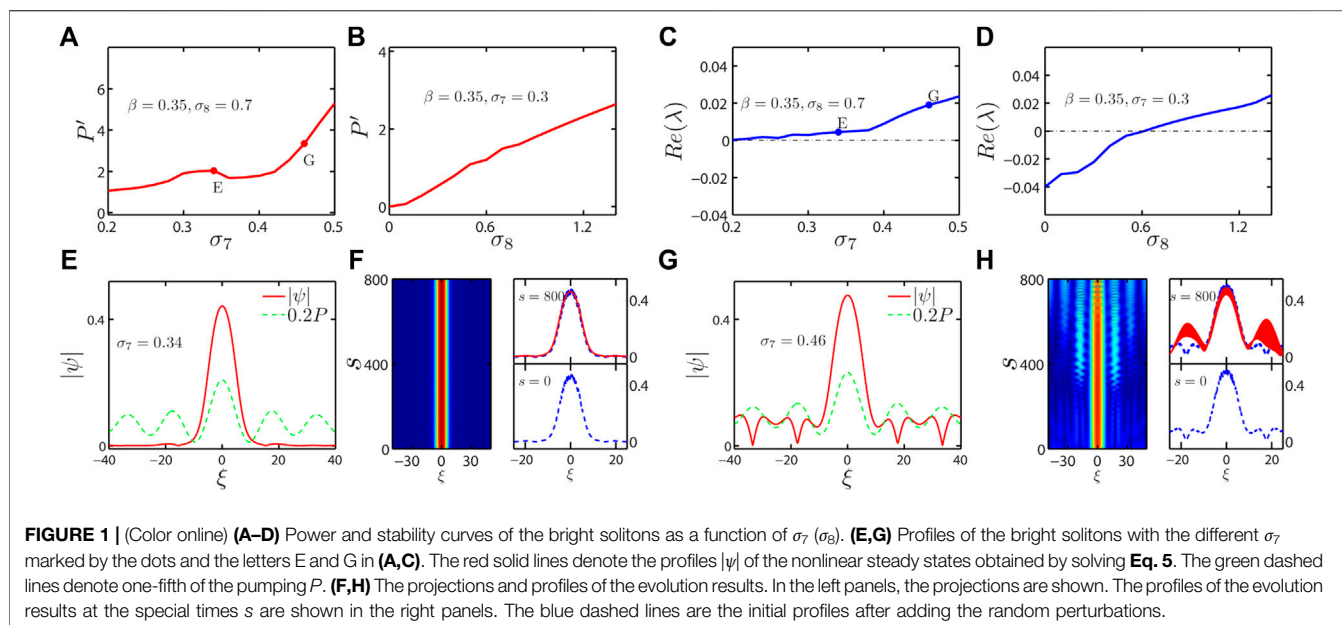
The linear properties of Eqs 3, 4 have been discussed in Ref. [38]. To obtain the stationary soliton solutions, we substitute $u(\xi, \eta, s) = \psi(\xi, \eta) \exp(i\beta s)$ and $n(\xi, \eta, s) = n'(\xi, \eta)$ into Eqs 3, 4, and obtain

$$-\beta \psi + \nabla_{\perp}^2 \psi + \sigma_1 |\psi|^2 \psi + \sigma_2 n' \psi - i(\sigma_3 n' - \sigma_4) \psi = 0, \tag{5}$$

where $n' = \frac{P(\mathbf{r})}{1 + \sigma_6 |\psi|^2}$. By using the Newton conjugate gradient method [51], the profiles and power $P' = \int \int_{-\infty}^{+\infty} |\psi|^2 d\xi d\eta$ or the renormalized power $P' = \int \int_{-\infty}^{+\infty} \|\psi\|^2 - |\psi_0|^2 d\xi d\eta$ (mainly for the solitons with background) of the soliton solutions are obtained, and ψ_0 is the amplitude of the background. The stability of the soliton solutions ψ can also be analyzed by introducing the perturbations $u(\xi, \eta, s) = \{\psi(\xi, \eta) + \epsilon [v_1(\xi, \eta) e^{\lambda s} + v_2^*(\xi, \eta) e^{\lambda^* s}]\} e^{i\beta s}$, $n(\xi, \eta, s) = n' + \epsilon [v_3(\xi, \eta) e^{\lambda s} + v_3^*(\xi, \eta) e^{\lambda^* s}]$, where v_1 , v_2 , and v_3 are the normal modes and λ is the corresponding eigenvalue of the perturbations, and solving the eigenvalue problem as shown in Ref. [38]. In general, we use the parameters $\sigma_1 = -1$, $\sigma_2 = 0.3$, $\sigma_3 = 0.15$, $\sigma_4 = 0.1$, $\sigma_5 = 1$, and $\sigma_6 = 4$, which are obtained by substituting the parameters in [32] into these formulas given Eq. 4.

3.1 The Soliton Solutions for One-Dimensional System

We discuss the one-dimensional soliton solutions of Eq. 5 and their stability first. The nonlinear modes are excited on a



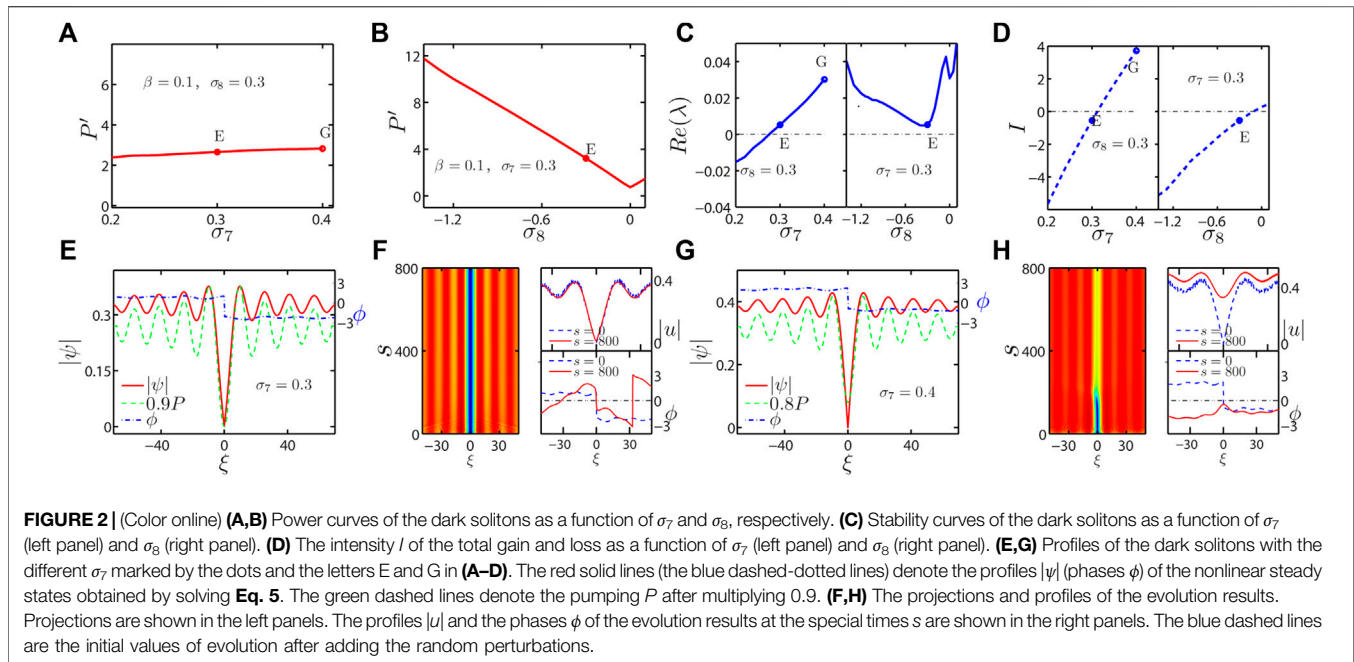
homogeneous background of the steady condensate wave function for the bright and dark solitons. The bright solitons are with a zero background, and the dark solitons are with a nonzero homogeneous background, so the physical mechanisms to generate and stabilize these bright solitons and dark solitons are different. For them, the nonlinear saturated gain term cannot be balanced by the constant loss directly, so an inhomogeneous pumping is necessary to stabilize them. For the dark solitons, the inhomogeneous pumping can not only be used as an external potential but also suppress the nonlinear saturated gain caused by the nonlinear excitation. Meanwhile, the homogeneous pumping suppresses the constant loss for the soliton. For the bright ones, the competitions between the inhomogeneous pumping, nonzero homogeneous background of the steady condensate wave function, and the gain and loss are complicated. Here, we choose the 0-order Bessel inhomogeneous pumping and $w = 2.5$ to find the one-dimensional bright and dark solitons.

Using the Newton conjugate gradient method [51], we first find the soliton solutions of Eq. 5, then make the linear stability analyses, and evolve the obtained solitons by Eqs 3, 4. The bright solitons with $\beta = 0.35$ and their stability are shown in Figure 1. From the profiles of bright solitons, since a dip contributes to the nonlinear saturated gain term, $\sigma_8 > 0$ is taken to compensate the loss. Figures 1A–D illustrate the power and stability curves as a function of the homogeneous pumping σ_7 (the inhomogeneous pumping σ_8). From the power curves in Figure 1B, one can find positive inhomogeneous pumping is necessary to generate the bright solitons. Although there are stable bright solitons, the stable ranges for the parameters $\sigma_{7,8}$ are narrow, as shown in Figures 1C, D. In Figures 1E, G, the red solid lines denote the profiles $|\psi|$ of the steady states by solving Eq. 5, the green dashed lines denote the profiles of pumping. From them, we find the pumping can modulate the profiles of the bright solitons. In the center of Figure 1E, the highest peak of the soliton profile is fully in agreement with the highest peak of the pump, and the two side

peaks of the soliton profile are also in agreement with the side peaks of pumping. And in the center of Figure 1G, the highest peaks of the soliton profile and pump are also concordant, but the second and third side peaks of pumping are coincident with the second and third side valleys of the soliton profile. Thus, the profiles of pumping can be used to reconstruct the shapes of bright solitons.

The stability is proved further by a numerical evolution of Eqs 3, 4, and adding the random perturbations into the initial values of evolution, that is, the initial value is taken as $u(s = 0, \xi, \eta) = \psi(\xi, \eta) (1 + \varepsilon\rho_1)$ and $n(s = 0, \xi, \eta) = n'(\xi, \eta) (1 + \varepsilon\rho_2)$, where $\varepsilon = 0.1$, $\rho_{1,2}$ are the random variables uniformly distributed in the interval $[0, 1]$, and $s = 100$ denotes 54.5 ns. In Figures 1F, H, the projections of the evolution results are shown in the left panels, and the blue dashed lines and the red solid lines denote the profiles of the evolution results at the times $s = 0$ and $s = 800$ in the right panels, respectively. The evolution results are in agreement with the results of the stability analyses. The profile of the bright soliton can conserve perfectly even after evolution 436 ns from the numerical results in Figure 1F.

There are the Kerr nonlinearity terms, saturated nonlinearity terms, and the composite pumping terms in Eqs 3, 4, so it is possible to support the bright solitons and dark solitons simultaneously in the exciton–polariton condensate system. Since a hump contributes to the nonlinear saturated gain term due to the profiles of the dark solitons, the coefficient of inhomogeneous pumping should be $\sigma_8 < 0$ for reducing the gain. Figures 2A, B illustrate the power curves as a function of the homogeneous pumping σ_7 and the inhomogeneous pumping σ_8 , respectively, here, $\beta = 0.1$. Figure 2C shows the stability curves as a function of σ_7 and σ_8 . For explaining the reason of soliton stability, we introduce $I = \int_{-\infty}^{+\infty} (\sigma_3 n - \sigma_4) d\xi$ to denote the intensity of the total gain and loss. In Figure 2D, the intensity I as a function of σ_7 and σ_8 is shown. We find that the value of I is close to zero when the soliton is stable. The small value means



that the balance or near balance between the nonlinear gain and constant loss is realized. From the power curves in **Figure 2B**, one can find that the negative inhomogeneous pumping supports the dark solitons, very narrow intervals of $\sigma_8 > 0$ also support the dark solitons. And the stable ranges of parameters $\sigma_{7,8}$ are both narrow. In **Figures 2E,G**, the red solid lines (the blue dashed dotted lines) denote the profiles $|\psi|$ (the phases ϕ) of soliton solutions by solving Eq. 5. And the green dashed lines denote the intensity of pumping. From them, it is obvious that the profiles of dark solitons are modulated by pumping.

The evolution results are shown in **Figures 2F,H**, the projections of evolution results are shown in the left panels, the evolution results of $s = 0$ ($s = 800$) are denoted by the red solid lines (the blue dashed lines) in the right panels. From the phase ϕ of the right panels in **Figure 2F**, the profiles of dark solitons in $s = 0$ and $s = 800$ are in agreement very well, and the phase jump still keeps obviously after evolution 436 ns. The phase jump will disappear with the increasing evolution times when the dark solitons are unstable, as shown in **Figure 2H**.

From the aforementioned results, the homogeneous and inhomogeneous parts of pumping are both very important to generate and stabilize the bright and dark solitons. Furthermore, the inhomogeneous part can also be used to modulate the shapes of nonlinear solitons. And the small intensity I is very important for the stability of dark solitons, that is, the balance between the nonlinear gain and constant loss is very important to stabilize dark solitons.

3.2 The Soliton Solutions for Two-Dimensional System

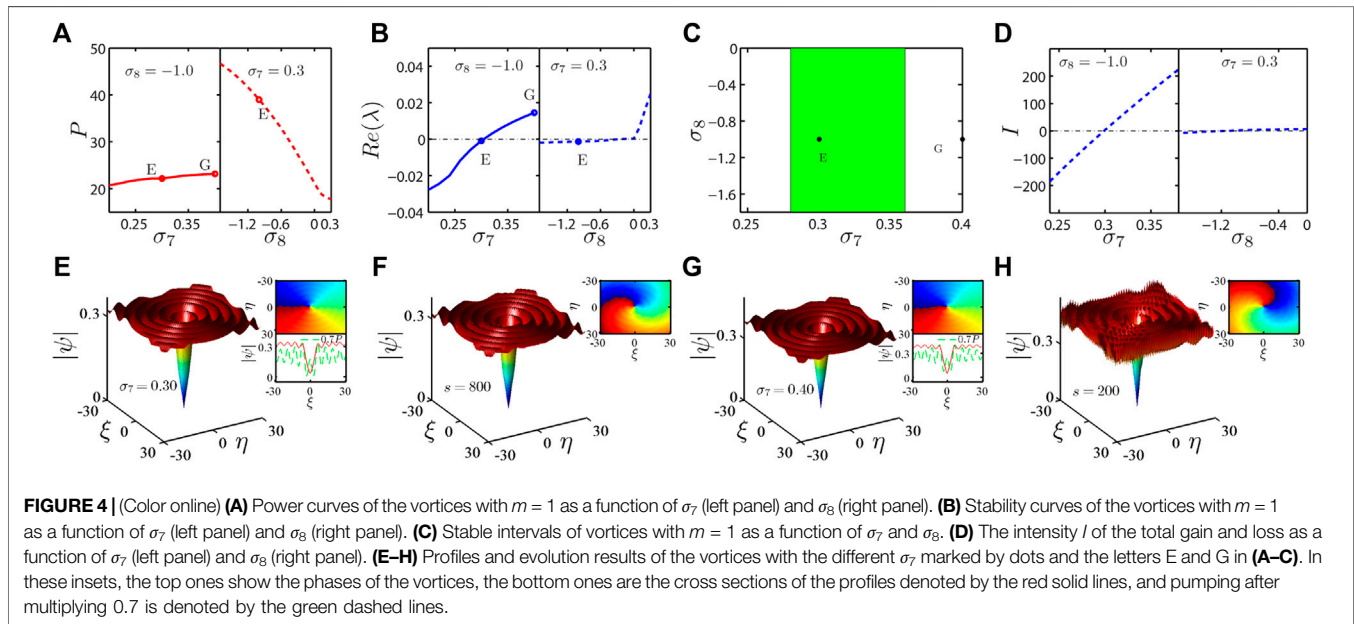
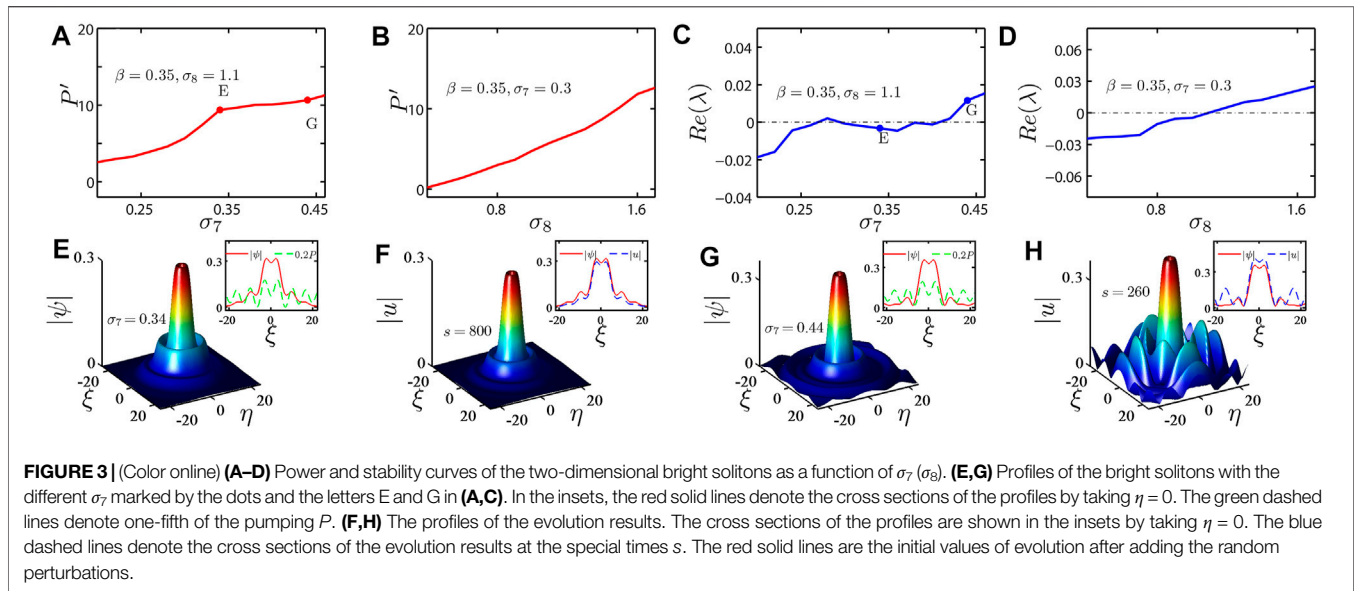
It is very interesting to find stable high-dimensional solitons and control them in exciton–polariton models (3) and (4). In this subsection, we study the properties of high-dimensional spatial

solitons. Here, we take the second-order Bessel-type pumping and $w = 1$ in the inhomogeneous pumping.

The two-dimensional bright solitons are shown in **Figure 3** with $\beta = 0.35$. **Figures 3A–D** show the power and stability curves as a function of σ_7 (σ_8). From them, we not only obtain the parameters ranges supporting the two-dimensional bright solitons but also show the stability of these solitons. **Figures 3E–H** are the profiles and evolutions of the two-dimensional bright solitons, respectively. In these insets, the red solid lines, the green dashed lines, and the blue dashed lines are the cross sections of the profiles of the initial bright solitons, the profiles of the pumping, and the profiles after the evolution s , respectively. From **Figure 3E**, we find that there are three rings in the profile of the bright soliton, which we do not find in the previous reports about soliton solutions of the exciton–polariton system, and the positions of the rings are consistent with those of the ring of pumping by the insets. After taking the different parameters as shown in **Figure 3G**, many more rings appear in the profile of the bright soliton.

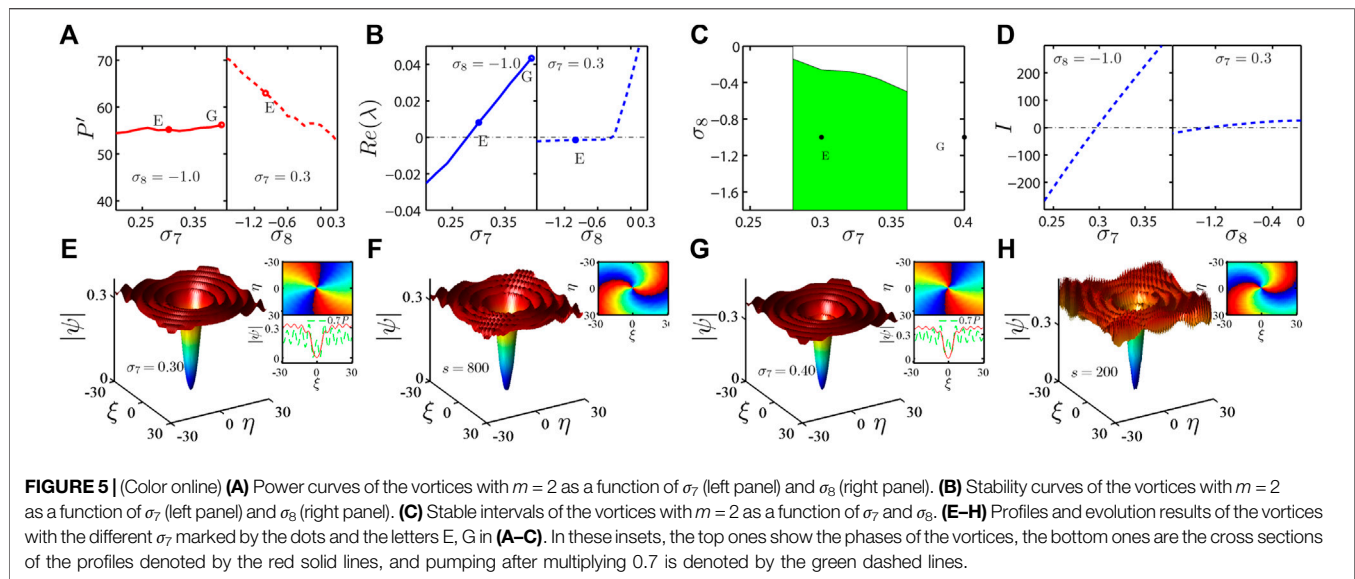
We also find the vortex solitons by imprinting a phase factor $\exp(im\theta)$ ($\theta = \arctan \frac{y}{x}$ is the azimuthal coordinate) onto the initial trial solutions. Although there exist the vortices in the exciton–polariton system, it is still interesting to study how to enhance the stability of the vortices and control the shapes of the vortices.

In **Figure 4**, we show the vortices with $m = 1$ and their stability. The power and stability curves as a function of $\sigma_{7,8}$ are shown in **Figures 4A–B**. **Figure 4C** shows the stable intervals of the vortices with $m = 1$ as a function of σ_7 and σ_8 ; **Figure 4D** is the intensity I of the total gain and loss as a function of σ_7 and σ_8 , and **Figures 4E–H** are the profiles and the evolutions, respectively. Comparing **Figure 4A** with **Figure 4B**, despite there are the wide parameter intervals of σ_7 to support the



vortices, the parameter ranges to stabilize them are very narrow. However, the situation is different for parameter σ_8 , we find the vortices with $m = 1$ are stable in the whole ranges of $\sigma_8 < 0$, as shown in **Figure 4B**, and unstable for $\sigma_8 > 0$. And the vortex is also stable even in the absence of the inhomogeneous pumping (in the case of $\sigma_8 = 0$). Thus, for the case of $m = 1$, all these vortices are stable as long as $\sigma_8 \leq 0$. The results are also consistent with the intensity curves, as shown in **Figure 4D**. From the insets of **Figure 4E**, the top one is the phase of vortex, and in the bottom one, the red solid line and the green dashed line are the cross sections of the profiles of the initial vortex and one of the pumping. From this, we also find every fluctuation of the profile is consistent with that of the pump.

Furthermore, we also investigate the vortices with $m = 2$ in **Figure 5**. The power and stability curves as a function of σ_7, σ_8 are shown in **Figures 5A–B**. **Figure 5C** shows the stable intervals of the vortices with $m = 2$ as a function of σ_7 and σ_8 , and **Figure 5D** shows the intensity I of the total gain and loss as a function of σ_7 and σ_8 . **Figures 5E–H** are the profiles and the evolutions, respectively. Comparing **Figure 5** with **Figure 4**, one can find the existence ranges of the vortices are the same, and the tendency of the stability curves is also the same, whereas the stability is different. Through the stability analyses, it is obvious that the vortices of $\sigma_8 < -0.3$ are stable for $m = 2$. However, from the intensity I curves as shown in the right panel of **Figure 5D**, the value of I is small in the whole intervals of σ_8 , but there is only the



stable interval of $\sigma_8 < -0.3$. It is obvious that the small value of I is necessary and is not a sufficient condition of soliton stability. Thus, we see that the inhomogeneous pumping is very important for stabilizing the higher charged vortices.

4 SUMMARY

In conclusion, the stability and controllability of one- and two-dimensional spatial solitons have been discussed by introducing the nonresonant pumping in exciton–polariton condensates. The introduced pumping contains the homogeneous part that balances the constant loss, in addition to the inhomogeneous Bessel-type spatially modulating part that compensates the gain or loss caused by the denominator of the nonlinear saturated gain term. The bright and dark solitons in the one-dimensional system, and the bright solitons and vortices with $m = 1$ and $m = 2$ in the two-dimensional system have been found. These solitons could be stabilized by engineering the homogeneous and inhomogeneous pumping. The intensity of the total gain and loss could be used as the necessary condition of soliton stability. And in the parameter regions of the stable soliton, the bright solitons can be excited by initial Gaussian or sech functions with the similar amplitude and width to the stable bright solitons, the dark solitons can be excited by the initial tanh function with the similar background and width to the stable black solitons, or by the phase imprinting method. And the vortex solitons can be excited by imprinting a phase factor $\exp(im\theta)$ onto a trial function with the similar amplitude and width to the stable vortex solitons. In

addition, we have also found that the Bessel-type inhomogeneous pumping could be used to control the profiles of nonlinear modes. The results presented here may be useful for understanding the physical properties of the condensates out of equilibrium and guiding the experimental studying of the condensate solitons, which may have potential applications in polariton condensates for information storages and processing or quantum simulators.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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