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Enhancement of the DOA detection performance through optimization of the steering matrix of the array

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Direction Of Arrival (DOA) of signals detection technology is an important vehicle in the field of in remote sensing, radar, wireless communication. In this study, we elaborate on an enhanced method to detect the DOA. In the developed scheme, we mainly focus on solving the steering matrix of the array which contains all the information of the signals. The iterative relation between the steering matrix and the signal vector is first established on the basis of the equation of the array output. Then, to get a more accurate of steering matrix, we construct a cost function that aims to minimize some signal subspace error. In the optimization process of the developed scheme, we also set a constraint for the steering matrix which can effectively eliminate convergence on local optimum and also reduce the number of iterations. Subsequently, the steering matrix of the array can be recovered faithfully. Finally, the DOA can be solved from the estimated steering matrix. Explicit analysis and derivation of the proposed scheme are presented.

KEYWORDS

signal processing, direction of arrival (DOA), partial noise subspace, multiple signal classification (MUSIC), sensors

Introduction

Array signal processing is an indispensable technique in signal processing with ubiquitous applications [1, 2]. The Direction Of Arrival (DOA) detection technology is a very popular topic in array signal processing [3, 4] in the field of in remote sensing, radar, wireless communication, *etc.* High-resolution subspace-based DOA methods have attracted considerable attention concerning the accurate detection of the DOA from observations of array output. The most representative high-resolution subspace-based approaches are the MUltiple SIgnal Classification (MUSIC) [5] and the Estimation Signal Parameter *via* Rotational Invariance Techniques (ESPRIT) [6]. The MUSIC method detects the DOA based on the orthogonality between the signal subspace and noise subspace [7, 8], and the ESPRIT algorithm builds on the rotational invariance of signal subspaces [9, 10]. The detection performance of this type of methods mainly depends on

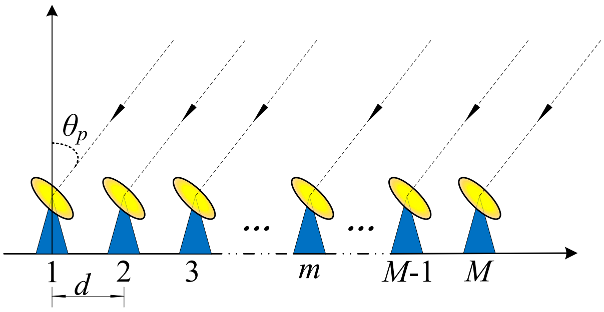

FIGURE 1
Signal model of the ULA.

the accuracy of the signal subspace. Thus, how to capture a high-precision signal subspace has always been the pursuit of these approaches [11, 12].

In this study, we design an enhanced scheme for the DOA detection. During the design process, a cost function by minimizing some signal subspace error is established to optimize the steering matrix of the array [13, 14]. In the optimization, a constraint is set to converge rapidly and eliminate converging on local optimum. Ultimately, the DOA can be solved from the obtained steering matrix of the array. We provide a series of simulations to demonstrate the superiority of the proposed method. To the best of our knowledge, the idea in this paper has not been considered in previous studies.

The organization of the paper reflects the key phases of the design process. The array signal model is first presented to formulate the problem. Next, we develop an enhanced DOA detection scheme through optimization of the steering matrix of the array. This is followed by the experimental results. Conclusions are covered in the last section.

Problem formulation

Without loss of generality, in this letter, we use a Uniform Linear Array (ULA) to illustrate the array signal model for DOA detection. We consider P narrow band noncoherent far field signals $\{s_p(t)\}_{p=1}^{P}$ [15, 16] with different DOAs impinging on the ULA which is composed of M antenna elements. Based on the above conditions, the array output is generally written in the following manner

$$\boldsymbol{X}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{n}(t)$ is the noise vector, and $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_p, \cdots, \boldsymbol{a}_P] \in C^{M\times P}$ contains the DOA information that is the so-called steering matrix of the array. For a given ULA, the steering vector in $\mathbf{A}$ is usually written as

$$\boldsymbol{a}_p = \exp\left[0, \cdots, j\frac{2\pi d}{\lambda}(m-1)\sin\theta_p, \cdots, j\frac{2\pi d}{\lambda}(M-1)\sin\theta_P\right]^T$$
$$m = 1, 2, \cdots, M \tag{2}$$

where T stands for the transpose operation, d denotes the spacing between adjacent antenna elements, λ and θ_p are the wavelength and the pth DOA of the signals, respectively. The array signal model is shown in Figure 1.

The high-resolution subspace-based approaches detect the DOA based on the accurate signal and the noise subspaces. Normally, the signal and the noise subspaces can be achieved through the Eigen decomposition of the array output covariance matrix [17]. Theoretically, the Eigen decomposition of the array output covariance matrix is computed in the following manner

$$\boldsymbol{R}_X = E\{\boldsymbol{X}\boldsymbol{X}^H\} = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \boldsymbol{\sigma}^2\mathbf{I} \tag{3}$$

where H stands for the complex conjugate transpose, R_s is the correlation matrix of the signal vector, and σ^2 means the noise power. The eigenvalue decomposition of the array output covariance matrix is expressed as

$$\boldsymbol{R}_X = \boldsymbol{U}_s\boldsymbol{A}_s\boldsymbol{U}_s^H + \sigma^2\boldsymbol{U}_n\boldsymbol{U}_s^H \tag{4}$$

where $\boldsymbol{A}_s$ is a diagonal matrix composed of P signal eigenvalues, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are respectively the signal and noise subspaces determined by the distribution of eigenvalues. Then, the DOA of the signals can be solved with the high-resolution subspace-based approaches.

Most of the existing subspace-based methods enhance the DOA detection performance through solving or optimizing an accurate signal subspace, which has always been a hot topic for scholars [18].

Optimization of the signal subspace

Based on the above array signal model, in this section, we develop a novel optimization scheme of the signal subspace. Mathematically, the detection of the DOA can be considered as the solution of the steering matrix of the array, and the corresponding problem is formulated as

$$\hat{\boldsymbol{A}} = \arg\min_{\boldsymbol{A}} \|\boldsymbol{X} - \boldsymbol{A}\boldsymbol{s}\|_2^2 \tag{5}$$

where $\|\bullet\|_2^2$ denotes the 2-norm. Normally, if we fix one of the variables, the other one can be solved through the method of least squares (by minimizing the standard squared error), which is expressed in the following manner

$$\begin{aligned} \hat{\boldsymbol{A}} &= \boldsymbol{X}\hat{\boldsymbol{s}}\left[\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^T\right]^{-1} \quad (a) \\ \hat{\boldsymbol{s}} &= \left[\hat{\boldsymbol{A}}^T\hat{\boldsymbol{A}}\right]^{-1}\hat{\boldsymbol{A}}\boldsymbol{X} \quad (b) \end{aligned} \tag{6}$$

It seems that the steering matrix of the array can be obtained in the above way (iteratively update the steering matrix and the signal vector). However, the array output contains not only signals but also noises, minimizing the standard squared error of (5) to produce the steering matrix is probably not desirable. To capture an accurate steering matrix so as to solve the DOA of the signals, we carry out the following design.

Assume that the steering matrix of the array computed by minimizing some cost function during the iteration is $\tilde{\mathbf{A}}_t$, where t denotes the index of the successive iteration. Then, we build up a signal subspace in the following form

$$\tilde{\boldsymbol{U}}_t = \tilde{\boldsymbol{A}}_t\left[\tilde{\boldsymbol{A}}_t^H\tilde{\boldsymbol{A}}_t\right]^{-\frac{1}{2}} \tag{7}$$

and the projection matrix [19, 20] of the signal subspace is defined as

$$\boldsymbol{Q}_t = \tilde{\boldsymbol{A}}_t\left[\tilde{\boldsymbol{A}}_t^H\tilde{\boldsymbol{A}}_t\right]^{-1}\tilde{\boldsymbol{A}}_t^H \tag{8}$$

Ideally, this reconstructed signal subspace and the estimated signal subspace through the covariance matrix of the array output should be equal. Thus, from this point of view, we establish such a cost function

$$J = \tilde{\boldsymbol{A}}\left[\tilde{\boldsymbol{A}}^H\tilde{\boldsymbol{A}}\right]^{-1}\tilde{\boldsymbol{A}}^H - \hat{\boldsymbol{U}}_s\hat{\boldsymbol{U}}_s^H \tag{9}$$

and combine it with (5) to optimize the signal subspace to determine the DOA.

Proceeding with more details, the developed scheme starts by computing the covariance matrix of the array output. Then, a set of initial DOAs is estimated using some classical approaches (say, MUSIC, ESPRIT, *etc.*) to form an initial steering matrix of the array $\tilde{\mathbf{A}}_0$ to promote the implementation of the algorithm. Subsequently, the steering matrix of the array and the signal vector update according to (6), and then minimize the constructed cost function. The entire process is repeated until there are no significant changes to the entries of the cost function reported in the two successive iterations of the method. Finally, the DOA can be solved from the resulting steering matrix of the array.

In order to avoid the algorithm falling into a local optimum, we set a constraint for the steering matrix of the array. Let $U(\theta_{p0}, \delta)$ denote the δ-neighborhood of θ_{p0} (the pth initial DOA), which is expressed in the following form

$$U\left(\theta_{p0}, \delta\right) = \left\{\chi \mid \theta_{p0} - \delta < \chi < \theta_{p0} + \delta\right\} \tag{10}$$

That is, during the iteration process, we limit the steering matrix of the array to a certain range by keeping the DOA to be detected to a certain range, which can effectively eliminate convergence on local optimum and also reduce the number of iterations of the algorithm. Obviously, this strategy can not only ensure the detection accuracy of DOA, but also accelerate the convergence speed of the method.

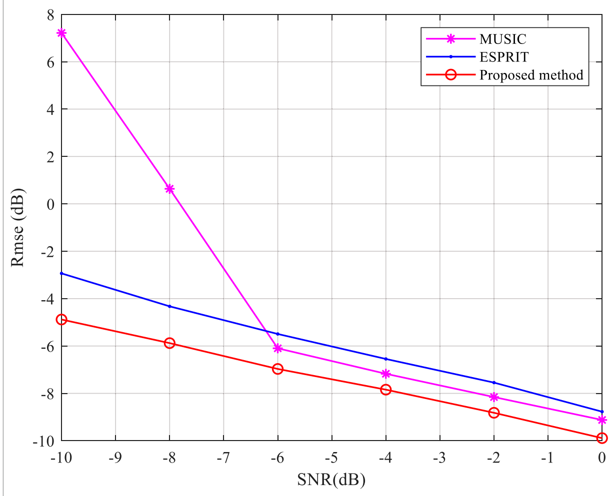

FIGURE 2
RMSE *versus* SNR.

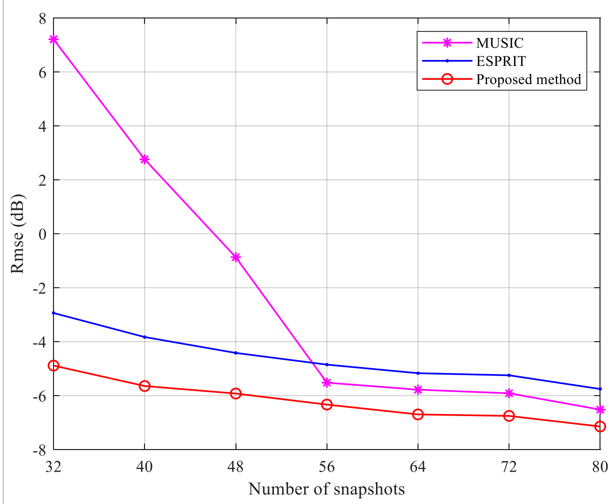

FIGURE 3
RMSE *versus* number of snapshots.

Experimental studies

We offer a series of simulations to demonstrate the Root-Mean-Square Error (RMSE) [20] performance of the approach compared with the MUSIC and the ESPRIT methods. In all simulations, a 15 elements ULA with a relative interelement spacing of $d = \lambda/2$ is used, and four narrowband signals with the DOAs [5°, 10°, 15°, 30°] impinge on the array. In this letter, the RMSE is defined as [21, 22].

$$10 \log 10 \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left\{ \frac{1}{P} \sum_{p=1}^{P} \left[\hat{\theta}_P(n) - \theta_p \right]^2 \right\}} \quad (\text{dB}) \qquad (11)$$

where N denotes the independent trials, and in the following simulations we set it as 200; $\hat{\theta}_P$ is the pth estimated DOA of the θ_P.

First, we test the RMSE performance of the methods *versus* the SNR, where the number of snapshots is fixed at 32, and the SNR varies from −10 to 0 with two intervals. The means of the simulation results are plotted in Figure 2. It is apparent that the performance of DOA detection is enhanced compared with the MUSIC and the ESPRIT methods with the developed method, and the developed method is also not very sensitive to the low SNR.

After that, we test the RMSE performance of the methods *versus* the number of snapshots. In the simulation, the SNR is fixed as −10 dB, and the number of snapshots varies from 32 to 80 with eight intervals. Figure 3 shows the simulation results. It is noticeable that the proposed method outperforms the MUSIC method and becomes insensitive to the changes of the number of snapshots. As previously mentioned in this letter, the detection performance of these subspace-based methods mainly depends on the accuracy of the signal subspace. The developed scheme optimizes the signal subspace through constructing a cost function and determining an optimal solution of the steering matrix of the array so as to solve the DOA. During this process, the signal subspace is optimized and the performance of the DOA detection becomes enhanced.

Conclusion

A scheme for DOA detection is put forward in this paper. The proposed scheme mainly involves the construction of the cost function of the steering matrix and the design of the steering matrix optimization. A constraint for the steering matrix is also set to make the method converge fast and eliminate the convergence on local optimum. The DOA is solved from the resulting steering matrix of the array. The simulation results indicate that the developed scheme achieves much better estimation performance than the traditional algorithms.

Hence, this paper proposes a fresh way to detect the DOA and also poses a problem of reducing the complexity of the method, as the developed scheme includes a series of iterations. Furthermore, hardware design and consideration of a real noise environment would also be interesting topics for research.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

All the authors made significant contributions to the work. The idea was proposed by GL; GX simulated the algorithm, analysed the data designed the experiments and polish the English, and wrote the paper. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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