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# Interaction of spin-orbit angular momentum in the tight focusing of structured light

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As an intrinsic property of light, angular momentum has always been an important research object of light field. In the past few years, the interactions between spin angular momentum and orbital angular momentum in tightly focused structured light have attracted much attention. Different from the independent conservation in the paraxial condition, the polarization-dependent spin angular momentum and the phase-dependent orbital angular momentum are coupled under tight focusing condition based on different physical mechanisms. The research on spin-orbit interaction will be helpful to deeply understand the nature of photon as well as extend the applications of light. Here, different forms of spin-orbit interaction during the tight focusing of structured light have been briefly introduced and classified. Besides, the existing problems and development prospects in the research about spin-orbit interaction of light are discussed, including the quantitative detection of the local distribution of optical spin and orbital angular momentum in experiments and the further applications of spinorbit interaction.

#### KEYWORDS

structured light, tight focus, spin-orbit interaction, spin angular momentum, orbital angular momentum

# Introduction

Besides the momentum related to the local wavevector, photon with wave-particle duality can also carry angular momentum (AM), which includes the polarization-dependent spin angular momentum (SAM) [1–5] and phase-dependent orbital angular momentum (OAM) [6, 7]. In 1909, Poynting first predicted that left- and right-circularly polarized light carries the SAM of  $\pm \hbar$  [5], which was experimentally proved by measuring the torque of the half-wave plate when conversing the right-hand circularly polarized light to left-hand circular polarization [8]. In 1992, Allen first proposed that the vortex beam containing a helical phase distribution exp (*ilθ*) carries OAM where *l* is the phase topological charge and the OAM of a single photon is  $l\hbar$  [7]. In the frame of paraxial optical fields, SAM and OAM are mostly independent of each other and conserved separately during the diffraction of light in free space. While in the cases

like waveguide structures [9], subwavelength metallic structures [10, 11] and tight focusing [12–16], coupling conversion between SAM and OAM can be achieved. In the absence of light-matter interaction, the significant spin-orbital interaction (SOI) during tight focusing provides a flexible and effective approach to manipulate the AM behavior of optical field.

Previous research has established that tightly focused circularly polarized light can induce the spin-to-orbit conversion (STOC). In 2006, Bomzon et al. presented that the focusing of circularly polarized plane wave can generate vortex phase with a topological charge of 2 in the focal plane, indicating the OAM can be derived from the conversion of SAM [17]. In 2007, Zhao et al. experimentally demonstrated the STOC in focused circularly polarized (CP) Gaussian beam by observing the orbital motion of micron-sized metal particles at the focal plane [18, 19]. In 2016, Bauer et al. found that pure transversely spinning of light can be derived by tightly focusing linearly polarized Gaussian beam, where the longitudinal spin component is zero [20]. For the structured light with spatially inhomogeneous distribution of amplitude, phase and polarization where the SAM and OAM may be coexisting in single optical beam [21-32], the interaction between SAM and OAM during its tight focusing exhibits diverse forms and has attracted more and more attention in recent years [33-38]. Related investigations not only deepen the understanding of the nature of photon, but the conversion of SAM and OAM also bring additional manipulation degree of freedom for the AM of light, leading to new applications including optical micromanipulation [39-44], information storage [45, 46] microscopic imaging [47, 48] and optical communication [49-52].

Here, we first briefly introduce the classification and calculations of AM. Then, the types of spin-orbit interaction (SOI) in tightly focused structured light are classified and summarized, including STOC, catalystlike effect of OAM, coupling of SAM and OAM, and SAM induced novel OAM. In addition, the problem of the quantitative measurement of SAM and OAM local distributions and the potential application of SOI are discussed.

# The classification and calculations of AM

The classification of light AM is shown in Figure 1. The AM can be divided into SAM that induces the spin motion of trapped dielectric particles and OAM that induces the orbital motion of trapped dielectric particles. SAM can be distinguished according to the direction of AM: the longitudinal-type SAM (l-SAM) is in the same direction of beam propagation and transverse-type SAM (t-SAM) is perpendicular to the direction of propagation, similar to the spinning movement of the rotor blades of an aircraft and the rolling bicycle wheel [53-56] respectively. Different from the polarization-related SAM, the phasedependent OAM is usually associated with the orbital energy flux. OAM can be divided into intrinsic-OAM (i-OAM) and extrinsic-OAM (e-OAM). The i-OAM is correlated with the spiral phase wavefront, whose magnitude and direction can be defined by the topological charge. The e-OAM is related to the transverse coordinate of the beam centroid, similar to AM of the classical particle, which is derived from the cross-product of the



spatial coordinates of the beam centroid and its linear momentum [38].

For the arbitrary optical field in non-magnetic media, the temporally averaged three-dimensional SAM and OAM density distribution can be generally expressed as [57, 58].

1 -

$$\vec{S} = \frac{1}{4\omega_0} \operatorname{Im} \left[ \varepsilon_0 \vec{E}^* \times \vec{E} \right]$$
$$\vec{L} = \frac{1}{4\omega_0} \vec{r} \times \operatorname{Im} \left[ \varepsilon_0 \left( \vec{E}^* \cdot (\nabla) \vec{E} \right) \right]$$

where  $\varepsilon_0$  is the permittivity in vacuum,  $\omega_0$  is the angular frequency, and  $\vec{E}^*$  denotes the complex conjugate of the electric field  $\vec{E}$ .  $\vec{S}$  denotes the SAM that contains the *l*-SAM and *t*-SAM, and  $\vec{L}$  denotes the OAM including *i*-OAM and *e*-OAM associated with optical vortex and optical trajectory, respectively [34].

# SOI in tightly focused structured light

The spatial manipulation of amplitude, phase and polarization in optical field leads to the generation of diverse structured light. SOI has been observed in the focusing of structured lights including linearly polarized vortex beams (LPVBs) [57], radially polarized vortex beams (RPVBs) [59], azimuthally polarized vortex beams (APVBs) [60], cylindrical vortex vector beams (CVVBs) [61], circularly polarized vortex beams (CPVBs) [62], azimuthal-variant hybridly polarized vector beams [63], radial-variant hybridly polarized vector beams [64], circularly polarized Bessel beams [65], higherorder Poincaré sphere beams (HOPBs) [66], etc. Different forms of SOI have been demonstrated including SAM to i-OAM conversion, OAM catalyzes the local distribution of t-SAM and l-SAM, coupling of SAM and OAM, and SAM induced novel OAM. The phenomena with complex behaviors of SOI are not only related to the numerical aperture (NA) of the focusing system, but also influenced by the spatial degrees of freedom of structured light.

## Spin-to-orbit conversion

In 2011, Bliokh et al. presented a general theory of STOC in focusing optical systems based on the Debye-Wolf theory [12]. The conversion efficiency of STOC can be estimated by 1–cos $\theta$ , where  $\theta$  is the aperture angle of the objective, indicating that the STOC becomes more significant as the field is focused at larger NA. In 2016, Yan et al. demonstrated the STOC by tightly focusing right-hand CPVBs and RPVBs [62]. The focal fields all exhibit hollow intensity distributions with cylindrical symmetry. The radius of the ring-like field reflects the OAM of light. The larger the absolute value of OAM, the larger the radius of focusing at the focal plane is. It was found that the OAM of the focal field is not only related to the sign and quantity of topological charge of the incident light, but also its polarization distribution. The incident right-hand CPVBs with positive OAM lead to a larger radius of the ring-like focal field than that with negative OAM, and the annulus radius of the RPVBs with the same focusing parameters falls in between these two situations. This indicates that the magnitudes of OAM change during tight focusing which is attributed to the conversion of the SAM to OAM. In 2017, Chen et al. investigated the STOC of azimuthalvariant hybridly polarized vector beams under a highly nonparaxial condition [63]. It was revealed that the density distribution of the converted OAM in the focal field is influenced by the spatial polarization distribution of the incident light. Larger polarization topological charge corresponds to more irregular OAM distribution, which is caused by the superposition of the converted OAM from SAM. In 2012, Pu et al. investigated the conversion of the radial-variant SAM to OAM in tightly focused radial-variant hybridly polarized vector beams based on vectorial Debye theory [64]. The intensity distribution of the focal field changes from an elliptical spot to an annular distribution with increasing the radial index, and the phase of the z-polarization component associated with the local SAM of vector beam varies with azimuthal angle from  $-\pi$  to  $\pi$  in the focus ring. Considering the SAM is radially varied, the OAM of the z-polarization component in the focal field also exhibits a radial distribution. This result lies in the fact that the conversion of local SAM in the radial-variant hybridly polarized vector beams to OAM under tight focusing leads to the helical phase profile.

### Catalyst effect of OAM

In general, the impact of OAM on the SAM in the focal field is not reflected in conversion, but more likely performs as a catalyst for the redistribution of SAM. In 2018, Yan et al. predicted in theory that the OAM carried by the APVBs can induce local redistribution of the SAM under tight focusing conditions [60]. The focal field exhibits a sharp central focal spot unrelated to the sign of OAM carried by the incident light. It was demonstrated that the phase of the radial and azimuthal components vary periodically along the azimuthal direction, indicating the magnitude of OAM is unchanged during focusing. A  $\pi$ -phase jump can be found in the radial direction of the azimuthal component, which causes the inhomogeneous distribution of SAM density. The distribution of SAM and OAM with the same orientation are concentrated in the center of the focus and surrounding the annulus respectively, while the total value of SAM and OAM in the focal field is constant. These results suggest that no OAM carried by the incident APVBs is converted to SAM after tight focusing, but OAM catalyzes the local redistribution of the SAM. In the same year, Liu et al. demonstrated the catalytic effect of OAM during the focusing of RPVBs. The OAM carried by the RPVBs can induce the conversion of transverse to three-dimensional SAM [59]. The RPBs with the total OAM of zero can induce the local t-SAM redistribution which is caused by the phase difference between the radial and longitudinal components. The *l*-SAM will appear when using the RPVBs with non-zero OAM as the incident light, which is attributed to the variation of spin flux caused by OAM. In 2022, Gu et al. theoretically investigated the three-dimensional distribution of SAM and OAM in tightly focused LPVBs [57]. Similarly, the incident light without carrying SAM exhibits both t-SAM and l-SAM after tight focusing. OAM is acting as a special catalyst to influence the local distribution of SAM, and the total quantity of SAM remains unchanged. In 2019, Man et al. theoretically investigated the angular momentum properties of tightly focused generalized cylindrical vortex vector beams (CVVBs) [61]. They found that existing of OAM in cylindrical vector beam leads to the appearance of longitudinal SAM in the tightly focused field. The increase of initial phase difference between the superposed beams with orthogonal polarization for generating vector beam causes the increase of the absolute value of longitudinal SAM in the focal field, and the sign of initial phase difference only affects the radial SAM density distribution.

### Coupling of SAM and OAM

As the variation of total quantity and the local redistribution are both the manifestations of SOI the bidirectional influence of SAM and OAM has also been observed during the tight focusing of some structured light. In 2020, Gong et al. theoretically studied the unique SOI phenomena by tightly focused HOPBs with both the vortex phase and complex polarization distribution [66]. The OAM converted from the SAM during tight focus combined with the origin OAM jointly affects the local distribution of SAM which will in turn influence the STOC. Thus, the coupling of SAM and OAM exhibits a complex behavior in the tight focusing of HOPBs. Besides, it has been demonstrated that by controlling the superposition weights of two vortex beams with orthogonal polarization and the NA of the focusing system, specific modulation of the local SAM and OAM density distribution in the focal field can be achieved.

### SAM induced novel OAM

Most of the current research considers the OAM associated with the helical phase profile, while the novel OAM induced by inhomogeneous distribution of SAM provided a new understanding of SOI in tightly focused structured light. In 2010, Wang et al. predicted the novel OAM relevant to the curl of polarization by tightly focusing the radial-variant hybridly polarized vector beams [67]. The focal field is donut-shaped distribution with a non-zero central intensity, and the maximum OAM is located at the radial of maximum intensity. They predicted that the novel OAM has a positive correlation with the gradient of radial variation of polarization. The existence of OAM is experimentally proved by observing the circular motion of the trapped isotropic particles in the focal plane. In 2021, Wang et al. presented a new strategy to produce OAM by constructing radial intensity gradient (RIG) in the focal plane of a circularly polarized light [65]. The spin-dependent local OAM obtained by focusing the CP with RIG can also trap the microparticles and realize the counterintuitive orbital motion of the microparticles in the experiment, which is attributed to the nonzero macroscopic spin flow along the azimuth. However, the changes of AM in the quantity and the local density distribution of SAM during the focusing process are still unclear.

## Discussion

As summarized in Table 1, the SOI in the tight focusing of structured light can be divided into four forms as STOC, catalystlike effect of OAM, coupling of SAM and OAM, and SAM induced novel OAM. The tight focusing of different structured light can lead to different forms of SOI. For STOC, the total SAM will decrease while the OAM will also change in total quantity. The increase or decrease of OAM is determined by the directions of initial SAM and OAM. For the forms of catalystlike effect of OAM, the quantity of OAM and SAM during tight focusing remain unchanged while the local distribution of SAM will change. The coupling of SAM and OAM, where the STOC and catalystlike effect of OAM may happen simultaneously, leads to the complex variation of AM including its quantity and local distribution. For the situation of SAM induced novel OAM, the variation of SAM is still indetermined and the quantity of OAM increases from 0.

For the experimental detection of SOI, there are mainly two methods. The one mostly used method focus on the observation of the spin and orbital motion behavior of microparticles which can be trapped and driven by AM of light. This method can only act as a semiquantitative approach to evaluate the magnitude and direction of AM. The measurement of amplitude, phase and polarization based on the near field scanning optical microscopy (NMOS) system is another experimental method which has the limitations of lower accuracy, poorer stability and sensitivity [68]. It is also difficult for a single nanoprobe to achieve the measurement of multiple degrees of freedom in the focal field. Therefore, the quantitative detection of SAM and OAM distribution in the tightly focal field remains a challenge and this has actually caused arguments in the study of SOI as no rigorous and reliable experimental data can prove some of the theoretical results. Efficient and accurate experimental methods for quantitatively detecting the conversion and the localized density of SAM and OAM should be further developed.

SOI	Structured light	SAM	OAM
STOC	CPVBs, azimuthal-variant hybridly polarized vector beams	Quantity decreases	Quantity changes
Catalystlike effect of OAM	LPVBs, CVVBs	Quantity unchanged Redistribution	Unchanged
Coupling of SAM and OAM	CPVBs, HOPBs	Quantity decreases Redistribution	Quantity changes
SAM induced novel OAM	CP with RIG, radial-variant hybridly polarized vector beams	Indetermination	Quantity increases

TABLE 1 Different forms of SOI.

Benefiting from in-depth research about the SOI, the potential of structured light has been considered in many applications including optical communications, optical trapping and optical nano-probing. From another perspective, the extending of the structured light may also provide opportunities to explore new forms of SOI. It is believed that further investigation of the SOI of structured light in tightly focused systems may bring new insights into light-matter interactions and promote the development of related applications. The special distribution of SAM and OAM in the focal field region based on spin-orbit interaction has been demonstrated as an effective way to achieve particle capture and optical micromanipulation. Besides, the combination of AM and other fundamental degrees of freedom of photon may drive the development of multiplexing technology, which has the advantages of ultra-high capacity, ultra-fast speed, high security and low bit error rate, and can be used in optical communication, optical information storage, information security encryption, and high-dimensional quantum information. Furthermore, the intrinsic coupling between the polarization, phase and position of the light in the focal field makes the SOI to act as an extremely sensitive probe to the position and scattering properties of the nanoparticle. By capturing and analyzing the scattered radiation of the nanoparticle located in the tightly focused field, the variation and conversion of AM distribution can be analyzed to obtain the information about nanoparticle, which is highly attractive for optical nano-probing [69].

# Conclusion

In conclusion, the interactions of spin-orbit AM in the tight focusing of structured light have been reviewed. Four different forms of SOI including STOC, catalyst like effect of OAM, coupling of SAM and OAM and SAM induced novel OAM can be summarized. With the optimization and upgrading of the detection method of local AM of light and modulation technology of structured light, the mechanism of spin-orbit interaction during tight focusing will be more clear and new forms of SOI may be demonstrated. Related research will further promote the application of structured light in optical communications, optical trapping and optical nano-probing.

## Author contributions

J-XG wrote the main manuscript and prepared figure, W-YW and T-YC finished the references collection and provided valuable suggestions for the work, J-QL finished the whole manuscript modification and finalization. All authors reviewed the manuscript.

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# Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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