



Diverse Novel Stable Traveling Wave Solutions of the Advanced or Voltage Spectrum of Electrified Transmission Through Fractional Non-linear Model

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This study analyzes the exact solutions of the compliance fractional non-linear time–space telegraph (FNLST) equation by Oliver Heaviside in 1880 via three non-applied analytical schemes. The solutions obtained to define the advanced or voltage spectrum of electrified transmission with day-to-day distance from electrical communication or the application of electromagnetic waves. Many new solutions are obtained, and three distinct styles of drawings are introduced (two-dimensional, three-dimensional, and density plots). Furthermore, stability characterization of the solutions is addressed using the properties of the Hamiltonian system. The originality of this study is shown by matching the solutions built with solutions produced previously using various analytical methods. Overall, the success of the three systems demonstrates their quality, intensity, and capacity to cope with several different types of non-linear evolutionary equations.

Keywords: electrical transmission, electromagnetic waves, fractional non-linear time–space telegraph equation, computational simulation, conformable fractional derivative

AMS classification: 35Q55, 33F05, 65D07, 37K10, 35C08

1. INTRODUCTION

The new generators of complicated systems, namely infectious disease epidemiology [1], neural network [2], genetics [3], fluid mechanic [4], community ecology [5], Solid-State Physics [6], plasma wave spread [7], thermodynamics [8], matter condensation [9], non-linear optimal [10], were employed. Formulating these phenomena through non-linear evolution equations relies on real testing to define parameters and to provide empirically unmistakable evidence that describes the intricate mechanisms of their physical and dynamic actions [11–13]. This implies that the key activity of these models is specified as functions, while the success factors for this behavior become [14] parameters. Many separate mathematicians and physicists also concentrate extensively on researching these mathematical models to understand more about their undiscovered properties [15, 16]. On the other hand, computational, analytical, and semi-analytical structures have been derived from fulfilling the same function [17–20].

Nowadays, fractional computation receives great attention in several science branches as it is able to discuss the non-local property of the mathematical models in detail [21, 22].

Thus, many fractional derivative operators have been derived to convert the non-linear partial differential equations into the ordinary differential equations, such as Riemann–Liouville, Caputo, the conformal fractional, and Atangana–Baleanu derivative operators [23–26]. These fractional operators have been employed for several non-linear evolution equations and have proved their effectiveness and powerfulness [27, 28].

This study handles a fundamental non-linear construction equation in electromagnetic waves, namely the compatible non-linear time-space equation. This equation model illustrates the advanced or voltage spectrum of an electrical transmission range with day-to-day distance from the electrical or wave application. This model is given mathematically by [29–31]

$$D_{tt}^{2\alpha} \mathcal{V} - D_{xx}^{2\alpha} \mathcal{V} + D_t^\alpha \mathcal{V} + b\mathcal{V} + d\mathcal{V}^3 = 0, \quad 0 < \alpha \leq 1, \quad (1)$$

where b, d are arbitrary constants to be evaluated through the considered analytical schemes. Mixing between conformable fractional derivative and traveling wave transformation as following $\mathcal{V}(x, t) = \mathcal{Q}(\Xi)$, $\Xi = \frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha}$, where a, c are arbitrary constants, and using it along with Equation (1) converts the non-linear partial differential equation (NLPDE) into the next ordinary differential equation (ODE).

$$(c^2 - a^2) \mathcal{Q}'' - c\mathcal{Q}' + b\mathcal{Q} + d\mathcal{Q}^3 = 0. \quad (2)$$

Using the homogeneous balance principle along the auxiliary equation of extended simplest equation method [32–35] [$\Phi'(\Xi) = \alpha_* + \lambda \Phi(\Xi) + \mu \Phi(\Xi)^2$], with $n = 1$, where n is the obtained value of homogeneous principle between the highest order derivative term and non-linear term in Equation (2), we get the following [$\mathcal{Q}'' \& \mathcal{Q}^3 \mapsto n + 2 = 3n$].

The section of the rest of the study are ordered as follows: in section 2, the above analytical schemes [32–35] are employed to achieve the solitary solution of the non-linear fractional time-space equation. Besides, the dynamical behavior of the solutions obtained is investigated through two-dimensional, three-dimensional, and density plots. The originality of the research is clarified in section 3. The conclusion of the entire study is presented in section 4.

2. SOLITARY WAVE SOLUTIONS

In this section, the solitary wave solutions of fractional non-linear time-space telegraph (FNLST) equation are investigated through the abovementioned computational schemes as following:

2.1. Extended Sech–Tanh Expansion Method’s Solution

The general solution of the FNLST equation is formulated by the suggested structure and measured balance value by

$$\mathcal{Q}(\Xi) = \sum_{i=1}^n \operatorname{sech}^{i-1}(\Xi) \left(a_i \operatorname{sech}(\Xi) + b_i \tanh(\Xi) \right) + a_0 = a_1 \operatorname{sech}(\Xi) + a_0 + b_1 \tanh(\Xi), \quad (3)$$

where a_0, a_1 , and b_1 are arbitrary constants to be determined later. Using the framework of the suggested method, produces the value of above-shown parameters is given as follows

$$a_0 = -\frac{\sqrt{b}}{2\sqrt{d}}, a_1 = 0, b_1 = -\frac{\sqrt{b}}{2\sqrt{d}}, c = \frac{3b}{4}, a = -\frac{1}{4}\sqrt{b}\sqrt{9b-2}, \text{ where } (b > \frac{2}{9} \& d > 0).$$

Hence, the solutions of Equation (1) in the next form is

$$\mathcal{V}_1(x, t) = \frac{1}{2} \sqrt{\frac{b}{d}} \left(\tanh \left(\frac{3bt^\alpha + \sqrt{b}\sqrt{9b-2}x^\alpha}{4\alpha} \right) - 1 \right). \quad (4)$$

2.2. The Solutions of Extended sinh-Gordon Equation Expansion Method

The general solution of the FNLST equation is formulated by the suggested structure and the measured balance value by

$$\mathcal{V}(\Xi) = \sum_{i=0}^n \cosh^{i-1}(w(\Xi)) \left(A_i \cosh(w(\Xi)) + B_i \sinh(w(\Xi)) \right) + A_0 = A_1 \cosh(w(\Xi)) + A_0 + B_1 \sinh(w(\Xi)), \quad (5)$$

where A_0, A_1 , and B_1 are arbitrary constants to be evaluated later. Using the framework of the suggested method, the value of above shown parameters can be obtained as follows

Case I. For $w'(\theta) = \sinh(\theta)$ and by substituting Equation (5) and its derivatives into Equation (2) in the framework of the suggested scheme, we obtained the following values of the abovementioned arbitrary constants:

Family I.

$$A_0 = \frac{\sqrt{-b}}{2\sqrt{d}}, A_1 = -\frac{\sqrt{-b}}{2\sqrt{d}}, B_1 = 0, a = -\frac{1}{4}\sqrt{b}\sqrt{9b-2}, c = \frac{3b}{4}, \text{ where } d < 0 \& b > \frac{2}{9}.$$

Hence, the solutions of Equation (1) take the next forms

$$\mathcal{V}_2(x, t) = \frac{-1}{2} \sqrt{\frac{-b}{d}} \left(\tanh \left(\frac{3bt^\alpha + \sqrt{b}\sqrt{9b-2}x^\alpha}{4\alpha} \right) - 1 \right), \quad (6)$$

$$\mathcal{V}_3(x, t) = \frac{-1}{2} \sqrt{\frac{-b}{d}} \left(\coth \left(\frac{3bt^\alpha + \sqrt{b}\sqrt{9b-2}x^\alpha}{4\alpha} \right) - 1 \right). \quad (7)$$

Family II.

$$A_0 = 0, A_1 = 0, B_1 = -\frac{\sqrt{2b-1}}{\sqrt{d}}, a = -\frac{\sqrt{2b-1}}{\sqrt{2}}, c = 1 \text{ where } (d > 0 \& \frac{1}{2} \neq b > 0).$$

Thus, the solutions of Equation (1) are constructed by

$$\mathcal{V}_4(x, t) = -\sqrt{\frac{1-2b}{d}} \operatorname{sech} \left(\frac{\sqrt{2b-1}x^\alpha}{\sqrt{2}\alpha} + \frac{t^\alpha}{\alpha} \right), \quad (8)$$

$$\mathcal{V}_5(x, t) = -\sqrt{\frac{1-2b}{d}} \operatorname{csch} \left(\frac{\sqrt{2b-1}x^\alpha}{\sqrt{2}\alpha} + \frac{t^\alpha}{\alpha} \right). \quad (9)$$

Family III.

$$A_0 = \frac{-1}{2} \sqrt{\frac{-b}{d}}, A_1 = \frac{-1}{2} \sqrt{\frac{-b}{d}}, B_1 = 0, a = -\frac{1}{4} \sqrt{b} \sqrt{9b-2},$$

$$c = -\frac{3b}{4}, \text{ where } (b < 0 \ \& \ d > 0).$$

Therefore, the solutions of Equation (1) are given by

$$\mathcal{V}_6(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \left(\tanh \left(\frac{3bt^\alpha - \sqrt{b}\sqrt{9b-2}x^\alpha}{4\alpha} \right) - 1 \right), \quad (10)$$

$$\mathcal{V}_7(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \left(\coth \left(\frac{3bt^\alpha - \sqrt{b}\sqrt{9b-2}x^\alpha}{4\alpha} \right) - 1 \right). \quad (11)$$

Case 2. For $w'(\theta) = \cosh(\theta)$ and by substituting Equation (5) and its derivatives into Equation (2) in the framework of the suggested scheme, we obtained the following values of the abovementioned arbitrary constants:

Family I.

$$A_0 = -\frac{i\sqrt{b}}{2\sqrt{d}}, A_1 = 0, B_1 = -\frac{\sqrt{b}}{2\sqrt{d}}, a = -\frac{1}{4} \sqrt{2b-9b^2},$$

$$c = \frac{3ib}{4}, \text{ where } (d < 0, i = \sqrt{-1}).$$

Thus, the solutions of Equation (1) take the following formulas

$$\mathcal{V}_8(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \left(-1 + \tanh \left(\frac{3bt^\alpha - \sqrt{(9b-2)b}x^\alpha}{4\alpha} \right) \right), \quad (12)$$

$$\mathcal{V}_9(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \left(-1 + \coth \left(\frac{3bt^\alpha - \sqrt{(9b-2)b}x^\alpha}{4\alpha} \right) \right). \quad (13)$$

Family II.

$$A_0 = -\frac{i\sqrt{b}}{2\sqrt{d}}, A_1 = -\frac{\sqrt{b}}{2\sqrt{d}}, B_1 = -\frac{\sqrt{b}}{2\sqrt{d}},$$

$$a = -\frac{1}{2} \sqrt{2b-9b^2}, c = \frac{3ib}{2}, \text{ where } (d < 0, i = \sqrt{-1}).$$

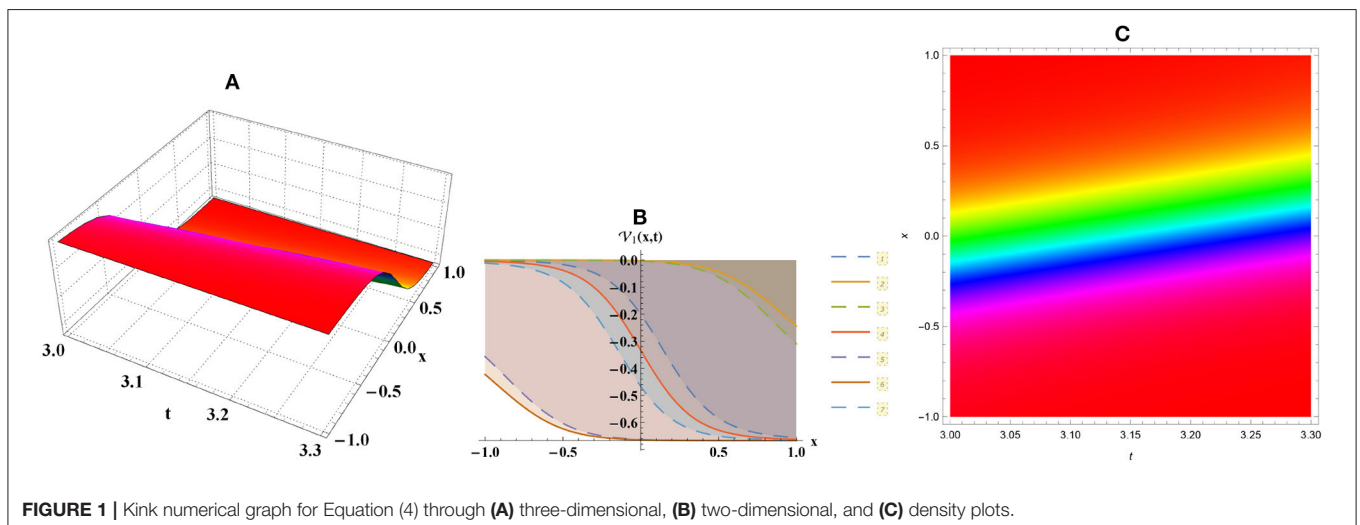
Hence, the solutions of Equation (1) are formulated in the following forms

$$\mathcal{V}_{10}(x, t) = \left(\frac{1}{2} \sqrt{\frac{-b}{d}} \tanh \left(\frac{3bt^\alpha}{2\alpha} - \frac{\sqrt{9b^2-2b}x^\alpha}{2\alpha} \right) \right.$$

$$\left. \mp \frac{1}{2} \sqrt{\frac{b}{d}} \operatorname{sech} \left(\frac{3bt^\alpha}{2\alpha} - \frac{\sqrt{9b^2-2b}x^\alpha}{2\alpha} \right) \right) - \frac{1}{2} \sqrt{\frac{-b}{d}}, \quad (14)$$

$$\mathcal{V}_{11}(x, t) = \left(\frac{1}{2} \sqrt{\frac{-b}{d}} \coth \left(\frac{3bt^\alpha}{2\alpha} - \frac{\sqrt{9b^2-2b}x^\alpha}{2\alpha} \right) \right.$$

$$\left. \mp \frac{1}{2} \sqrt{\frac{b}{d}} \operatorname{csch} \left(\frac{3bt^\alpha}{2\alpha} - \frac{\sqrt{9b^2-2b}x^\alpha}{2\alpha} \right) \right) - \frac{1}{2} \sqrt{\frac{-b}{d}}. \quad (15)$$



Family III.

$$A_0 = \frac{-1}{2} \sqrt{\frac{-b}{d}}, A_1 = 0, B_1 = \frac{\sqrt{b}}{2\sqrt{d}}, a = \frac{1}{4} \sqrt{2b - 9b^2},$$

$$c = -\frac{3ib}{4}, \text{ where } (d < 0, b > \frac{2}{9}).$$

Hence, the solutions of Equation (1) are constructed by

$$V_{12}(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \tanh \left(\frac{3bt^\alpha}{4\alpha} - \frac{i\sqrt{2b - 9b^2}x^\alpha}{4\alpha} \right) - \frac{1}{2} \sqrt{\frac{-b}{d}},$$

(16)

$$V_{13}(x, t) = \frac{1}{2} \sqrt{\frac{-b}{d}} \coth \left(\frac{3bt^\alpha}{4\alpha} - \frac{i\sqrt{2b - 9b^2}x^\alpha}{4\alpha} \right) - \frac{1}{2} \sqrt{\frac{-b}{d}}.$$

(17)

2.3. Extended Simplest Equation Method's Solutions

The general solution of the FNLST equation is formulated by the suggested structure and the measured balance value by

$$V(\Xi) = \sum_{i=-n}^n a_i \Phi(\Xi)^i = \frac{a_{-1}}{\Phi(\Xi)} + a_0 + a_1 \Phi(\Xi), \quad (18)$$

where a_{-1} , a_0 , and a_1 are arbitrary constants to be determined later, while $\Phi(\Xi)$ is the solution function of the next ODE

$$\Phi'(\Xi) = \alpha_* + \lambda\Phi(\Xi) + \mu\Phi(\Xi)^2, \quad (19)$$

where α_* , λ , and μ are arbitrary constants to be calculated later. Using the framework of the suggested method, gets the value of above shown parameters as follows:

Family I.

$$b = \frac{-2c^2}{9(a+c)(a-c)}, d = \frac{(3a^2\lambda - 3c^2\lambda + c)^2}{18a_0^2(a-c)(a+c)},$$

$$\mu = \frac{(3a^2\lambda - 3c^2\lambda - c)(3a^2\lambda - 3c^2\lambda + c)}{36\alpha_*(a-c)^2(a+c)^2},$$

$$a_{-1} = 0, a_1 = \frac{a_0(3a^2\lambda - 3c^2\lambda - c)}{6\alpha_*(a-c)(a+c)}.$$

Hence, the solitary solutions of Equation (1) take the following formulas

When $\lambda = 0$,

for $\alpha_* \mu > 0$, we obtain

$$V_{14}(x, t) = a_0 - \frac{a_0 c \sqrt{\alpha_* \mu}}{6 \mu \alpha_* (a - c) (a + c)} \tan \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right),$$

(20)

$$V_{15}(x, t) = a_0 - \frac{a_0 c \sqrt{\alpha_* \mu}}{6 \mu \alpha_* (a - c) (a + c)} \cot \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right).$$

(21)

for $\alpha_* \mu < 0$, we obtain

$$V_{16}(x, t) = a_0 - \frac{a_0 c \sqrt{-\alpha_* \mu}}{6 \mu \alpha_* (a - c) (a + c)} \tanh \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right),$$

(22)

$$V_{17}(x, t) = a_0 - \frac{a_0 c \sqrt{-\alpha_* \mu}}{6 \mu \alpha_* (a - c) (a + c)} \coth \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right).$$

(23)

When $\alpha_* = 0$,

for $\lambda > 0$, we obtain

$$V_{18}(x, t) = a_0 + \frac{a_0 \lambda e^{\lambda \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right)} (3 a^2 \lambda - 3 c^2 \lambda - c)}{6 \alpha \left(1 - \mu e^{\lambda \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right)} \right) (a - c) (a + c)}.$$

(24)

for $\lambda < 0$, we obtain

$$V_{19}(x, t) = a_0 - \frac{a_0 \mu e^{\lambda \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right)} (3 a^2 \lambda - 3 c^2 \lambda - c)}{6 \alpha \left(1 + \mu e^{\lambda \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right)} \right) (a - c) (a + c)}.$$

(25)

The general solutions has the next form:

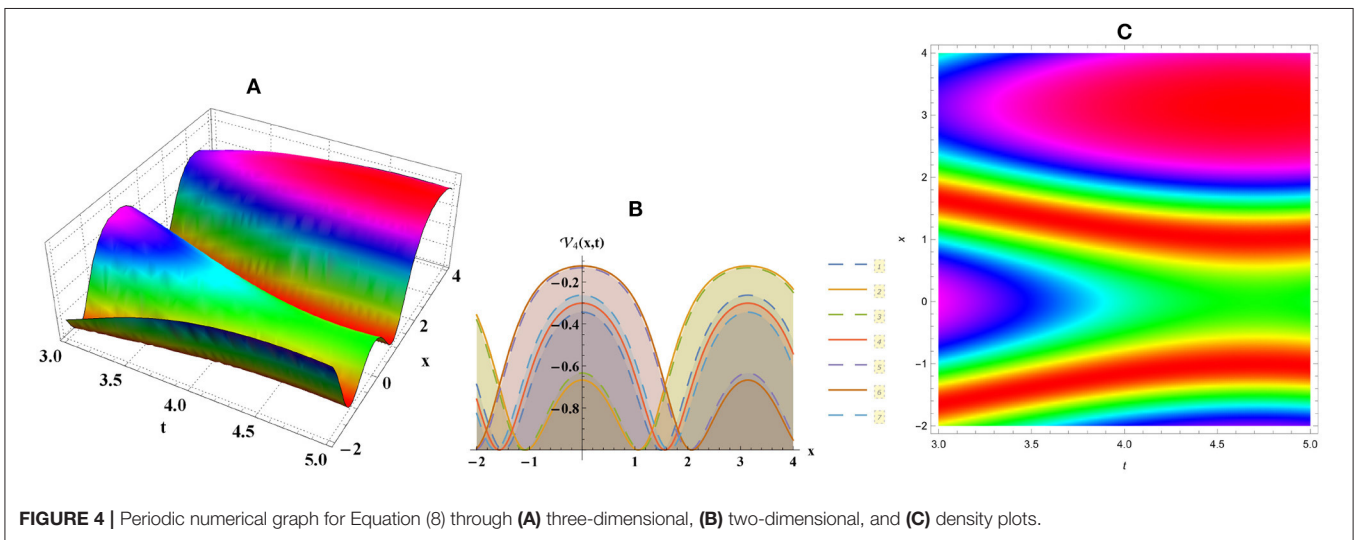
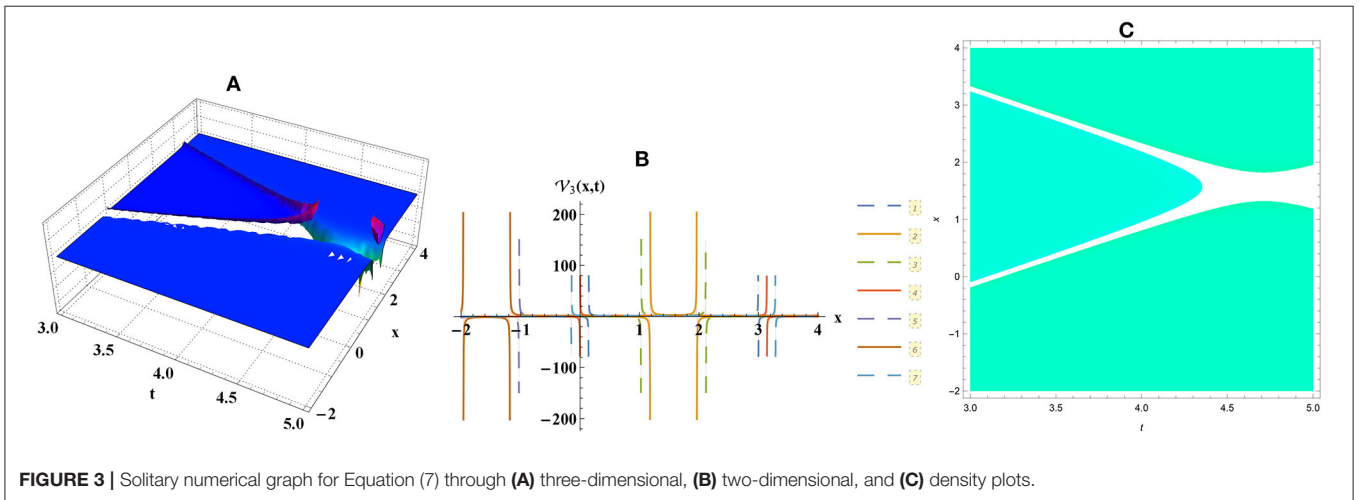
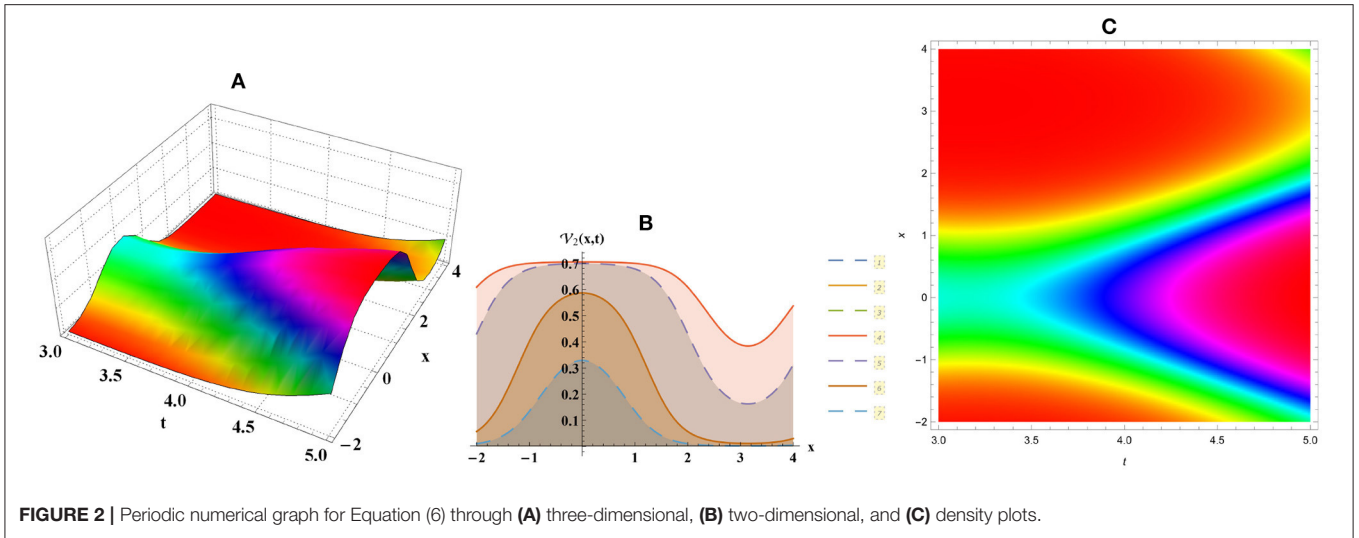
When $4\alpha_* \mu > \lambda^2$ and $\mu > 0$, we get

$$V_{20}(x, t) = a_0 + \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda - c)}{12 \mu \alpha_* (a - c) (a + c)} \sqrt{4 \alpha_* \mu - \lambda^2} \tan \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right) - \lambda,$$

(26)

$$V_{21}(x, t) = a_0 + \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda - c)}{12 \mu \alpha_* (a - c) (a + c)} \sqrt{4 \alpha_* \mu - \lambda^2} \cot \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right) - \lambda.$$

(27)



When $4\alpha_*\mu > \lambda^2$ and $\mu < 0$, we get

$$\mathcal{V}_{22}(x, t) = a_0 + \frac{a_0(3a^2\lambda - 3c^2\lambda - c)}{12\mu\alpha_*(a-c)(a+c)}\sqrt{4\alpha_*\mu - \lambda^2} \tan\left(\frac{\sqrt{4\alpha_*\mu - \lambda^2}}{2}\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)\right) + \lambda, \quad (28)$$

$$\mathcal{V}_{23}(x, t) = a_0 + \frac{a_0(3a^2\lambda - 3c^2\lambda - c)}{12\mu\alpha_*(a-c)(a+c)}\sqrt{4\alpha_*\mu - \lambda^2} \cot\left(\frac{\sqrt{4\alpha_*\mu - \lambda^2}}{2}\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)\right) + \lambda. \quad (29)$$

Family II.

$$b = \frac{-2c^2}{9(a+c)(a-c)}, d = \frac{2c^2}{9a_0^2(a-c)(a+c)}, \lambda = \frac{c}{3(a+c)(a-c)}, \alpha_* = a_{-1} = 0.$$

Hence, the solitary solutions of Equation (1) are constructed in the next forms for $\lambda > 0$, as

$$\mathcal{V}_{24}(x, t) = a_0 + \frac{a_1\lambda e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}{1 - \mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}} \text{ and} \quad (30)$$

for $\lambda < 0$, as

$$\mathcal{V}_{25}(x, t) = a_0 - \frac{a_1\mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}{1 + \mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}. \quad (31)$$

Family III.

$$b = \frac{-2c^2}{9(a+c)(a-c)}, d = 2\frac{\mu^2(a^2 - c^2)}{a_1^2}, \lambda = \frac{c}{3(a+c)(a-c)}, \alpha_* = a_{-1} = 0.$$

Hence, the solitary solutions of Equation (1) are given, for $\lambda > 0$, as

$$\mathcal{V}_{26}(x, t) = a_0 + \frac{a_1\lambda e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}{1 - \mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}} \text{ and} \quad (32)$$

for $\lambda < 0$, as

$$\mathcal{V}_{27}(x, t) = a_0 - \frac{a_1\mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}{1 + \mu e^{\lambda\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)}}. \quad (33)$$

Family IV.

$$\alpha = \frac{(3a^2\lambda - 3c^2\lambda - c)(3a^2\lambda - 3c^2\lambda + c)}{36\mu(a-c)^2(a+c)^2}, b = \frac{-2c^2}{9(a+c)(a-c)}, d = \frac{(3a^2\lambda - 3c^2\lambda - c)^2}{18a_0^2(a-c)(a+c)},$$

$$a_{-1} = \frac{a_0(3a^2\lambda - 3c^2\lambda + c)}{6\mu(a-c)(a+c)}, a_1 = 0.$$

Hence, the solitary solutions of Equation (1) are evaluated in the following formulas:

When $\lambda = 0$,
for $\alpha_*\mu > 0$, we obtain

$$\mathcal{V}_{28}(x, t) = \frac{a_0c}{6\sqrt{\alpha_*\mu}(a-c)(a+c)\tan\left(\sqrt{\alpha_*\mu}\left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C\right)\right)} + a_0, \quad (34)$$

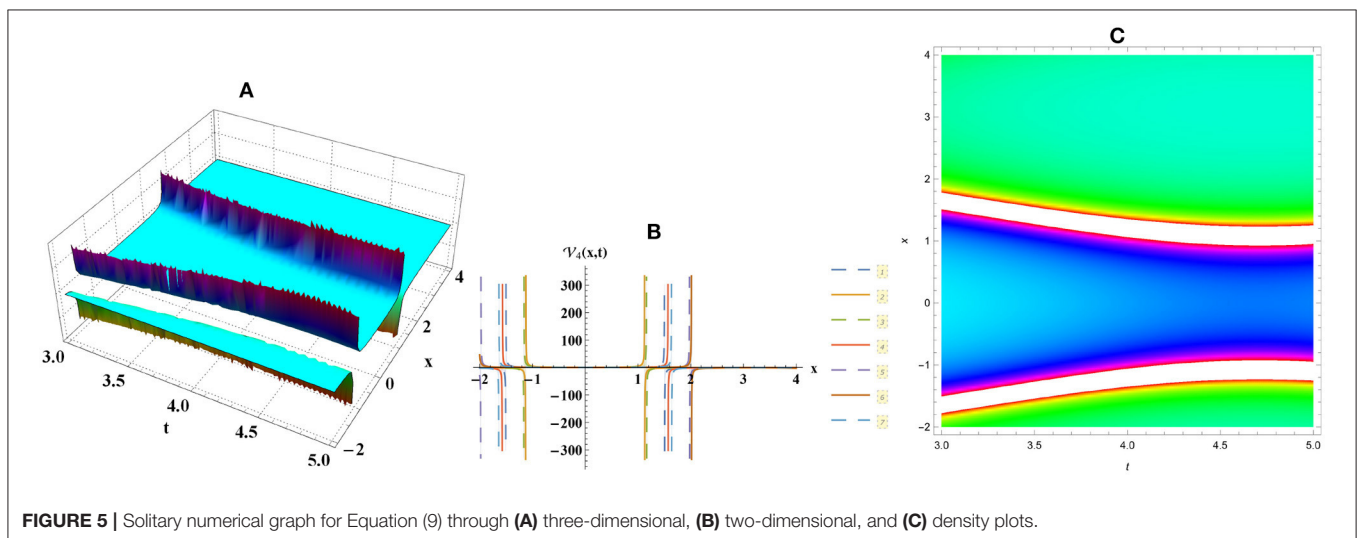


FIGURE 5 | Solitary numerical graph for Equation (9) through (A) three-dimensional, (B) two-dimensional, and (C) density plots.

$$\mathcal{V}_{29}(x, t) = \frac{a_0 c}{6 \sqrt{\alpha_* \mu} (a - c) (a + c) \cot \left(\sqrt{\alpha_* \mu} \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right) \right)} + a_0. \quad (35)$$

for $\alpha_* \mu < 0$, we obtain

$$\mathcal{V}_{30}(x, t) = \frac{a_0 c}{6 \sqrt{-\alpha_* \mu} (a - c) (a + c) \tanh \left(\sqrt{-\alpha_* \mu} \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right)} + a_0, \quad (36)$$

$$\mathcal{V}_{31}(x, t) = \frac{a_0 c}{6 \sqrt{-\alpha_* \mu} (a - c) (a + c) \coth \left(\sqrt{-\alpha_* \mu} \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right)} + a_0. \quad (37)$$

When $\alpha_* = 0$,

for $\lambda > 0$, we obtain

$$\mathcal{V}_{32}(x, t) = \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c) \left(1 - \mu e^{\lambda \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right)} \right)}{6 \mu (a - c) (a + c) \left(\lambda e^{\lambda \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right)} \right)} + a_0. \quad (38)$$

for $\lambda < 0$, we get

$$\mathcal{V}_{33}(x, t) = \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c) \left(1 + \mu e^{\lambda \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right)} \right)}{6 \mu (a - c) (a + c) \left(-\mu e^{\lambda \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right)} \right)} + a_0. \quad (39)$$

The general solutions has the next form:

For $4 \alpha_* \mu > \lambda^2$ and $\mu > 0$, we get

$$\mathcal{V}_{34}(x, t) = \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c)}{3 (a - c) (a + c) \left(\sqrt{4 \alpha_* \mu - \lambda^2} \tan \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{ax^\alpha}{\alpha} - \frac{ct^\alpha}{\alpha} + C \right) - \lambda \right) \right)} + a_0. \quad (40)$$

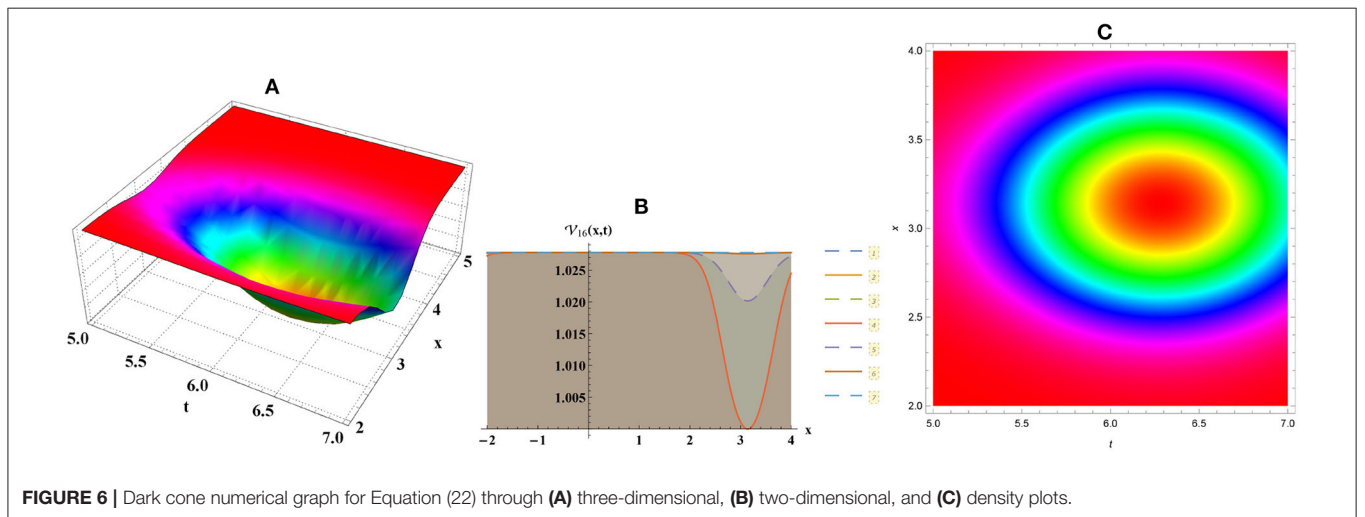


FIGURE 6 | Dark cone numerical graph for Equation (22) through (A) three-dimensional, (B) two-dimensional, and (C) density plots.

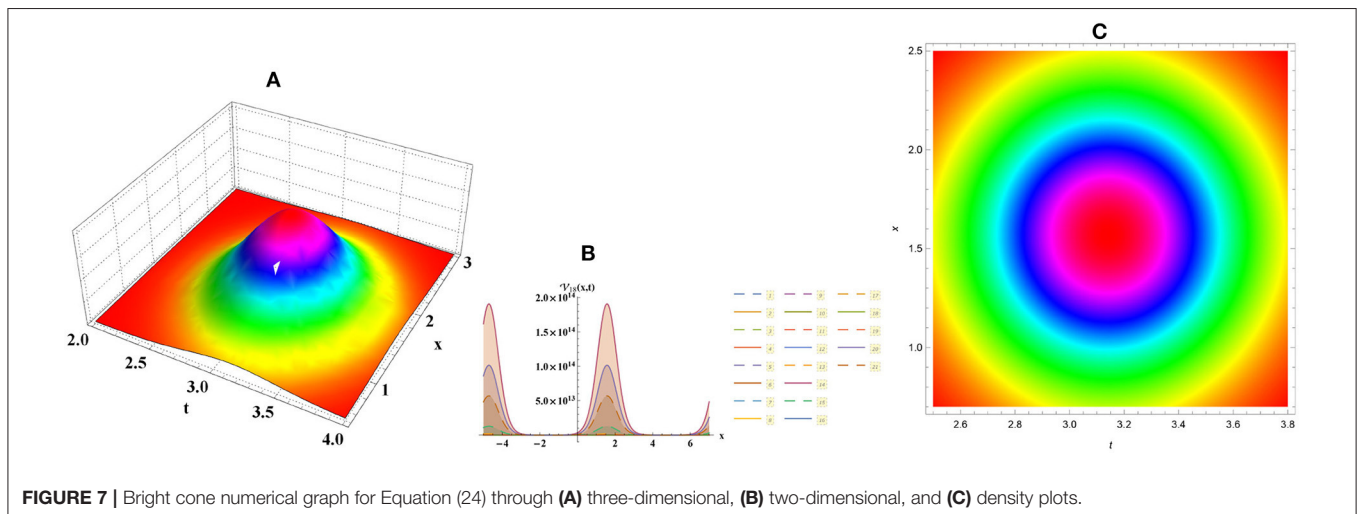


FIGURE 7 | Bright cone numerical graph for Equation (24) through (A) three-dimensional, (B) two-dimensional, and (C) density plots.

$$\begin{aligned} & \mathcal{V}_{35}(x, t) \\ &= \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c)}{3 (a - c) (a + c) \left(\sqrt{4 \alpha_* \mu - \lambda^2} \cot \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) - \lambda \right) \right)} + a_0, \end{aligned} \tag{41}$$

For $4 \alpha_* \mu > \lambda^2$ and $\mu < 0$, we get

$$\begin{aligned} & \mathcal{V}_{36}(x, t) \\ &= \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c)}{3 \left(\sqrt{4 \alpha_* \mu - \lambda^2} \tan \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) + \lambda \right) \right) (a - c) (a + c)} + a_0, \end{aligned} \tag{42}$$

$$\begin{aligned} & \mathcal{V}_{37}(x, t) \\ &= \frac{a_0 (3 a^2 \lambda - 3 c^2 \lambda + c)}{3 \left(\sqrt{4 \alpha_* \mu - \lambda^2} \cot \left(\frac{\sqrt{4 \alpha_* \mu - \lambda^2}}{2} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) + \lambda \right) \right) (a - c) (a + c)} + a_0. \end{aligned} \tag{43}$$

Family V.

$$\begin{aligned} \alpha &= -\frac{c^2}{144 \mu (a - c)^2 (a + c)^2}, \quad b = \frac{-2c^2}{9 (a + c) (a - c)}, \\ d &= \frac{c^2}{18 a_0^2 (a - c) (a + c)}, \quad \lambda = 0, \\ a_{-1} &= \frac{c a_0}{24 \mu (a - c) (a + c)}, \quad a_1 = \frac{6 \mu (a^2 - c^2) a_0}{c}. \end{aligned}$$

Therefore, the solitary solutions of Equation (1) are evaluated for $\alpha_* \mu > 0$, as

$$\begin{aligned} \mathcal{V}_{38}(x, t) &= \frac{c a_0}{24 \sqrt{\alpha_* \mu} (a - c) (a + c) \tan \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right)} \\ &+ \frac{6 \sqrt{\alpha_* \mu} (a^2 - c^2) a_0}{c} \tan \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right), \end{aligned} \tag{44}$$

$$\begin{aligned} \mathcal{V}_{39}(x, t) &= \frac{c a_0}{24 \sqrt{\alpha_* \mu} (a - c) (a + c) \tan \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right)} \\ &+ \frac{6 \sqrt{\alpha_* \mu} (a^2 - c^2) a_0}{c} \cot \left(\sqrt{\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} + C \right) \right). \end{aligned} \tag{45}$$

for $\alpha_* \mu < 0$, as

$$\begin{aligned} \mathcal{V}_{40}(x, t) &= \frac{c a_0}{24 \sqrt{-\alpha_* \mu} (a - c) (a + c) \tanh \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right)} \\ &+ \frac{6 \sqrt{-\alpha_* \mu} (a^2 - c^2) a_0}{c} \tanh \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right), \end{aligned} \tag{46}$$

$$\begin{aligned} \mathcal{V}_{41}(x, t) &= \frac{c a_0}{24 \sqrt{-\alpha_* \mu} (a - c) (a + c) \coth \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right)} \\ &+ \frac{6 \sqrt{-\alpha_* \mu} (a^2 - c^2) a_0}{c} \coth \left(\sqrt{-\alpha_* \mu} \left(\frac{a x^\alpha}{\alpha} - \frac{c t^\alpha}{\alpha} \right) \mp \frac{\ln(C)}{2} \right). \end{aligned} \tag{47}$$

3. RESULTS AND DISCUSSION

This section studies the constructed traveling wave solutions of the FNLST equation through the above applied schemes:

1. The obtained solution by using sech-tanh expansion method (4) is equal to the solution that obtained by using extended sinh-Gorden expansion method (6).
2. All other solutions are different. This shows the powerfulness and effectiveness of these three different techniques.
3. Comparing the constructed solutions with that which have been obtained by Ozkan Guner and Ahmet Bekir, who used the Exp-function method [36], shows the novelty and originality of the study, where all our solutions are completely different from their solutions.
4. Interpretation of figures:
We have explained some of the evaluated solutions through three different schemes (three-dimensional, two-dimensional, and density plots).

- **Figure 1** shows kink wave of Equation (4) along with $b = 4, d = 9$.
- **Figure 2** explains periodic wave of Equation (6) along with $b = 2, d = -4$.
- **Figure 3** demonstrates solitary wave of Equation (7) along with $b = 7, d = -3$.
- **Figure 4** figures out periodic wave of Equation (8) along with $b = 4, d = -7$.
- **Figure 5** explains solitary wave of Equation (9) along with $b = 5, d = -6$.
- **Figure 6** shows dark cone wave of Equation (22) along with $a_0 = 1, a = 2, \alpha_* = -1, c = 1, C = 3, \mu = 4$.
- **Figure 7** explains bright cone wave of Equation (24) along with $a_0 = 3, a = 5, c = 2, C = 4, \lambda = 4$.

4. CONCLUSION

This study effectively applied three non-applied methods to the FNLST equation. The compatible fractional operator is used to transform the non-linear fractional partial differential equation into an ordinary differential equation of integer order. Various modern numerical options have been developed to illustrate the cutting-edge or voltage of an electrical transmission spectrum with a day yet distance across two-dimensional, three-dimensional, and density plots. The physical interpretation of these sketches have been explained in Equation (3) to show more novel properties of the considered model. The newness of this study is explored by contrasting the observations in the previous study.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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