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Equitable Domination in Vague Graphs With Application in Medical Sciences

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Considering all physical, biological, and social systems, fuzzy graph (FG) models serve the elemental processes of all natural and artificial structures. As the indeterminate information is an essential real-life problem, which is mostly uncertain, modeling the problems based on FGs is highly demanding for an expert. Vague graphs (VGs) can manage the uncertainty relevant to the inconsistent and indeterminate information of all real-world problems, in which FGs possibly will not succeed in bringing about satisfactory results. In addition, VGs are a very useful tool to examine many issues such as networking, social systems, geometry, biology, clustering, medical science, and traffic plan. The previous definition restrictions in FGs have made us present new definitions in VGs. A wide range of applications has been attributed to the domination in graph theory for several fields such as facility location problems, school bus routing, modeling biological networks, and coding theory. Concepts from domination also exist in problems involving finding the set of representatives, in monitoring communication and electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take the height measurement for an entire region). Hence, in this article, we introduce different concepts of dominating, equitable dominating, total equitable dominating, weak (strong) equitable dominating, equitable independent, and perfect dominating sets in VGs and also investigate their properties by some examples. Finally, we present an application in medical sciences to show the importance of domination in VGs.

Keywords: vague set, vague graph, equitable dominating set, equitable neighborhood, medical science, Mathematics Subject Classification: 05C99, 03E72

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1 INTRODUCTION

Many real-world situations can accessibly be explained by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. Notice that in such diagrams one is mainly interested in whether two given points are joined by a line; the manner in which they are joined is immaterial. A mathematical abstraction of situations of this type gives rise to the concept of a graph. To exemplify the objects and the connection between them, the graph nodes and edges are being employed accordingly. FGs are intended to demonstrate the connection structure among objects so that the concrete object existence (node) and the relationship between two objects (edge) are matters of degree. FG models are advantageous mathematical tools for addressing the combinatorial problems in several fields integrating research, algebra, computing, environmental science, and topology. Owing to the vagueness and ambiguity of natural existence, fuzzy graphical models outperform other graphical models. In 1965, Zadeh [44] proposed fuzzy set (FS) theory as a model for the exemplification of uncertainty and vagueness in real-world systems. FS theory is an exceedingly influential mathematical tool for resolving approximate reasoning-related problems. By defining the VS notion through changing the value of an element in a set with a subinterval of [0,1], Gau and Buehrer [13] introduced the VS theory. More probabilities are illustrated by VSs compared to FSs. A VS is more effective for explaining the false membership degree existence. Many events in the real world provided the incentive for introducing FGs. Kauffman [15] described FGs based on Zadeh's fuzzy relation [44]. Kosari et al. [16] defined vague graph structure. Fuzzy Graph was introduced by Rosenfeld [32]. Akram et al. [1–6] proposed new definitions on FGs. Mordeson et al. [17–19] studied some results in FGs. Borzooie and Rashmanlou [7–11] analyzed several concepts of VGs. Samanta et al. [33–38] defined fuzzy competition graphs and some bipolar fuzzy graph results. Shao et al. [25, 26, 39–41] introduced new results in FGs and intuitionistic fuzzy graphs. Ramakrishna [24] presented VG concepts and examined their properties. Rashmanlou et al. [27–31] advanced new concepts in VGs.

A VG is a generalized structure of a FG that provides more exactness, adaptability, and compatibility to a system when matched with systems that run on FGs. In addition, a VG is capable of concentrating on determining the uncertainty coupled with the inconsistent and indeterminate information of any real-world problem, where FGs may not lead to adequate results. There exist an extensive array of applications for domination in graph theory in several fields such as school bus routing, facility location problems, and electrical networks. The domination idea was introduced first in the chessboard problem. In 1962, Ore [22] pioneered to apply the expression "domination" for undirected graphs. Somasundaram [42] presented the domination and independent domination in FGs. Gani and Chandrasekaran [20, 21] investigated the fuzzy-DS and independent-DS notion utilizing strong arcs. Cockayne [12] and Hedetniemi [14] described the independent and irredundance domination number in graphs. The domination concept in intuitionistic fuzzy graphs was examined by Parvathi and Thamizhendhi [23]. Talebi and Rashmanlou [43] studied new applications of domination in VGs. Domination in VGs has several uses in different fields. Hence, this study seeks to consider different concepts of dominating, equitable dominating, total equitable dominating, weak (strong) equitable dominating, equitable independent, and perfect dominating sets in VGs and investigate their properties by some examples.

Previously, many emergency patients died due to delays in transportation to the hospital; therefore, we introduce an application in the transportation system to show the importance of domination in VGs.

2 PRELIMINARIES

In this section, to consider the stage for our analysis and to facilitate the following of our discussion, a brief overview of some of the basic definitions is introduced. A graph denotes a pair $G^* = (V, E)$ satisfying $E \subseteq V \times V$. The elements of V and E are the nodes and edges of the graph G^*, correspondingly.

An FG has the form of $\xi = (\gamma, \nu)$, where $\gamma : V \to [0,1]$ and $\nu : V \times V \to [0,1]$ are defined as $\nu(ab) \leq \gamma(a) \wedge \gamma(b)$, $\forall a, b \in V$, and ν is a symmetric fuzzy relation on γ and $\wedge$ denotes the minimum.

Definition 2.1. [13] A VS A is a pair (t_A, f_A)on set V where t_A and f_A are used as real valued functions which can be defined on $V \to [0,1]$, so that $t_A(a) + f_A(a) \leq 1$, for all a belongs V. The interval $[t_A(a), 1 - f_A(a)]$is considered as the vague value of a in A.G^* will be a crisp graph (V, E) and ζ a VG (A, B) throughout this article.

Definition 2.2. [13] The support of a vague set $A = (t_A, f_A)$, denoted by $supp(A)$, is defined as $supp(A) = supp^t(A) \cup supp^f(A)$, where $\mathrm{supp}^t(A) = \{a | t_A(a) > 0\}$, $supp^f(A) = \{a | f_A(a) > 0\}$.

Definition 2.3. [24] A pair $\zeta = (A, B)$is called a VG on a crisp graph G^* that $A = (t_A, f_A)$is a VS on V and $B = (t_B, f_B)$ is a VS on $E \subseteq V \times V$such that $t_B(ab) \leq \min(t_A(a), t_A(b))$ and $f_B(ab) \geq \max(f_A(a), f_A(b))$, for each edge $ab \in E$.

Definition 2.4. [8] Let $\zeta = (A, B)$be a VG. Then, (i) the vertex cardinality of ζ is described by $|A|$ and defined as $|A| = \sum_{a_i \in V} (t_A(a_i), f_A(a_i))$. and (ii) the edge cardinality of ζ is described by $|B|$ and defined as

$$|B| = \sum_{a_i, a_j \in V} \left(t_B\left(a_i a_j\right), f_B\left(a_i a_j\right)\right).$$

Definition 2.5. [8] Let $\zeta = (A, B)$ be a VG. If $a_i, a_j \in V$, then the t-strength of connectedness between a_i and a_j is defined as $t_B^\infty(a_i, a_j) = \sup\{t_B^k(a_i, a_j) | k = 1, 2, \ldots, n\}$ and f-strength of connectedness is as $f_B^\infty(a_i, a_j) = \inf\{f_B^k(a_i, a_j) | k = 1, 2, \ldots, n\}$. In addition, we have

$$t_B^k(ab) = \sup\{t_B(a, b_1) \wedge t_B(b_1, b_2) \wedge t_B(b_2, b_3) \wedge \cdots \wedge t_B(b_{k-1}, b) | \times (a, b_1, b_2, \ldots, b_{k-1}, b) \in V\}.$$

and

$$f_B^k(ab)() = \inf\{f_B(a, b_1) \vee f_B(b_1, b_2) \vee f_B(b_2, b_3) \vee \cdots \vee f_B(b_{k-1}, b) | \times (a, b_1, b_2, \ldots, b_{k-1}, b) \in V\}.$$

Definition 2.6. [8] An edge abin a VG $\zeta = (A, B)$is called strong edge if $t_B(ab) \geq (t_B)^\infty(ab)$and $f_B(ab) \leq (f_B)^\infty(ab)$.

Definition 2.7. [8] Two nodes a_i and a_j in a VG $\zeta = (A, B)$ are called to be adjacent if either one of the following conditions holds. (i) $t_B(a_i a_j) > 0$ and $f_B(a_i a_j) \geq 0$. (ii) $t_B(a_i a_j) \geq 0$ and $f_B(a_i a_j) > 0$, $a_i, a_j \in V$. (iii) A node a in a VG ζ is called an isolated node if $t_B(ab) = 0$and $f_B(ab) = 0$, $\forall b \in V$, $a \neq b$. That is, $N(a) = \varnothing$.

Definition 2.8. [9] The degree of a node a in a VG ζ is defined as the sum of weights of edges incident to a. It is defined by $d_\zeta(a) = (\deg^t(a), \deg^f(a))$. The minimum degree of ζ is $\delta(\zeta) = \min\{d_\zeta(a) | a \in V\}$. The maximum degree of ζ is $\Delta(\zeta) = \max\{d_\zeta(a) | a \in V\}$.

Definition 2.9. [8] Let $\zeta = (A, B)$be a VG. Suppose that $a, b \in V$; then, a dominates b in ζ if ∃a strong edge between a and b.

Definition 2.10. [8] A subset S of V is called a DS in ζ if for each $a \in V - S$, $\exists b \in S$so that a dominates b. A DS S of a VG ζ is referred to as a Minimal DS if no proper subset of S is a DS.

Definition 2.11. [8] If ζ is a VG, then the vertex cardinality of $S \subseteq V$ is defined as follows:

TABLE 1 | Some basic notations.

Notation	Meaning
FG	Fuzzy graph
VS	Vague set
ζ	Vague graph
DS	Dominating set
EN	Equitable neighborhood
EDS	Equitable dominating set
END	Equitable neighborhood degree
DEVG	Degree equitable vague graph
EIS	Equitable independent set
EDN	Equitable dominating number
TEDS	Total equitable dominating set
EIN	Equitable isolated node
EIDS	Equitable independent dominating set
PDS	Perfect dominating set
PDN	Perfect domination number
MI-EDS	Minimal equitable dominating set
MA-EDS	Maximal equitable dominating set
MA-EIS	Maximal equitable independent set

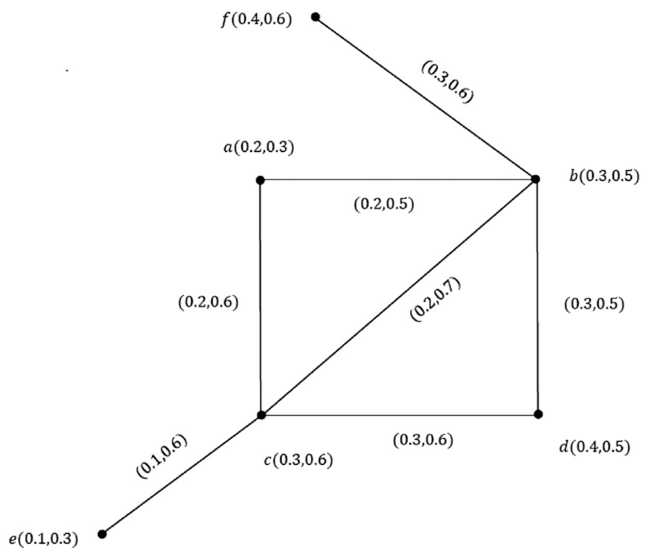

FIGURE 1 | EN degree of nodes in ζ.

TABLE 2 | Vague set A on set V.

A	a	b	c	d	e	f
t_A	0.2	0.3	0.3	0.4	0.1	0.4
f_A	0.3	0.5	0.6	0.5	0.3	0.6

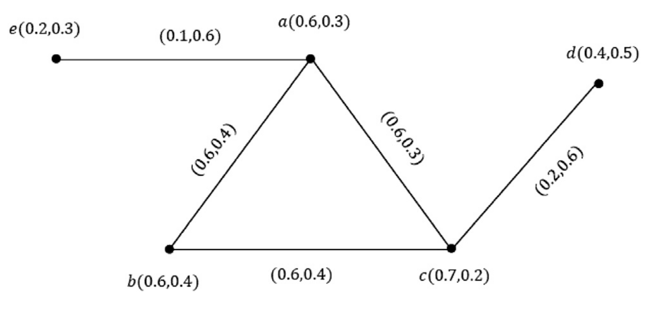

FIGURE 2 | Equitable isolated node.

TABLE 3 | Vague set B in $V \times V$.

B	ab	bd	ac	cd	bf	ce
t_B	0.2	0.3	0.2	0.3	0.3	0.1
f_B	0.5	0.5	0.6	0.6	0.6	0.6

$$|S| = \left|\sum_{a \in S} \frac{1 + t_A(a) - f_A(a)}{2}\right|.$$

All the basic notations are shown in **Table 1**.

3 DOMINATION IN VGS

Definition 3.1. Let $\zeta = (A, B)$be a VG. The equitable neighborhood (EN) of a node $a \in V$, described by $EN(a)$ and defined as $EN(a) = (EN^t(a), EN^f(a))$, where $EN^t(a) = \{b \in V | b \in N(a), |\deg^t(b) - \deg^t(a)| \le 1\}$, $t_B(ab) = \min\{t_A(b), t_A(a)\}$, $EN^f(a) = \{b \in V | b \in N(a), |\deg^f(b) - \deg^f(a)| \ge 1\}$, $f_B(ab) = \max\{f_A(b), f_A(a)\}$.

Definition 3.2. The END of a node $a \in V$, denoted by $\deg_{EN}(a)$, is defined as $\deg_{EN}(a) = (\deg^t_{EN}(a), \deg^f_{EN}(a))$, where $\deg^t_{EN}(a) = \sum_{b \in EN(a)} t_B(ab)$and $\deg^f_{EN}(a) = \sum_{b \in EN(a)} f_B(ab)$. The minimum END, denoted by $\delta_{EN}(\zeta)$, is defined as $\delta_{EN}(\zeta) = (\delta^t_{EN}(\zeta), \delta^f_{EN}(\zeta))$, where $\delta^t_{EN}(\zeta) = \min\{\deg^t_{EN}(a) | a \in V\}$ and $\delta^f_{EN}(\zeta) = \min\{\deg^f_{EN}(a) | a \in V\}$.The maximum END, denoted by $\Delta_{EN}(\zeta)$, is defined as $\Delta_{EN}(\zeta) = (\Delta^t_{EN}(\zeta), \Delta^f_{EN}(\zeta))$, where $\Delta^t_{EN}(\zeta) = \max\{\deg^t_{EN}(a) | a \in V\}$ and $\Delta^f_{EN}(\zeta) = \max\{\deg^f_{EN}(a) | a \in V\}$.

Example 3.3. Let $\zeta = (A, B)$ be a VG on $V = \{a, b, c, d, e, f\}$ so that $A = (t_A, f_A)$ is a vague subset of V, in **Table 2**, and $B = (t_B, f_B)$ is a vague subset of $V \times V$ defined in **Table 3**. The VG is shown in **Figure 1**. By simple calculation, we have $EN(a) = \{b\}$, $EN(b) = \{a, f, d\}$, $EN(c) = \{e\}$, $EN(d) = \{b\}$, $EN(e) = \{c\}$, and $EN(f) = \{b\}$ .The ENDs of nodes are calculated as $\deg_{EN}(a) = (0.2, 0.5)$, $\deg_{EN}(b) = (0.8, 1.6)$, $\deg_{EN}(c) = (0.1, 0.6)$, $\deg_{EN}(d) = (0.3, 0.5)$, $\deg_{EN}(e) = (0.1, 0.6)$, and $\deg_{EN}(f) = (0.3, 0.6)$. The minimum END of VG ζ is $\delta_{EN}(\zeta) = (0.1, 0.5)$ and the maximum END of a VG ζ is $\Delta_{EN}(\zeta) = (0.8, 1.6)$.

Definition 3.4. Let $\zeta = (A, B)$ be a VG. A node $b \in V$ is called an EIN in ζ if, for each $a \in V$, $|\deg^t(a) - \deg^t(b)| > 1$, $t_B(ab) < \min\{t_A(a), t_A(b)\}$, and $|\deg^f(a) - \deg^f(b)| < 1$, $f_B(ab) > \max\{f_A(a), f_A(b)\}$, i.e., $EN(b) = \varnothing$.

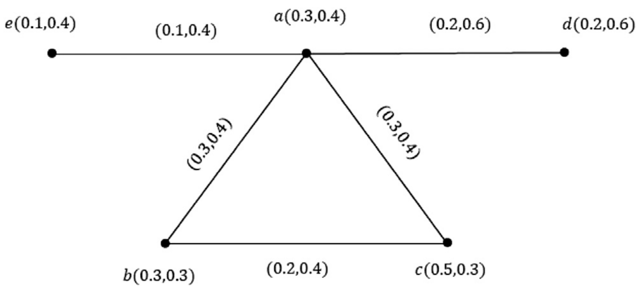

FIGURE 3 | Equitable dominating set of ζ.

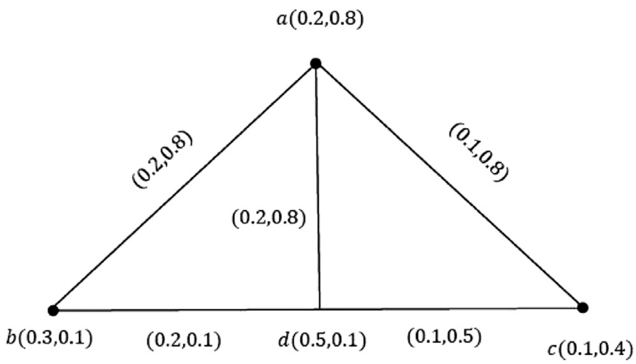

FIGURE 4 | Degree equitable VG.

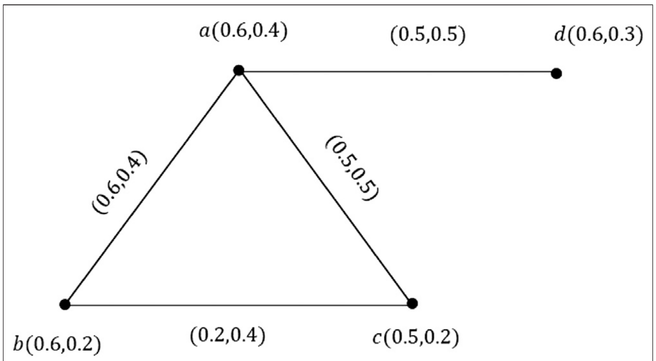

FIGURE 5 | EIS of ζ.

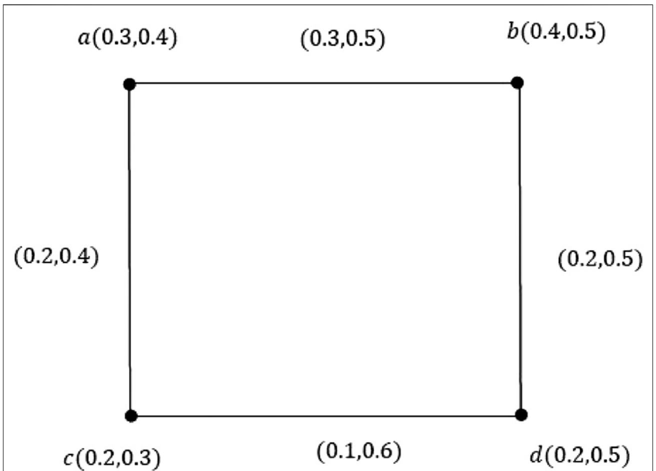

FIGURE 6 | Strong EDS of ζ.

Example 3.5. Consider a VG $\zeta = (A, B)$ on $V = \{a, b, c, d, e\}$ which is shown in **Figure 2**. From **Figure 2**, we have $\deg(a) = (1.3, 1.3)$, $\deg(b) = (1.2, 0.8)$, $\deg(c) = (1.4, 1.3)$, $\deg(d) = (0.2, 0.6)$, and $\deg(e) = (0.1, 0.6)$. Since $|\deg^t(a) - \deg^t(e)| > 1$, $t_B(ae) < \min\{t_A(a), t_A(e)\}$, and $|\deg^f(a) - \deg^f(e)| < 1$, $f_B(ae) > \max\{f_A(a), f_A(e)\}$, i.e., $EN(e) = \varnothing$. Also, $|\deg^t(c) - \deg^t(d)| > 1$, $t_B(cd) < \min\{t_A(c), t_A(d)\}$, and $|\deg^f(c) - \deg^f(d)| < 1$, $f_B(cd) > \max\{f_A(c), f_A(d)\}$, i.e., $EN(d) = \varnothing$. Hence, e and d are isolated nodes in ζ.

Definition 3.6. Let ζ be a VG. A subset $S \subseteq V$ is called an EDS of ζ if for each node $b \in V - S$, ∃ a node $a \in S$ so that $ab \in E$, $|\deg^t(a) - \deg^t(b)| \le 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$, and $|\deg^f(a) - \deg^f(b)| \ge 1$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$. The EDN of ζ, denoted by $\gamma_e(\zeta)$, is defined as the minimum cardinality of an EDS of S.

Definition 3.7. An EDS S of a VG ζ is called a MI-EDS of ζ if for each node $b \in S$, the set $S - \{b\}$ is not an EDS; i.e., no proper subset of S is an EDS of ζ.

Example 3.8. Consider a VG $\zeta = (A, B)$, as shown in **Figure 3**. It is easy to show that the MI-EDS of VG ζ is $S = \{a\}$. The EDN of ζ is $\gamma_e(\zeta) = 0.45$.

Definition 3.9. Let $\zeta = (A, B)$ be a VG. ζ is called a DEVG if, for each $b \in V$, ∃ a node $a \in V$ so that $ab \in supp(B)$, $|\deg^t(a) - \deg^t(b)| \le 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$, and $|\deg^f(a) - \deg^f(b)| \ge 1$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$.

Example 3.10. Consider a VG ζ, as shown in **Figure 4**. Simple calculations show that ζ is a degree equitable VG.

Definition 3.11. A subset $I \subseteq V$ is called an EIS of a VG $\zeta = (A, B)$ if $|\deg^t(a) - \deg^t(b)| > 1$, $t_B(ab) < \min\{t_A(a), t_A(b)\}$, and $|\deg^f(a) - \deg^f(b)| < 1$, $f_B(ab) > \max\{f_A(a), f_A(b)\}$, for all $a, b \in I$. The EIN of ζ, denoted by $\gamma_{ie}(\zeta)$, is defined as the minimum cardinality of an EIS of ζ.

Definition 3.12. An EIS I is called a MA-EIS of ζ if, for each node $a \in V - I$, the set $I \cup \{a\}$ is not an EIS.

Example 3.13. Consider a VG $\zeta = (A, B)$ given in **Figure 5**. It is clear that $S = \{a, d\}$ is maximal EIS. The EIN is $\gamma_{ie}(\zeta) = 1.25$.

Definition 3.14. Let $\zeta = (A, B)$ be a VG. For any two nodes $a, b \in V$, a strongly dominates b in ζ if $t_B(ab) = \min\{t_A(a), t_A(b)\}$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$, and $deg^t_\zeta(a) \ge deg^t_\zeta(b)$, $deg^f_\zeta(a) \le deg^f_\zeta(b)$. Similarly, a weakly dominates b if $t_B(ab) = \min\{t_A(a), t_A(b)\}$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$, $deg^t_\zeta(b) \ge deg^t_\zeta(a)$, and $deg^f_\zeta(b) \le deg^f_\zeta(a)$.

Definition 3.15. An EDS $S \subseteq V$ is called a weak (strong) EDS of ζ if, for each node $b \in V - S$, ∃ at least one node $a \in S$ so that a weakly (strongly) dominates b. The weak (strong) EDN of ζ,

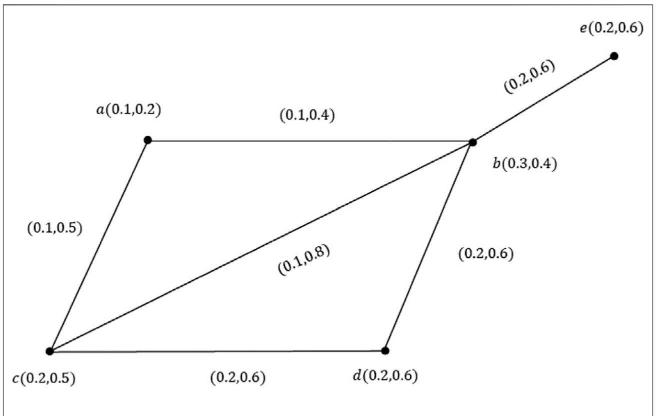

FIGURE 7 | TEDS of ζ.

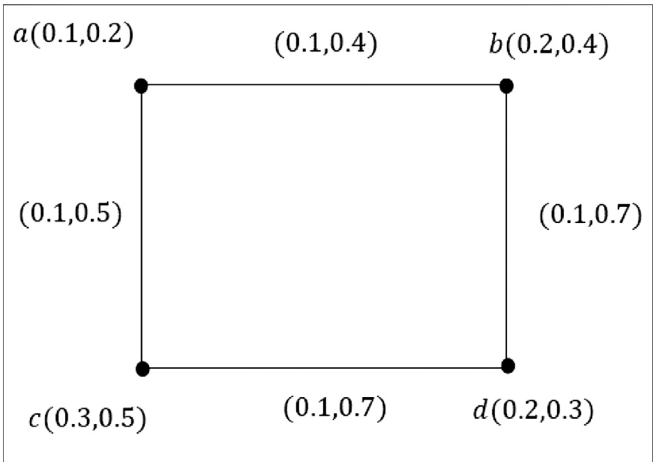

FIGURE 8 | VG ξ.

denoted by $\gamma_{we}(\zeta)$ $(\gamma_{se}(\zeta))$, is called as the minimum cardinality of a weak (strong) EDS of ζ.

Example 3.16. Consider the VG $\zeta = (A, B)$ given in **Figure 6**. It is easy to see that the strong EDSs of ζ are $S_1 = \{a, b\}$ and $S_2 = \{a, d\}$. The strong EDN of ζ is $\gamma_{se}(\zeta) = 0.8$.

Theorem 3.17. Let $\zeta = (A, B)$ be a VG. An EDS S of ζ is a minimal EDS if and only if, for every $b \in S$, one of the following conditions holds: (i) b is an isolated node in S, (ii) $EN(b) \cap (V - S) \neq \varnothing$, or (iii) $\exists$ a node $a \in V - S$ so that $EN(a) \cap S = \{b\}$.

Proof. Let ζ be a VG with minimal EDS S; then, for each node $b \in S$, the set $S' = S - \{b\}$ is not an EDS. Hence, ∃ at least one node $a \in V - S'$ so that a is not dominated by any node in S'. So, we have two cases.

If $a = b$, then b is an isolated node in S; i.e., b is not neighbor to any node $c \in S$ so that $t_B(bc) = 0 = f_B(bc)$. Thus, $EN(b) \cap (V - S) \neq \varnothing$; that is, every node in S has a neighbor in $V - S$.

If $a \neq b$, i.e., $a \in V - S$, then a is dominated by some node of S but not dominated by any node in S'. Hence, a is neighbor only to one node $b \in S$, so $EN(a) \cap S = \{b\}$.

Conversely, suppose that S is an EDS of a VG ζ and, for every node $b \in S$, one of the given conditions holds. Assume that S is not a MI-EDS, then clearly ∃ a node $b \in S$ so that $S - \{b\}$ is an EDS of ζ. Therefore, b is neighbor to at least one node of set $S - \{b\}$; i.e., b is not an isolated node in S, and thus condition (i) is false. In addition, if we get $S' = S - \{b\}$ as an EDS of ζ, then each node of $V - S'$ is neighbor to at least one node in S'. Hence, conditions (ii) and (iii) are also false which is a contradiction. ∎

Theorem 3.18. Let $\zeta = (A, B)$ be a VG with order $o(\zeta)$, then: (i) $\gamma_e(\zeta) \leq \gamma_{se}(\zeta) \leq o(G^*) - \Delta^t_{EN}(\zeta)$, (ii) $\gamma_e(\zeta) \leq \gamma_{we}(\zeta) \leq o(G^*) - \delta^t_{EN}(\zeta)$.

Proof. According to definition, every weak (strong) EDS of a VG ζ is an EDS of ζ, $\gamma_e(\zeta) \leq \gamma_{se}(\zeta)$ and $\gamma_e(\zeta) \leq \gamma_{we}(\zeta)$. Let a and b be two arbitrary nodes of ζ. If $\deg^t_{EN}(b) = \Delta^t_{EN}(\zeta)$ and $\deg^t_{EN}(a) = \delta^t_{EN}(\zeta)$, then $V - EN(b)$ is a strong EDS of ζ and $V - EN(a)$ is a weak EDS of ζ. Hence, $\gamma_{se}(\zeta) \leq |V - EN(b)|$ and $\gamma^t_{we}(\zeta) \leq |V - EN(a)|$, i.e.,

$$\gamma_{se}(\zeta) \leq o(G^*) - \Delta^t_{EN}(\zeta)$$

and

$$\gamma_{we}(\zeta) \leq o(G^*) - \delta^t_{EN}(\zeta).$$

∎

Theorem 3.19. Let ζ be a VG without single nodes and S be a MI-EDS of ζ; then $V - S$is an EDS of ζ.

Proof. Let ζ be a VG with MI-EDS S; then, for each node $b \in S$, there is at least one node $a \in V - S$ so that $|\deg^t(a) - \deg^t(b)| \leq 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$ and $|\deg^f(a) - \deg^f(b)| \geq 1$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$. Hence, $V - S$ dominates each element of S. So, $V - S$ is an EDS of ζ. ∎

Theorem 3.20. Let ζ be a VG with EIDS I; then I is both a MI-EDS and a MA-EIS of ζ. Conversely, any MA-EIS I of a VG ζ is an EIDS of ζ.

Proof. Let ζ be a VG with EIDS K; then, for each node $b \in V - K$, the set $K \cup \{b\}$ is not an EIS and the set $K - \{b\}$is not an EDS of ζ. So, K is both a MI-EDS and a MA-EIS of ζ.Conversely, assume that K is a MA-EIS of ζ; then, for each node $b \in V - K$, the set $K \cup \{b\}$ is not an EIS of ζ. Hence, the set K dominates each node $a \in V - K$ and so K is an EDS of ζ. Therefore, K is an EIDS of ζ. ∎

Theorem 3.21. A subset $I \subseteq V$is an EIS and EDS of a VG ζ if and only if I is a MA-EIS of ζ.

Proof. Assume that K is both an EDS and an EIS of a VG ζ. Suppose that K is not a MA-EIS of ζ, then clearly there exists a node $a \in V - K$ so that $K \cup \{a\}$ is an EIS; namely, a is not dominated by any node $b \in K$ that shows K is not an EDS of ζ, a contradiction, so K is a MA-EIS of ζ. Conversely, let K be a MA-EIS of ζ; then, for each node $a \in V - K$, the set $K \cup \{a\}$ is not an EIS of ζ. Hence, the set K dominates each node $a \in V - K$; that is, K is an EDS of ζ. So, K is both an EDS and an EIS of ζ. ∎

Definition 3.22. A total-EDS (TEDS) of a VG $\zeta = (A, B)$ is a subset $S \subseteq V$if for each node $b \in V$, ∃ at least one node $a \in S$ so that $ab \in E(\zeta)$, $|\deg^t(a) - \deg^t(b)| \leq 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$, and $|\deg^f(a) - \deg^f(b)| \geq 1$, $f_B(ab) = \max\{f_A(a), f_A(b)\}$. The TEDN of ζ, denoted by $\gamma_{te}(\zeta)$, is defined as the minimum cardinality of a TEDN S.

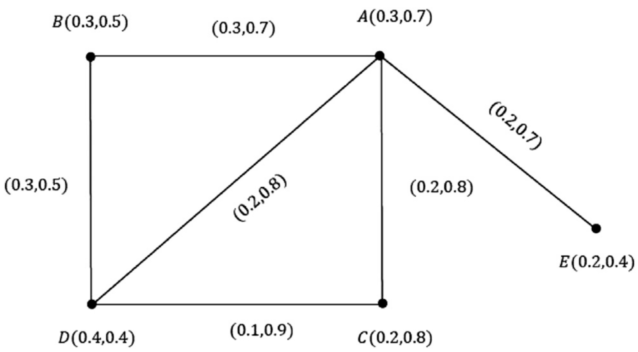

FIGURE 9 | VG ζ.

Definition 3.23. A TEDS S of a VG ζ is called a minimal TEDS if, for each node $b \in S$, the set $S - \{b\}$is not a TEDS; i.e., no proper subset of S is a TEDS of ζ.

Example 3.24. Consider a VG $\zeta = (A, B)$, as shown in **Figure 7**. It is clear that $\{a, b\}$ and $\{b, c\}$ are TEDSs of ζ.

Theorem 3.25. Let ζ be a VG with no isolated nodes; then $\gamma_t(\zeta) \le \gamma_{te}(\zeta)$.

Proof. As each TEDS of a VG ζ is a total dominating set, so $\gamma_t(\zeta) \le \gamma_{te}(\zeta)$. ∎

Definition 3.26. Let $\zeta = (A, B)$ be a VG. A subset $S \subseteq V$ is called a PDS of ζ if, for each node $b \in V - S$, there exists exactly one vertex $a \in S$so that a dominates b.

Definition 3.27. A PDS S of a VG ζ is called a minimal PDS if, for each $a \in S$, the set $S - \{a\}$ is not a PDS in ζ. The minimum cardinality between all MI-PDSs is called the PDN of ζ and it is denoted by $\gamma_{pif}(\zeta)$ or simply γ_{pif}.

Example 3.28. Consider a VG $\zeta = (A, B)$ given in **Figure 8**. By simple computation, it is clear that $S = \{a, d\}$is a MI-PDS. The PDN of ζ is $\gamma_{pif} = 0.9$.

4 THE APPLICATION OF VDS IN MEDICAL SCIENCES

In the past, many emergency patients died due to the delays in transportation to the hospital, but today the number has dropped dramatically. Traffic problems in cities are one of the factors influencing this delay. In addition, the specialization of hospitals has meant that each patient must be transferred to the relevant hospital based on the main complaint, even though this specialized hospital is further away than other available hospitals. Therefore, in this study, we have tried to identify the nearest hospital based on distance, traffic load, and patient complaints. For this purpose, we consider four hospitals located in one city. We show hospitals as B, C, D, and E. In this vague graph, one vertex represents the patient's home and other vertices are related to the hospitals in the city. The edges indicate the accumulation of cars in the city (See **Figure 9**).

The node $B(0.3, 0.5)$ means that it has 30% of the necessary facilities for treating the patient and unfortunately lacks 50% of the necessary equipment.

The edge AB shows that only 30% of the patient's transport route to the hospital by ambulance has a low traffic load, and unfortunately 70% of the route between these two points has a heavy traffic load during most hours of the day. The EDSs for **Figure 8** are as follows:

$$
\begin{aligned}
S_1 &= \{A, B\},\\
S_2 &= \{A, D\},\\
S_3 &= \{A, B, C\},\\
S_4 &= \{A, B, D\},\\
S_5 &= \{A, B, E\},\\
S_6 &= \{A, C, D\},\\
S_7 &= \{A, D, E\},\\
S_8 &= \{B, C, E\},\\
S_9 &= \{C, D, E\},\\
S_{10} &= \{A, B, D, E\},\\
S_{11} &= \{A, C, D, E\},\\
S_{12} &= \{B, C, D, E\},\\
S_{13} &= \{A, B, C, E\},\\
S_{14} &= \{A, B, C, D\}.
\end{aligned}
$$

After calculating the cardinality of $S_1, \cdots, S_{14}$, we obtain

$$
\begin{aligned}
|S_1| &= 0.7,\\
|S_2| &= 0.8,\\
|S_3| &= 0.9,\\
|S_4| &= 1.2,\\
|S_5| &= 1.1,\\
|S_6| &= 1,\\
|S_7| &= 1.3,\\
|S_8| &= 1,\\
|S_9| &= 1.1,\\
|S_{10}| &= 1.6,\\
|S_{11}| &= 1.4,\\
|S_{12}| &= 1.5,\\
|S_{13}| &= 1.3,\\
|S_{14}| &= 1.4.
\end{aligned}
$$

It is obvious that S_1 has the smallest size between other DSs; hence, we conclude that it can be the best choice because first there is more free space for the ambulance from the patient's home to hospital B, so that it can get the patient to the desired location faster, saving time and money. Second, hospital B has more medical services compared to other hospitals. So, the government should invest more on widening roads and controlling traffic between cities so that ambulances can transport patients to the relevant specialized hospitals faster.

5 CONCLUSION

Considering the precision, elasticity, and compatibility in a system, vague models outweigh the other FGs. The VG concept generally has a large variety of applications in different areas such as computer science, operation research, topology, and natural networks. Domination in graph theory has a wide range of applications in several fields such as facility location problems, school bus routing, and coding theory. Therefore, in this research, we described several concepts of dominating sets, ED, TED, weak (strong) ED, EISs, and PDS, in VGs and also studied their properties incorporating some basic examples. Finally, we introduced an application of domination in the transportation system. Future research will hold the investigation of new concepts of vague planer graphs, vague bridges, vague cycles, and vague competition graphs and represent their applications in medical sciences and social networks.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors have contributed equally to this work. All authors have read and agreed to the possible publication of the manuscript.

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