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Investigating Inventory Strategy Under Demand Distribution Fluctuation in Dual-Channel Supply Chain

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When manufacturers construct a dual-channel distribution system, which includes online and offline sales channels, they need to solve the inventory management problem to ensure supply and reduce inventory costs of the supply chain system. The dual-channel supply chain is the research object, and the inventory decision model is designed to achieve optimal profit when market demand is divided into online and offline demands. The results of the numerical analysis and simulations, conducted using MATLAB, indicate that both the manufacturer and the retailer increase their inventories and that their profits decrease when demand uncertainty increases. Besides, the increase in the online demand ratio causes the increase in the manufacturer's inventory and reduces the profits of the retailer and the entire supply chain.

Keywords: dual channel, inventory optimization, demand distribution, supply chain, online demand ratio

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INTRODUCTION

Rapid development in electronic commerce, based on advanced Internet technologies and logistics systems, has led physical economy enterprises to increasingly develop online sales channels when they develop offline sales channels. This challenges the traditional business relationship between manufacturers and retailers, adding competition to the cooperation relationship between the upstream and the downstream in the supply chain. In other words, it creates a channel conflict problem [1]. The supply chain operation strategy and coordination mechanism under the dual-channel structure have been studied by many scholars. R. H. Niu et al. consider joint pricing and inventory and production decision problems for members in a monopoly two-stage dual-channel retailer supply chain. For a dual-channel retailer, pricing in one channel will affect the demand in the other channel. This subsequently affects the retailer's replenishment ordering decisions, which has an impact on the producer's inventory and production plans and the wholesale price decisions. They analyze joint pricing, inventory, and production problems under three scenarios by incorporating intra-product line price interaction in the economic order quantity model and present a unique equilibrium under certain realistic conditions [2]. Chiang studies the inventory competition and coordination of the dual-channel supply chain in the case of permit stockout. The author designs a coordination contract of revenue sharing and cost sharing of inventory holding to achieve the coordination of the dual-channel supply chain [3]. Cai et al. [4] study the impact of price discount contracts on dual-channel profit under consistent or inconsistent prices and find that the dual-channel pricing agreement or the implementation of price discount contracts can ease the channel conflict. Xu et al. [5] use the mean–variance method to measure risk in the risk-averse dual-channel supply chain. They design the revenue sharing contract of both sides to realize the coordination of the dual-channel supply chain. Mahdi Shafieezadeh et al. propose an integrated inventory management model within a multi-item multi-echelon supply chain. Three inventory models are developed because of different layers of the supply chain in an integrated manner to optimize the total cost of the entire supply chain. They design an evolution algorithm to cope with the computational complexity caused by the nonlinear structure of the proposed model [6]. Yan et al. calculate the maximum profit of the supply chain system under centralized decision-making and evaluate the impact of both sales service and retailer loyalty on the member enterprises' decision and profit. In the situation of decentralized decision-making, they analyze each member's performance level and profit behavior under four different cases, namely, wholesale price, buyback, revenue sharing, and advanced purchase discount contract. Numerical results reveal that the advanced purchase discount contract can maximize the integrated profit of the supply chain system because it can fairly diversify the inventory risk between the manufacturer and the retailer [7]. The articles [8, 9] study the problems of pricing and coordination of dual channel. Rhee and Park [10] discuss the relationship between service level and channel selection. Dumrongsiri et al. [11] highlight that product price and service are important factors in attracting consumers to dual channel. However, the logistics conflict that exists in the dual channel has been studied rarely. Furthermore, logistics conflict in the dual-channel supply chain focuses on the inventory problem of online sales and traditional retail. Because the demand uncertainty and channel selection have an impact on the inventory and profits of manufacturers and retailers, how to adjust their inventory levels is especially important for them. This study examines the inventory optimization strategy of the dual-channel supply chain and reveals the change in trends of inventory levels and profits of the dual-channel supply chain and its node enterprises.

The research on inventory control of the dual-channel supply chain has achieved several research results. Zhao and Cao provide an insight into the impact of two-cycle inventory strategies of the dual-channel supply chain on pricing and profit. Their research indicates that the commodity price of the network retailer with zero stock is lower than that of the retailer with a certain stock and that the price gap decreases with rapid market expansion over time [12]. Chiang and Monahan study a hybrid dual channel comprising a retailer with a traditional retail channel and a manufacturer with an online channel. The manufacturer and the retailer manage their inventory separately. When customer demand cannot be satisfied by the stock in one channel, the customers will turn to another channel in a certain proportion. Using a numerical example, they find that the mixed structure of dual channel is more dominant than the single-channel structure [13]. Boyaci studies the distribution system in dual channel, assuming the products as substitutes and price as an exogenous variable. He analyzes the decision-making problem on the order quantity of the dual channel and the impact of the product substitution rate on channel efficiency [14]. Geng and Mallik discuss the inventory competition and distribution problems of the dual channel and separately design the game equilibrium on the limited and unlimited production capacity of the manufacturer. They find that the manufacturer is likely to reject the retailer's orders even if the production capacity is not limited [15]. Xu et al. [16] construct a two-tier dual-channel supply chain model considering the manufacturer's shortage cost and present a coordination mechanism for decentralized inventory management in which the optimal system can be obtained only when a small amount of global information can be shared between the manufacturer and the retailer. Wang et al. study the decision-making problem on the optimal pricing of the network channel in the dual-channel supply chain and the optimal inventory of the hybrid channel in the case of the market demand being random and affected by the price. Their research indicates that the existence of the dual-channel model improves the expected returns of retailers, manufacturers, and the whole supply chain when the change in market demand distribution is within a reasonable range in the two channels [17].

The existing research is based on the assumption that the inventory is under centralized management. This is not entirely consistent with reality where both manufacturers and retailers often manage their own inventories, and the impact of the change in the demand ratio from online and offline channels on the inventory and profit of the supply chain is not a concern. Therefore, our study begins by analyzing the inventory and profit of the manufacturer and the retailer based on the existing literature, constructs the inventory model under the dual-channel environment, investigates the dual-channel optimal inventory decision-making under decentralized management, and analyzes—through numerical experiments—the impacts of the fluctuation in demand distribution on the optimal inventory levels and profits of the manufacturer and the retailer, respectively.

CONSTRUCTION OF THE INVENTORY MODEL OF THE DUAL-CHANNEL SUPPLY CHAIN

Inventory Operation Model of the Dual-Channel Supply Chain

We assume that there are two types of channels in the dual-channel supply chain system, namely, the retailer's traditional product sales channel and the manufacturer's online sales channel. Under this assumption, the consumers consider not only the price of the product but also other factors, such as the purchase convenience and time and travel costs. Accordingly, there are two different channel choices—offline channel and online channel; the consumer demands for which are assumed to be mutually exclusive random variables. Both the manufacturer and retailer keep inventories. The manufacturer's inventory is used to meet the needs of the online sales channel's consumers, and the retailer's inventory is used to meet the needs of the retailer's consumers. Demand information come from retail channel and online channel and flow to the upstream and finally to the manufacturer. The operational model is illustrated in **Figure 1**.

Construction of the Model

Because this study focuses on the inventory strategy of manufacturers and retailers, the model treats the wholesale

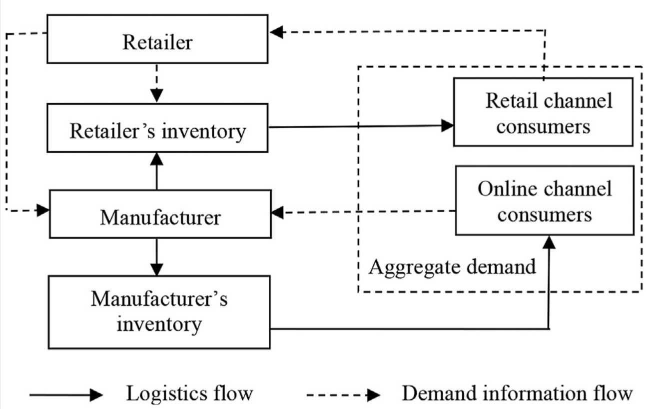

FIGURE 1 | Schematic diagram of inventory operation.

and retail prices of the product as exogenous variables. The manufacturer and the retailer decide the optimal inventory strategy after comprehensively considering revenues and costs. A manufacturer's revenue includes sales revenue from the online channel, wholesale revenue from retailers, and residual value of the inventory. The manufacturer's cost includes the production cost, stockholding cost, distribution cost of products sold through the online channel, and shortage cost of the online sales channel. A retailer's revenue includes sales revenue from the retail channel and the residual value of inventory. The retailer's cost includes the retailer's ordering cost, stockholding cost, and shortage cost of the retail channel. The symbols used in this model and their meanings are presented below:

δ is the online demand ratio and represents the ratio of consumers shopping online to the total number of consumers.
D_r, D_d, and D represent the retail channel, online channel, and aggregate demands, respectively. Furthermore, $D_d = \delta D$, $D_r = (1-\delta)D$.
R_{π_m}, R_{π_r}, and R_π represent the decrease in rates of the optimal profits of the retailer, the manufacturer, and the total supply chain, respectively.
$g(g)$ and $f(g)$ represent distribution densities of the retail and online channel demands, respectively.
$G(g)$ and $F(g)$ represent the cumulative distribution function of the retail and online channel demands, respectively.
p_r and p_d represent product prices of the retailer's traditional retail channel and the manufacturer's online channel, respectively. Both of these are exogenous variables.
w is the wholesale price of the product from the manufacturer.
c is the production cost per unit product.
α is the residual value coefficient of the unit surplus inventory of the retailers, αp_r is the residual value of the unit surplus inventory of the retailers, and $\alpha p_r < c < w < p_d < p_r$.
β is the residual value coefficient of the unit surplus inventory of the manufacturer, βp_d is the residual value of the unit surplus inventory of the manufacturer, and $\beta p_d < c < w < p_d < p_r$.
l_r and l_d represent shortage costs per unit of the product from the retail and online channel, respectively.
t_d is the distribution cost per unit of the product from the manufacturers' online channel.
Q_r, Q_d, and Q represent the inventory levels of the retailer, the manufacturer, and the total supply chain, respectively; $Q = Q_d + Q_r$.
a_r, and a_d are the fixed stockholding cost of the retailer and manufacturer, respectively.
b_r and b_d are the variable stockholding costs of the retailer and manufacturer, respectively, and these change with the inventory level.

It is assumed that a linear relationship exists between the stockholding cost and the inventory level in the model.

λ_r and λ_d are the fixed operating costs of the retailer and manufacturer, respectively.
μ_r and μ_d are the variable operating cost of the retailer and manufacturer, respectively, and these change with the demand level.
π_r, π_m, and π are the profit of the retailer, manufacturer, and supply chain, respectively.

The decision goal of the retailer and the manufacturer is to maximize their respective profits. The decision variables of the model include Q_r and Q_d. The determination of optimal inventory is closely related to the profit. The expression formula of the retailer's profit is as follows:

$$\begin{aligned}\pi_r = {} & \min[Q_r, D_r]\times p_r + \max[Q_r - D_r, 0]\times \alpha p_r \\ & - \max[D_r - Q_r, 0]\times l_r - Q_r\times w - (a_r + b_r\times Q_r) \\ & - (\lambda_r + \mu_r D_r)\end{aligned} \tag{1}$$

In the equation, $\min[Q_r, D_r]\times p_r$ represents the product sales revenue of the retailer, $\max[Q_r - D_r, 0]\times \alpha p_r$ represents the residual value of the surplus inventory, $\max[D_r - Q_r, 0]\times l_r$ represents the shortage cost of the retailers, $Q_r \times w$ represents the retailer's purchasing cost, $a_r + b_r Q_r$ represents the retailer's stockholding cost, and $\lambda_r + \mu_r D_r$ represents the operation cost of the retailer, including the order processing cost, after-sales service, and administrative expense.

Equation 1 illustrates the structure of the revenue and the cost of the retailer.

If D_r is a continuous random variable and $G(g)$ is the cumulative distribution function of D_r, the profit of the retailer can be described as the following formula:

$$\begin{aligned}\pi_r(Q_r) = {} & \int_0^{Q_r} [p_r g + \alpha p_r (Q_r - g) - wQ_r - (a_r + b_r Q_r) \\ & - (\lambda_r + \mu_r g)]dG(g) + \int_{Q_r}^{\infty} [Q_r p_r - l_r (g - Q_r) - wQ_r \\ & - (a_r + b_r Q_r) - (\lambda_r + \mu_r Q_r)]dG(g).\end{aligned} \tag{2}$$

The first integral part of **Eq. 2** indicates the revenue of the retailer when the market demand from the retail channel is less than or equal to the inventory level of the retailer $(D_r \le Q_r)$. The second integral part of **Eq. 2** indicates the revenue of the retailer when the market demand from the retail channel is greater than the inventory level of the retailer $(D_r > Q_r)$.

In the same way, the formula of the manufacturer's profit can be expressed as follows:

$$\pi_m = \min[Q_d, D_d] \times p_d + \max[Q_d - D_d, 0] \times \beta p_d + Q_r \times w - \max[D_d - Q_d, 0] \times l_d - \min[Q_d, D_d] \times t_d - (Q_d + Q_r) \times c - (a_d + b_d Q_d) - (\lambda_d + \mu_d D_d). \quad (3)$$

In the equation, $\min[Q_d + D_d] \times p_d$ represents the manufacturer's online product sales revenue, $\max[Q_d - D_d, 0] \times \beta p_d$ represents the residual value of the surplus inventory of the manufacturer, $\max[D_d - Q_d, 0] \times l_d$ represents the manufacturer' s shortage cost, $\min[Q_d, D_d] \times t_d$ represents the manufacturer's distribution cost, $Q_r \times w$ represents the manufacturer's sales revenue from the retailer, $(Q_d + Q_r) \times c$ represents the production cost, $a_d + b_d Q_d$ represents the manufacturer's inventory cost, and $\lambda_d + \mu_d D_d$ represents the manufacturer's operation cost, including the order processing cost, after-sales service, and administrative expenses.

Eq. 3 illustrates the structure of the revenue and the cost of the manufacturer. Similarly, if D_d is a continuous random variable and $F(g)$ is the cumulative distribution function of D_d, the profit of the manufacturer can be described as the following formula:

$$\pi_m(Q_d) = \int_0^{Q_d} [p_d g + \beta p_d (Q_d - g) - t_d g + wQ_r - c(Q_d + Q_r) - (a_d + b_d Q_d) - (\lambda_d + \mu_d g)]dF(g) + \int_{Q_d}^{\infty} [Q_d p_d - l_d(g - Q_d) - Q_d t_d + wQ_r - c(Q_d + Q_r) - (a_d + b_d Q_d) - (\lambda_d + \mu_d Q_d)]dF(g). \quad (4)$$

The first integral part of the **Eq. 4** indicates the revenue of the manufacturer when the market demand from the online channel is less than or equal to the inventory level of the manufacturers $(D_d \le Q_d)$. The second integral part of **Eq. 4** indicates the revenue of the manufacturers when the market demand from the online channel is greater than the inventory level of the manufacturer $(D_d > Q_d)$.

Suppose Q_r^* and Q_d^* are the optimal inventory levels of the retailer and manufacturer, respectively. Then, the optimal inventory strategy of the model is as follows:

$$G(Q_r^*) = \frac{p_r + l_r - w - b_r - \mu_r}{(1-\alpha)p_r + l_r - \mu_r}, \quad (5)$$

$$F(Q_d^*) = \frac{p_d + l_d - t_d - c - b_d - \mu_d}{(1-\beta)p_d + l_d - t_d - \mu_d}, \quad (6)$$

Proof: According to **Eqs 2** and **4**, it is known that the optimal inventory of the retailer and manufacturer should satisfy the condition that the first derivative is zero:

TABLE 1 | Assignment of the model parameters.

p_d	p_r	l_d	l_r	c	w	α	β	t_d	a_r,a_d	b_r,b_d	λ_r,λ_d	μ_r,μ_d
13	17	2	2.5	4.5	6.5	0.1	0.13	1.5	0	0.75	0	0.85

$$A = \int_0^{Q_r} [p_r x + \alpha p_r (Q_r - x) - wQ_r - (a_r + b_r Q_r) - (\lambda_r + \mu_r x)]dG(x)$$
$$= \int_0^{Q_r} [(p_r - \alpha p_r - \mu_r)x + (\alpha p_r - w - b_r)Q_r - (a_r + \lambda_r)]dG(x)$$
$$= [(1-\alpha)p_r - \mu_r] \int_0^{Q_r} xdG(x) + [(\alpha p_r - w - b_r)Q_r - (a_r + \lambda_r)] \int_0^{Q_r} dG(x)$$
$$= [(1-\alpha)p_r - \mu_r] \int_0^{Q_r} xg(x)dx + [(\alpha p_r - w - b_r)Q_r - (a_r + \lambda_r)]G(Q_r),$$

$$B = \int_{Q_r}^{\infty} [Q_r p_r - l_r(x - Q_r) - wQ_r - (a_r + b_r Q_r) - (\lambda_r + \mu_r Q_r)]dG(x)$$
$$= -l_r \int_{Q_r}^{\infty} xdG(x) + [(p_r + l_r - w - b_r - \mu_r)Q_r - (a_r + \lambda_r)] \int_{Q_r}^{\infty} dG(x)$$
$$= -l_r \left[\int_0^{\infty} xdG(x) - \int_0^{Q_r} xdG(x)\right] + [(p_r + l_r - w - b_r - \mu_r)Q_r - (a_r + \lambda_r)][1 - G(Q_r)]$$
$$= -l_r \int_0^{\infty} xg(x)d(x) + l_r \int_0^{Q_r} xg(x)d(x) + [(p_r + l_r - w - b_r - \mu_r)Q_r - (a_r + \lambda_r)][1 - G(Q_r)],$$

$$\pi_r(Q_r) = A + B$$
$$= [(1-\alpha)p_r + l_r - \mu_r] \int_0^{Q_r} xg(x)dx + [\mu_r - l_r - (1-\alpha)p_r]Q_r G(Q_r) + (p_r + l_r - w - b_r - \mu_r)Q_r - \left[(a_r + \lambda_r) + l_r \int_0^{\infty} xg(x)dx\right]$$

Because $(a_r + \lambda_r) + l_r \int_0^{\infty} xg(x)dx$ is a constant, and $[\int_0^{Q_r} xg(x)dx]' = xg(x)|0Q_r = Q_r g(Q_r)$,
therefore,

$$\frac{\partial \pi_r(Q_r)}{\partial Q_r} = [w - l_r - (1-\alpha)p_r]G(Q_r) + p_r + l_r - w - b_r - \mu_r,$$

$$\frac{\partial \pi_r(Q_r)}{\partial Q_r} = 0 \Rightarrow G(Q_r^*) = \frac{p_r + l_r - w - b_r - \mu_r}{(1-\alpha)p_r + l_r - \mu_r}.$$

Furthermore, $\frac{\partial^2 \pi_r(Q_r)}{\partial^2 Q_r} = [\mu_r - l_r - (1-\alpha)p_r] \frac{\partial G(Q_r)}{\partial Q_r} = [\mu_r - l_r - (1-\alpha)p_r]g(Q_r) = -[(1-\alpha)p_r + l_r - \mu_r]g(Q_r) < 0$,
Proving by the same method：

$$F(Q_d^*) = \frac{p_d + l_d - t_d - c - b_d - \mu_d}{(1-\beta)p_d + l_d - t_d - \mu_d},$$

Proof finished.

Equation 5 shows that the optimal inventory of the retail channel is affected by the retail channel price, shortage cost, purchase price, and residual value of the surplus inventory. **Equation 6** demonstrates that the optimal inventory of online channels is affected by the online channel price, shortage cost, distribution cost, residual value of the surplus inventory, variable inventory holding cost, and production cost.

NUMERICAL ANALYSIS

Because the expression of the optimal inventory is an implicit function, it becomes difficult to observe the impact of the changes in various parameters on the adjustment of the inventory strategy of the manufacturer and retailer. Thus,

TABLE 2 | Results of the calculation.

σ	Q_d^*	Q_r^*	Q^*	π_m	R_{π_m}	π_r	R_{π_r}	π	R_π
10	121.56	182.68	304.24	998.22	−0.0000	1566.79	−0.0000	2565.01	−0.0000
30	124.69	188.05	312.74	976.85	−0.0214	1492.26	−0.0476	2469.11	−0.0374
50	127.81	193.41	321.22	955.87	−0.0215	1418.62	−0.0493	2374.49	−0.0383
70	130.94	198.77	329.71	934.95	−0.0219	1345.11	−0.0518	2280.06	−0.0398
90	134.06	204.14	338.20	914.09	−0.0223	1271.73	−0.0546	2185.82	−0.0413
110	137.19	209.50	346.69	893.61	−0.0224	1199.26	−0.0570	2092.87	−0.0425

this section assigns values to the parameters of the model. The specific values are presented in **Table 1** to illustrate the fluctuation of the optimal inventory level and profit of the manufacturer and retailer under uncertain demand, and the impact of the change in the online demand ratio on the inventory strategy and profit of the manufacturer and retailer in the supply chain.

Impact of Demand Uncertainty on Inventory

The total demand of the dual channel follows the normal distribution with a mean value of 300. Assume the following conditions: $\delta = 0.4$, $D \sim N(300, \sigma^2)$, $D_d \sim N(120, 0.16\sigma^2)$, and $D_r \sim N(180, 0.36\sigma^2)$[18–20]. We examine the optimal inventory level of the manufacturer and retailer, as well as the corresponding overall profit of the manufacturer, the retailer, and the supply chain when the variance increases, that is, when demand uncertainty increases. The calculated results are presented in **Table 2**. This provides a quantitative conclusion for their inventory decision under the condition of the online demand ratio $\delta = 0.4$ also provides a quantitative solution to determine the inventory level under different online demand rates.

The results of the numerical analysis demonstrate the following:

(1) With the increase in demand uncertainty, the manufacturer and retailer tend to increase inventory, inventory changes are more obvious, and the manufacturer's inventory increases at the same speed as that of the retailer. It means that the manufacturer and retailer need to make an accurate forecast of the market demand when they decide their inventory levels. The retailer has to make great efforts on the collection and research of market demand information, as well as product sales because of the channel price gap. The manufacturer also needs to make an accurate forecast of the entire market demand, the cost of which is relatively high because of the logistics expense.

(2) As the demand uncertainty increases, the profits of the manufacturer, the retailer, and the entire supply chain decrease, which is aligned with reality. The increase in the inventory level will inevitably lead to an increase in inventory costs. The changes in decrease rates of the optimal profits R_{π_m} and R_{π_r} illustrate that the retailer is more affected by uncertainty than the manufacturer because the fixed and variable costs of the traditional channel are much higher than the manufacturer's cost.

(3) The online demand ratio $\delta = 0.4$, which leads to the manufacturer's profit, is smaller than the retailer's profit. Besides, their profit gap is due to the existence of the channel price gap and distribution cost.

Impact of Online Demand Ratio on Inventory

It is assumed that the total demand of the dual channel follows the normal distribution with a mean value of 300 and a standard deviation of 50, that is, $D \sim N(300, 50^2)$, $D_d \sim N(300\alpha, (50\alpha)^2)$, and $D_r \sim N(300(1-\alpha), [50(1-\alpha)]^2)$. When the online demand ratio δ changes in the interval (0, 1), the change trends of Q_d^*, Q_r^*, and Q^* are presented in **Figure 2**, while the change trends of π_m, π_r, and π are illustrated in **Figure 3**.

The following conclusions can be drawn from the numerical analysis:

(1) With the increase in the online demand ratio δ, the total inventory of the supply chain remains unchanged, although the inventory of the retailer initially reduces gradually and then follows a logarithmic reduction, finally reducing to zero. In contrast, the manufacturer's inventory grows at an increasing pace and eventually equals the total inventory of the entire supply chain.

(2) With the increase in the online demand ratio δ, the profit of the whole supply chain initially reduces gradually and then follows a logarithmic reduction. The profit of the retailer follows a similar path, although the pace of decline is slightly faster than that of the entire supply chain. In contrast, the manufacturer's profit initially increases gradually and then exponentially and eventually becomes equal to the profit of the entire supply chain.

(3) With the increase in the online demand ratio δ, the manufacturer's inventory curve does not completely coincide with the profit curve because the greater the δ, the higher is the cost of information acquisition for the manufacturer and the greater is the decline in the retailer's profit. The sum of these two items is bigger than the profit of the manufacturer. Thus, to improve the manufacturer's profit, while avoiding damage to the profit of the entire supply chain, the contract mechanism should be built between the manufacturer and the retailer to achieve mutually beneficial results.

CONCLUSION

In this article, we discuss the inventory strategy of products under the dual-channel supply chain. The manufacturer sells its

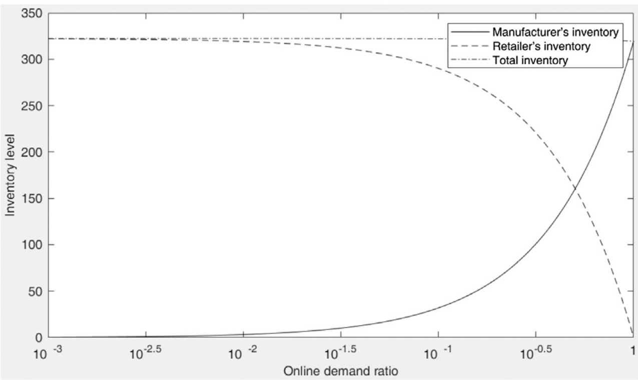

FIGURE 2 | Inventory trends.

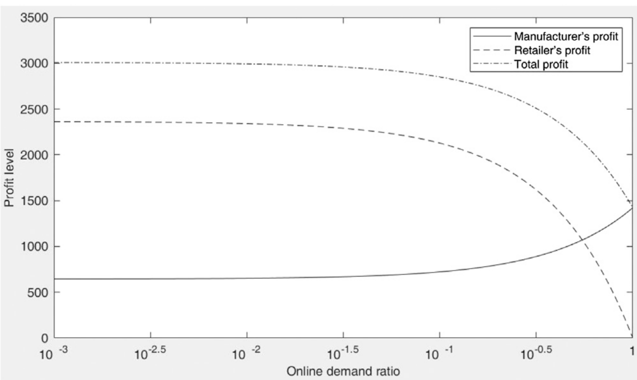

FIGURE 3 | Profit trends.

products through both the retail and online direct sales channels. The consumers in the market are divided into two types according to the channel type. We construct the optimal inventory strategy model for the manufacturer and retailer. We then create a numerical experiment to analyze the impact of demand uncertainty on the optimal inventory level and profits of the manufacturer and retailer. We find that both the manufacturer and retailer need to increase their inventory and their profits decline, with the retailer's profit declining faster than the manufacturer's. The reason for this phenomenon is the existence of the channel price gap and distribution cost. We also discuss the impact of the online demand ratio on the optimal inventory and profits of the manufacturer and retailer. The results of the numerical analysis show that increasing the online demand ratio leads to an increase in the manufacturer's inventory and the loss of the retailer's profit and that of the entire supply chain. The reason is that the cost of the retailer will not decrease linearly in the case of fewer consumers, and the hidden benefit produced by the retailer is freely transferred to the manufacturer. How to establish a mutually beneficial mechanism between the manufacturer and the retailer on the allocation of the profit from the online channel is a future research direction. Furthermore, we can study the impact of competition between online and offline sales on inventory in dual-channel supply chain from the perspective of the game theory [21–23].

DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

AUTHOR CONTRIBUTIONS

NX, JC, and ZC contributed the conception and design of the study. JC organized the literature. ZC performed the design of figures. NX wrote the first draft of the manuscript. All authors contributed to the manuscript revision, read, and approved the submitted version.

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