Investigation of Electromagnetic Wave Structures for a Coupled Model in Anti-ferromagnetic Spin Ladder Medium

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The article studies the extraction of electromagnetic wave structures in a spin ladder anti-ferromagnetic medium with a coupled generalized non-linear Schrodinger model. The direct algebraic technique is used to extract the wave solutions. The solutions are obtained in the form of dark, singular, kink, and dark-singular under different constraint conditions. Moreover, the dynamic behavior of the structures have depicted in 3D graphs and their corresponding counterplots. The results are helpful for the understanding of wave propagation study and are also vital for numerical and experimental verifications in the field of electromagnetic wave theory.

Keywords: electromagnetic waves, coupled Schrödinger model, anti-ferromagnetic medium, integrability, direct algebraic technique

1. INTRODUCTION

The theory of solitons is an attractive and exciting area of research. It is interlinked with many branches of mathematics and engineering [1–9]. Its aspects are charming and amazing because soliton travels with a steady speed and maintains its shape while propagating. It arises by balancing dispersive and non-linear terms. Solitons are discussed in different fields, and for references see [10–18].

The magnetic moments of molecules and atoms, normally linked to the spins of electrons, adjusted in an ordinary pattern with neighboring spins spell in reverse directions. This is like ferrimagnetism and ferromagnetism, a manifestation of ordered magnetism. Theoretical and mathematical theories argue that the spin ladder system is an excellent medium through which the interaction between different spins can be mapped to an approximate Heisinberg-type coupling with a coupling parameter that is inversely proportional to the distance between two separated spins. The dynamics of electromagnetic solitons with the coupled model in an anti-ferromagnetic spin ladder medium is of great interest among researchers due to its variety of applications. The spin ladder systems are a great source with which to develop significant interest in both experimental and theoretical points of view. Anti-ferromagnets test different ideas that involve a strong correlated system [6–9]. Spin ladders have many applications in different fields of quark physics, superconductors, and ultra-cold atoms, etc. The study of anti-ferromagnetic is still in its early stages. In anti-ferromagnets, the staggered magnetization variable M contributes the first

derivative in this manner, i.e., (∇M). This is because, in the case of anti-ferromagnets, we have taken limits in specified intervals for two different sublattices individually. The parent cuprate insulators are the best example of anti-ferromagnets with isotropic and predominantly nearest-neighbor coupling. They satisfied the theory that gives an ordered ground state for $(s = \frac{1}{2} AF)$ by showing simple long range anti-ferromagnetic order at low temperature [19–26].

In this article, a coupled generalized derivative non-linear Schrödinger that describes the dynamical behavior of electromagnetic waves in a spin ladder antiferromagnetic medium system is considered and under investigation. The Heisenberg model was studied in Chen et al. [15] and Xu et al. [16] and considered with an anisotropic spin ladder and two ferromagnetic lattices. This lattices consist of N spins and are directed in the same direction? For more details see also Kavitha et al. [26]. The system is read as

$$i\frac{\partial q_j}{\partial t} + \frac{\partial^2 q_j}{\partial x^2} + i\alpha_1 \frac{\partial}{\partial x}(|q_j|^2 q_j) - \alpha_2 \frac{\partial^3 q_j}{\partial x^3} + \alpha_3 \frac{\partial}{\partial x}(q_n - q_j)$$
$$+ \alpha_4 \frac{\partial^3 q_n}{\partial x^3} - \alpha_5 \frac{\partial q_j}{\partial x} = 0,$$

(1. 1)

where q_j for $j = 1, 2$, and $n = 3 - j$, are the wave profiles, and α_k for $k = 1, 2, \cdots, 5$, represent the real coefficients and are defined by $\alpha_1 = \frac{1+g}{2gM_0^2}\alpha$, $\alpha_2 = \frac{ij_a \alpha^3}{g\gamma}$, $\alpha_3 = \frac{ij_b \alpha}{g\gamma}$, $\alpha_4 = \frac{ij_c \alpha^3}{g\gamma}$ and $\alpha_5 = \frac{iA}{g\gamma}\alpha$. Where j_a is ferromagnetic spin exchange interaction, j_b is antiferromagnetic coupling, j_c is the exchange coupling, and A represents the single-ion uniaxial anisotropy. The last equation is reduced to generalized coupled derivative non-linear Schrodinger system by considering $\alpha_m = 0$, for $m = 2, \cdots, 5$. In the following section, the considered model is analyzed.

2. THE MODIFIED DIRECT ALGEBRAIC METHOD

The section studies, MDAM [27] to investigate the wave structures of NLPDs. Thus, we consider NLPDs in following form:

$$H\left(q, q_t, q_x, q_{tx}, q_{xx}, \cdots\right) = 0,$$

where q is a profile of wave structure and H is called a polynomial of q and its partial derivatives along with non-linear terms.

To extract wave structures, the method is followed by using the steps as discussed under.

Step 1: First, the NPDEs is converted into non-linear ODEs using the following transformation.

$$q(x, t) = U(\xi), \text{ and } \xi = B(x - wt),$$

where B and w are arbitrary parameters. It allows us to reduce the above equation in an ODE of U and have the form

$$Q(U, U', U'', U''', \cdots) = 0.$$

Step 2: It is supposed that the solution of above equation satisfies the following ansatze:

$$U(\xi) = A_0 + \sum_{j=0}^{m} \left(A_j \varphi^j + B_j \varphi^{-j}\right)^i,$$
$$\varphi' = \gamma + \varphi^2,$$

where γ is a parameter and its value is determined, $\varphi = \varphi(\xi)$, $\varphi' = \frac{d\varphi}{d\xi}$.

Step 3: The homogeneous balance technique is followed where the highest order derivative is balanced with non-linear terms, to find the value of m, and where $m \in Z_+$.

Step 4: The use of the above equation and collecting the terms of the same order of φ^j together. Equate each term of φ^j to zero, which produce the system of algebraic equations.

Step 5: The solution of the system of algebraic equations along with the following wave structures are general solutions.

(i) If $\gamma < 0$

$$\varphi = -\sqrt{-\gamma} \tanh(\sqrt{-\gamma}\xi), \quad or \quad \varphi = -\sqrt{-\gamma} \coth(\sqrt{-\gamma}\xi),$$

it depends on condition.

(ii) If $\gamma > 0$

$$\varphi = \sqrt{\gamma} \tan(\sqrt{\gamma}\xi), \quad or \quad \varphi = -\sqrt{\gamma} \cot(\sqrt{\gamma}\xi),$$

it depends on condition.

(iii) If $\gamma = 0$

$$\varphi = \frac{-1}{\xi}.$$

In the following section, the exact wave structures of Equation (1.1) can be obtained.

3. ANALYTICAL ANALYSIS

We consider the complex transformation $q_j(x, t) = U_j(\xi) \times e^{i\phi}$, where $\xi = B(x - vt)$ and $\phi = -kx + wt + \theta$. It reduces the partial differential equation to an ordinary differential equation. After some mathematical work, the following real part of Equation (1.1) is obtained.

$$-\omega U_j + B^2 U_j'' - k^2 U_j + k\alpha_1 U_j^3 + 3\alpha_2 Bk^2 U_j' - B^3 \alpha_2 U_j'''$$
$$+ B\alpha_3 U_n' - B\alpha_3 U_j' - 3Bk^2 \alpha_4 U_n' + B^3 \alpha_4 U_n'' - \alpha_5 B U_j' = 0. \quad (3. 2)$$

The imaginary part equation gives the constraint condition

$$v = 2k = \pm 2\sqrt{\frac{\alpha_3 + \alpha_5}{\alpha_2}}$$

for $\alpha_2(\alpha_3 + \alpha_5) > 0$. To find the solution of Equation (1.1), let us consider the $U(\xi) = a_0 + \sum_{i=1}^{m}(a_i Z^i + b_i Z^{-i})$ form of the

solution (see also [18]), where $Z' = \gamma + Z^2$, $a_i's$, $b_i's$, and γ are the real parameters. The parameters are to be determined later. It is also noted that $Z = Z(\xi)$, and so does $Z' = \frac{dZ}{d\xi}$.

To investigate the electromagnetic waves of the system, we find the solution of Equation (3.2) by finding the homogenous balance $m = 1$ between the non-linear term and highest order derivatives present in this equation. We have the following value of U after substituting the homogenous balance:

$$U = a_0 + a_1 Z + b_1 Z^{-1}, \qquad (3.3)$$

where a_0, a_1 and b_1 are real parameters. To calculate real parameters, we put U and required derivatives in Equation (3.2). After simplification and by equating the coefficients of same power of Z, the system of equations is obtained. To get the values of parameters and the solutions against these different parameters, we solve this system by using *Maple*. Thus, different cases along with solutions are discussed below.

For case 1: The values of parameters are

$$a_0 = 0, \quad a_1 = 0, \quad b_1 = r\sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}},$$

$$B = -\left(\frac{\omega\alpha_2^2 + \alpha_1^2(\alpha_3 + \alpha_5)^2}{2\alpha_1^2\alpha_2^2 r}\right)$$

and the corresponding dark and singular wave structures can be obtained for different values of γ. The constraint condition, for the existence of these solutions, is given by

$$\gamma^2 b(b^2\delta + 2\gamma^2) < 0.$$

For $\gamma < 0$, one may have the following wave structures of Equation (1.1)

$$q_{j_1} = q_1 = -\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \coth\left(i\sqrt{r}\xi\right) \times e^{i\phi}$$

and

$$q_{j_2} = q_2 = -\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh\left(i\sqrt{r}\xi\right) \times e^{i\phi}.$$

The following two cases are obtained from the above solution and considered as the diagonal components of the spin ladder.

$$q_{j_{2,1}} = \left[-\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh\left(i\sqrt{r}B(x - vt)\right)\right]$$
$$\cos\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right),$$

and

$$q_{j_{2,1}} = \left[-\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh\left(i\sqrt{r}B(x - vt)\right)\right]$$
$$\sin\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right).$$

For $\gamma > 0$, one may have the following periodic solutions.

$$q_{j_3} = q_3 = i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \cot(\sqrt{r}\xi) \times e^{i\phi},$$

and

$$q_{j_4} = q_4 = -i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tan(\sqrt{r}\xi) \times e^{i\phi}.$$

For $\gamma = 0$, one may have the following periodic solutions

$$q_{j_5} = q_5 = -ir\xi\sqrt{\frac{2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \times e^{i\phi},$$

where,

$$\xi = -\frac{\omega\alpha_2^2 + \alpha_1^2(\alpha_3 + \alpha_5)^2}{2\alpha_1^2\alpha_2^2 r}\left(x - vt\right).$$

The graphical representations and contour plots of the solutions for q_1 to q_4 are shown in **Figure 1** for different values of parameters $\alpha_1 = 1$, $r = 1.25$, $B = 5$, $k = 0.1$, $\xi = 0.01$, $\theta = 0.2$, and $b = 0.5$.

For case 2: The values of the parameters are

$$a_0 = 0, \quad a_1 = \sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}, \quad b_1 = 0, \quad B = -\frac{1}{2\alpha_2}(\alpha_3 + \alpha_5)\alpha_1 a_1^2$$

and the corresponding combined dark-singular wave structures are constructed.

For $\gamma < 0$, the following type of exact solutions of Equation (1.1) is written:

$$q_{j_6} = q_6 = \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}\xi) \times e^{i\phi},$$

The following two cases are obtained from the above solution and considered as the diagonal components of the spin ladder.

$$q_{6,1} = \left[\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))\right]$$
$$\cos\left(-\left(\frac{\alpha_3 + \alpha_5}{\alpha_2}\right)x + wt + \theta\right)$$

$$q_{6,1} = \left[\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))\right]$$
$$\sin\left(-\left(\frac{\alpha_3 + \alpha_5}{\alpha_2}\right)x + wt + \theta\right)$$

and we also have

$$q_{j_7} = q_7 = \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \coth(i\sqrt{r}\xi) \times e^{i\phi}.$$
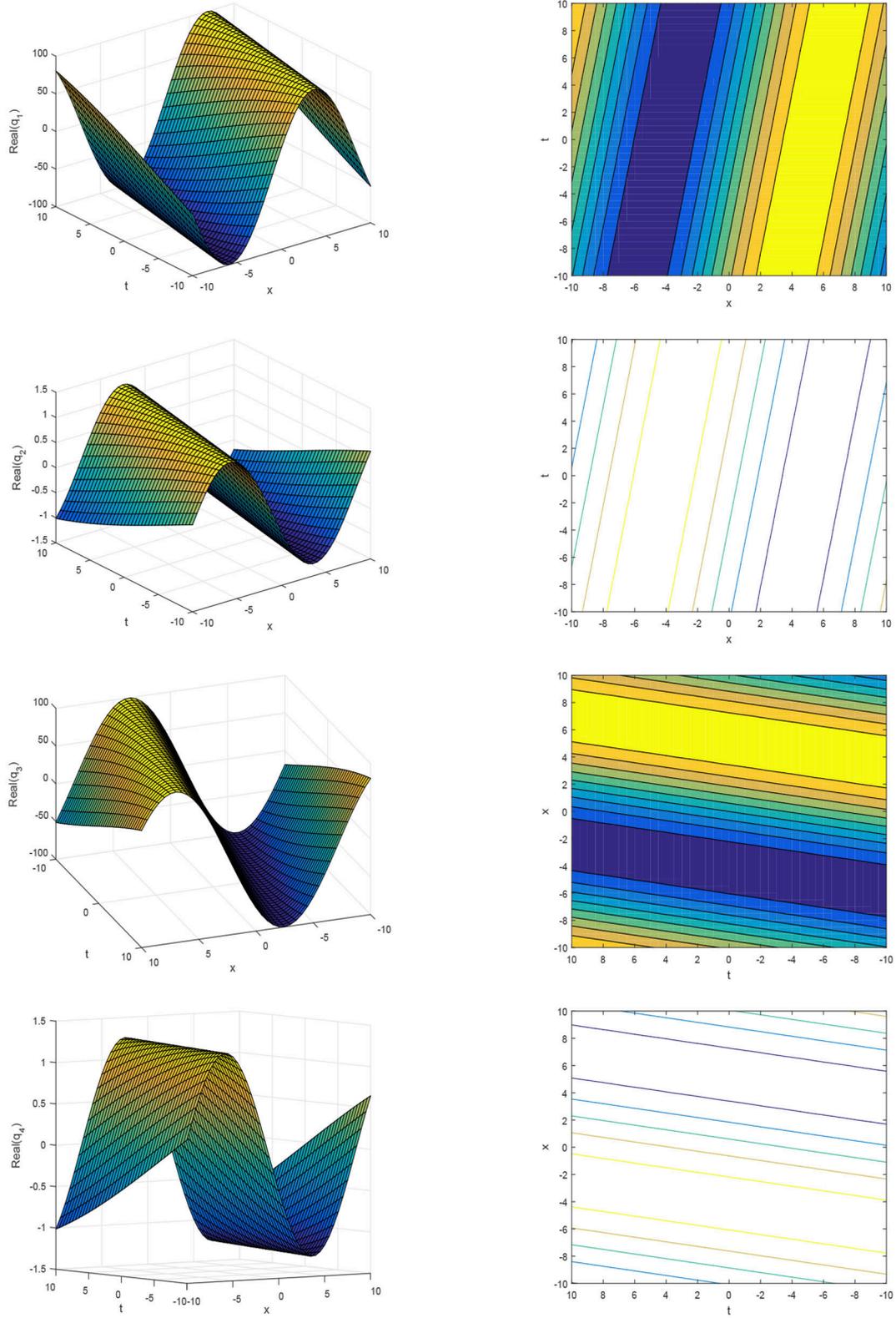
FIGURE 1 | The 3D plots and contour plots of the real part of solution $q_1(x, t)$ to $q_4(x, t)$ for different parameters.

For $\gamma > 0$, one may have the following periodic solutions

$$q_{j_8} = q_8 = i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tan(\sqrt{r}\xi) \times e^{i\phi},$$

and

$$q_{j_9} = q_9 = -i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \cot(\sqrt{r}\xi) \times e^{i\phi}.$$

For $\gamma = 0$, one may have the following periodic solutions

$$q_{j_{10}} = q_{10} = -\frac{i}{\xi}\sqrt{\frac{2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \times e^{i\phi},$$

where,

$$\xi = -\frac{1}{2\alpha_2}(\alpha_3 + \alpha_5)\alpha_1 a_1^2 (x - vt).$$

The pattern of the solutions for q_6 to q_9 are shown in **Figure 2** for the values of parameters $\alpha_1 = 0.007$, $\alpha_2 = 0.98$, $\alpha_3 = 0.01$, $r = 0.76$, $B = 0.98$, $k = 0.98$, $\xi = 0.01$, $\theta = 0.2$, and $b = 1.5$.

For case 3: The values of parameters are

$$a_0 = b_1\sqrt{\frac{2}{r}}, \quad a_1 = 0, \quad b_1 = r\sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}},$$

$$B = -\frac{(\alpha_3 + \alpha_5)\alpha_1 b_1^2}{2r^2\alpha_2}$$

and the corresponding dark and singular wave structures can be obtained.

For $\gamma < 0$, the following forms of the exact solutions to Equation (1.1) are obtained.

$$q_{j_{11}} = q_{11} = [\sqrt{\frac{2b_1^2}{r}} - \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \coth(i\sqrt{r}\xi)] \times e^{i\phi},$$

and

$$q_{j_{12}} = q_{12} = [\sqrt{\frac{2b_1^2}{r}} - \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}\xi)] \times e^{i\phi}.$$

The following two cases are obtained from the above solution and considered as the diagonal components of the spin ladder.

$$q_{12,1} = [\sqrt{\frac{2b_1^2}{r}} - \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))]$$

$$\cos\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right)$$

$$q_{12,1} = [\sqrt{\frac{2b_1^2}{r}} - \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))]$$

$$\sin\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right).$$

For $\gamma > 0$, one may have the following periodic solutions

$$q_{j_{13}} = q_{13} = [\sqrt{\frac{2b_1^2}{r}} + i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \cot(\sqrt{r}\xi)] \times e^{i\phi},$$

and

$$q_{j_{14}} = q_{14} = [\sqrt{\frac{2b_1^2}{r}} - i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tan(\sqrt{r}\xi)] \times e^{i\phi},.$$

For $\gamma = 0$, one may have the following periodic solutions

$$q_{j_{15}} = q_{15} = [\sqrt{\frac{2b_1^2}{r}} - ir\xi\sqrt{\frac{2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}] \times e^{i\phi},$$

where,

$$\xi = -\frac{(\alpha_3 + \alpha_5)\alpha_1 b_1^2}{2r^2\alpha_2}(x - vt)$$

$$\phi = \frac{2Br^2}{\alpha_1 b_1^2}x + (\frac{5(\alpha_3 + \alpha_5)\alpha_1 b_1^2}{r\alpha_2} - (\frac{\alpha_3 + \alpha_5}{\alpha_2})^2)t + \theta.$$

The pattern of the solutions for q_{11} to q_{14} are shown in **Figure 3** for the values of parameters $\alpha_1 = 2$, $r = 1.5$, $B = 3.9$, $k = 0.98$, $\xi = 0.01$, $\theta = 0.2$, and $b = 0.25$.

For case 4: The values of parameters are

$$a_0 = 0, \quad a_1 = 0, \quad b_1 = r\sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}, \quad B = -\frac{b_1^2(\alpha_3 + \alpha_5)\alpha_1}{2\alpha_2 r^2}$$

and the corresponding dark and singular wave structures can be obtained.

For $\gamma < 0$, the following exact solutions to Equation (1.1) are obtained.

$$q_{j_{16}} = q_{16} = -\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \coth(i\sqrt{r}\xi) \times e^{i\phi}$$

and

$$q_{j_{17}} = q_{17} = -\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}\xi) \times e^{i\phi}.$$

The following two cases are obtained from the above solution and are considered the diagonal components of the spin ladder.

$$q_{17,1} = [-\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))]$$

$$\cos\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right)$$

$$q_{17,1} = [-\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \tanh(i\sqrt{r}B(x - vt))]$$

$$\sin\left(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x + wt + \theta\right)$$

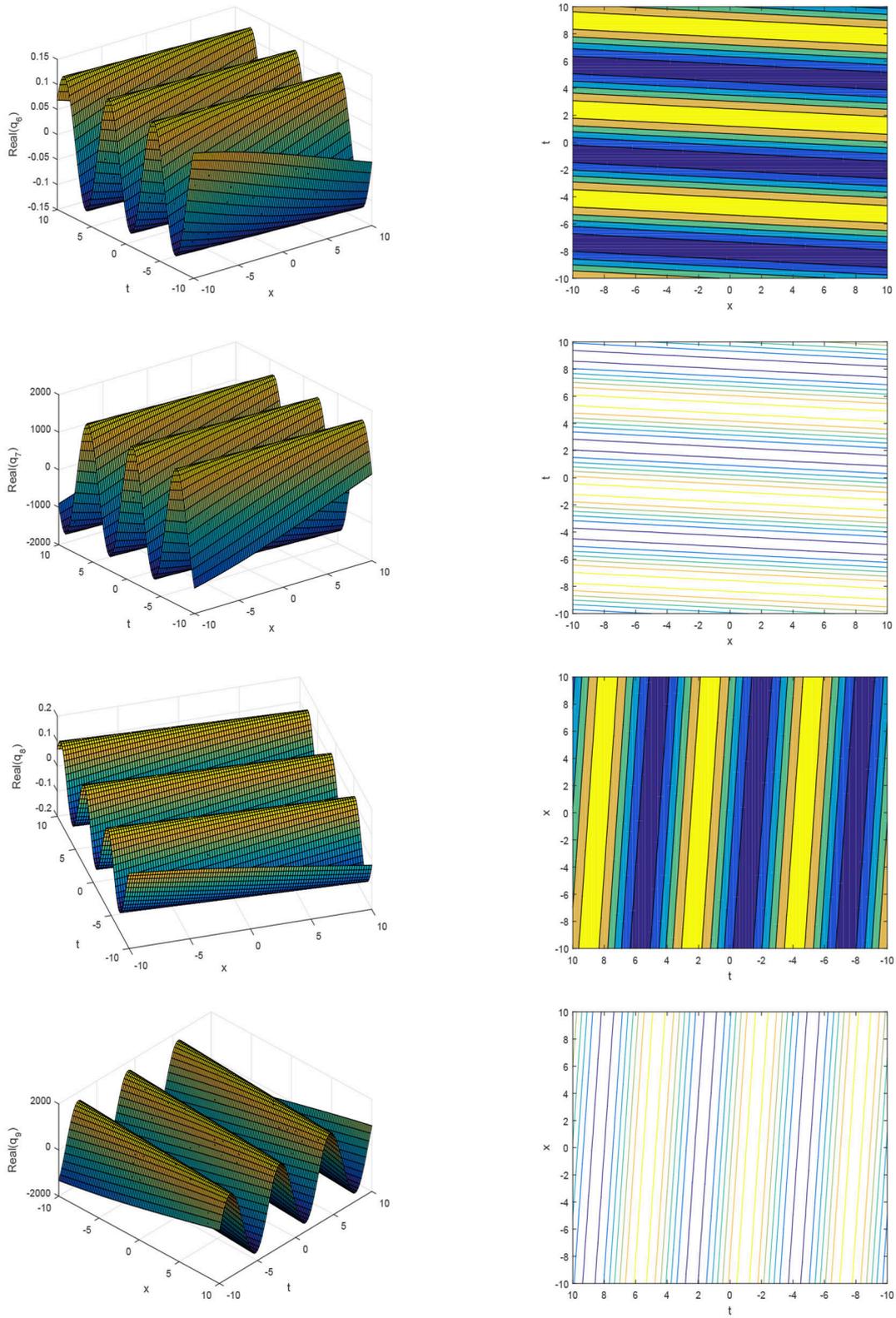For $\gamma > 0$, one may have the following periodic solutions

FIGURE 2 | The 3D plots and contour plots of the real part of solution $q_6(x, t)$ to $q_9(x, t)$ for different parameters.
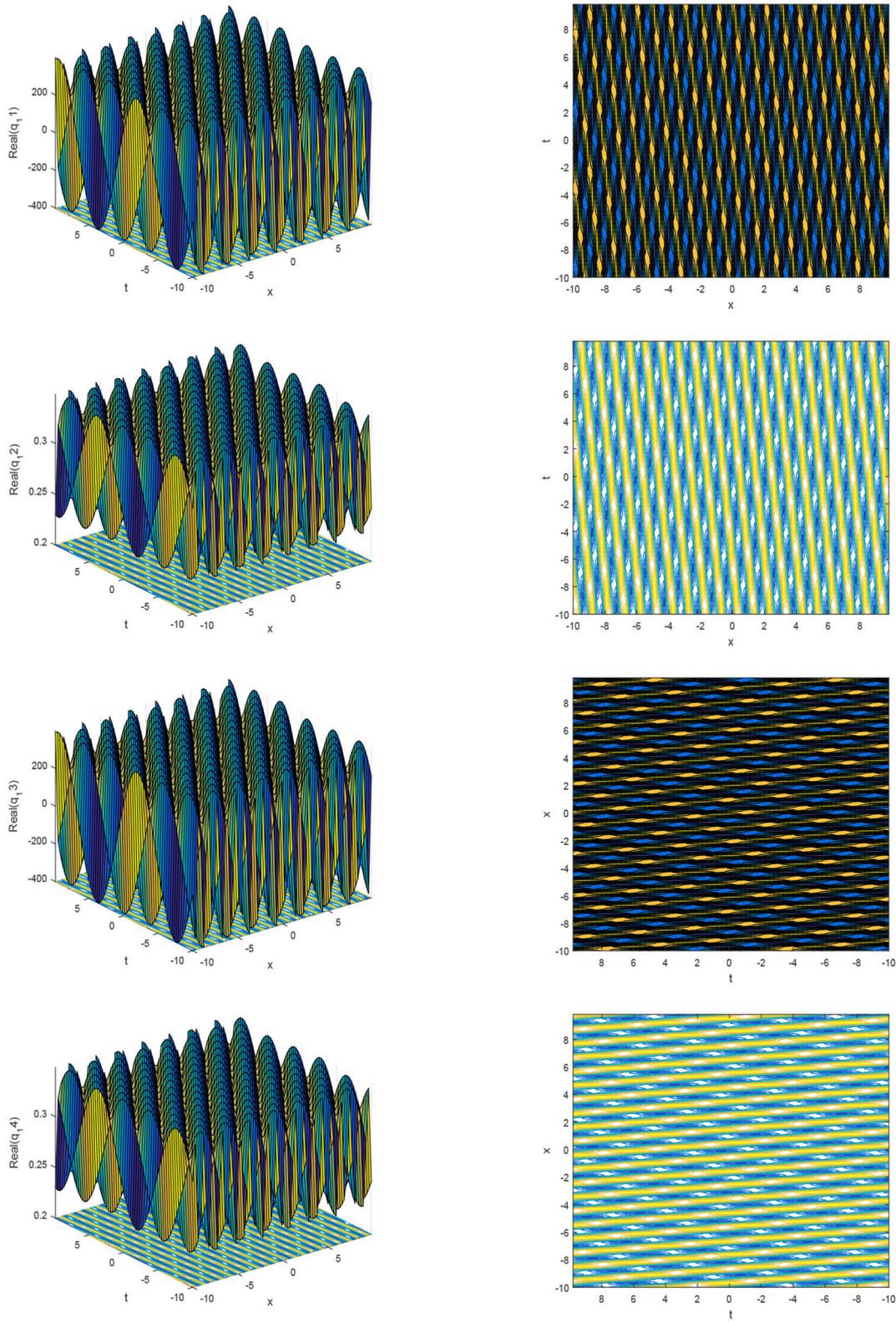
FIGURE 3 | The 3D plots and contour plots of the real part of solution $q_{11}(x, t)$ to $q_{14}(x, t)$ for different parameters.

$$q_{j_{18}} = q_{18} = i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\cot(\sqrt{r}\xi) \times e^{i\phi},$$

and

$$q_{j_{19}} = q_{19} = -i\sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\tan(\sqrt{r}\xi) \times e^{i\phi}.$$

For $\gamma = 0$, one may have the following periodic solutions

$$q_{j_{20}} = q_{20} = -ir\xi\sqrt{\frac{2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}} \times e^{i\phi},$$

where

$$\xi = -\frac{b_1^2(\alpha_3 + \alpha_5)\alpha_1}{2\alpha_2 r^2}\left(x - vt\right).$$

The pattern of the solutions for q_{16} to q_{19} are shown in **Figure 4** for the values of parameters $\alpha_1 = 0.002$, $r = 0.5$, $B = 0.9$, $k = 0.98$, $\xi = 0.01$, $\theta = 0.2$, and $b = 0.25$.

For case 5: The values of parameters are

$$a_0 = 0, \quad b_1 = r\sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}},$$

$$a_1 = \frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)},$$

$$B = \frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 3a_1b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}{2r\alpha_2^2}$$

and the corresponding dark and singular wave structures are obtained.

For $\gamma < 0$, one can obtain the following exact solutions to Equation (1.1).

$$q_{j_{21}} = q_{21} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}i\sqrt{r}\tanh(i\sqrt{r}\xi)$$
$$- \sqrt{\frac{2Br\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\coth(i\sqrt{r}\xi)] \times e^{i\phi}.$$

The following two cases are obtained from the above solution and considered as the diagonal components of the spin ladder.

$$q_{21,1} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}i\sqrt{r}\tanh(i\sqrt{r}B(x - vt)$$
$$- \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\coth(i\sqrt{r}B(x - vt))]\cos(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x$$
$$+ wt + \theta)$$

$$q_{21,1} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}i\sqrt{r}\tanh(i\sqrt{r}B(x - vt)$$
$$- \sqrt{\frac{2rB\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\coth(i\sqrt{r}B(x - vt))]\sin(-(\frac{\alpha_3 + \alpha_5}{\alpha_2})x$$
$$+ wt + \theta),$$

and

$$q_{j_{22}} = q_{22} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}i\sqrt{r}\coth(i\sqrt{r}\xi)$$
$$- \sqrt{\frac{2Br\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\tanh(i\sqrt{r}\xi)] \times e^{i\phi}.$$

For $\gamma > 0$, one may have the following periodic solutions

$$q_{j_{23}} = q_{23} = [\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}\sqrt{r}\tan(\sqrt{r}\xi)$$
$$+ \sqrt{\frac{-2Br\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\cot(\sqrt{r}\xi)] \times e^{i\phi}$$

$$q_{j_{24}} = q_{24} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}\sqrt{r}\cot(\sqrt{r}\xi)$$
$$- \sqrt{\frac{-2Br\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}\tan(\sqrt{r}\xi)] \times e^{i\phi}$$

For $\gamma = 0$, one may have the following periodic solutions

$$q_{j_{25}} = q_{25} = [-\frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 2\alpha_2^2 Br}{3b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}(\frac{1}{\xi})$$
$$- r\xi\sqrt{\frac{-2B\alpha_2}{\alpha_1(\alpha_3 + \alpha_5)}}] \times e^{i\phi}$$

where

$$\xi = \frac{\omega\alpha_2^2 + (\alpha_3 + \alpha_5)^2 - 3a_1b_1\alpha_1\alpha_2(\alpha_3 + \alpha_5)}{2r\alpha_2^2}\left(x - vt\right).$$

These are the new solitons and periodic wave structures.

4. CONCLUSIONS

The article gives single and combined electromagnetic wave structures for the coupled non-linear Schrödinger equations along with the coefficients of ferromagnetic spin exchange interaction, antiferromagnetic coupling, exchange coupling, and single-ion uniaxial anisotropy. The model under investigation describes the dynamic behavior of electromagnetic waves in a spin ladder antiferromagnetic medium. First the complex transformation is used and then modified extended direct algebraic method is utilized to find dark, singular, and dark-singular wave structures. Some other solutions (singular periodic) are also fall out during the analytical analysis. The constraint conditions for the existence of wave structures for different parameters are also observed. Moreover, the 3D plots and corresponding contour plots of the real part of solutions are drawn by choosing suitable parameters.

It is also observed that the method used is effective, powerful, reliable, and much more practical in obtaining the exact wave structures for non-linear phenomena that arise in fields like telecommunication engineering, mathematical
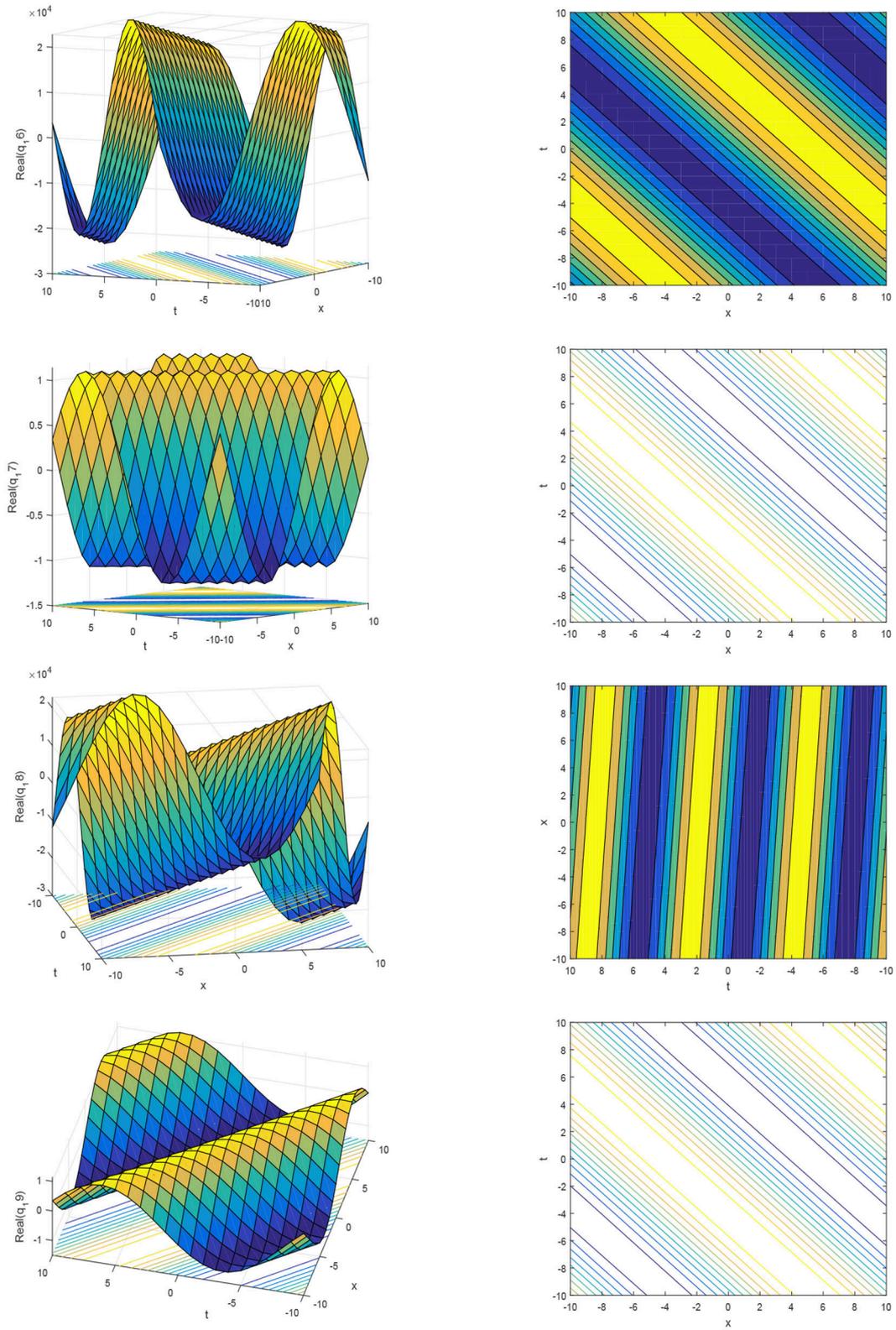
FIGURE 4 | The 3D plots and contourplots of the real part of solution $q_{16}(x, t)$ to $q_{19}(x, t)$, for different parameters.

biology, mathematical physics, an ocean engineering and vice versa.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary materials, further inquiries can be directed to the corresponding author/s.

AUTHOR CONTRIBUTIONS

MY, SR, and DB contributed to conception and design of the study. MY and UY performed the analytical analysis. NA and MI established the results. NA and UY draw the graphs using mathematical. MI and UY wrote the first draft of manuscript. MY, SR, NA, and DB wrote sections of the manuscript. All authors contributed to manuscript revision, read, and approved the submitted version.

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