Analyses of Daily Market Impact Using Execution and Order Book Information

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We analyzed the Tokyo Stock Exchange (TSE) for a 29-month period from August 2014 to December 2016, including every transaction and order book snapshot, and confirmed through a simple statistical test that the market impact depends on each stock. Based on a correlation analysis, we found that the market impact slowly changes over time. From an order book analysis, negative correlations were found between the market impact and the averaged limit order volumes in the order book. We also clarified that one of the factors of market impact is the volume of limit orders in the order book.

Keywords: financial markets, stock price, market impact, market microstructure, order book analysis

1. INTRODUCTION

Many previous studies have addressed market impact from theoretical [1–8] and empirical [9–13] viewpoints since it is related to market efficiency and trading costs, especially for trading large volumes.

The analyses of market impact using order book data has since the 2000s gradually become popular [14–18] thanks to the provision of datasets of the order book from stock exchange markets such as the London Stock Exchange (LSE), New York Stock Exchange (NYSE), and Tokyo Stock Exchange (TSE) and developments of computer technology that enable us to analyze huge size of order book data.

These studies mainly focused on short time scale market impact such as a single transaction, a few seconds, or a few minutes. It is useful to discuss how large volume order affects the market price over a long time scale. However long time scale of it, such as daily market impact, has not been studied enough compared to the short time scale. In order to estimate transaction costs, daily market impact analysis is important for asset management companies that manage their position daily.

In a previous study of a historical dataset of the Tokyo Stock Exchange (TSE) for a 17-month period from August 2014 to December 2015 [19], we identified the following two relationships: (i) a proportional relationship between the return of the market price and the order imbalance of the executed volume and (ii) an inverse proportional relationship between the market impact and the averaged order book volume. We also introduced a transaction cost model by calculating the spread cost and applying the relationship between the return and the order imbalance of execution.

We focused on the TOPIX Core 30 where the issues have very high market capitalization and liquidity, such as Toyota and Sony. However, in this study, we analyzed not only the TOPIX Core 30 but also such smaller stocks as the TOPIX Large 70 and Mid 400 and identified the following results about market impact: (i) it depends on issues; (ii) it slowly changes over time; and (iii) one of its factors is order book volume. These findings are useful to estimate transaction costs.

Our paper is organized as follows. In section 2, we present the empirical analysis results of the execution and order book data. Section 3 is our conclusion.

2. EMPIRICAL ANALYSIS

We analyzed the Tokyo Stock Exchange data for a 29-month period from August 2014 to December 2016, which recorded every transaction and every order book from the best price (0-th level) to the 7-th level for the TOPIX Core 30, the Large 70, and the Mid 400. The datasets can be purchased from Japan Exchange Group (https://www.jpx.co.jp/english/markets/paid-info-equities/historical/01.html).

In this section, the dataset is analyzed, and we show the market impact depends on each stock by a simple statistical test. We clarify the market impact slowly changes over time based on a correlation analysis of it. Additionally, dependencies of market impact on order book volume are investigated.

Figure 1 shows an example of Sony's market price (stock code: 6758) in the upper figure and the cumulative transaction volume of deal-ask (a market order hits the best-ask price) and deal-bid (a market order hits the best-bid price) in the lower figure from 1 second past 9 a.m. to 1 second before 3 p.m. on June 21, 2016. Execution of the opening and closing transactions usually tends to have very large volume. We focused on continuous sessions in this study, removed the opening and closing transactions, and defined the return of the market price for 1 day:

$$r = \frac{P_e - P_s}{P_s}, \tag{1}$$

FIGURE 1 | Time series of market price and cumulative transaction volume of Sony (stock code: 6758) from 1 second past 9 a.m. to 1 second before 3 p.m. on June 21, 2016.
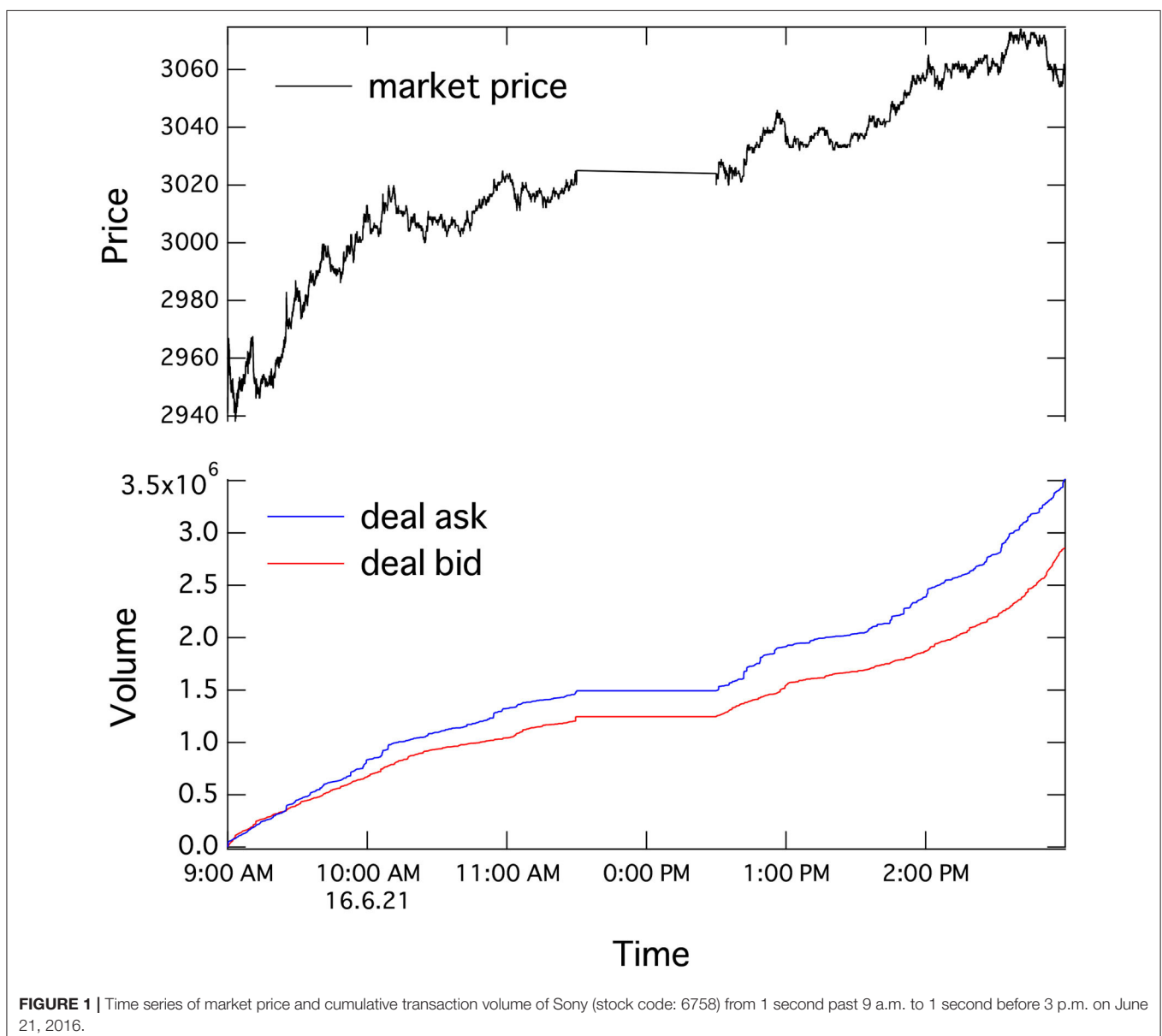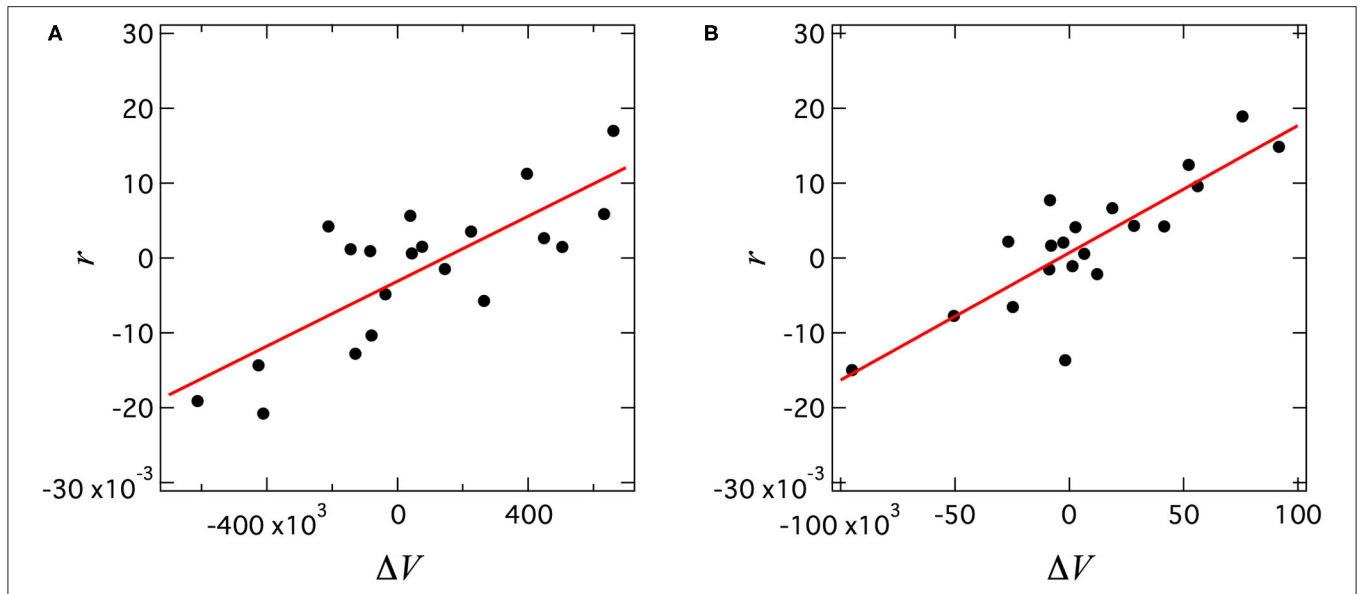
FIGURE 2 | Scatter plots between return (*r*) and executed order imbalance (Δ*V*) from October 1 to 31, 2016. **(A)** Sony corporation (stock code: 6758) in Core 30, **(B)** Nippon Paper Industries Co., Ltd. (stock code: 3863).
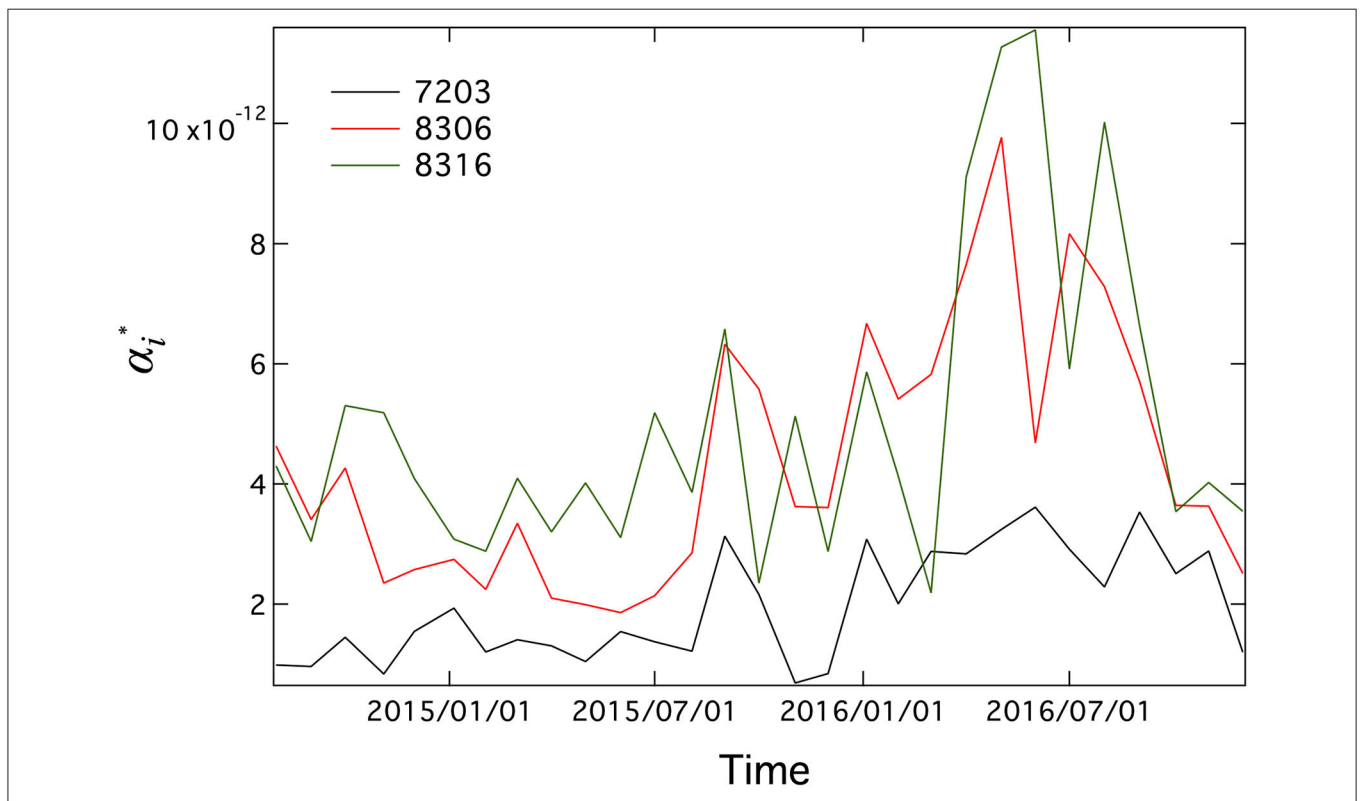
FIGURE 3 | Monthly time series of α_i^* during 29-month period for three companies: Toyota Motor Corp. (7203), Mitsubishi UFJ Financial Group, Inc. (8306), and Sumitomo Mitsui Financial Group, Inc. (8316).

where P_s and P_e are the prices at 9:00:01 and 14:59:59. The executed order imbalance (Δ*V*) is calculated as the cumulative deal-ask minus deal-bid 1 s before 3 p.m.

Figure 2 shows a scatter plot of the return (*r*) and the executed order imbalance (Δ*V*) of the Sony Corporation (stock code: 6758) in the Core 30 and Nippon Paper Industries Co., Ltd.

FIGURE 4 | Cumulative distributions of p-values for Core 30, Large 70, and Mid 400.
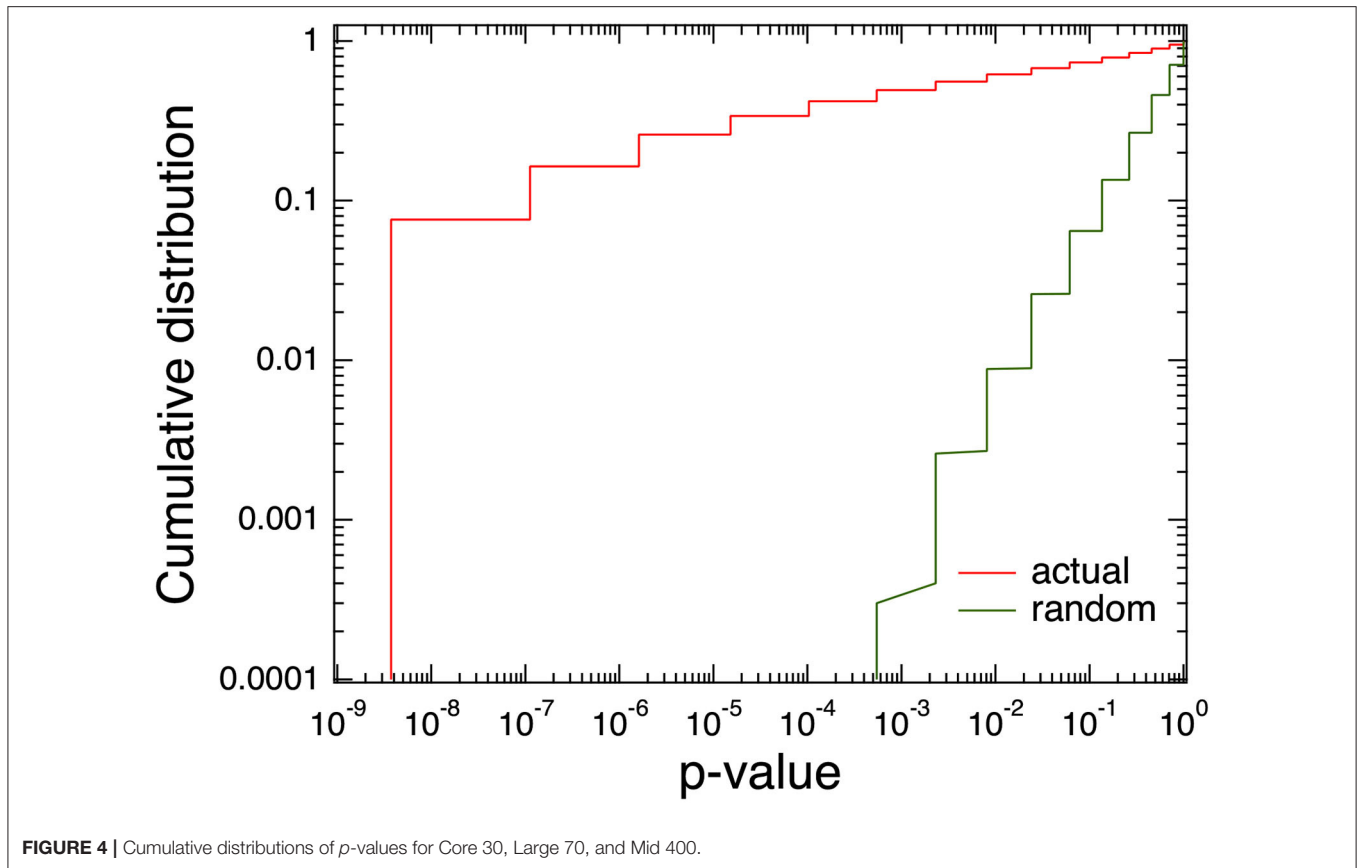
(stock code : 3863) in the Mid 400 from October 1 to 31, 2016. Each point is calculated by the data of a single day. We found a clear linear correlation between return (r) and executed order imbalance (ΔV):

$$r = \alpha_l \Delta V + \beta_l, \tag{2}$$

where gradient α is a measurement of market impact. In the case of Sony Corporation (stock code: 6758), $\alpha_l = 2.17 \times 10^{-8}$, standard error of α_l is 4.07×10^{-9}, and p-value for α_l is 4.49×10^{-5}. In the case of Nippon Paper Industries Co., Ltd. (stock code: 3863), $\alpha_l = 1.70 \times 10^{-7}$, standard error is 2.59×10^{-8}, and p-value is 3.78×10^{-6}.

To statistically check the existence of the differences of the market impact among stocks, we define the market impact for one yen as follows:

$$\alpha_i^* = \frac{\alpha_{l,i}}{P_i}, \tag{3}$$

where $\alpha_{l,i}$ describes the market impact for unit volume, P_i is the last price in a month, and i is the index of the stock. We show examples of monthly time series of α_i^* during 29-month period for three companies in **Figure 3**. We compare the monthly market impact, α_i^*, of companies A and B, and execute the a binomial test. A null hypothesis is the market impacts between company A and B is the same. In other words the probability where the market impact of company A is larger than B is half. If company A has a larger market impact n_A times than B, on the

other hand company B has a larger market impact n_B times than A, a p-value is calculated:

$$p_{val} = \begin{cases} 2 \times \displaystyle\sum_{k=0}^{\min(n_A, n_B)} \binom{n}{k}\left(\frac{1}{2}\right)^n & (n_A \neq n_B) \\ 1 & (n_A = n_B) \end{cases} \tag{4}$$

where two times describes a two-tailed test and $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1}$.

We applied a statistical test for all the combinations of Core 30, Large 70, and Mid 400. We are interested in small p-values and show a cumulative distribution of the p-values in a log-log scale (**Figure 4**) and found the actual data have a much bigger proportion of small p-values than a random case. For example, in the actual case, about 68% of the p-values are <0.05, implying the market impact depends on the issues.

To check the robustness of the market impact size, we show a scatter plot of the $\log_{10} \alpha^*$ at the M-th and $M + 1$-th months in **Figure 5**. We added a median as well as 25 and 75% percentiles in each box and identified a clear correlation. The correlation coefficients have large values (0.67) in the Pearson product-moment correlation case and 0.71 in the Spearman rank-order correlation case.

We also checked the ΔM dependencies for the correlation shown in **Figure 6**. The correlation decreases until $\Delta M = 12$ (1 year) and saturates. For the Spearman rank-order correlation, it is
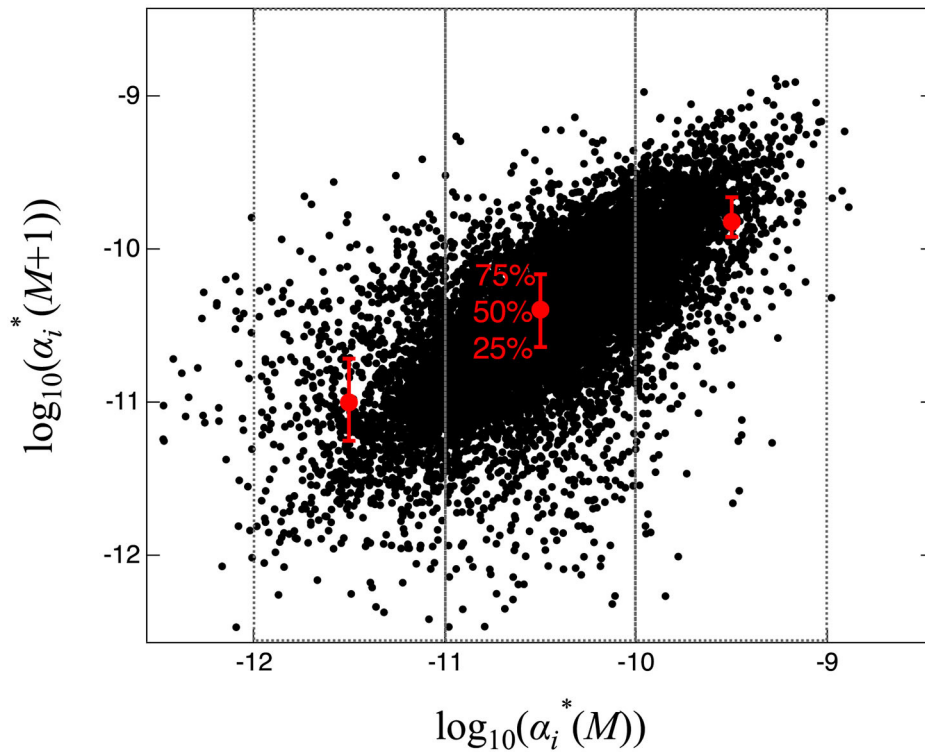
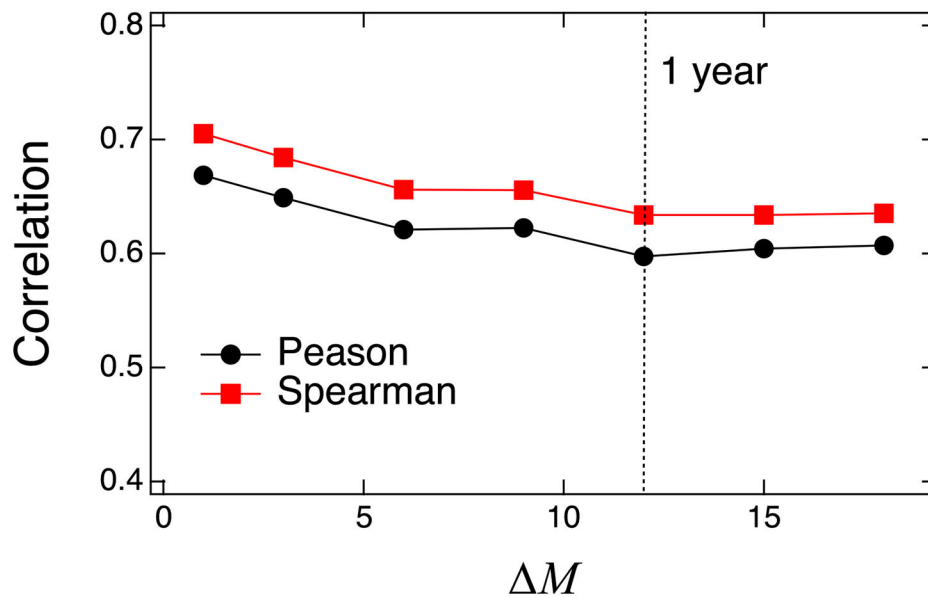**FIGURE 5 |** Scatter plot of log α^* at M-th and $M + 1$-th months.

FIGURE 6 | Pearson product-moment correlation and Spearman rank-order correlation for $\log(\alpha_i^*)$ at M and ΔM months.

approximately 0.64 when $\Delta M = 18$. The market impact, which does not change drastically, can be estimated using past market impact values.

Next we focused on the relationships between the market impact characterized by α_i^* and the order book volume. We

defined the i-th best-ask and bid-side volumes at t as $V_{a,i}(t)$ and $V_{b,i}(t)$. For example, the 0th (best) ask-side volume at t is written as $V_{a,0}(t)$. and the average of both the bid- and ask-sides order book volumes from the 0-th to the 7-th levels at $t = t_s$ are given:
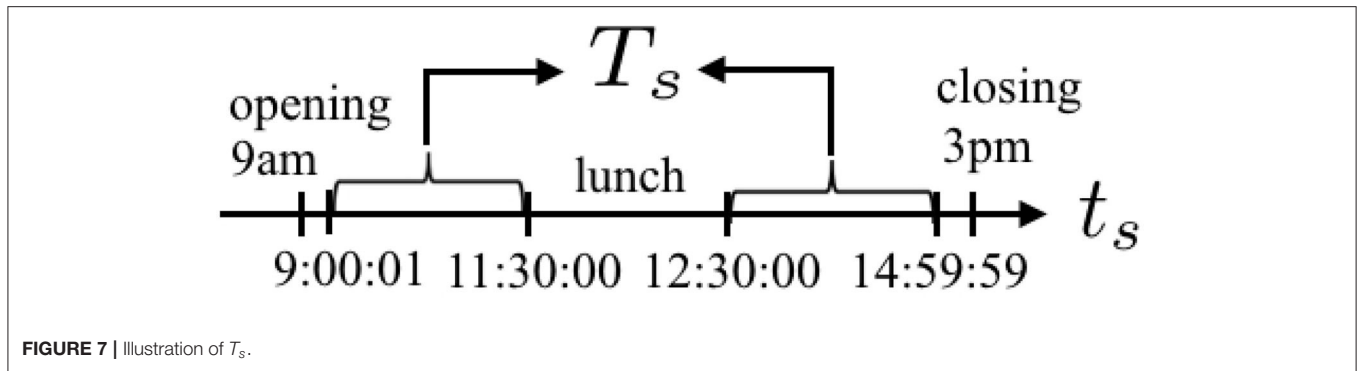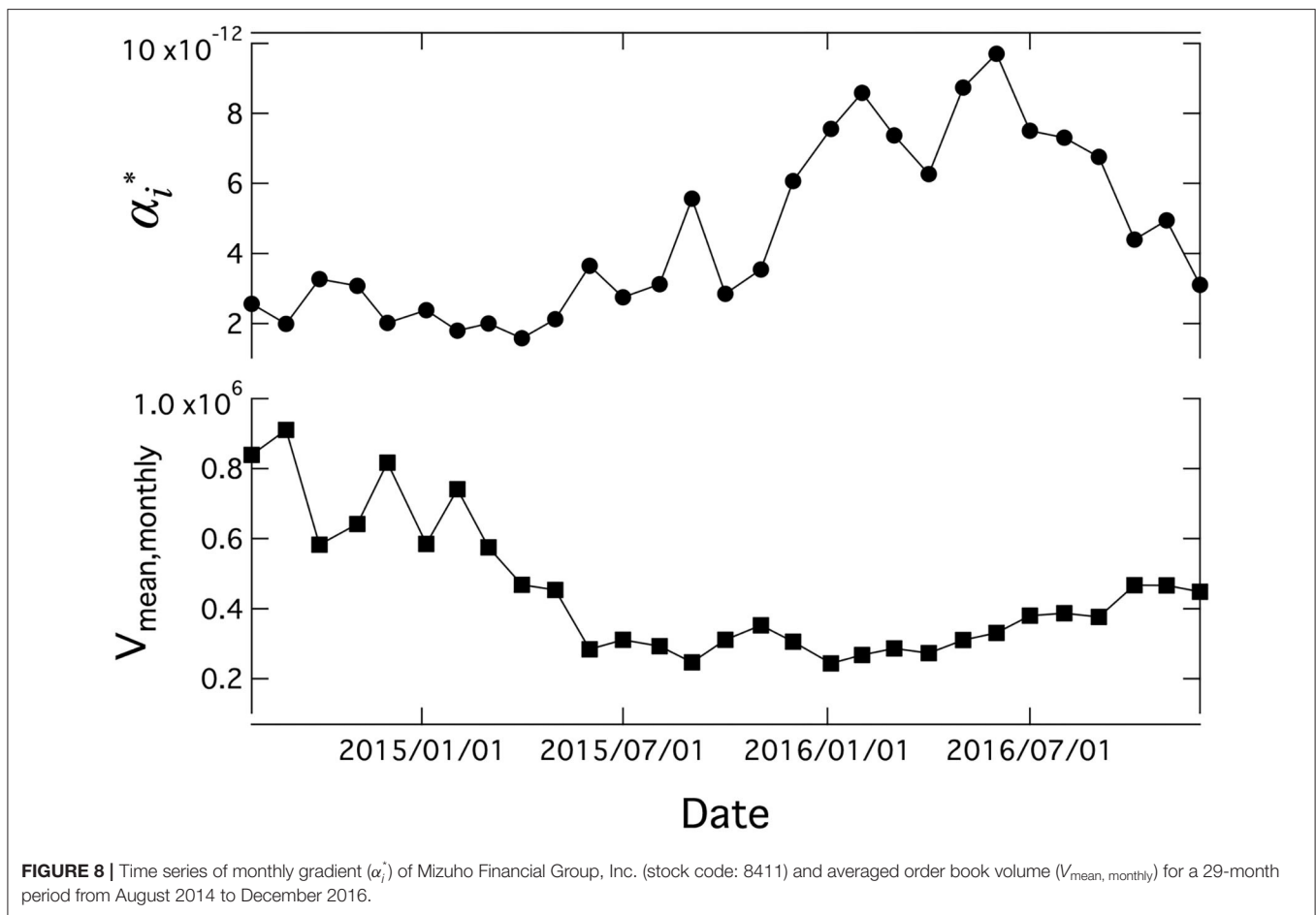
FIGURE 7 | Illustration of T_s.

FIGURE 8 | Time series of monthly gradient (α_i^*) of Mizuho Financial Group, Inc. (stock code: 8411) and averaged order book volume ($V_{\text{mean, monthly}}$) for a 29-month period from August 2014 to December 2016.

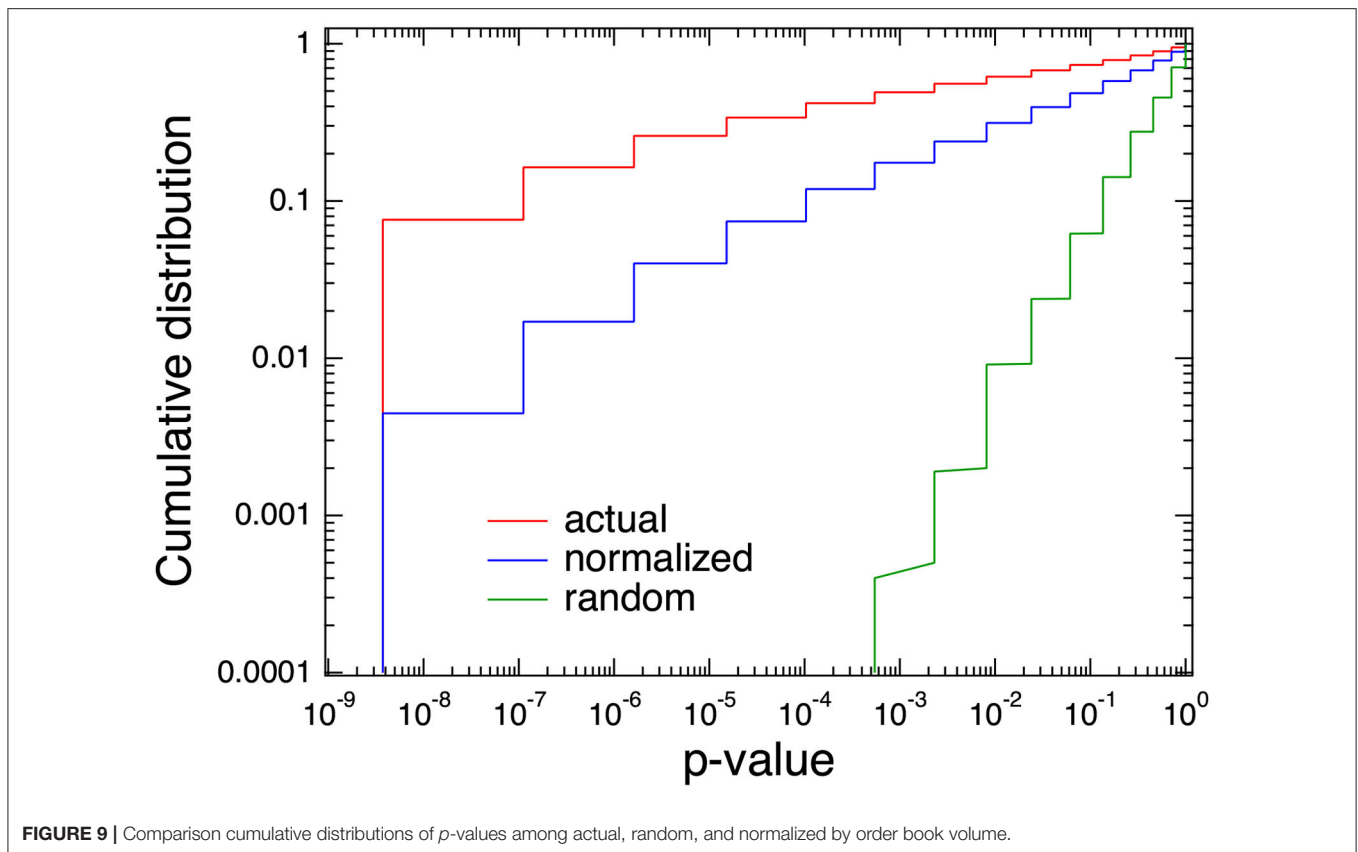$$V_m(t_s) = \frac{1}{2} \mathbb{E}_{i \in \{0,\cdots,7\}} \left(V_{b,i}(t_s) + V_{a,i}(t_s) \right), \tag{5}$$

where $\mathbb{E}$ represents the expectation value over the samples. We then defined the daily averaged order book volume:

$$V_{\text{mean,daily}}(t_d) = \mathbb{E}_{t_s \in T_s} \left(V_m(t_s) \right). \tag{6}$$

Here we ignore the opening (9 a.m.), closing (3 p.m.), and lunch (11:30 a.m.–0:30 p.m.) times (**Figure 7**) and finally calculate the

monthly averaged order book volume:

$$V_{\text{mean,monthly}}(t_m) = \mathbb{E}_{t_d \in T_d} \left(V_m(t_d) \right), \tag{7}$$

where T_d represents trading days of TSE market in 1 month (excluding weekends and holidays).

The upper time series in **Figure 8** shows the monthly α_i^* of the Mizuho Financial Group, Inc. (stock code: 8411), and the lower time series shows the monthly averaged order book volume defined in Equation (7). We observed a negative correlation between these two values.

FIGURE 9 | Comparison cumulative distributions of *p*-values among actual, random, and normalized by order book volume.

One reason for the dependencies is the order book volume; if we normalized the market impact by order book volume, the cumulative distribution of the *p*-values shifted more to random cases than to unnormalized cases (**Figure 9**). Here, we focused on order book volumes; if we normalized the market impact by such other effects as order flow and the configuration of the stock holders, the *p*-value distribution would probably shift more to the random case.

3. CONCLUSION

We showed that the daily market impact depends on stocks by executing a binomial test for the Tokyo Stock Exchange (TSE) during a 29-month period from August 2014 to December 2016. We focused on a longer time scale (such as 1 day) than previous studies that used a single transaction or 5 min. We verified the market impact's robustness and found it does not change drastically and realized that we can estimate it using past market impact values. However, we need to pay attention to big news, such as the Lehman shock, that change market conditions drastically, causing sudden variation in terms of market impact. We also investigated the dependencies of the market impact on order book volume and concluded that if a stock has a larger order book volume, the market impact tends to be smaller. We expect our findings to be useful for discussions about controlling transaction costs.

DATA AVAILABILITY STATEMENT

Publicly available datasets were analyzed in this study. The datasets analyzed for this study can be purchased from Japan Exchange Group (https://www.jpx.co.jp/english/markets/paid-info-equities/historical/01.html).

AUTHOR CONTRIBUTIONS

KY and TM carried out the conceptualization, methodology, investigation, and validation. KY carried out the formal analysis and prepared and wrote the original draft. TM carried out the project administration and funding acquisition. Both authors contributed to the article and approved the submitted version.

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