



Information Entropy Squeezing and Non-local Correlation Between a Two-Level Atom and Two-Mode Field Under the Classical Field Effect

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This paper examines the squeezing on information entropy and entanglement on the basis of a quantum scheme that contains two-mode interaction with time dependence of a two-level atom, including the external field. The proposed model is formulated by eliminating the external field after applying a certain transformation. The dynamics of information entropy squeezing and non-local correlation information for the system Hamiltonian are investigated. The effect of the evolution of the squeezing and entanglement by the classical field are estimated. Based on the results, the obtained dynamic behavior of squeezing and entanglement is controlled using the external field.

Keywords: external field, non-local correlation, two level atom, information entropy squeezing, two mode

INTRODUCTION

The NLC (non-local correlations) between bipartite systems, such as the atom field, performs different tasks in quantum information [1–6]. In this way, the atomic von Neumann entropy (ANE) has usually been measured by the NLC between two-partite systems [7]. The measure of NLC for the complex physical systems cannot be achieved by the ANE. Therefore, different investigations have been done to quantify the NLC for different quantum schemes. The correlation function for a TLA that interacts with a field mode prepared at the coherent state within the evolving Wehrl space entropy has been investigated [8]. Additionally, the classical field's influence in the quantum scheme belonging to a two TLA based on the evolution of quantum correlations between two TLA through quantum discord (QD) has been studied [9]. On the contrary, the study of the dynamics of NLC of a two-TLA Interaction, which leads to the exploration of many useful phenomena within quantum information, such as sudden death (ESD) and sudden birth (ESB), has been carried out [10]. For example, the ESD and ESB of two TLA for many conditions for the atomic state's position have been examined [11]. In addition, two-level atoms that were expanded into three two-level atoms prepared in a squeezed state has been presented [12]. This work has been extended to include the ESD of $su(1,1)$ set with two non-linear qubits [13].

Recently, different effects on the quantum schemes have been established based on the JCM (Jaynes-Cummings model). These schemes have been modified by examining different effects, such as atomic motion, intrinsic damping, and the external classical field [14]. Also, the effect of the

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cavity damping on the model described by the JCM based on approximation has been studied based on dispersive Approximation [15]. It has been shown that the system is sensitive to the decay and Stark-shift parameters. Furthermore, the type of interaction between two TLA and the N-level atom has been explored [16]. The results show that the non-local correlation depends crucially on the detuning.

Measuring the squeezing on the quantum system assumed from information entropy fulfills diverse tasks in quantum information processing (QIP). It employs the density matrix and entropic uncertainty relation (EUR) of the suggested model [17, 18]. The EURs were introduced as the basic component in the quantum key distribution, quantum cryptographic protocols, and quantum memory [19]. The Shannon entropy-a classical measure on the point of view of QIP-has been employed to quantify the squeezing based on the EUR. Further directions include efforts to quantify entropy squeezing with multi-level atomic models because it is hard to get an analytical formula of the density matrix. The atomic entropy squeezing for a quantum scheme of TLA that interacts with field mode in the coherent state [20] when having non-linear medium [21, 22] and two coupled modes under frequency converter type has been explored [23]. More recently, the effects of decaying terms and external fields on the performance of entropy squeezing have been explored [15, 24].

It is known that adding the classical field gives us the permittivity of exploring the way a strong field affects the interacting one or two atoms with it. The effect of the strong field on two atoms in a time-independent fashion was studied [25]. Furthermore, it was concluded that the dynamical performance of AFI is organized by the NLC of time-dependent coupling [26]. In this paper, we consider a two-level atom dependent on time. We obtain a solution through the condition of integration. This gives us the opportunity to study and compare the state of time dependence and independence.

Our main goal in this paper is to determine the dynamics of the concurrence and skew information correlations for the interaction between two qubits with a pair of fields of the amplifier type. The paper is ordered as follows: the physical scheme and its solution of the field that interact with two TLA are presented in section II; the atomic population is discussed in section III; the TLA degree of entanglement is provided in section IV; the entropy squeezing is discussed in section V; and our main results are presented in the final section.

QUANTUM SCHEME AND SOLUTION

This section introduces a quantum scheme of a TLA that interacts with a field of two modes in a cavity containing a classical field. The system Hamiltonian takes the form

$$H = \sum_{j=1}^2 \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\omega_0}{2} \hat{\sigma}_z + i\lambda \xi(t) (\hat{\sigma}_+ - \hat{\sigma}_-) (\hat{a}_1 \hat{a}_2 \exp(i\phi(t)) + \hat{a}_1^\dagger \hat{a}_2^\dagger \exp(-i\phi(t))) + \lambda_3 (\hat{\sigma}_+ + \hat{\sigma}_-), \quad (1)$$

with \hat{a} denoting annihilation operator, whereas \hat{a}^\dagger denotes creation operator and obeys the commutations relations:

$$[\hat{a}_j^\dagger, \hat{a}_k] = \delta_{jk}, \quad j, k = 1, 2 \quad (2)$$

such that δ_{jk} suggests the Kronecker delta, which satisfies that $\delta_{jk} = 1 \forall j = k, \delta_{jk} = 0$. Otherwise, ω_i and ω_0 denote the frequencies of the field and atomic transition, respectively. But $\hat{\sigma}_+, \hat{\sigma}_-$ and $\hat{\sigma}_z$ are the Pauli matrix. They are given by existing state ($|e\rangle$) and ground state($|g\rangle$), such as $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, and $\hat{\sigma}_- = |g\rangle\langle e|$, where $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$ and $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$. $\xi(t)$ is the coupling parameter for the atom and is the coupling constant for the electromagnetic field λ is the coupling constant for the electromagnetic field. λ_3 is the external classical field coupling.

We are currently going to make a detuning to remove the last two terms in the last equation. In their presence, we cannot obtain a general solution to this system. Therefore, a conversion must be made in the atomic space as follows

$$|+\rangle = \cos \theta |e\rangle + \sin \theta |g\rangle, \quad (3)$$

$$|-\rangle = -\sin \theta |e\rangle + \cos \theta |g\rangle, \quad (4)$$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\lambda_3}{\omega_0} \right) \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}. \quad (5)$$

If we define the operators $\hat{S}_{\pm,z}$ where $\hat{S}_z = |+\rangle\langle +| - |-\rangle\langle -|$, $\hat{S}_+ = |+\rangle\langle -|$, $\hat{S}_- = |-\rangle\langle +|$, they have the same properties of commutation relation, such as $\hat{\sigma}_z, \hat{\sigma}_+$, and $\hat{\sigma}_-$. Through previous transformations, we can calculate the connection between $\hat{\sigma}_s$ operators and \hat{S}_s operators as follows

$$\begin{bmatrix} \hat{\sigma}_z \\ \hat{\sigma}_x \\ \hat{\sigma}_y \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta & 0 \\ -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{S}_z \\ \hat{S}_x \\ \hat{S}_y \end{bmatrix}. \quad (6)$$

At $\lambda_3 = 0$, i.e., $\theta = 0$, it is easy to see $\langle +| = \langle e|$ and $\langle -| = \langle g|$. After performing the transformation (6) and using the rotating wave approximation, the system (1) becomes

$$H = \sum_{j=1}^2 \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\Omega_0}{2} \hat{S}_z + \lambda \xi(t) (\hat{a}_1 \hat{a}_2 \hat{S}_+ \exp(i\phi(t)) + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{S}_- \exp(-i\phi(t))), \quad (7)$$

where $\Omega_0 = \sqrt{\omega_0^2 + 4\lambda_3^2}$. We note that the effective external field introduces the frequency definition for the atom Ω_0 .

For the calculation of the wave function, it is necessary to calculate the constant of motion by writing the equations of motion by using the Heisenberg equation as follows

$$\frac{d\hat{E}}{dt} = \frac{1}{i\hbar} [\hat{E}, \hat{H}] + \frac{\partial \hat{E}}{\partial t}. \quad (8)$$

Consequently, the operators' motion equations in the model (7) are calculated as follows:

$$\frac{id\hat{S}_z}{dt} = [\hat{S}_z, \hat{H}] = 2\lambda\xi(t)(\hat{a}_1\hat{a}_2\hat{S}_+ \exp(i\phi(t)) - \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{S}_- \exp(-i\phi(t))), \quad (9)$$

$$\frac{id(\hat{a}_j^\dagger\hat{a}_j)}{dt} = [\hat{a}_j^\dagger\hat{a}_j, \hat{H}] = -\lambda\xi(t)(\hat{a}_1\hat{a}_2\hat{S}_+ \exp(i\phi(t)) - \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{S}_- \exp(-i\phi(t))), j = 1, 2 \quad (10)$$

Thus, we deduce that operators $N_j = \frac{\hat{S}_z}{2} + \hat{a}_j^\dagger\hat{a}_j, j = 1, 2$ is the constant of motion. The Hamiltonian (7) turns into:

$$\hat{H} = \omega_1N_1 + \omega_2N_2 + \frac{\Delta}{2}\hat{S}_z + \lambda\xi(t)(\hat{a}_1\hat{a}_2\hat{S}_+ \exp(i\phi(t)) + \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{S}_- \exp(-i\phi(t))), \quad (11)$$

where $\Delta = (\Omega_0 - \omega_1 - \omega_2)$

Wave Function

We are now calculating the wave function to describe the features of this physical model. The initial state of the atom in superposition from the atomic states:

$$|\varphi(0)\rangle_a = c_+|+\rangle + c_-|-\rangle, \text{ with } |c_+|^2 + |c_-|^2 = 1. \quad (12)$$

The field prepared in pair coherent states,

$$|\varphi(0)\rangle_f = \sum_{n=0}^{\infty} r_{n,\eta}|n, q + n\rangle, \quad (13)$$

$$r_{n,\eta} = \sqrt{\left(\frac{1}{|\eta|^q B_q(2|\eta|)}\right) \frac{\eta^n}{\sqrt{n!(q+n)!}}}$$

where $B_q(2\eta)$ is the modified Bessel function. So the formula of the wave function becomes,

$$|\varphi(t)\rangle = \sum_{n=0}^{\infty} r_{n,\eta} \{A(n, t)|n, q + n, +\rangle + B(n, t)|n + 1, q + n + 1, -\rangle\}, \quad (14)$$

Some techniques have been introduced for solving the Schrödinger equation $i\frac{\partial}{\partial t}|\varphi(t)\rangle = \hat{H}|\varphi(t)\rangle$ [27]. For calculating factors $A(n, t)$ and $B(n, t)$ as follows,

$$i\frac{\partial A(n, t)}{\partial t} = \frac{\Delta}{2}A(n, t) + \lambda\Gamma B(n, t)\xi(t) \exp(i\phi(t)),$$

$$i\frac{\partial B(n, t)}{\partial t} = -\frac{\Delta}{2}B(n, t) + \lambda\Gamma A(n, t)\xi(t) \exp(-i\phi(t)), \quad (15)$$

where $\Gamma = \sqrt{(n+1)(n+q+1)}$. Suppose that

$$\chi(t) = \phi(t) - \Delta t,$$

$$X(n, t) = A(n, t) \exp\left(\frac{i\Delta}{2}t\right),$$

$$Y(n, t) = B(n, t) \exp\left(-\frac{i\Delta}{2}t\right),$$

$$\frac{dX(n, t)}{dt} = -i\Gamma\mu \xi(t) \exp(i\chi)Y(n, t),$$

$$\frac{dY(n, t)}{dt} = -i\Gamma\mu \xi(t) \exp(-i\chi)X(n, t), \quad (16)$$

using the integrability condition $\dot{\chi}(t) = c\lambda\xi(t)$, where c is constant, the general solution of the aforementioned equations (16) is

$$A(n, t) = \left\{ c_+ - i\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)^{-\frac{1}{2}} \left(\frac{c-\Gamma}{c} + \frac{c_+}{2}\right) \tan\left(\chi(t)\sqrt{\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)}\right) \right\}$$

$$\times \cos\left(\chi(t)\sqrt{\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)}\right) \exp\left\{-\frac{i}{2}(\Delta t - \chi(t))\right\}, \quad (17)$$

$$B(n, t) = \left\{ c_- + i\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)^{-\frac{1}{2}} \left(\frac{c-\Gamma}{2} - \frac{c_+\Gamma}{c}\right) \tan\left(\chi(t)\sqrt{\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)}\right) \right\}$$

$$\times \cos\left(\chi(t)\sqrt{\left(\frac{\Gamma^2}{c^2} + \frac{1}{4}\right)}\right) \exp\left\{\frac{i}{2}(\Delta t - \chi(t))\right\}. \quad (18)$$

It should be noted that this solution is general, and we will allocate time dependence in the case of velocity and acceleration as follows:

$$\chi(t) = \alpha t + \beta t^2 + \phi \quad (19)$$

After solving the problem generally, we may calculate some of the statistical quantities, which explain and analyze the performance of the system in the presence and absence of the external field. This task can be accomplished in the next sections.

ATOMIC INVERSION

Studying the phenomena of collapse and revival provides us with more details about the behavior of the physical system during periods of strong and weak interaction between the parts of the system. The study of atomic populations is therefore an important quantity in quantum information. It may be perceived as

$$W(t) = \langle \hat{S}_z(t) \rangle \cos 2\theta - \langle \hat{S}_x(t) \rangle \sin 2\theta,$$

$$= \sum_{n=0}^{\infty} |r_{n,\eta}|^2 (|A(n, t)|^2 - |B(n, t)|^2) \cos 2\theta$$

$$- \text{Re} \left\{ \sum_{n=0}^{\infty} r_{n,\eta} r_{n+1,\eta}^* A(n, t) B^*(n+1, t) \right\} \sin 2\theta. \quad (20)$$

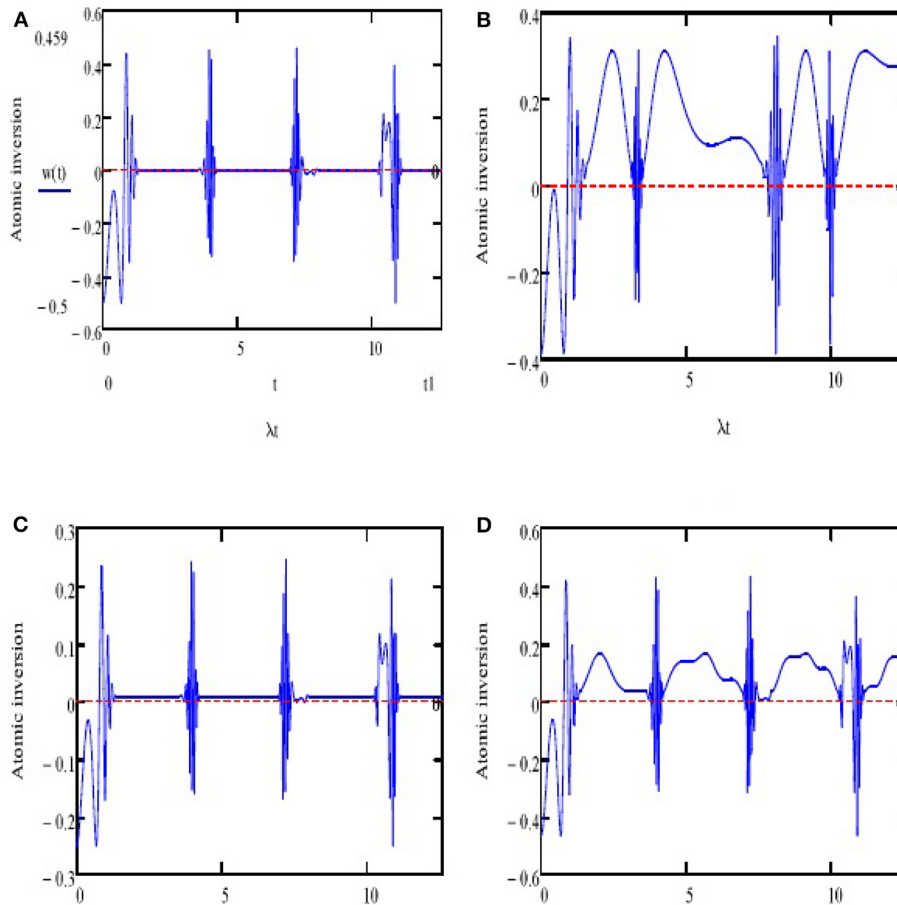


FIGURE 1 | $W(t)$ for the parameters $\eta = 25$, $\phi = \pi/2$ and $\Delta = 0.9$, $c = \alpha = 1$, $\beta = 0.3$. The atom is initially in the upper state $c_+ = 0$ in **(A,B)** and in a superposition state $c_+ = 0.5$ in **(C,D)**. The classical field effect is ignored $\lambda_3 = 0$ in **(A,C)** and is considered for $\lambda_3 = 0.3\lambda$ in **(B,D)**.

where B^* is the complex conjugate of B .

For plotting the atomic inversion, consider the parameters $\phi = \frac{\pi}{2}$, $\Delta = 0.9\lambda$, $c = \alpha = 1$, $\beta = 0.3$, $\eta = 25$. First, neglect the external field ($\lambda_3 = 0$) and $c_+ = 0$; the function shows symmetric oscillation around the horizontal axis, and the phenomena of revivals and collapses are achieved in many intervals (see **Figure 1A**). Note that the collapses almost occur in consideration of the time interaction except for some squeezed intervals for the revivals. We note that the symmetry around the horizontal axis in the aforementioned case fades after adding the external field of the interaction cavity. The fluctuations among the ground and excited states are increased. Therefore, the interactive atom and the field become sturdier (see **Figure 1B**). We also find that the period of collapse oscillates among the ground and excited conditions, which reflect the impact of the external field terms. For case $c_+ = 0.5$ and ($\lambda_3 = 0$), the phenomena of collapse and revival occur occasionally when reducing the amplitude of oscillations as seen in **Figure 1C**. After putting the parameter $\lambda_3 = 0.3\lambda$, we see that the amplitude of oscillations increased and the collapses phenomena never occurred, while the revivals occurred periodically within a period as seen in **Figure 1D**.

TLA-FIELD DEGREE OF ENTANGLEMENT

The field or atomic entropy is one of the most important criteria used to measure the entanglement of the parts of a closed system (no external effects). We therefore calculate the atomic entropy of the previously obtained condition (14).

We consider the ANE formula concerning the eigenvalues of the atomic density matrix $\hat{\rho}_a = \text{tr}_f(|\varphi(t)\rangle\langle\varphi(t)|)$ as

$$S(t) = -\lambda_1 \ln(\lambda_1) - \lambda_2 \ln(\lambda_2). \tag{21}$$

The degree of NLC is obtained according to the value of $S(t)$ when the ANE has a zero value for the atom-field system in the separable state, while the system takes a maximally entangled state for $S = \ln(2)$. The same conditions are used. In the first case, in absence of the classical external field and fixing the parameter $c_+ = 0$, the entanglement function $S(t)$ shows periodical weak entanglement with a higher value, but the lower value is almost the pure state at $n\pi$ (**Figure 2A**). In the second case, consider the classical external field $\lambda_3 = 0.3\lambda$, the function displays consistent fluctuations, and the minimum values reduce (see **Figure 2B**). The function shows a different behavior when we take $\lambda_3 = 0$

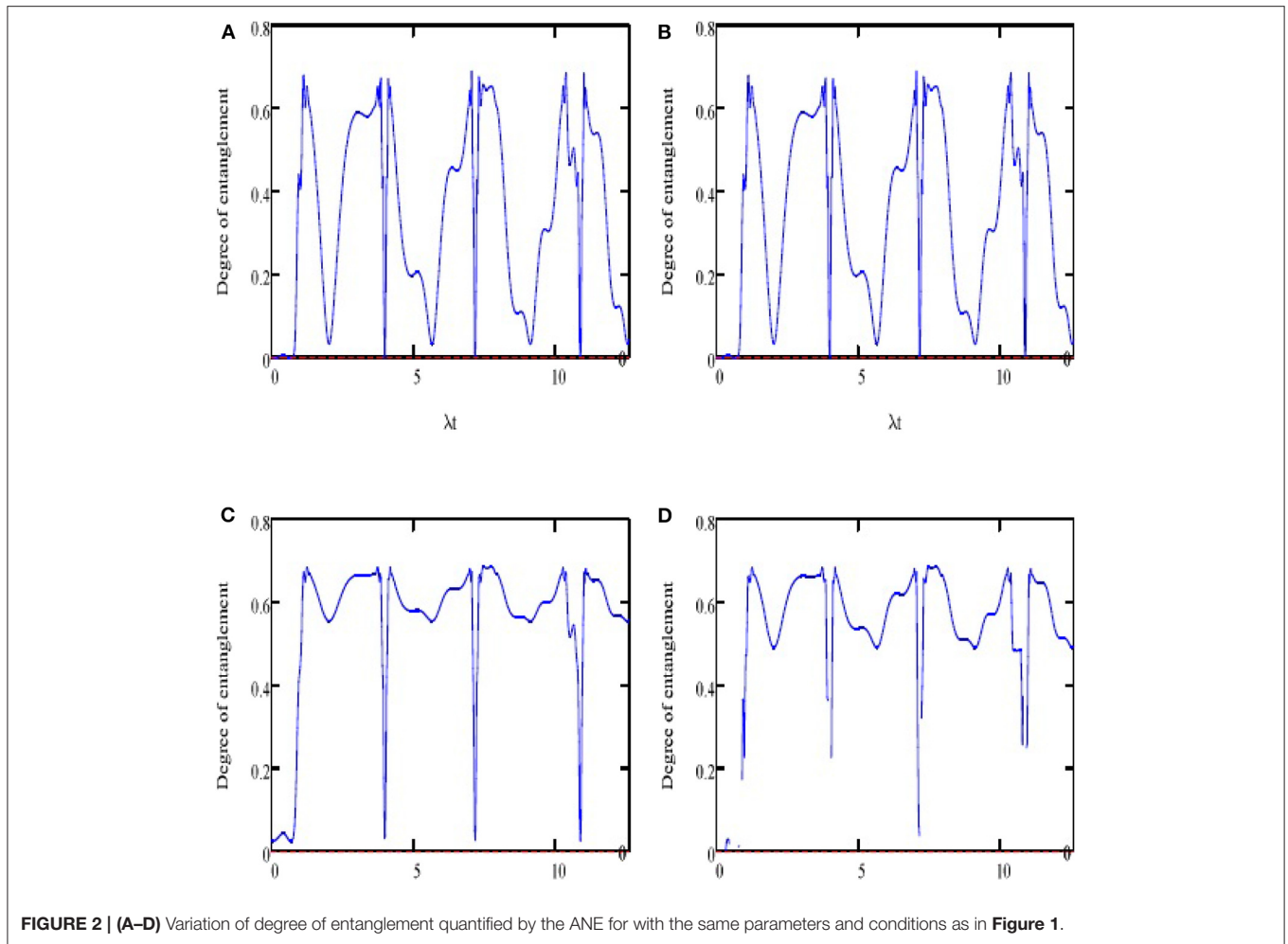


FIGURE 2 | (A–D) Variation of degree of entanglement quantified by the ANE for with the same parameters and conditions as in **Figure 1**.

and $c_+ = 0.5$. The function shows more regular oscillations and reaches pure states at $n\pi$, as shown in **Figure 2C**). When the parameter λ_3 is added to the cavity interaction, the function has the minimum values for the intervals of revivals (see **Figure 2D**).

PHENOMENA OF INFORMATION ENTROPY SQUEEZING

This section investigates the squeezing of classical entropy of the atomic system based on Shannon information entropy $H(\hat{S}_r)$ with regard to the atomic operator of Pauli matrix $\hat{\sigma}_r$ as

$$H(\hat{\sigma}_r) = -\frac{1}{2}(\hat{\rho}_r + 1)\ln\frac{1}{2}(\hat{\rho}_r + 1) - \frac{1}{2}(1 - \hat{\rho}_r)\ln\frac{1}{2}(1 - \hat{\rho}_r), \quad r = x, y, z \tag{22}$$

where $H(\hat{\sigma}_r)$ indicates the Shannon information entropy [23, 28–30], ρ_r is the reduced atomic-density matrix. The squeezing is achieved for the component $\hat{\sigma}_r$ when

$$E_r(t) = \left(\delta H(\hat{\sigma}_\alpha) - \frac{2}{\sqrt{\delta H(\hat{\sigma}_z)}} \right) < 0, \quad \delta H(\hat{\sigma}_r) = \exp(H(\hat{\sigma}_r)), \tag{23}$$

$r = x \text{ or } y,$

the uncertainty relation becomes

$$\Delta(\hat{\sigma}_x) \cdot \Delta(\hat{\sigma}_y) \geq \frac{|\langle \hat{\sigma}_z \rangle|}{2} \tag{24}$$

Figure 3 shows the entropy squeezing when applying the aforementioned conditions. When $A(0) = 0$, irregular squeezing achieves both quadratures $E_x(t)$ and $E_y(t)$. The highest squeezing for quadrature is achieved in the center of the areas of collapse. Quadrature $E_y(t)$ is achieved in the center of the areas of revivals (see **Figure 3A**). In contrast, for $\lambda_3 = 0.3\lambda$ and $c_+ = 0$, the amount of squeezing rises considerably in this case (see **Figure 3B**). When we considered the case in which $\lambda_3 = 0$ and $c_+ = 0.5$, the squeezing is achieved in the quadrature $E_y(t)$ but not on the other. Moreover, the areas of the squeezing reach the lowest values (see **Figure 3C**). The reduction continues in the intervals of squeezing after considering the acceleration ($\lambda_3 = 0.3\lambda$ and $c_+ = 0.5$) (see **Figure 3D**). We thus conclude that the acceleration plays a role in the reduction of the squeezing in $E_x(t)$. In all cases, the squeezing in quadrature $E_y(t)$ is achieved according to the revival intervals.

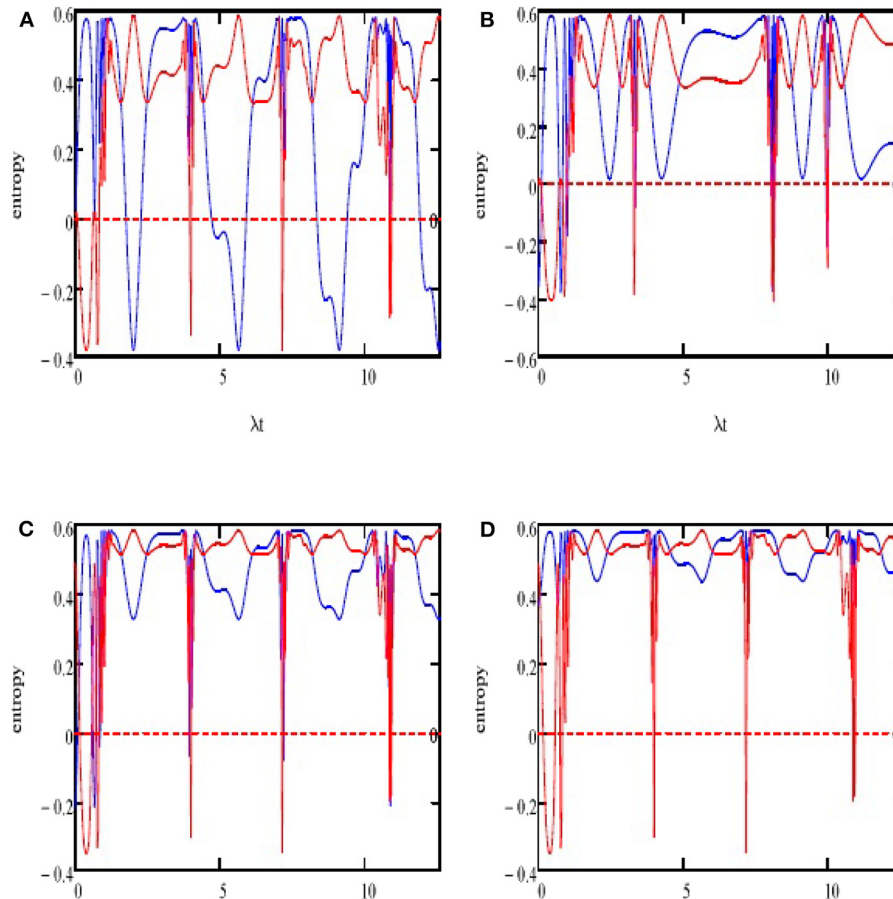


FIGURE 3 | (A–D) Variation of the entropy squeezing components $E_x(t)$ (blue curve) and $E_y(t)$ (red curve) of classical entropy for with the same parameters and conditions as in **Figure 1**.

CONCLUSION

In summary, we have used a quantum model of the interaction between a time-dependent two-level atom and two-mode field in the cavity containing an external field. The general solution is obtained after using the integrability condition. The resulting wave function and atomic density matrix are used to investigate atomic inversion and atomic entropy, respectively. We have shown that the amplitude of oscillations is increased, and the collapsed phenomena never occur, while the revivals occur periodically. Furthermore, we have discussed the effect of the external classical field on the phenomenon of the degree of entanglement the function illustrates systematic fluctuations; the minimum values, however, reduce. Finally, we have found that the external classical field does not affect the entropy squeezing phenomena. Such phenomena benefit in various uses of quantum physics and optics.

DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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