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On Optical Solitons of the Fractional (3+1)-Dimensional NLSE With Conformable Derivatives

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In this article, the fractional (3+1)-dimensional nonlinear Shrödinger equation is analyzed with kerr law nonlinearity. The extended direct algebraic method (EDAM) is applied to obtain the optical solitons of this equation with the aid of the conformable derivative. Optical solitons are investigated for this equation with the aid of the EDAM after the nonlinear Shrödinger equation transforms an ordinary differential equation using the wave variables transformation.

Keywords: optical solitons, nonlinear Shrödinger equation, conformable derivative, Kerr law nonlinearity, the extended direct algebraic method

INTRODUCTION

Over the past few decades, there have been many studies on optical solitons [1–10]. The nonlinear wave process can be viewed in several scientific fields, such as optical fiber, quantum theory, plasma physics, fluid dynamics [11, 12], etc. Solitons are one pulse forms which are created due to the proportion between nonlinearity and wave stage speed dispersal impacts in the system. The envelope soliton, which holds both fast and slow vibrations, performs for nonlinearity proportions with the wave group dispersal impacts in the physical systems. The envelope soliton is controlled by a small field adjusted wave package whose dynamics are controlled via the nonlinear Schrödinger equation (NSE) [1–12]. The analytical solutions of these NPDEs plays a significant part in the analysis of nonlinear phenomena. Over the past few decades, numerous methods were developed to obtain analytical solutions of NPDEs such as the inverse scattering method [13], the Sine—cosine function method [14], the tanh-expansion method, and the Kudryashov-expansion method [15], etc.

There has also been considerable interest and significant theoretical improvements in fractional calculus, applied in many fields, and in fractional differential equations and its applications [16–25]. Nonlinear fractional partial differential equations (FPDEs) are a special type of NPDEs. Several studies have discussed these equations. Additionally, FPDEs are significant in several analyses because of the iterative reporting and the probability explanation process in water wave hypothesis, nonlinear optics, fluid dynamics, plasma physics, optical fiber, quantum mechanics, signal processing, and so on. Several researchers have investigated the wave solutions of NPDEs with the aid of some mathematical algorithms. Besides, one advantage of the conformable fractional derivative is that it is easy to apply [26–34].

The conformable derivative of order $\alpha \in (0,1)$ is defined as the following expression [28]

$$_tD^\alpha f(t) = \lim_{\vartheta \to 0} \frac{f(t + \vartheta\, t^{1-\alpha}) - f(t)}{\vartheta}, \quad f:(0,\infty) \to \text{R}.$$

A few properties for the conformable derivative are given by [28, 31].

$$\begin{aligned}
a)\, _tD^\alpha t^\eta &= \eta t^{\alpha - \eta}, \quad \forall \alpha \in \text{R}, \\
b)\, _tD^\alpha(fg) &= f\,_tD^\alpha g + g\,_tD^\alpha f, \\
c)\, _tD^\alpha(fog) &= t^{1-\alpha} g'(t) f'(g(t)), \\
d)\, _tD^\alpha\left(\frac{f}{g}\right) &= \frac{g\,_tD^\alpha f - f\,_tD^\alpha g}{g^2}.
\end{aligned}$$

Recently, there have been about the conformable model of fractional computations [25–33].

The (3+1)-dimensional dependent NLSE is given by:

$$iq_t + \nabla^2 q + \lambda F(|q|^2)q = 0. \tag{1.1}$$

in [10–12], there are analyzed symmetry reductions for the (3 + 1)-dimensional NLSE.

Then, Equation (1.1) can be scripted for fractional (3+1)-dimensional NLSE with conformable derivatives as:

$$iD_t^\alpha q + \nabla^2 q + \lambda F(|q|^2)q = 0, \ t > 0, \ 0 < \alpha \leq 1. \tag{1.2}$$

where F is a real-valued function and has the fluency of the complex function $F(|q|^2)q : C \to C$. When the $F(|q|^2)q$ is k times continuously differentiable, the following situation can be written,

$$F(|q|^2)q \in \bigcup_{m,n=1}^\infty C^k((-n,n) \times (-m,m); R^2).$$

For Kerr law nonlinearity, Equation (1.2) is converted to

$$iD_t^\alpha q + \nabla^2 q + \lambda |q|^2 q = 0, \ t > 0, \ 0 < \alpha \leq 1. \tag{1.3}$$

In (1.3), the first expression describes the evolution condition, the second expression, describes the dispersal in x, y, and z directions while the third expression describes nonlinearity. Solitons are the consequence of an attentive adjust between dispersal and nonlinearity.

In this work, we analyze the fractional (3+1)-dimensional nonlinear Shrödinger's equation with the aid of a conformable derivative operator to find solitons using the extended direct algebraic method (EDAM) [8, 26].

This method is a powerful in solving nonlinear evolution equations and it can be applied to solve the above mentioned equations. This has led to the innovation of many modern techniques to solve these equations. There are several advantages and disadvantages of this modern method. Although a closed type soliton solution can be found with the aid of this process, the disadvantage of this method is that this technique cannot calculate the conserved quantities of nonlinear evolution equations.

DESCRIPTION FOR THE EXTENDED DIRECT ALGEBRAIC METHOD

Suppose the general nonlinear partial differential equation,

$$U(q, q_t^{(\alpha)}, q_x, q_y, q_z, q_t^{(2\alpha)}, q_{xx}, ...) = 0. \tag{2.1}$$

where $q = q(x, y, z)$, U is a polynomial in $q = q(x, y, z, t)$ and the x, y, z, t define the partial fractional derivatives.

- Assume the traveling wave transformation:

$$q(x, y, z, t) = v(\phi), \qquad \phi = x\cos\xi + y\cos\kappa + z\cos\chi + Q\frac{t^\alpha}{\alpha}, \tag{2.2}$$

where $\cos^2\xi + \cos^2\kappa + \cos^2\chi = 1$.

With the aid of (2.2) wave transformation, Equation (2.1) is changed into an ordinary differential equation for $v(\phi)$:

$$B(v, v_\phi, v_{\phi\phi}, v_{\phi\phi\phi}, ...) = 0. \tag{2.3}$$

where the sub-indices define the ordinary derivatives with respect to ϕ.

- Suppose the solution of Equation (2.3),

$$v(\phi) = \sum_{j=0}^M a_j G^j(\phi), \tag{2.4}$$

where $a_M \neq 0$ and $G(\phi)$ can be satisfied as follows:

$$G'(\phi) = \ln(E)(fG^2(\phi) + gG(\phi) + h), \ E \neq 0, 1, \tag{2.5}$$

where f, g, h are arbitrary constants.

- M is obtained by balancing between the highest order derivatives and the nonlinear terms in Equation (2.3).
- First, Equation (2.4) and Equation (2.5) are placed into Equation (2.3). Then each coefficient of the polynomials are synchronized to zero ve algebraic equations of a_j $(j = 1, 2, ..., M)$, Q and f, g, h are obtained.
- The obtained system is solved and parameters a_j $(j = 1, 2, ..., M)$ and Q are found. Thus, solutions of Equation (2.3) are found.

Where a few specific solutions of Equation (2.3) are given by;

1) When $\Psi = g^2 - 4hf < 0$ and $f \neq 0$,

$$\begin{aligned}
G_1(\phi) &= -\frac{g}{2f} + \frac{\sqrt{-\Psi}}{2f}\tan_E\left(\frac{\sqrt{-\Psi}}{2}\phi\right), \\
G_2(\phi) &= -\frac{g}{2f} + \frac{\sqrt{-\Psi}}{2f}\cot_E\left(\frac{\sqrt{-\Psi}}{2}\phi\right),
\end{aligned}$$

2) When $\Psi = g^2 - 4hf > 0$ and $f \neq 0$,

$$G_3(\phi) = -\frac{g}{2f} + \frac{\sqrt{\Psi}}{2f}\left(-\tanh_E(\sqrt{\Psi}\phi) \pm i\sqrt{\Delta\Omega}\sec h_E(\sqrt{\Psi}\phi)\right),$$

$$G_4(\phi) = -\frac{g}{2f} + \frac{\sqrt{\Psi}}{2f}\left(-\coth_E(\sqrt{\Psi}\phi) \pm \sqrt{\Delta\Omega}\csc h_E(\sqrt{\Psi}\phi)\right),$$

3) When $fh > 0$ and $g = 0$,

$$G_5(\phi) = \sqrt{\frac{h}{f}}(\tan_E(2\sqrt{hf}\phi) \pm \sqrt{\Delta\Omega}\,\sec_E(2\sqrt{hf}\phi)),$$
$$G_6(\phi) = \sqrt{\frac{h}{f}}(-\cot_E(2\sqrt{hf}\phi) \pm \sqrt{\Delta\Omega}\,\csc_E(2\sqrt{hf}\phi)),$$

4) When $fh < 0$ and $g = 0$,

$$G_7(\phi) = -\sqrt{-\frac{h}{f}}\tanh_E(\sqrt{-hf}\phi),$$
$$G_8(\phi) = -\sqrt{-\frac{h}{f}}\coth_E(\sqrt{-hf}\phi),$$

5) When $g = 0$ and $f = h$,

$$G_9(\phi) = \frac{1}{2}(\tan_E(\frac{h}{2}\phi) - \cot_E(\frac{h}{2}\phi)).$$

6) When $g = 0$ and $f = -h$,

$$G_{10}(\phi) = -\frac{1}{2}(\tanh_E(\frac{h}{2}\phi) + \coth_E(\frac{h}{2}\phi)).$$

7) When $g^2 = 4hf$,

$$G_{11}(\phi) = -2h\frac{g\phi\ln(E) + 2}{g^2\phi\ln(E)}.$$

8) When $g \neq 0$ and $h = 0$,

$$G_{12}(\phi) = -\frac{\Delta g}{f(\cosh_E(g\phi) - \sinh_E(g\phi) + \Delta)},$$

Remark. The generalized trigonometric and hyperbolic functions are defined as Ghosh and Nandy [13];

$$\sin_E(\phi) = \frac{\Delta E^{i\phi} - \Omega E^{-i\phi}}{2i}, \cos_E(\phi) = \frac{\Delta E^{i\phi} + \Omega E^{-i\phi}}{2},$$
$$\tan_E(\phi) = -i\frac{\Delta E^{i\phi} - \Omega E^{-i\phi}}{\Delta E^{i\phi} + \Omega E^{-i\phi}}, \cot_E(\phi) = i\frac{\Delta E^{i\phi} + \Omega E^{-i\phi}}{\Delta E^{i\phi} - \Omega E^{-i\phi}},$$
$$\sec_E(\phi) = \frac{2}{\Delta E^{i\phi} + \Omega E^{-i\phi}}, \csc_E(\phi) = \frac{2i}{\Delta E^{i\phi} - \Omega E^{-i\phi}},$$
$$\sinh_E(\phi) = \frac{\Delta E^{\phi} - \Omega E^{-\phi}}{2}, \cosh_E(\phi) = \frac{\Delta E^{\phi} + \Omega E^{-\phi}}{2},$$
$$\tanh_E(\phi) = \frac{\Delta E^{\phi} - \Omega E^{-\phi}}{\Delta E^{\phi} + \Omega E^{-\phi}}, \coth_E(\phi) = \frac{\Delta E^{\phi} + \Omega E^{-\phi}}{\Delta E^{\phi} - \Omega E^{-\phi}},$$
$$\sec h_E(\phi) = \frac{2}{\Delta E^{\phi} + \Omega E^{-\phi}}, \csc h_E(\phi) = \frac{2}{\Delta E^{\phi} - \Omega E^{-\phi}}.$$

where ϕ is an independent variable, $\Delta \neq 0$ and $\Omega \neq 0$ are called deformation parameters.

SOLUTIONS OF TIME FRACTIONAL (3+1)-DIMENSIONAL NLSE WITH KERR LAW NONLINEARITY USING CONFORMABLE DERIVATIVES

Now, suppose the wave variable transform:

$$q(x, y, z, t) = e^{i(a(x\cos\xi + y\cos\kappa + z\cos\chi) + w\frac{t^\alpha}{\alpha})}v(\phi),$$
$$\phi = x\cos\xi + y\cos\kappa + z\cos\chi + Q\frac{t^\alpha}{\alpha}, \quad (3.1)$$

By placing Equation (3.1) into Equation (1.3) and taking the properties of conformable time fractional derivatives into account, the following nonlinear equation is obtained,

$$v''(\phi) + \lambda v(\phi)^3 - (a^2 + w)v(\phi) = 0, \quad (3.2)$$

with $Q = -2a$.

Suppose the solution of Equation (3.2) is expressed as a finite series. We can write this solution as follows,

$$v(\phi) = \sum_{k=0}^{M} a_k G^k(\phi) \quad (3.3)$$

where $G(\phi)$ satisfies Equation (2.5), $\phi = x\cos\xi + y\cos\kappa + z\cos\chi + Q\frac{t^\alpha}{\alpha}$ and a_k for $k = \overline{1, M}$ are values to be described.

With the aid of balance $v''(\phi)$ with $v(\phi)^3$ in Equation (3.3), is found $M = 1$.

We can write the solution of Equation (3.3) in the following form:

$$v(\phi) = a_0 + a_1 G(\phi), \quad (3.4)$$

First, Equation (3.4) and Equation (2.5) are placed into the Equation (3.2). Then each coefficient of the $G(\phi)$ synchronized to zero ve from algebraic equations and the following values are found:

$$a_0 = \frac{ig\ln(E)}{\sqrt{2\lambda}}, a_1 = \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}},$$
$$w = -a^2 - \frac{1}{2}(g^2 - 4fh)\ln^2(E). \quad (3.5)$$

The solutions of Equation (1.3) are found as follows; ($\Upsilon = e^{i(a(x\cos\xi + y\cos\kappa + z\cos\chi) - (a^2 + \frac{1}{2}(g^2 - 4fh)\ln^2(E))\frac{t^\alpha}{\alpha})}$ and $\Psi = g^2 - 4hf$)

1) When $\Psi < 0$ and $f \neq 0$, the singular periodic solutions are obtained as follows

$$q_1 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\frac{g}{2f} + \frac{\sqrt{-\Psi}}{2f}\tan_E(\frac{\sqrt{-\Psi}}{2}\phi))),$$
$$q_2 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\frac{g}{2f} + \frac{\sqrt{-\Psi}}{2f}\cot_E(\frac{\sqrt{-\Psi}}{2}\phi))),$$

2) When $\Psi > 0$ and $f \neq 0$, the singular soliton solutions are obtained as follows

$$q_3 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\tanh_E(\sqrt{\Psi}\phi)$$
$$\pm i\sqrt{\Delta\Omega}\,\sec h_E(\sqrt{\Psi}\phi))),$$
$$q_4 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\coth_E(\sqrt{\Psi}\phi)$$
$$\pm \sqrt{\Delta\Omega}\,\csc h_E(\sqrt{\Psi}\phi))),$$

3) When $g = 0$ and $fh > 0$, the singular periodic solutions are obtained as follows

$$q_5 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(\sqrt{\frac{h}{f}}(\tan_E(2\sqrt{hf}\phi)$$
$$\pm \sqrt{\Delta\Omega}\,\sec_E(2\sqrt{hf}\phi)))),$$
$$q_6 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(\sqrt{\frac{h}{f}}(-\cot_E(2\sqrt{hf}\phi)$$
$$\pm \sqrt{\Delta\Omega}\,\csc_E(2\sqrt{hf}\phi)))),$$

4) When $g = 0$ and $fh < 0$, the singular and dark soliton solutions are obtained as follows

$$q_7 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\sqrt{-\frac{h}{f}}\tanh_E(\sqrt{-hf}\phi))),$$

$$q_8 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\sqrt{-\frac{h}{f}}\coth_E(\sqrt{-hf}\phi))),$$

5) When $g = 0$ and $f = h$, the singular periodic solution is obtained as follows

$$q_9 = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(\frac{1}{2}(\tan_E(\frac{h}{2}\phi) - \cot_E(\frac{h}{2}\phi)))),$$

6) When $g = 0$ and $f = -h$, the combined soliton solution is obtained as follows

$$q_{10} = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\frac{1}{2}(\tanh_E(\frac{h}{2}\phi) + \coth_E(\frac{h}{2}\phi)))),$$

7) When $g^2 = 4hf$, the rational solution is obtained as follows

$$q_{11} = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} + \frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-2h\frac{\phi\ln(E)g + 2}{g^2\phi\ln(E)})),$$

8) When $h = 0$ and $g \neq 0$, the singular soliton is obtained as follows

$$q_{12} = \Upsilon(\frac{ig\ln(E)}{\sqrt{2\lambda}} +$$
$$\frac{i\sqrt{2}f\ln(E)}{\sqrt{\lambda}}(-\frac{\Delta g}{f(\cosh_E(g\phi) - \sinh_E(g\phi) + \Delta)})),$$

GRAPHICAL EXPRESSION OF THE SOLUTIONS

In this section we draw 2D and 3D graphics for some of the solutions obtained in the previous section. We obtained these graphics using Matlab. In **Figures 1**, **2**, we show some numerical models and q_1 and q_4. 3D plots are drawn for $-10 \leq x \leq 10$, $-10 \leq t \leq 10$. 2D plots are drawn for $x = 0.1$.

The above graphics were drawn for $h = 1, f = 2, g = 1, E = 2.7, a = 0.5, \lambda = 2, \xi = \kappa = \frac{\pi}{2}, \alpha = 0.8, \Delta = \Omega = 1$ in (a) and for $h = 1, f = -1, g = 0, E = 2.7, a = 0.5, \lambda = 2, \xi = \kappa = \frac{\pi}{2}, \alpha = 0.8, \Delta = \Omega = 1$ in (b).

We obtained the sum of solutions found for the fractional (3+1)-dimensional NLSE with kerr law nonlinearities via the conformable fractional derivative operator. In addition, we presented some graphics of solutions in **Figures 1**, **2**.

CONCLUSION

In this article, the EDAM is applied to find new soliton solutions for the (3+1)-dimensional NLSE with kerr law
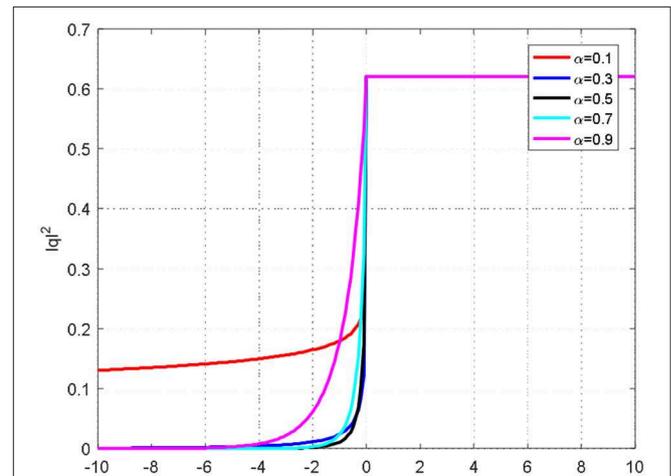

FIGURE 2 | The 2D graphic of the (3+1)-dimensional NLSE with kerr law non-linearities for a different value of α.
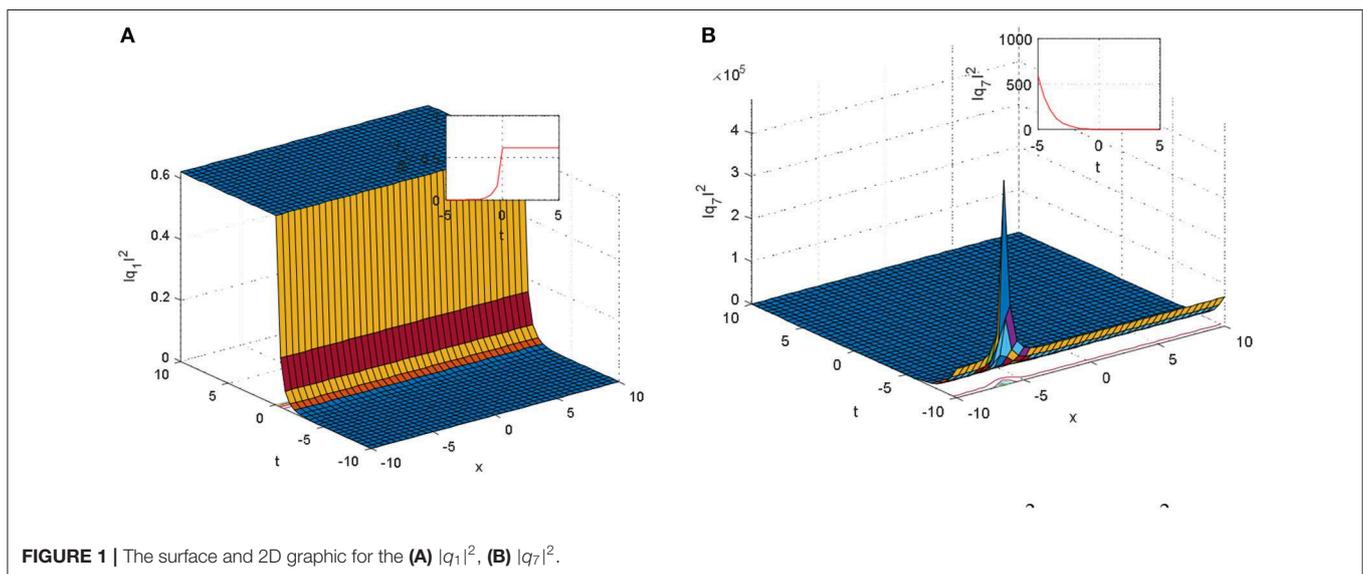

FIGURE 1 | The surface and 2D graphic for the **(A)** $|q_1|^2$, **(B)** $|q_7|^2$.

nonlinearities, with the aid of the conformable fractional derivative operator. The dark, bright, and combined optical solitons are obtained. There are 12 different situations in these solutions. The existence of solutions obtained from these functions are all stipulated through limitation states that are also listed in addition to the solutions. Some interesting figures are also presented in **Figures 1**, **2**. The method applied in this article is appropriate to investigate several problems that are face in the fields of engineering and science.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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