



# **Corrigendum: Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File**

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### A Corrigendum on

Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File by Lapolla, A., and Godec, A. (2019). Front. Phys. 7:182. doi: 10.3389/fphy.2019.00182

In the original article, there was an error. In section 2.1 the diffusion matrix **D** the in-line equation was defined with a factor of 2 instead of 1/2, i.e.,  $\mathbf{D} = 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T$  instead of  $\mathbf{D} = \boldsymbol{\sigma}\boldsymbol{\sigma}^T/2$ .

In section 2.1 in the paragraph following Equation (4), a copy-paste error occurred in the sentence "... where for reversible system (i.e., those obeying detailed balance) we have  $\hat{\mathcal{L}}\hat{\mathcal{L}}^{\dagger} = \hat{\mathcal{L}}^{\dagger}\hat{\mathcal{L}} = 0$ ."

In section 2.2. in the paragraph following Equation (13) there is an obvious redundant additional factor  $\Psi_{00}^{-1}(\mathbf{q})d\mathbf{q}$  present immediately after the in-line equation:  $\langle \Psi_{l0}|\Psi_{k0}\rangle_{\Psi_{00}^{-1}} \equiv$ 

 $\int_{\Xi} d\mathbf{q}' \Psi_{00}(\mathbf{q}')^{-1} \Psi_{0l}(\mathbf{q}') \Psi_{k0}(\mathbf{q}').$ 

A correction has been made to section 2.1 [paragraph following Equation (1)]. The paragraph now reads:

"where **D** is the symmetric positive-definite diffusion matrix.  $\hat{\mathcal{L}}$  propagates probability measures  $\mu_t(\mathbf{x})$  in time, which will throughout be assumed to posses well-behaved probability density functions  $P(\mathbf{x}, t)$ , i.e.,  $d\mu_t(\mathbf{x}) = P(\mathbf{x}, t)d\mathbf{x}$  [thereby posing some restrictions on  $\mathbf{F}(\mathbf{x})$ ]. On the level of individual trajectories Equation (1) corresponds to the Itô equation  $d\mathbf{x}_t = \mathbf{F}(\mathbf{x}_t)dt + \boldsymbol{\sigma} d\mathbf{W}_t$  with  $\mathbf{W}_t$  being a *d*-dimensional vector of independent Wiener processes whose increments have a Gaussian distribution with zero mean and variance dt, i.e.,  $\langle dW_{t,i}dW_{t',j}\rangle = \delta_{ij}\delta(t-t')dt$ , and where  $\boldsymbol{\sigma}$  is a  $d \times d$  symmetric noise matrix such that  $\mathbf{D} = \boldsymbol{\sigma} \boldsymbol{\sigma}^T/2$ . Moreover, we assume that  $\mathbf{F}(\mathbf{x})$  admits the following decomposition into a potential (irrotational) field  $-\mathbf{D}\nabla\varphi(\mathbf{x})$  and a non-conservative component  $\vartheta(\mathbf{x})$ ,  $\mathbf{F}(\mathbf{x}) = -\mathbf{D}\nabla\varphi(\mathbf{x}) + \vartheta(\mathbf{x})$  with the two fields being mutually orthogonal  $\nabla\varphi(\mathbf{x}) \cdot \vartheta(\mathbf{x}) = 0$  [73]. By insertion into Equation (1) one can now easily check that  $\hat{\mathcal{L}}e^{-\varphi(\mathbf{x})} = 0$ , such that the stationary solution of the Fokker-Planck equation (also referred to as the steady state [74, 75], which is the terminology we adopt here) by construction does not depend on the non-conservative part  $\vartheta(\mathbf{x})$ ."

A correction has been made to the aforementioned sentence in section 2.1, in the paragraph following Equation (4), which now reads:

"such that the conditional probability density starting from a general initial condition  $|p_0\rangle$ becomes  $P(\mathbf{x}, t|p_0, 0) = \langle \mathbf{x} | \hat{U}(t) | p_0 \rangle \equiv \int d\mathbf{x}_0 p_0(\mathbf{x}_0) G(\mathbf{x}, t | \mathbf{x}_0, 0)$ . Moreover, as  $\mathbf{F}(\mathbf{x})$  is assumed to be sufficiently confining (i.e.,  $\lim_{\mathbf{x}\to\infty} P(\mathbf{x}, t) = 0$ ,  $\forall t$  sufficiently fast), such that  $\hat{\mathcal{L}}$  corresponds to a coercive and densely defined operator on V (and  $\hat{\mathcal{L}}^{\dagger}$  on W, respectively) [76–78]. Finally,  $\hat{\mathcal{L}}$  is throughout assumed to be *normal*, i.e.,  $\hat{\mathcal{L}}^{\dagger}\hat{\mathcal{L}} - \hat{\mathcal{L}}\hat{\mathcal{L}}^{\dagger} = 0$  and thus henceforth V = W, where for reversible system (i.e., those obeying detailed balance) we have  $\hat{\mathcal{L}} \Leftrightarrow \hat{\mathcal{L}}^{\dagger}$ ".

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Lapolla A and Godec A (2020) Corrigendum: Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File. Front. Phys. 8:7. doi: 10.3389/fphy.2020.00007 Finally, the redundant factor  $\Psi_{00}^{-1}(\mathbf{q})d\mathbf{q}$  has been deleted in section 2.2 in the paragraph following Equation (13).

"can be equal to  $Q_{p_{ss}}(\mathbf{q}, t|\mathbf{q}_0, 0)$ . As this will generally not be the case this essentially means that the projected dynamics is in general non-Markovian. The proof is established by noticing that  $\Psi_{kl}(\mathbf{q}') = \Psi_{lk}^{\dagger}(\mathbf{q}')$  such that  $\langle \Psi_{l0}|\Psi_{k0}\rangle_{\Psi_{00}^{-1}} \equiv \int_{\Xi} d\mathbf{q}' \Psi_{00}(\mathbf{q}')^{-1} \Psi_{0l}(\mathbf{q}') \Psi_{k0}(\mathbf{q}')$ ."

The authors apologize for this error and state that this does not change the scientific conclusions

of the article in any way. The original article has been updated.

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