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Dark-Bright Optical Soliton and Conserved Vectors to the Biswas-Arshed Equation With Third-Order Dispersions in the Absence of Self-Phase Modulation

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The form-I version of the new celebrated Biswas-Arshed equation is studied in this work with the aid of complex envelope ansatz method. The equation is considered when self-phase is absent and velocity dispersion is negligibly small. New Dark-bright optical soliton solution of the equation emerge from the integration. The acquired solution combines the features of dark and bright solitons in one expression. The solution obtained are not yet reported in the literature. Moreover, we showed that the equation possess conservation laws (Cls).

Keywords: complex envelope ansatz, dark-bright optical soliton, Biswas-Arshed equation, conserved vectors, multiplier, numerical simulations

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1. INTRODUCTION

The study of dynamical systems in non-linear physical models plays an important role in optical fibers, electrical transmission lines, plasma physics, mathematical biology, and many more [1]. This is motivated by the capacity to model the behavior of these systems and other under different physical conditions [2]. These systems are represented by non-linear equations. Seeking the exact solutions of non-linear evolution equations has been an interesting topic in mathematical physics, and the solutions of corresponding models are the ways to well describe their dynamics. Several results have been reported in the last few decades [3–24]. The main principle for the existence of solitons in metamaterials, optical fibers, and crystals is the existence of a balance between non-linearity and dispersion. It is obvious that some situations may lead to. Recently, Biswas and Arshed [3] put forward a new model for soliton transmission in optical fibers in the event when self-phase modulation is neglible in the absence of non-linearity.

The third-order model in the absence of self-phase modulation that will be studied in this paper is given by [3, 4]:

$$i\psi_t + \alpha\psi_{xx} + \gamma\psi_{xt} + i[\sigma\psi_{xxx} + \delta\psi_{xxt}] - i[\Omega(|\psi|^2\psi)_x + \mu(|\psi|^2)_x\psi + \theta|\psi|^2\psi_x] = 0. \tag{1}$$

The function $\psi(x,t)$ representing the dependent involving t an x which denotes the temporal and spatial components. The first term represents the temporal evolution of the wave, γ represents the(STD) coefficient, α is the coefficient of GVD and σ is the coefficient of the third order dispersion, δ is the coefficient of spatio-temporal 3OD (ST-3OD), Ω is the effect of self-steepening. Finally, μ and θ provide the effect of non-linear dispersion. Dark, bright, combo and singular soliton solutions of Equation (1) have been reported in Biswas and Arshed [3] and Ekici and Sonmezoglu [4]. But to the best of our knowledge, the dark-bright optical soliton and Cls of the equation have not been reported. In this work, this special solution combining the features of dark and bright optical soliton in one expression will be recovered by applying a suitable ansatz. The Cls of the equation will be derived using the multiplier method [8, 9].

2. DARK-BRIGHT OPTICAL SOLITON

In order derive the dark-bright soliton solution of the equation, we consider the ansatz solution given by Li et al. [5]:

$$\psi(x,t) = A(x,t) \times e^{i\Psi(x,t)}, \tag{2}$$

with

$$\Psi(x,t) = -kx + \omega t + \nu. \tag{3}$$

In Equation (2), Ψ denotes the phase shift, k denotes the wave number, ω represents the frequency and ν is the phase constant. We now utilize the ansatz put forward from Li et al. [5]:

$$A(x,t) = \mathrm{i}\beta + \lambda \tanh[\eta(x-vt)] + \mathrm{i}\rho \operatorname{sech}[\eta(x-vt)], \tag{4}$$

where v represents the velocity and η is the pulse width. In the even when $\lambda \to 0$ or $\rho \to 0$, the Equation (4) transforms to a bright or dark soliton solution. The intensity of $A(x,t)$ is given by

$$|A(x,t)| = \left\{\lambda^2 + \beta^2 + 2\beta\rho \operatorname{sech}[\eta(x-vt)] + (\rho^2 - \lambda^2)\operatorname{sech}^2[\eta(x-vt)]\right\}^{\frac{1}{2}}. \tag{5}$$

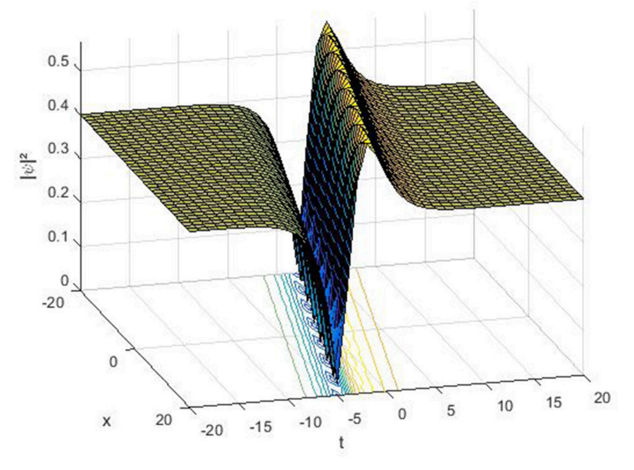

FIGURE 1 | 3D surface of solution Equation(24) by selecting the parameter values of $\eta = 0.1, \lambda = 0.1, \theta = 1, \Omega = 0.1$.

The non-linear phase shift ψ_{NL} is represented by

$$\psi_{NL} = \arctan\left[\frac{\beta + \rho \operatorname{sech}[\eta(x-vt)]}{\lambda \tanh[\eta(x-vt)]}\right]. \tag{6}$$

Putting Equation (2) into Equation (1) leads to

$$\begin{aligned} &-A\left(\omega + k(k(\alpha+k\sigma) - (\gamma+k\delta)\omega) + k(\theta+\mu+\Omega)\,|A|^2 + 2i\Omega A A_x\right) - \\ &i\left((-1+k(\gamma+k\delta))A_t + \left(k(2\alpha+3k\sigma) - (\gamma+2k\delta)\omega + (\theta+\mu+\Omega)\,|A|^2\right)A_x + \right. \\ &\left. i\left((\gamma+2k\delta)A_{xt} + (\alpha+3k\sigma-\delta\omega)A_{xx} + i\left(\delta A_{xxt} + \sigma A_{xxx}\right)\right)\right) = 0. \end{aligned} \tag{7}$$

Now, putting Equation(4) into Equation(7), expanding the result and equating the combination of coefficients of $\operatorname{sech}(\tau)$ and $\tanh(\tau)$, we acquire the independent parametric equations represented by:

Constants:

$$\begin{aligned} &-i\beta\left(\omega + k\left(-\gamma\omega + k(\alpha+k\sigma-\delta\omega) + \left(\beta^2+\lambda^2\right)\right.\right. \\ &\left.\left.(\theta+\mu+\Omega)\right)\right) = 0, \end{aligned} \tag{8}$$

sech(τ) :

$$\begin{aligned} &-i\rho\left(\omega + k\left(-\gamma\omega + k(\alpha+k\sigma-\delta\omega) + \left(3\beta^2+\lambda^2\right)\right.\right. \\ &\left.\left.(\theta+\mu+\Omega)\right)\right) = 0, \end{aligned} \tag{9}$$

sech$^2(\tau)$:

$$\begin{aligned} &i(\nu(-1+k(\gamma+k\delta))\eta\lambda - 3k^2\eta\lambda\sigma - \eta\lambda \\ &(-\gamma\omega + (\beta^2+\lambda^2)(\theta+\mu+\Omega)) + \\ &k(-2\alpha\eta\lambda + 2\delta\eta\lambda\omega + \beta(\lambda^2 - 3\rho^2)(\theta+\mu+\Omega))) = 0, \end{aligned} \tag{10}$$

sech$^3(\tau)$:

$$\begin{aligned} &i\rho(-\alpha\eta^2 + \nu(\gamma+2k\delta)\eta^2 - 2\beta\eta\theta\lambda + k\theta\lambda^2 - 2\beta\eta\lambda\mu \\ &+k\lambda^2\mu - k\theta\rho^2 - k\mu\rho^2 - 3k\eta^2\sigma + \\ &\delta\eta^2\omega - 2\beta\eta\lambda\Omega + k\lambda^2\Omega - k\rho^2\Omega) = 0, \end{aligned} \tag{11}$$

sech$^4(\tau)$:

$$i\eta\lambda\left(2\eta^2(\nu\delta-\sigma) + (\lambda-\rho)(\lambda+\rho)(\theta+\mu+\Omega)\right) = 0, \tag{12}$$

tanh(τ) :

$$\begin{aligned} &-\lambda\left(\omega + k\left(-\gamma\omega + k(\alpha+k\sigma-\delta\omega) + \left(\beta^2+\lambda^2\right)\right.\right. \\ &\left.\left.(\theta+\mu+\Omega)\right)\right) = 0, \end{aligned} \tag{13}$$

tanh(τ)sech(τ) :

$$\begin{aligned} &\rho(\nu(-1+k(\gamma+k\delta))\eta - 3k^2\eta\sigma - 2k(\alpha\eta - \delta\eta\omega \\ &+\beta\lambda(\theta+\mu+\Omega)) - \eta(-\gamma\omega + \\ &\lambda^2(\theta+\mu+\Omega) + \beta^2(\theta+\mu+3\Omega))) = 0, \end{aligned} \tag{14}$$

tanh(τ)**sech**$^2(\tau)$ **:**

$$(-2\alpha\eta^2\lambda + 2\nu(\gamma + 2k\delta)\eta^2\lambda + k\theta\lambda^3 + k\lambda^3\mu - 2\beta\eta\theta\rho^2 - k\theta\lambda\rho^2 - 2\beta\eta\mu\rho^2 - k\lambda\mu\rho^2 - 6k\eta^2\lambda\sigma + 2\delta\eta^2\lambda\omega + \left(\lambda^2(2\beta\eta + k\lambda) - (6\beta\eta + k\lambda)\rho^2)\Omega\right) = 0, \tag{15}$$

tanh(τ)**sech**$^3(\tau)$ **:**

$$\eta\rho\left(5\eta^2(\nu\delta - \sigma) + (\lambda - \rho)(\lambda + \rho)(\theta + \mu + 3\Omega)\right) = 0, \tag{16}$$

tanh$^2(\tau)$**sech**(τ) **:**

$$-i\eta\rho(\eta(-\alpha + \nu(\gamma + 2k\delta) - 3k\sigma + \delta\omega) - 2\beta\lambda\Omega) = 0, \tag{17}$$

tanh$^2(\tau)$**sech**$^2(\tau)$ **:**

$$-2i\eta\lambda\left(2\eta^2(\nu\delta - \sigma) + (\lambda - \rho)(\lambda + \rho)\Omega\right) = 0, \tag{18}$$

tanh$^3(\tau)$**sech**(τ) **:**

$$\eta^3\rho(-\nu\delta + \sigma) = 0, \tag{19}$$

where $\tau = \eta(x - \nu t)$. From the solution of Equations(8)–(19),we observed that $\beta = 0$. but, for a dark-bright optical soliton to exist, we require both $\rho \neq 0$ and $\lambda \neq 0$. For the sake of compatibility, we considered the case when $\rho = \lambda$ from the solutions of Equations(8)–(19). We acquire the velocity as

$$\nu = -\rho^2(\theta + \mu + \Omega), \tag{20}$$

the wave number is represented by

$$k = -\frac{\omega}{\rho^2(\theta + \mu + \Omega)}. \tag{21}$$

We also acquire the value of δ and α as

$$\delta = -\frac{\sigma}{\rho^2(\theta + \mu + \Omega)}, \tag{22}$$

$$\alpha = -\gamma\rho^2(\theta + \mu + \Omega). \tag{23}$$

The dark-bright optical soliton to the model reads:

$$\psi(x,t) = \left\{i\lambda\mathrm{sech}[\eta(x + t\lambda^2(\theta + \mu + \Omega))] + \lambda\tanh[\eta(x + t\lambda^2(\theta + \mu + \Omega))]\right\} \times e^{i(\theta + t\omega + \frac{x\omega}{\lambda^2(\theta+\mu+\Omega)})}. \tag{24}$$

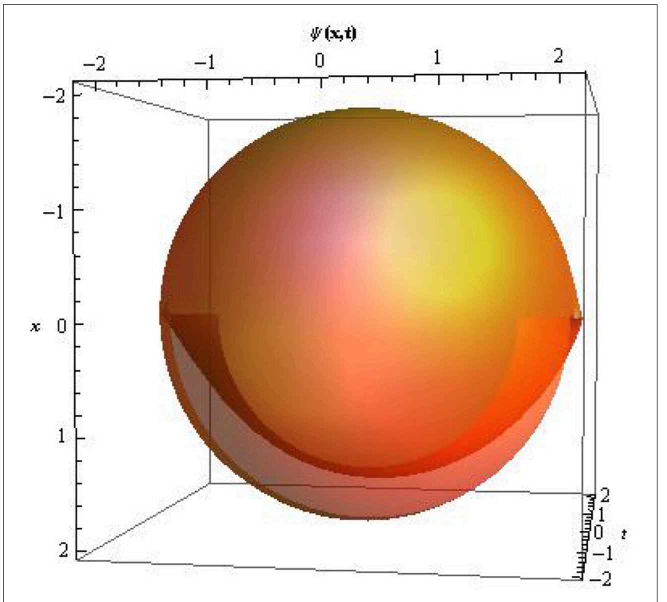

FIGURE 3 | contour plot in spherical coordinates of solution Equation (24) by selecting the parameter values of $\eta = 0.1, \lambda = 0.1, \theta = 1, \Omega = 0.1$.

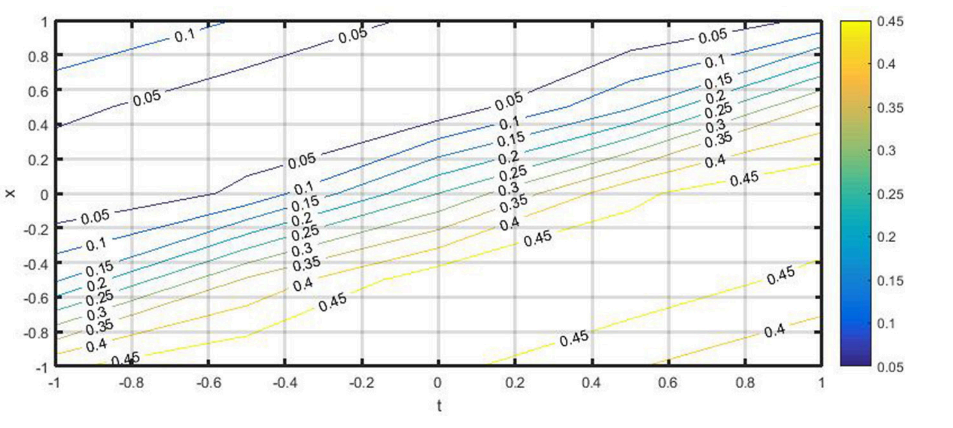

FIGURE 2 | The contour surface of solution Equation (24) by selecting the parameter values of $\eta = 0.1, \lambda = 0.1, \theta = 1, \Omega = 0.1$.

and the intensity gives

$$|\psi(x,t)|^2 = \lambda^2. \tag{25}$$

The phase shift is represented by

$$\psi_{NL} = \arctan\left[\frac{\operatorname{sech}[\eta(x+t\lambda^2(\theta+\mu+\Omega)]}{\tanh[\eta(x+t\lambda^2(\theta+\mu+\Omega)]}\right]. \tag{26}$$

The dark-bright soliton Equation (24) represents a soliton combining the features of dark and bright solitons in one expression. The constant $\beta = 0$ implies a pronounced "platform" underneath the soliton under non-zero boundary conditions and its asymptotic value approaches λ as $|t| \to \infty$. To analyze the dynamics behavior of the soliton solution Equation (24), we have made numerical evolutions for some perturbations to show the evolution of the dark-bright optical soliton solution. **Figures 1-3** shows the profiles surfaces of the dark-bright soliton Equation (24). The obtained soliton Equation (24) possesses the structure of the physical properties of dark and bright optical solitons in the same expression. These solitons appear temporal solitons observed in optical fibers.

3. CONSERVATION LAWS

In this part, we will utilize the multiplier to derive the Cls [8, 9]. To achieve this aim, we apply

$$\psi(x,t) = u(x,t) + i\upsilon(x,t), \tag{27}$$

to transform Equation (1) to a system of PDEs. Putting Equation (27) into Equation (1), we acquire:

$$\begin{cases} -\upsilon_t + 2(\mu+\Omega)u\upsilon u_x + (\theta+\Omega)u^2\upsilon_x + (\theta+2\mu+3\Omega)\upsilon^2\upsilon_x \\ +\gamma u_{xt} + \alpha u_{xx} - \delta\upsilon_{xxt} - \sigma\upsilon_{xxx} = 0. \\ \\ u_t + (-\theta-2\mu-3\Omega)u^2u_x + (-\theta-\Omega)\upsilon^2u_x - 2(\mu+\Omega)u\upsilon\upsilon_x \\ +\gamma\upsilon_{xt} + \alpha\upsilon_{xx} + \delta u_{xxt} + \sigma u_{xxx} = 0. \end{cases} \tag{28}$$

Applying the formula for determining equations in [9], we acquires the multipliers of zeroth-order $\Lambda_1(x,t,u,\upsilon), \Lambda_2(x,t,u,\upsilon)$ for Equation (1)

$$\begin{aligned} \Lambda_1 &= c_1 u, \\ \Lambda_2 &= c_1 \upsilon, \end{aligned} \tag{29}$$

where c_1 is a constant.

1. If $c_1 = 1$ in Equation(29), then we have the following multipliers:

$$\Lambda^1 = u, \Lambda^2 = \upsilon. \tag{30}$$

Subsequently, we acquire the fluxes given by:

$$\begin{aligned} \mathbf{T^x} &= \frac{-\delta(uu_{xx}-\upsilon\upsilon_{xx})}{\sigma}, \\ \mathbf{T^t} &= \frac{u^3tu_t(3\Omega+2\mu+\theta)+\sigma(uu_{xx}+\upsilon\upsilon_{xx})}{\sigma}. \end{aligned} \tag{31}$$

4. CONCLUDING REMARKS

In this article, we have explored a suitable ansatz solution to derive a dark-bright soliton solution of the new celebrated Biswas-Arshed equation. observing the solutions derived in Biswas and Arshed [3] and Ekici and Sonmezoglu [4], we observed that the solution of the equation acquired in this manuscript is new. The method used here has been proved to be efficient in investigating the combined soliton solution of non-linear models. We finally showed that the equation has conservation laws and we reported the conserved vectors. We hope to apply other techniques to extract additional new forms of solutions of the new model in the future.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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