



# On Electron Adiabaticity in Collisionless Shocks

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## 1. INTRODUCTION

Collisionless shocks are considered as one of the most efficient phenomena for the electron acceleration in space and astrophysical plasmas, yet the detailed acceleration process and heating mechanism remain unsolved. Various acceleration and heating mechanisms have so far been proposed and discussed to explain the spacecraft measurements in situ at Earth's bow shock, involving the large-scale electromagnetic fields, plasma turbulence, and wave-particle interactions. A key element of the acceleration and heating mechanisms is the spatial scale of the shock transition with respect to the gyro-radius of the electrons [1]. In the simplest picture, since the shock transition occurs on a spatial scale of ion gyro-radius, the electrons behave as magnetized at the shock transition.

The cross-shock potential (electrostatic potential drop across the shock transition) plays two distinct roles in electron dynamics [2]. First, the cross-shock potential reduces a certain amount of the upstream flow kinetic energy (mostly carried by the ions due to their mass); Second, the kinetic energy is used to heat the electrons (DC heating). On the other hand, the electrons can also be scattered (or demagnetized) by high-amplitude, randomly-oscillating electromagnetic fields such as turbulence or strong inhomogeneities (AC heating). Space turbulence can potentially scatter the electrons randomly leaving upstream or downstream of the shock ramp region. For a quasi-perpendicular shock, the electrons become scattered in a non-adiabatic sense perpendicular to the mean magnetic field (Figure 1).

In this paper, the adiabaticity in collisionless shocks is addressed to the studies based on particle-in-cell-simulation (PIC). Two important parameters enter a PIC simulation carried out for the study of a collisionless shock: the mass ratio  $m_p/m_e$  between ions and electrons and the ratio of the electron plasma frequency to gyrofrequency  $\omega_{pe}/\Omega_{ce}$ , see e.g., [3]. Due to the computational limitations, the usage of the realistic values for the both parameters is not yet possible. The common way to overcome this problem is to find a compromise for the two parameters that can mostly optimize the purpose of the study. The impact of using low frequency ratio of the electron plasma frequency to gyrofrequency in numerical simulations has been discussed in Krasnoselskikh et al. [4]: the electric field is considerably overestimated than it should be for a comparison with the Earth bow shock. Here, I address the question, "What is the consequence of using a smaller frequency ratio upon the adiabatic motion of the electrons"?

## 2. AUTHOR'S OPINION

### 2.1. Adiabaticity and PIC (Particle-in-Cell) Simulations

Adiabatic electrons are characterized by a constant magnetic moment  $\mu = \frac{mv_{\perp}^2}{2B}$  and a particle energy (which is a sum of the electrostatic potential and the kinetic energy)  $\mathcal{E} = e\Phi + \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2$ , where  $m$  is the electron mass,  $v_{\parallel}$  and  $v_{\perp}$  the particle velocity parallel and perpendicular to the

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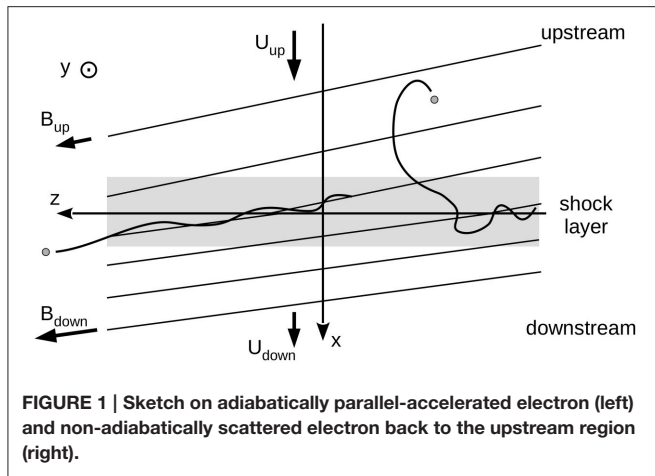
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local magnetic field with a magnitude of the magnetic field  $B$ , respectively,  $e$  the elementary charge, and  $\Phi$  the potential. The potential is electrostatic, and therefore, depends on the frame of Goodrich and Scudder [1]. Convenient choices are the references attached to the shock front such as the shock is in rest. In particular, the normal incident frame (NIF) and the de Hoffmann-Teller frame (HT) are useful in analyzing the electron dynamics in the shock transition layer. The both frames are inertial. On the assumption of a planar geometry and time stationarity, the shock transition reduces to one-dimensional variations along the shock normal direction. NIF sets the incoming flow aligned with the normal direction of the shock. HT minimizes the motional (or convective) electric field by introducing a slide with a speed tangential to the shock.

The gyro-radius of a magnetized electron changes with the magnitude of the magnetic field as  $\rho \propto B^{-1/2}$ , where  $\rho$  denotes the electron gyro-radius. A sharp gradient of the magnetic field can demagnetize the electrons when the gradient scale is smaller than the gyro-radius. The particle becomes unmagnetized and it is perpendicular accelerated by the cross-shock potential.

Naively speaking, since a lower value of the frequency ratio  $\omega_{pe}/\Omega_{ce}$  means a higher gyro-frequency, the electrons should be more magnetized. Here I use a condition for the non-adiabatic electrons as [5],

$$\tilde{R}_{NA} = \frac{m_e c^2}{e B^2} \left| \frac{dE_x}{dx} \right| \geq 1 \quad (1)$$

where  $E_x$  is the electric field along the shock normal in a suitable inertial frame of reference (NIF, HT, or any intermediate). The magnetic field  $B$  can be derived by using the centimetre-gram-second system of units (CGS) in terms of the electron plasma- and gyro-frequencies as,

$$B = \frac{m_e c \Omega_{ce}}{e} = \frac{m_e c \Omega_{ce}}{\omega_{pe} \sqrt{\frac{m_e}{4\pi}}} = \sqrt{\frac{4\pi n_e}{\tau}} m_e c^2 \quad (2)$$

where  $n_e$  is the upstream electron particle density,  $c$  is the speed of light, and the frequency ratio is denoted by,

$$\tau = \left( \frac{\omega_{pe}}{\Omega_{ce}} \right)^2 \quad (3)$$

The electric field  $E$  can be estimated by using Faraday's law for an electromagnetic whistler wave of frequency  $\omega$  and wave number  $k$  standing on the shock front (as a precursor) and co-moving with the shock velocity  $v_{sh}$ , [4],

$$E = \frac{1}{c} \frac{\omega}{k} B = \frac{1}{c} v_{sh} B = \frac{1}{c} M_A V_A B = M_A \sqrt{\frac{m_e}{m_p} \frac{4\pi n_e}{\tau^2}} m_e c^2 \quad (4)$$

where,  $M_A$  is the shock Mach number, and  $V_A$  is the Alfvén velocity,  $V_A = \sqrt{m_e/m_p} V_{Ae} = \sqrt{m_e/m_p} c/\tau^{1/2}$ .

The magnetic field  $\tilde{B}_{sim}$  in the simulation code “em1D” [6], is defined in a dimensionless way by normalizing the field energy in the volume of the Debye cube  $\lambda_D^3$  (or a Debye sphere without the prefactor  $4\pi/3$ ) to the rest energy of a single electron  $m_e c^2$ ,

$$\tilde{B}_{sim}^2 \equiv \left( \frac{B^2 \lambda_D^3}{m_e c^2} \right)_{sim} = \left( \frac{B^2 \lambda_D^3}{m_e c^2} \right)_{obs} \quad (5)$$

By comparing Equation (2) and Equation (5), the magnetic field obtained from the simulation is related to the observed one as,

$$B_{sim} = \sqrt{\frac{\tau_{obs}}{\tau_{sim}}} B_{obs} \quad (6)$$

The magnitude of the electric field is normalized in the same fashion as in Equation (5) by replacing the magnetic field  $B$  by the electric field  $E$ . Then, by using Equation (4),

$$E_{sim} = \frac{\tau_{obs}}{\tau_{sim}} \sqrt{\frac{(m_p/m_e)_{real}}{(m_p/m_e)_{sim}}} E_{obs} \quad (7)$$

where  $(m_p/m_e)_{real}$  is the realistic mass ratio and  $(m/m_e)_{sim}$  is the mass ratio used in the simulation. Assuming a spatial periodic structure with the wave number  $k$ , one may evaluate as  $\frac{dE_x}{dx} = k \delta E_x \approx kE$ . The left hand side of Equation (1) can be written as,

$$\tilde{R}_{NA} \approx \frac{m_e c^2}{e B^2} kE = \frac{k M_A}{e} \sqrt{\frac{m_e}{m_p} \frac{m_e c^2}{4\pi n_e}} = k \lambda_D M_A \sqrt{\frac{m_e}{m_p} \frac{2\tau}{\beta_e}} \quad (8)$$

The normalization relation for the non-adiabaticity condition is obtained as,

$$(\tilde{R}_{NA})_{sim} = \sqrt{\frac{\tau_{sim}}{\tau_{obs}}} \sqrt{\frac{(m_p/m_e)_{real}}{(m_p/m_e)_{sim}}} (\tilde{R}_{NA})_{obs} \quad (9)$$

A typical *in-situ* value for the electron plasma frequency to the electron gyrofrequency in the solar wind is  $(\omega_{pe}/\Omega_{ce})_{obs} \approx 200$ , see e.g., [4], while for a common particle-in-cell simulation (using the realistic value for proton-to-electron mass ratio) is

$(\omega_{pe}/\Omega_{ce})_{\text{sim}} \approx 10$ . Equations (6) and (7) show that the electromagnetic field is overestimated in the simulation by a factor of 20 for the magnitude of the magnetic field and by a factor of 400 for the magnitude of the electric field. Similarly, the non-adiabaticity condition in the simulation is underestimated by a factor of 20. Accordingly to Equation (1) and Equation (9), the electron in simulation is 20 times stronger magnetized than it usually has to be in the solar wind plasma.

Let us now briefly discuss the simulation results of Comișel et al. [7]. Here, the Liouville mapping has been applied to the results obtained in a particle-in-cell simulation of a high Mach number quasi-perpendicular shock. The following input parameters have been used:  $\omega_{pe}/\Omega_{ce} = 8$ ,  $m_p/m_e = 1800$ ,  $\beta_e = \beta_i = 0.2$ ,  $M_A = 8$ , and the angle between the upstream magnetic field direction and the normal to the shock is  $\Theta_{Bn} = 81^\circ$ . Due to strong residual electric fields, the cross-shock potential resulted from the Liouville mapping was compared with the integrated electric field in an *ad-hoc* de Hoffmann-Teller frame where the additional convective field components have been removed. I raise the question why Liouville mapping can provide relatively consistent results with the data from the simulation, although the electrons have to work against the existing motional electric field in the de Hoffmann-Teller frame.

Here, I argue that the low pitch angle electrons could cross the shock bound on the magnetic field line experiencing only the electrostatic component of the total electric field. Due to the simulation setup (low  $\omega_{pe}/\Omega_{ce}$  ratio or high gyrofrequency  $\Omega_{ce}$ ), these electrons should have small gyroradii and they consequently lose their adiabaticity. However, as long as the condition for breaking the adiabaticity is not fulfilled, the particles remain adiabatic and get parallel energy from the electrostatic field.

The energy gain for an adiabatic electron moving along the magnetic field line (or at low pitch angles) can be evaluated in a one-dimensional treatment of the two-fluid model of plasma as,

$$\begin{aligned} \Delta\epsilon_e^{\text{fluid}} &\approx e \int \frac{\vec{b} \cdot \nabla \cdot \vec{P}_e}{n_e(\vec{n} \cdot \vec{b})} dx \\ &= \int \frac{e}{n_e} \left( \frac{\partial P_{\text{exx}}}{\partial x} + \frac{b_y}{b_x} \frac{\partial P_{\text{exy}}}{\partial x} + \frac{b_z}{b_x} \frac{\partial P_{\text{exz}}}{\partial x} \right) dx \end{aligned} \quad (10)$$

where the electric field in the de Hoffmann-Teller frame is approximated by the ambipolar term of the electron momentum equation and by an additional motional electric field component,  $P_e$  is the electron pressure tensor,  $\vec{b}$  is the magnetic field unit vector, and  $\vec{n}$  is the unit vector normal to the shock surface. The first term in the above integral represents the electron energy gain associated with the fluid estimate of the cross shock potential in the de Hoffmann-Teller frame, see e.g., [8]. The next two terms are the energy gain contribution from the electron stress (the non-diagonal terms of the electron pressure tensor) and has the role of electron heating at local electron kinetic scales.

A good match between  $\Delta\epsilon_e^{\text{fluid}}$  and the energy gain provided by the Liouville mapping should support the proposed scenario. For highly energetic particles, the electromagnetic effect does not affect the energy gain irrespectively of the pitch angles, which should confirm the expectation that such highly energetic particles cross the shock layer so quickly and do not feel the variations of the electromagnetic field.

### 3. SUMMARY AND FUTURE PERSPECTIVE

I have analyzed the consequences of using low ratio values of the electron plasma frequency to gyrofrequency in particle-in-cell simulations. The electric field of electromagnetic origin tends to be overestimated due to the numerical setup of the electron Alfvén speed that is not representative for a typical solar wind condition and due to the overestimated magnetic field. The impact on the electron adiabaticity was discussed in the terms of the non-adiabaticity condition derived in Gedalin et al. [5]. The answer on the question addressed in the introduction is that the ratio between the electron plasma frequency to gyrofrequency can influence and significantly change the dynamics of the electron acceleration and heating in a quasi-perpendicular collisionless shock. By using small values of the frequency ratio  $\tau$ , the electrons strengthen their adiabaticity and could have a less physical behavior than in observation. Further studies on the electron adiabaticity by using the Liouville mapping and particle-in-cell-simulations should consider more realistic values for the plasma input parameters.

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The author confirms being the sole contributor of this work and approved it for publication.

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