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Aharonov-Bohm Phase for an Electric Dipole on a Noncommutative Space

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We study the non-relativistic behavior of a particle with electric dipole moment and interacting with external electromagnetic fields on a noncommutative space (NCS). For a special configuration of the field, the phase of an electric dipole is derived as an application of the Aharonov-Bohm effect to a system composed of two charges. We find that the quantum phase for an electric dipole obtains some corrections, and these corrections depend on the noncommutative parameter explicitly.

Keywords: noncommutative space, Aharonov-Bohm effect, electric dipole, quantum field theory, Lagrangian in noncommutative space

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1. INTRODUCTION

Noncommutative space-time is introduced to the standard model of particle physics through noncommutative quantum field theory based on star products and Seiberg-Witten maps [1]. Since the standard model of particle physics has many unsolved problems and quantum field theory on a noncommutative space-time may provide some answers to these puzzles, certain amount of work have been done concerning noncommutative quantum field theory [2–6]. In addition, because finding the measurable effects of space noncommutativity is the central topic in noncommutative quantum mechanics (NCQM), various aspects of quantum mechanics (QM) have been studied to a very great extent on noncommutative space (NCS) and noncommutative phase space (NCPS). For example, NCQM is applied to the hydrogen-like atom and the corrections to the Lamb shift are calculated accordingly and the constraint on θ is obtained to be $1/\sqrt{\theta} \geq 10^2 GeV$ [7–9]. Transitions in the helium atom is studied and the constraint: $1/\sqrt{\theta} \geq 3GeV$ is provided in Haghighat and Loran [10]. The authors of Zhang [11] have suggested the possibility of testing spatial noncommutativity via cold Rydberg atoms. In Chaichian et al. [12], Chaichian et al. [13], Falomir et al. [14], Li and Dulat [15], Mirza and Zarei [16], Li and Wang [17], Mirza et al. [18], Singlton and Vagenas [19], Dulat and Ma [20] and Singleton and Ulbricht [21], the Aharonov-Bohm type phase is studied on a NCS and a NCPS. A lower bound is shown to be $1/\sqrt{\theta} \geq 10^{-6} GeV$ for the space noncommutativity parameter in Chaichian et al. [12]. In Mirza and Zarei [16], Li and Wang [17] and Mirza et al. [18, 22], the Aharonov-Casher phase on a NCS and a NCPS has been studied for a spin-1/2 and a spin-1 particle. And in Mirza and Zarei [16] the limit for the space noncommutativity parameter is $1/\sqrt{\theta} \geq 10^{-7} GeV$. The authors of Harms and Micu [23], Dayi and Jellal [24] and Chakraborty et al. [25–30] have studied the noncommutative quantum Hall effect, and in Harms and Micu [23] a limit of $1/\sqrt{\theta} \geq 10 GeV$ is given. The noncommutative spin Hall effect (SHE) is discussed through a semiclassical constrained Hamiltonian and interesting results are obtained in Dayi and Elbistan [31]. By studying the SHE in the framework of NCQM a lower limit for the noncommutative parameter is shown to be $1/\sqrt{\theta} \geq 10^{-12} GeV$ in Ma and Dulat [32].

Also Yakup and Dulat [33] studied the Harmonic oscillator influenced by gravitational wave in noncommutative quantum phase space. Ma et al. [34] investigated the time-dependent Aharonov-Bohm effect on the NCS. Studies about the NCQM in three dimensions and rotational symmetry can be found in Sinha et al. [35].

In this paper, we focus on the quantum effect of an electric dipole on a NCS. First, the definition of noncommutative space-time is given in the following. NCS is a deformation of ordinary space in which the space coordinate operators $\hat{x}_i$ satisfy the following relation:

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij}, \tag{1}$$
$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}, \tag{2}$$
$$[\hat{p}_i, \hat{p}_j] = 0, \quad i, j = 1, 2, \cdots, n. \tag{3}$$

θ_{ij} is the totally antisymmetric real tensor, which represent the noncommutativity of the space. In addition, the product between the external fields on a NCS is

$$\hat{f}(\hat{\vec{x}})\,\hat{g}(\hat{\vec{x}}) \equiv f(\vec{x}) * g(\vec{x}) = \exp\left[\frac{i\theta_{ij}}{2}\partial_{x_i}\partial_{y_j}\right] f(\vec{x})g(\vec{y})|_{\vec{x}=\vec{y}}, \tag{4}$$

where $f(\vec{x})$ and $g(\vec{x})$ are two arbitrary, infinitely differentiable functions on a commutative space, and $\hat{f}(\hat{\vec{x}})$ and $\hat{g}(\hat{\vec{x}})$ are the corresponding functions on a NCS. In NCQM, one replaces the position and momentum operators with the θ-deformed one,

$$\begin{aligned} \hat{x}_i &= x_i + \frac{1}{\hbar}\theta_{ij}p_j\,, \\ \hat{p}_i &= p_i\,, \qquad i, j = 1, 2\cdots, n\,. \end{aligned} \tag{5}$$

Here x_i and p_i are coordinate and momentum operators in usual quantum mechanics. In the presence of the electro-magnetic field, we also need to replace the electromagnetic potentials with the shifts given as follows

$$\begin{aligned} \hat{\mathbf{A}}(\hat{x}_i) &= \mathbf{A}(x_i - \frac{1}{2\hbar}\theta_{ij}p_j) \\ &= \mathbf{A}(x_i) - \frac{1}{2\hbar}\partial_i\mathbf{A}(x_i)\,\theta_{ij}p_j + \mathcal{O}(\theta^2) \end{aligned} \tag{6}$$

$$\begin{aligned} \hat{\Phi}(\hat{x}_i) &= \Phi(x_i - \frac{1}{2\hbar}\theta_{ij}p_j) \\ &= \Phi(x_i) - \frac{1}{2\hbar}\partial_i\Phi(x_i)\,\theta_{ij}p_j + \mathcal{O}(\theta^2) \end{aligned} \tag{7}$$

and then applies the usual rules of QM.

This paper is organized as follows: in Section 2, on a NCS the phase of an electric dipole is obtained as an application of the AB effect to a system composed of two charges. In Section 3, the behavior of an electric dipole is analyzed on a NCS by using a Lagrangian approach. In Section 4, we derive the phase for electric dipole once more by using the Lagrangian approach, furthermore we prove the gauge invariance of the phase on a NCS. Conclusions are given in the last section.

2. AHARONOV-BOHM PHASE FOR AN ELECTRIC DIPOLE ON A NONCOMMUTATIVE SPACE

A charged particle moving around a magnetic flux in force-free regions acquires the Aharonov and Bohm [36] topological phase,

$$\phi_{AB} = \frac{q}{\hbar c}\oint \mathbf{A}\cdot d\mathbf{x}. \tag{8}$$

For an electric dipole with total mass $m = m_1 + m_2$ may be considered as being composed of two charges $\pm q$ of mass m_1, m_2 separated by small distance $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$. The phase for an electric dipole obtains through summing the AB phase of two charges $\pm q$ [37],

$$\begin{aligned} \phi &= \frac{q}{\hbar c}\oint \mathbf{A}(\mathbf{x}_1)\cdot d\mathbf{x}_1 - \frac{q}{\hbar c}\oint \mathbf{A}(\mathbf{x}_2)\cdot d\mathbf{x}_2 \\ &= \frac{1}{\hbar c}\oint(\mathbf{d}\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}\,, \end{aligned} \tag{9}$$

where $\mathbf{d} = \mathrm{q}\mathbf{r}$ is electric dipole moment, $\mathbf{x} = (m_1\mathbf{x}_1 + m_2\mathbf{x}_2)/m$ is the position of the center of mass.

We can obtain the topological AB phase for an electric dipole on a NCS by replacing $\mathbf{A}(x_i)$ in Equation (9) with the $\hat{\mathbf{A}}(\hat{x}_i)$,

$$\hat{\phi} = \frac{1}{\hbar c}\oint(\hat{\mathbf{d}}\cdot\nabla)\mathbf{A}(x)\cdot d\mathbf{x}, \tag{10}$$

where

$$\hat{d}_j = d_j - \frac{1}{2\hbar}d_j\,\theta_{ik}p_k\,\partial_i \tag{11}$$

is the electric dipole moment on a NCS; here the second term $-\frac{1}{2\hbar}d_j\theta_{ik}p_k\partial_i$ is the correction for the electric dipole moment on a NCS. Equation (12) can also be written as

$$\begin{aligned} \hat{\phi} &= \frac{1}{\hbar c}\oint(\hat{\mathbf{d}}\cdot\nabla)\mathbf{A}(x)\cdot d\mathbf{x} \\ &= \frac{1}{\hbar c}\oint d_j\,\partial_j A_m dx_m + \frac{1}{2\hbar^2 c}d_j\,\theta_{ik}p_k\oint \partial_i\,\partial_j A_m dx_m \\ &= \phi + \delta\phi_{NCS}, \end{aligned} \tag{12}$$

where the additional phase $\delta\phi_{NCS}$, related to the non-commutativity of space, is given by

$$\delta\phi_{NCS} = \frac{1}{2\hbar^2 c}d_j\theta_{ik}p_k\oint \partial_i\partial_j A_m dx_m. \tag{13}$$

The result (Equation 12) represents the noncommutative extension of the electric dipole phase (Equation 9). In the next section we will derive it by using a different approach.

3. LAGRANGIAN ON A NONCOMMUTATIVE SPACE

In this part we construct the Lagrangian for an electric dipole on NCS. The dipole Lagrangian on a commutative space is [37]

$$\begin{aligned} L = & \tfrac{1}{2} \ m_1\dot{\mathbf{x}}_1^2 + \frac{1}{2}m_2\dot{\mathbf{x}}_2^2 - q[\Phi(\mathbf{x}_1) - \Phi(\mathbf{x}_2)] \\ & + (q/c)[\dot{\mathbf{x}}_1 \cdot \mathbf{A}(\mathbf{x}_1) - \dot{\mathbf{x}}_2 \cdot \mathbf{A}(\mathbf{x}_2)] - U(\mathbf{r}), \end{aligned} \quad (14)$$

Using the expansion of $\Phi(\mathbf{x}_1) = \Phi(\mathbf{x}_2) + \mathbf{r}\nabla\Phi(\mathbf{x}_2)$ and $\mathbf{A}(\mathbf{x}_1) = \mathbf{A}(\mathbf{x}_2) + \mathbf{r}\nabla \cdot \mathbf{A}(\mathbf{x}_2)$, with dipole approximation $\mathbf{v} = (m_1\dot{\mathbf{x}}_1 + m_2\dot{\mathbf{x}}_2)/m$, Thus Equation (16) becomes

$$\begin{aligned} L = & \tfrac{1}{2} \ m\mathbf{v}^2 + \frac{1}{2}m_r\dot{\mathbf{r}}^2 + c^{-1}\mathbf{v}_1 \cdot [(q\mathbf{r} \cdot \nabla)\mathbf{A}] \\ & + c^{-1}q\dot{\mathbf{r}} \cdot \mathbf{A} - (q\mathbf{r} \cdot \nabla)\Phi - U(\mathbf{r}) \end{aligned} \quad (15)$$

where $m_r = m_1m_2/(m_1 + m_2)$ is the reduced mass and Φ and $\mathbf{A}$ are evaluated at $\mathbf{x}_2$. This result is different from Gianfranco Spavieri's work [37].

The Lagrangian for electric dipole in NCS can be obtained by using the Bopp's to scalar potential $\Phi(\mathbf{x}_2)$ and vector potential $\mathbf{A}(\mathbf{x}_2)$.

$$\begin{aligned} \hat{\mathbf{A}}(\hat{x}_i) &= \mathbf{A}(x_i - \frac{1}{2\hbar}\theta_{ij}p_j) \\ &= \mathbf{A}(x_i) - \frac{1}{2\hbar}\partial_i\mathbf{A}(x_i)\theta_{ij}p_j + \mathcal{O}(\theta^2) \end{aligned} \quad (16)$$

$$\begin{aligned} \hat{\Phi}(\hat{x}_i) &= \Phi(x_i - \frac{1}{2\hbar}\theta_{ij}p_j) \\ &= \Phi(x_i) - \frac{1}{2\hbar}\partial_i\Phi(x_i)\theta_{ij}p_j + \mathcal{O}(\theta^2) \end{aligned} \quad (17)$$

By inserting Equations (18) and Equation (19) into Equation (17), the Equation (17) becomes

$$\begin{aligned} \hat{L} = L - & \frac{m_2}{2\hbar c}\{(q\mathbf{r} \cdot \nabla)[(\mathbf{v}_2 \cdot \theta \cdot \nabla)(\mathbf{v}_1 \cdot \mathbf{A})] \\ & + q\dot{\mathbf{r}} \cdot [(\mathbf{v}_2 \cdot \theta \cdot \nabla)\mathbf{A}] - c(\mathbf{v}_2 \cdot \theta \cdot \nabla)(q\mathbf{r} \cdot \nabla\Phi)\} \\ & = L + \delta L_{NCS} \end{aligned} \quad (18)$$

with,

$$\begin{aligned} \delta L_{NCS} = - & \frac{m_2}{2\hbar c}\{(q\mathbf{r} \cdot \nabla)[(\mathbf{v}_2 \cdot \theta \cdot \nabla)(\mathbf{v}_1 \cdot \mathbf{A})] \\ & + q\dot{\mathbf{r}} \cdot [(\mathbf{v}_2 \cdot \theta \cdot \nabla)\mathbf{A}] - c(\mathbf{v}_2 \cdot \theta \cdot \nabla)(q\mathbf{r} \cdot \nabla\Phi)\} \end{aligned} \quad (19)$$

comparing with Gianfranco Spavieri's work [37], this is the correction term for Lagrangian in NCS. The difference depend on the non-commutativity θ of the space.

The canonical momentum for the center of mass in NCS is

$$\hat{\mathbf{P}} = \frac{\partial\hat{L}}{\partial\mathbf{v}} = \frac{\partial L}{\partial\mathbf{v}} + \frac{\partial\delta L_{NCS}}{\partial\mathbf{v}} = \mathbf{P} + \delta\mathbf{P}_{NCS} \quad (20)$$

where

$$\mathbf{P} = m\mathbf{v} + \frac{1}{c}(q\mathbf{r} \cdot \nabla)\mathbf{A} = m\mathbf{v} + \mathbf{Q}', \mathbf{Q}' = \frac{1}{c}(q\mathbf{r} \cdot \nabla)\,\mathbf{A} \quad (21)$$

$$\delta\mathbf{P}_{NCS} = -\frac{m}{2\hbar c}\{\frac{m}{m_1}(\mathbf{d} \cdot \nabla)(\mathbf{v} \cdot \theta \cdot \nabla)\mathbf{A} + (\dot{\mathbf{d}} \cdot \theta \cdot \nabla)\mathbf{A}\} \quad (22)$$

$\hat{\mathbf{P}}$ is the deformed momentum of electric dipole in NCS . The second term $\delta\mathbf{P}_{NCS}$ of Equation (22) is the correction momentum for electric dipole in NCS, and this also depend non-commutativity of space.

Expression (Equation 20) leads to the equation of motion

$$\begin{aligned} \frac{d}{dt}(m\mathbf{v}) = & \nabla[\mathbf{v}_1 \cdot \mathbf{Q}' + \frac{1}{c}\dot{\mathbf{d}} \cdot \mathbf{A} - (\mathbf{d} \cdot \nabla)\Phi] \\ & - \frac{m_2}{2\hbar c}\nabla[\mathbf{v}_1 \cdot (\mathbf{d} \cdot \nabla)(\mathbf{v}_2 \cdot \theta \cdot \nabla)\mathbf{A} \\ & + \dot{\mathbf{d}} \cdot (\mathbf{v}_2 \cdot \theta \cdot \nabla)\mathbf{A} - c(\mathbf{d} \cdot \nabla)(\mathbf{v}_2 \cdot \theta \cdot \nabla)\Phi] - \frac{d}{dt}\mathbf{Q}' \\ & + \frac{m}{2\hbar c}[\frac{m}{m_1}(\dot{\mathbf{d}} \cdot \nabla)(\mathbf{v} \cdot \theta \cdot \nabla)\mathbf{A} - c(\dot{\mathbf{d}} \cdot \nabla)(\theta \cdot \nabla)\Phi] \end{aligned} \quad (23)$$

with $d/dt = \partial_t + \mathbf{v} \cdot \nabla$, and $\nabla \times \nabla\Phi = 0$, and in terms of fields, Equation (25) reads

$$\begin{aligned} \frac{d}{dt}(m\mathbf{v}) = & [1 - \frac{m}{2\hbar}(\mathbf{v}_2 \cdot \theta \cdot \nabla)]\{(\mathbf{d} \cdot \nabla)\mathbf{E} \\ & + \frac{1}{c}\mathbf{v}_1 \times [\nabla \times (\mathbf{B} \times \mathbf{d})] - \frac{1}{c}\mathbf{B} \times \dot{\mathbf{d}}\} \\ & + \frac{m}{2\hbar c}\{\frac{m}{m_1}(\mathbf{v} \cdot \theta \cdot \nabla)[\nabla(\dot{\mathbf{d}} \cdot \mathbf{A}) + (\mathbf{B} \times \dot{\mathbf{d}})] \\ & + (\theta \cdot \nabla)(\dot{\mathbf{d}} \cdot \mathbf{E})\} \end{aligned} \quad (24)$$

The second, third term is also the correction term for equation of motion in NCS.

Dipole moves in force-free region where the fields are uniform, so the force free condition $\frac{d}{dt}(m\mathbf{v}) = 0$, and this gives $(\mathbf{d} \cdot \nabla)\mathbf{E} + c^{-1}\mathbf{v}_1 \times [\nabla \times (\mathbf{B} \times \mathbf{d})] = 0\,; \mathbf{v} \cdot \theta \cdot \nabla\mathbf{B} = 0$. This result is very important, we will make use of this in Section 4 to prove the gauge invariance of the phase.

The canonical momentum for relative coordinates is

$$\begin{aligned} \hat{\mathbf{P}}' = \frac{\partial\hat{L}}{\partial\dot{\mathbf{r}}} &= m_r\dot{\mathbf{r}} + \frac{q}{c}\mathbf{A} - \frac{m_2}{2\hbar c}q(\mathbf{v}_2 \cdot \theta \cdot \nabla)\mathbf{A} \\ &= m_r\dot{\mathbf{r}} + \frac{q}{c}\mathbf{A} + \delta\mathbf{P}'_{NCS} \end{aligned} \quad (25)$$

where $\delta\mathbf{P}'_{NCS} = -\frac{m_2}{2\hbar c}\,q(\mathbf{v}_2 \cdot \theta \cdot \nabla)\,\mathbf{A}$.

The equation of motion for relative coordinates is

$$\frac{d}{dt}(m_r\dot{\mathbf{r}}) = q(\mathbf{E} + \frac{\mathbf{v}_1}{c} \times \mathbf{B}) - \frac{qm_2}{2\hbar}(\mathbf{v}_2 \cdot \theta \cdot \nabla)(\mathbf{E} + \frac{\mathbf{v}_1}{c} \times \mathbf{B}) \quad (26)$$

for the angular momentum $\mathbf{L} = \mathbf{r} \times (m_r\dot{\mathbf{r}})$

$$\frac{d}{dt}\mathbf{L} = \mathbf{d} \times [(\mathbf{E} + \frac{\mathbf{v}_1}{c} \times \mathbf{B}) - \frac{m_2}{2\hbar}(\mathbf{v}_2 \cdot \theta \cdot \nabla)(\mathbf{E} + \frac{\mathbf{v}_1}{c} \times \mathbf{B})] \quad (27)$$

If the dipole moves in a region of space where fields are uniform, $(\mathbf{d} \cdot \nabla)\mathbf{E} + c^{-1}\mathbf{v} \times [\nabla \times (\mathbf{B} \times \mathbf{d})] = 0$ in Equation (26) and $(d/dt)(m\mathbf{v} + c^{-1}\mathbf{B} \times \mathbf{d})$. In this case, $\mathbf{v} = -(1/mc)\mathbf{B} \times \mathbf{d}) + const$ may be inserted in Equation (28), which becomes an equation in the variable $\mathbf{r}'$ only (here we assume $\mathbf{v} = \mathbf{v}_1 = \mathbf{v}_2$). In most cases the solution of this equation represents a bound oscillatory motion and will contain terms of the type $\sin(\omega t+\varphi)$ or $\cos(\omega t+\varphi)$, where ω is the frequency and φ is a constant phase. The average of a dynamical variable is obtained by performing the average over the phase constants and one may expect that the variable $\mathbf{d} = q\mathbf{r}$ oscillated about the constant average equilibrium position

$$\begin{aligned}\langle \mathbf{d}\rangle_\varphi \simeq \mathbf{d}_0(\theta) &\simeq \alpha[(\mathbf{E}+\frac{\mathbf{v}_1}{c}\times\mathbf{B}) \\ &- \frac{m_2}{2\hbar}(\mathbf{v}_2\cdot\theta\cdot\nabla)(\mathbf{E}+\frac{\mathbf{v}_1}{c}\times\mathbf{B})]+q\bar{\mathbf{x}} \\ &= \mathbf{d}_0+\mathbf{d}_0' \end{aligned} \tag{28}$$

where $\mathbf{d}_0 = \alpha(\mathbf{E}+c^{-1}\mathbf{v}_1\times\mathbf{B})+q\bar{\mathbf{x}}$; $\mathbf{d}_0' = -\alpha\frac{m_2}{2\hbar}(\mathbf{v}_2\cdot\theta\cdot\nabla)(\mathbf{E}+c^{-1}\mathbf{v}_1\times\mathbf{B})$ is the correction term in NCS; α is the polarizability and $\bar{\mathbf{x}}$ is the equilibrium position in the absence of external fields.

Because of Equation (30), $\hat{\mathbf{d}}_0$ is constant and independent of $\mathbf{x}$ only when the fields are uniform. In this case $\hat{\mathbf{d}}$ is no longer a dynamical variable and, with $\mathbf{E}_0 = -\nabla\Phi$, the Lagrangian assumes the simple form

$$\begin{aligned}\hat{L}_0 &= \frac{1}{2}m\mathbf{v}^2+\frac{1}{c}\mathbf{v}\cdot[(\mathbf{d}_0\cdot\nabla)\mathbf{A}]+\mathbf{d}_0\cdot\mathbf{E}_0 \\ &- \frac{m_2}{2\hbar c}\{\mathbf{v}_1\cdot(\mathbf{d}_0\cdot\nabla)(\mathbf{v}_2\cdot\theta\cdot\nabla)\mathbf{A}-c(\mathbf{d}_0\cdot\nabla)(\mathbf{v}_2\cdot\theta\cdot\nabla)\Phi\}\end{aligned} \tag{29}$$

4. THE HAMILTONIAN AND GAUGE INVARIANCE OF THE PHASE ON NONCOMMUTATIVE SPACE

Gauge invariance of the AB phase in the time dependent AB efect is discussed in MacDougall and Singleton [38]. In this paper, we discuss the quantum phase for an electric dipole on a NCS by solving the Schroedinger equation. Let $H(x,p)$ be the Hamiltonian operator of the usual quantum system, then the static Schrödinger equation on NC space is usually written as

$$H(x,p)*\psi = E\psi\ , \tag{30}$$

where the Moyal-Weyl (or star) product between two functions is defined in Equation (4). On a NCS the star product can be changed into the ordinary product by replacing $H(x,p)$ with $H(\hat{x},\hat{p})$. Thus the Schrödinger equation can be written as,

$$H(\hat{x}_i,\hat{p}_i)\psi = H(x_i-\frac{1}{2\hbar}\Theta_{ij}p_j,p_i)\psi = E\psi\ . \tag{31}$$

Thus the Equation (31) is actually defined on a commutative space, and the noncommutative effects can be evaluated through the Θ related terms. Note that the Θ term always can be treated as a perturbation in QM , since $\Theta_{ij} << 1$. When magnetic field is involved, the Schrödinger equation (Equation 30) becomes

$$H(x_i,p_i,A_i)*\psi = E\psi\ . \tag{32}$$

To replace the star product in Equation (32) with a usual product, we need to replace x_i, p_i, and A_i with the shifts given in Equations (5) and (7). Thus the Schrödinger Equation (32) in the presence of magnetic field becomes

$$H\big(x_i-\frac{1}{2\hbar}\Theta_{ij}p_j,p_i,A_i+\frac{1}{2}\Theta_{lj}p_l\partial_jA_i\big)\psi = E\psi\ . \tag{33}$$

Now let us consider a particle of mass m and charge q moving in a magnetic field with magnetic potential A_i, then the Schrödinger equation is (we choose unit of $\hbar = c = 1$),

$$\frac{1}{2m}\Big(p_i-qA_i-\frac{1}{2}q\Theta_{lj}p_l\partial_jA_i\Big)^2\psi = E\psi\ . \tag{34}$$

Using the expressions of the canonical momenta Equations (22) and (27), the Hamiltonian $\hat{H}$ may be derived from the Lagrangian Equation (20). the corresponding Schrödinger equation $\hat{H}(\hat{\mathbf{p}},\hat{\mathbf{x}})\psi(\mathbf{x}) = E\psi(\mathbf{x})$ in NCS reads

$$\begin{aligned}\{\frac{(\hat{\mathbf{P}}-\mathbf{Q}'-\delta\mathbf{P}_{NCS})^2}{2m} &+ \frac{(\hat{\mathbf{P}}'-\frac{q}{c}\mathbf{A}+\delta\mathbf{P}'_{NCS})^2}{2m_r} \\ &+ q\mathbf{r}\cdot\nabla\Phi+U(\mathbf{r})\}\Psi \\ &= E\Psi\end{aligned} \tag{35}$$

The main difference between classical and quantum behavior is due to the existence of the quantum phase ϕ of the wave function which, through the process of interference, may lead to an observable phase shift $\triangle\phi$. We obtain the solution of the Schrödinger equation (Equation 32) by direct substitution

$$\begin{aligned}\Psi &= exp(\frac{i}{\hbar}\int\mathbf{Q}'\cdot d\mathbf{x})exp(\frac{i}{\hbar}\int\delta\mathbf{P}_{NCS}\cdot d\mathbf{x})\Psi_0 \\ &= exp(i\phi)exp(i\delta\phi_{NCS})\Psi_0 \\ &= exp[i(\phi+\delta\phi_{NCS})]\Psi_0\end{aligned} \tag{36}$$

where $\delta\phi_{NCS} = \frac{i}{\hbar}\int\delta\mathbf{P}_{NCS}\cdot d\mathbf{x}$ is the correction term in NCS, $\mathbf{Q}' = \frac{1}{c}(\mathbf{d}\cdot\nabla)\mathbf{A}$, and Ψ_0 solves the equation with $\mathbf{A} = 0$, $\Phi = 0$. Thus, the quantum phase ϕ coincides with Equation (14) in the Previous section.

In the interference experiments with particles possessing an electric dipole moment, the observable quantity is the phase shift$(1/\hbar c)\oint\mathbf{Q}\cdot d\mathbf{x} = (1/\hbar c)\oint(\hat{\mathbf{d}}\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}$, Since

$$\begin{aligned}\langle\Psi|\oint(\hat{\mathbf{d}}\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}|\Psi\rangle &= \oint(\langle\Psi'|\hat{\mathbf{d}}|\Psi'\rangle\cdot\nabla)\mathbf{A}\cdot d\mathbf{x} \\ &= \oint(\mathbf{d}_0\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}+\oint(\mathbf{d}_0'\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}\end{aligned} \tag{37}$$

where $\oint(\mathbf{d}_0'\cdot\nabla)\mathbf{A}\cdot d\mathbf{x} = \oint(\langle\Psi'|\mathbf{d}'|\Psi'\rangle\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}$.
its expectation value reads

$$\begin{aligned}\triangle\hat{\phi} &= \frac{1}{\hbar c}\oint(\mathbf{d}_0\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}+\frac{1}{\hbar c}\oint(\mathbf{d}_0'\cdot\nabla)\mathbf{A}\cdot d\mathbf{x} \\ &= \frac{1}{\hbar c}\oint[\mathbf{B}\times\mathbf{d}_0+\nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x} \\ &+ \frac{1}{\hbar c}\oint(\mathbf{d}_0'\cdot\nabla)\,\mathbf{A}\cdot d\mathbf{x}\end{aligned} \tag{38}$$

the relevant term $\mathbf{B} \times \mathbf{d}_0$ known as Röntgen interaction. the second term $\frac{1}{\hbar c}\oint(\mathbf{d}_0' \cdot \nabla)\mathbf{A} \cdot d\mathbf{x}$ is our correction term for Gianfranco Spavieri's work [37] in NCS.

Our result (Equation 35) for the phase shift of an electric dipole in NCS differs from that proposed by other authors (Gianfranco Spavieri) for the presence of the extra term $\frac{1}{\hbar c}\oint(\mathbf{d}_0' \cdot \nabla)\mathbf{A} \cdot d\mathbf{x}$.

It can be shown that the phase shift $\triangle\hat{\phi}$ is also gauge independent in NCS. Gauge theory in NCS is different from the case in usual commutative space. Gauge transformation in NCS is $A'_\mu = A_\mu + \delta_\lambda A_\mu, \delta_\lambda A_\mu = \partial_\mu\lambda(x) + i[\lambda(x), A_\mu]_*$. Here we consider the three dimensional case: $\delta_\lambda\mathbf{A} = \nabla\lambda(x) + i[\lambda(x), \mathbf{A}]_*$.

Using the expression (Equation 35) in order to point out some properties of the AB phase shift, we write the contribution to the phase shift due to the gauge transformation as

$$\begin{aligned}\delta\hat{\phi} &= \frac{1}{\hbar c}\oint[(\mathbf{d}_0 + \mathbf{d}_0') \cdot \nabla]\delta_\lambda\mathbf{A} \cdot d\mathbf{x} \\ &= \frac{q}{\hbar c}\oint(\nabla\lambda) \cdot d\mathbf{x} + \frac{q}{\hbar c}\oint(\mathbf{v} \cdot \theta \cdot \nabla)(\nabla\lambda) \cdot d\mathbf{x} \\ &- \frac{1}{\hbar c}\oint(\mathbf{d}_0 \cdot \nabla)\nabla\lambda \cdot \theta \cdot (\nabla\mathbf{B}) \cdot d\mathbf{s} \\ &- \frac{1}{\hbar c}\oint[(\mathbf{v} \cdot \theta \cdot \nabla)\mathbf{d}_0 \cdot \nabla][\nabla\lambda \cdot \theta \cdot (\nabla\mathbf{B})] \cdot d\mathbf{s} \\ &= 0\end{aligned} \tag{39}$$

This gauge independence based on the fact that the scalar function λ is a monovalued function for which $\oint(\nabla\lambda) \cdot d\mathbf{x} = 0$, and $\theta \cdot (\nabla\mathbf{B}) = 0$ which we obtained under the force free condition.

5. CONCLUSION

In this paper we studied the noncommutative non-relativistic behavior of a neutral particle, which possessing electric dipole moments, in the presence of external electric and magnetic fields. For a special configuration of the field, we derived the phase of an electric dipole as an application of the AB effect to a system composed of two charges in NCS. We have shown that a topological phase of the AB type is a generic effect in dipole moment of neutral particles.

The result of this paper indicates that there is some difference between usual and NCQM studying same problem. For example, our Lagrangian, canonical momentum, phase, equation of motions are different from Gianfranco Spavieri's work [37] who studied the same problem in usual commutative space. The difference depend on the noncommutativity θ and $\bar{\theta}$ of the space. We may say that, our extra terms for theses quantities are the correction term for Gianfranco Spavieri's work [37] in NCS. Obviously,if noncommutative parameter $\theta = 0$, the phase in NCS is to be that in commutative space. If the present experimental situation is attainable, this effect in NCS might be tested at very high energy level, and the experimental observation of the effect remains to be further studied.

AUTHOR CONTRIBUTIONS

MA and RR designed the study; AA and MH carried out the theoretical calculations independently; MA performed the theoretical analysis; RR wrote the manuscript; MA edited the final manuscript.

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