Quality-Reliability Coupled Network Modeling and Importance Measure of Multi-Stage Manufacturing Systems *via* Network Controllability Analysis

Ding Zhang*, Yu Pei and Qiang Liu

State Key Laboratory of Precision Electronic Manufacturing Technology and Equipment, Guangdong University of Technology, Guangzhou, China

The fluctuation of product quality is closely related to the degradation of the equipment in multi-stage manufacturing systems. This paper proposes a critical measure approach for a quality-reliability coupled network of multi-stage manufacturing systems via network controllability. The impact of component degradation will be transmitted, expanded, and accumulated in multiple manufacturing stages, leading to quality flaws or even shutdowns of the entire system. An important measurement method *via* controllability analysis is provided by quantifying the impact of attacking the quality-reliability coupled network. By quantifying the control ability of the fault source node on the key quality attribute node, the weakness that affects the processing accuracy of a production line is identified. Case studies of real production lines are applied to verify the effectiveness, and comparative results show the method can guide the quality-reliability improvement of manufacturing systems.

1 INTRODUCTION

A highly reliable production line requires the system to maintain the specified production efficiency and the processed products to meet the quality inspection requirements. The quality-reliability of the production line describes the ability of the index value of the workpiece quality to keep in line with the technical requirements. The accuracy-reliability describes the ability of the production line to maintain the machining accuracy. As time goes by, the performance degradation of equipment parts will decrease machining accuracy. Although it may not be enough to cause the accuracy of a single piece of equipment to fail, its impact will be transmitted and expanded in multiple manufacturing stages. After accumulation, the production line will not process qualified workpieces, which will cause unnecessary downtime for maintenance activities and significantly reduce quality-reliability.

System reliability analysis identifies the impact of failures on the system. Its purpose is to measure the weakness of a system so that improvement measures can be implemented. For a large-scale processing system, accuracy fluctuations become extremely complicated after being transmitted in multiple stages. Its topological structure presents the characteristics of a large-scale network, which hides a wealth of uncertainty propagation information. Using graph theory or network knowledge to analyze topological structure is a new development. Therefore, this article starts from the perspective of the relationship of production line quality and reliability. By quantifying the control ability of the

fault source node on the key quality attribute node, the weakness that affects the processing accuracy of the production line is identified to improve the quality reliability of the system processing.

The paper is organized as follows. **Section 2** presents the state of this field. The logical flow of quality-reliability analysis via network controllability is provided in **section 3**. **Section 4** focuses on establishing a reliability-quality coupled network modeling of a production line. In **section 5**, based on the controllability analysis of complex networks and identifying key attribute nodes, the weakness that affects the processing accuracy of a production line is identified. **Section 6** shows two examples to verify the algorithm proposed in this paper. Finally, concluding remarks are summarized in **Section 7**.

2 LITERATURE REVIEW

At present, many scholars have gradually begun to study the relationship between the quality and reliability of production lines. Jiang and Murthy (Jiang and Murthy, 2009) measured the impact of manufacturing system product quality on machining process reliability. Jin et al. (Jin and Chen, 2001) first proposed the interactive effect of quality and reliability. Chen and Jin (Yong and Jin, 2005) proposed a QR chain (Quality-Reliability Chain) to describe a process of multi-station transfer of component reliability and workpiece quality along the production process. Sun (Sun et al., 2009) proposed an integrated model of quality and reliability for multi-station manufacturing systems. The model usually sets the strength of interaction relationship between quality and reliability as a certain interaction coefficient to evaluate the reliability under this coefficient. He et al. (2014) considered the interactive effects of quality and reliability, and proposed a reliability evaluation model based on the probability of product qualification and degradation of system components. Chen et al. (2021) proposed a reliability evaluation method that considers the quality-reliability dependency of a multi-state system, and use the quality state-space model to quantify the quality deviation. Zhou (Zhou and Lu, 2018) was focused on the bidirectional interaction mechanism between station reliability and product quality, and developed a station reliability evaluation method. Regarding the predictive failure probability of each machine and the production control policy, Wang et al. (2020) proposed predictive maintenance. Although the predecessors proposed appropriate reliability evaluation and design methods for the production line, they believed that the equipment of the production line was independent of each other, and each manufacturing stage was equally important and did not consider the impact of machining accuracy on the evaluation and design. However, the performance degradation of different equipment has different effects on the transmission of accuracy fluctuations. In this context, measuring the weaknesses that affect the quality-reliability of a production line from the perspective of accuracy is particularly important.

Researchers usually establish corresponding network models for large and complex systems to identify the quality-reliability coupling relationship and measure importance nodes, thereby

indirectly performing reliability analysis. For example, Zhang et al. (2013) proposed to use the in-degree, out-degree, and shortest path of a complex network to identify the importance of nodes in the material flow manufacturing network. Yang et al. (2013) identified brittle points that affect the stability of multi-product production lines by using network topology characteristics such as utilization, shortest path, and clustering coefficient. Haifeng (2010) used network to represent fault data structure and introduced the modular concept of complex network community structure analysis to diagnose compressor fault. The popular method (Datta and Goyal, 2019) to evaluate the network reliability is mostly through minimal paths (MP) or minimal cuts (MC) of a network. Lin and Chen (Tan et al., 2017) proposed a maximal flow method to research MP in a stochastic-flow network. For the controllability of dynamic complex networks, Zhou (Tan et al., 2017) selected an index to measure the controllability of networks quantitatively. According to this index, they compared the controllability between different network structures. Zhou et al. (2013) considered the role of nodes in network efficiency and degree distribution and the importance of adjacent edges, then provided a new method for measurement of node centrality in complex directed networks and made the evaluation of node centrality more accurate. Wang (Wang et al., 2016) introduced a new structural controllability analysis approach based on the reachability matrix to identify the minimum set of driver nodes, which could further control complex networks. The importance of nodes in a complex network has an important impact on the stability of the network. Based on previous ideas, Yang (Yang et al., 2019) considered degree centrality, tight centrality, and intermediate centrality and proposed a method for comprehensively evaluating the importance of nodes in complex networks. The entropy weight method that overcomes the influence of subjective factors is used to calculate the weight of each index. Mousavi and Haeri, (2016) introduced the zero expansion rule through the concept of a balanced set, providing a sufficient number of control nodes to ensure the controllability of some undirected networks. Chin et al. (2017) defined the notion of super driver nodes and discussed the effects that root and leaf nodes on driver and super driver nodes. Then they distinguished driver nodes from super driver nodes by calculating and evaluating node properties to control a network. Xiang (Xiang et al., 2019) had made great progress in the research of network controllability and has a comprehensive understanding of network controllability from many aspects, such as network topology and node-system dynamics.

However, it is difficult for a complex coupling network to summarize the importance of the fluctuation source node only with topological network indicators such as degree, betweenness centrality, and clustering coefficient. For example, the degree only judges the importance of a node from the number of connected edges. The clustering coefficient only determines its importance from the number of edges of its neighbor nodes, which lacks the overall consideration of the network. From the perspective of the network topology, the ability of nodes to intervene in the network is different, and attacks on different nodes have different effects
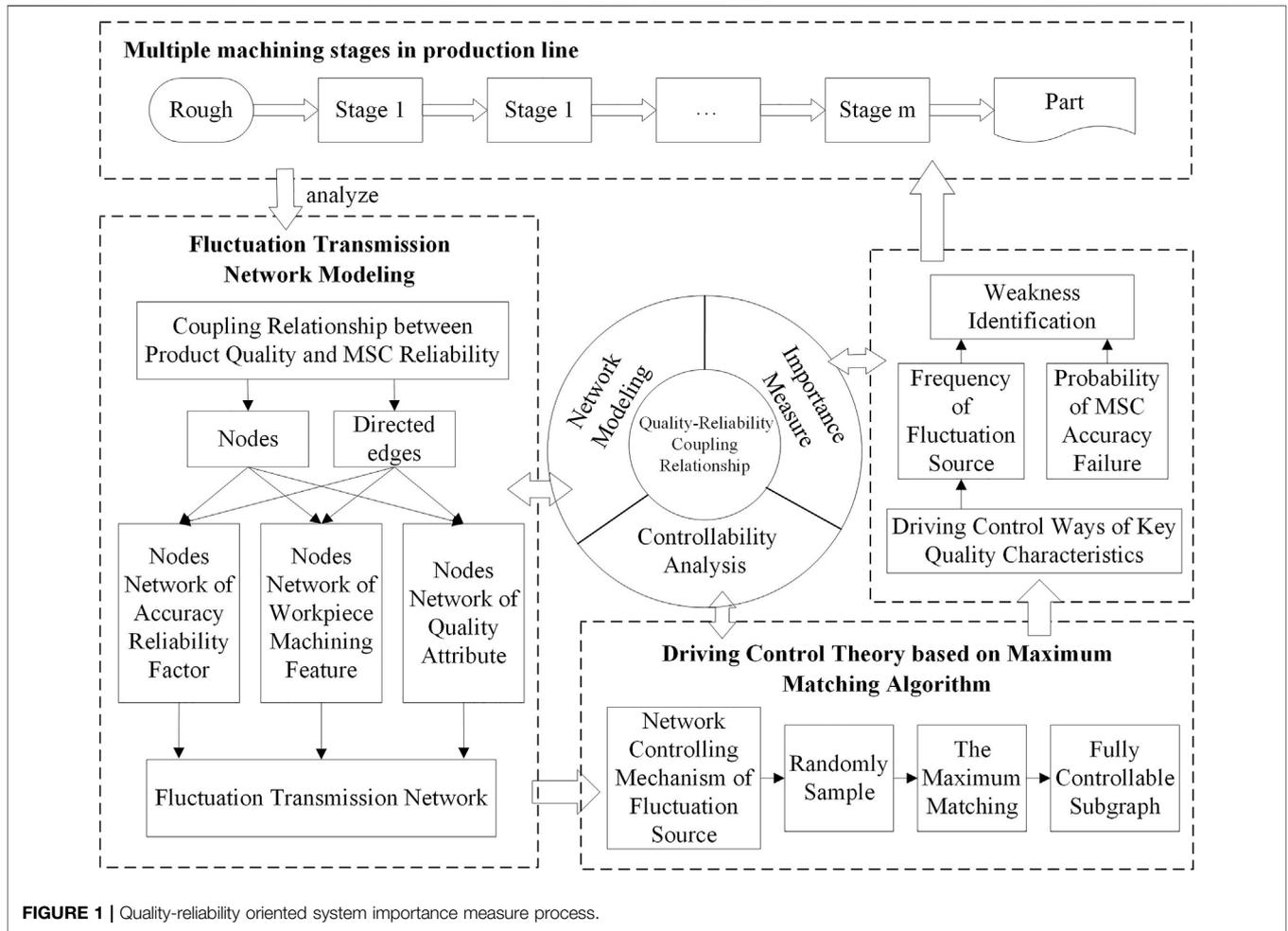
FIGURE 1 | Quality-reliability oriented system importance measure process.

on the network. This enlightens us to measure the importance of fluctuation sources based on their ability to control the network to overcome the limitations of the traditional degree, centrality, and other topological characteristic indicators.

Therefore, on the basis of previous research results, from the perspective of how to effectively control a network, this paper proposes a method for evaluating the importance of fluctuation sources based on network controllability analysis and then proposes an accuracy-oriented method for identifying production line weakness, and verifies the effectiveness of this method.

3 LOGICAL FLOW

In this section, the logic flow of the proposed method structure is mainly introduced. As shown in **Figure 1**, the method contains three parts, which are network modeling, network controllability analysis, and importance measure.

Specifically, it includes the following:

Step 1: Network modeling. There is a coupling between quality attributes of workpieces in the multi-manufacturing stages, performance degradation of machining system components

(MSC), and deviation of quality characteristics of workpieces. Based on this relationship, the nodes and directed edges of the network can be abstracted, thereby constructing a fluctuation propagation network model oriented to the quality-reliability of the production line.

Step 2: Network controllability analysis. From the perspective of the control network, the control ability of the node is measured, and the maximum matching algorithm obtains the driving control mode of the fluctuation source in graph theory.

Step 3: Importance measure. According to the control method of fluctuation, the source to quantify its ability to control key quality attributes. Comprehensively considers the probability of occurrence of accuracy failure to determine weakness so as to guide the increase of accuracy reliability or maintenance measures.

4 RELIABILITY-QUALITY COUPLED NETWORK MODELING

The influencing factors that lead to poor quality characteristics of workpieces in machining production lines have multiple sources.
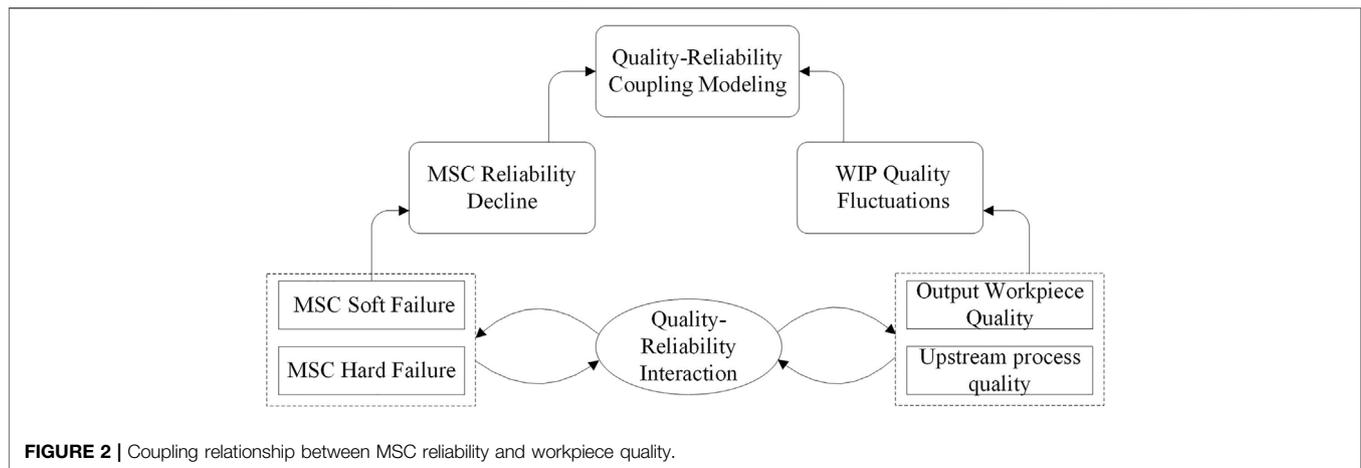
FIGURE 2 | Coupling relationship between MSC reliability and workpiece quality.

The specific sources include uncertain factors in processing equipment, measuring devices, personnel operations, external manufacturing environment, processing methods, workpiece materials, tools, fixtures, and so on (Riascos et al., 2007). When the performance of MSC declines in a certain process, this decline factor will become a source of fluctuations. It will not only cause fluctuations in its own accuracy, but also affect the processing accuracy of other processes related to it. Therefore, this section focuses on the coupling relationship between MSC, such as supporting fixtures, measuring devices, and tools of machining equipment and the deviation of workpiece quality characteristics.

4.1 Relationship Between Quality-Reliability and Error Fluctuation

In actual production, we hope that a highly reliable production line will run for a long time and produce high-quality workpieces. However, when the manufacturer configures the production line in the design stage, it often ensures processing efficiency and line balancing. With processing equipment as the main body, continuous running time is measured under the no-failure condition to evaluate the reliability of the production line. After the production line is actually delivered to a user, the reliability of use may be significantly lower than the reliability index given by the manufacturer. One of the reasons for this phenomenon is ignoring the coupling effect on MSC quality-reliability. The hard faults of MSC directly lead to equipment shut down for maintenance. Although MSC soft failure (decreased machine tool accuracy, wear of fixtures or tools, etc.) does not directly cause downtime, they cause poor product size, which may also trigger downtime detection and need to diagnose quality problems. **Figure 2** shows the coupling relationship between MSC soft failure, hard failure, and workpiece quality in the machining process.

Workpieces usually need to be processed in multiple manufacturing stages, as shown in **Figure 3**; they are affected by the hard failure and soft failure of MSC at this stage and the quality level of the upstream processing stage. Therefore, it is necessary further to study the quality-reliability coupling relationship after production line configuration.

The coupling relationship between quality-reliability and its transmission process along multiple manufacturing stages is shown in **Figure 4**. This effect can include the following aspects:

1) The effects of reliability factors on quality factors (Reliability-Quality Effects, R-Q Effects): MSC's soft failure and hard failure cause product quality problems in this manufacturing stage.
2) The effects of quality factors on reliability (Quality-Reliability Effects, Q-R Effects): refers to the influence of product quality problems on MSC reliability in the downstream manufacturing stage.
3) Quality effects transfer process caused by datum constraint (Datum Effects, DE): Refers to the processing of features in a certain manufacturing stage based on features processed in the upstream stage, resulting in error propagation.
4) Evolution Effects (ER): Refers to the process in which there is a sequential relationship between processing procedures, which leads to propagation of parts errors.

Figure 5 shows the structure of a piston machining production line, which mainly completes rough and finished processes of outer piston circles, ring grooves, combustion chamber, pinhole, and valve pit. Machining equipment includes special CNC lathes, vertical CNC milling machines, and precision combined boring machines.

The coupling relationship between equipment accuracy reliability factors of the piston production line and the error transmission process of the workpiece is shown in **Figure 6**. When degradation factors such as wear and fatigue occur in the machine tool equipment layer, key accuracy indicators will be unreliable, and fluctuations will be transmitted along multiple manufacturing stages. When there is a continuous deviation of critical workpiece dimensions, it needs to be shut down for maintenance, triggering quality diagnosis and maintenance activities. The sources of fluctuations are diverse, and network propagation occurs along multiple transmission paths. This relationship becomes more and more complex with the increase in the manufacturing stage of the production line and the types of workpieces to be processed.
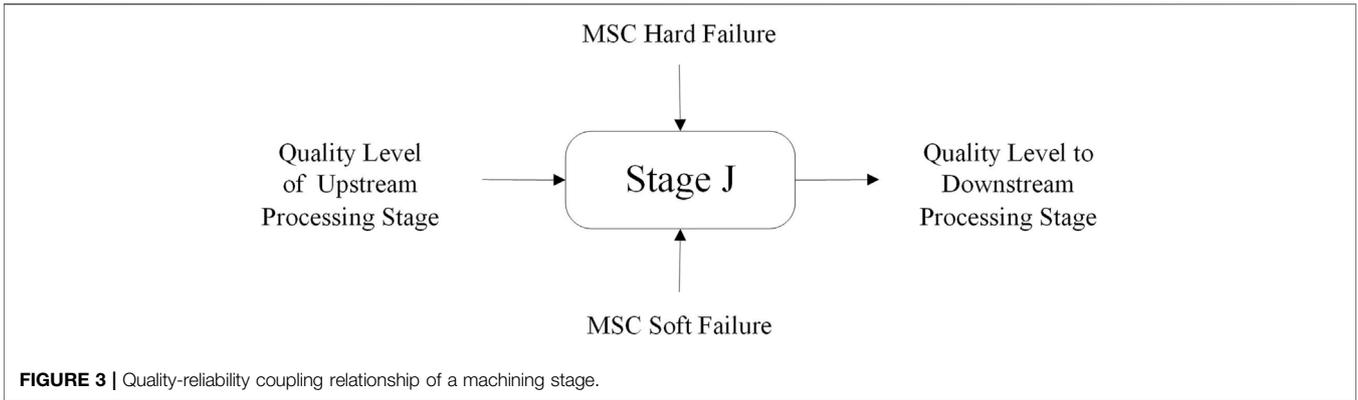
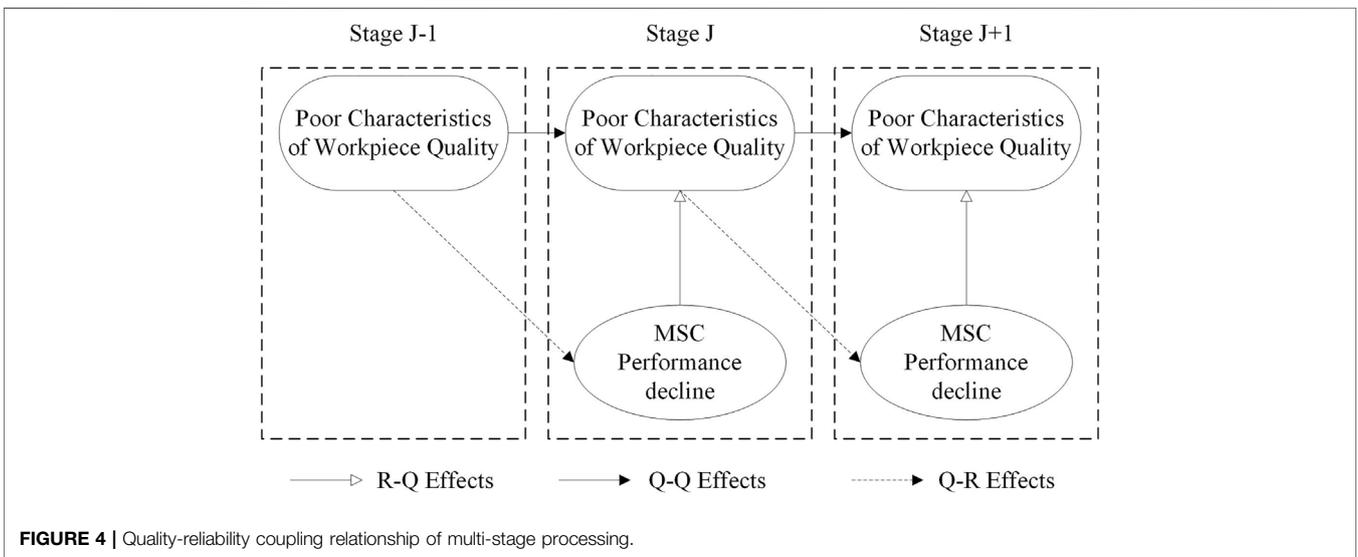FIGURE 3 | Quality-reliability coupling relationship of a machining stage.

FIGURE 4 | Quality-reliability coupling relationship of multi-stage processing.
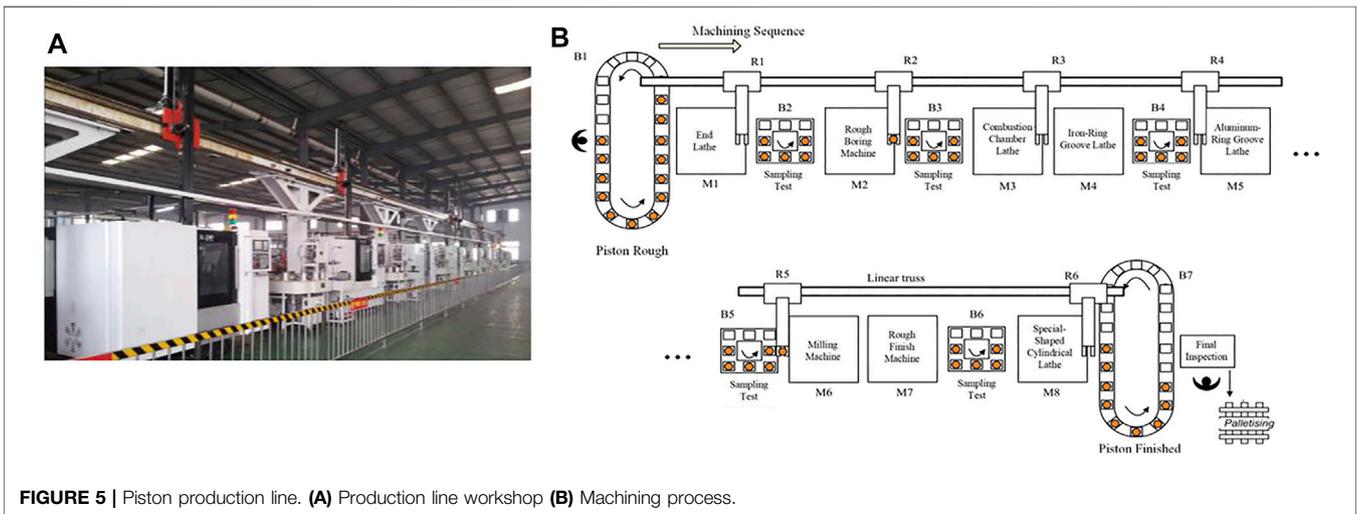

FIGURE 5 | Piston production line. (A) Production line workshop (B) Machining process.
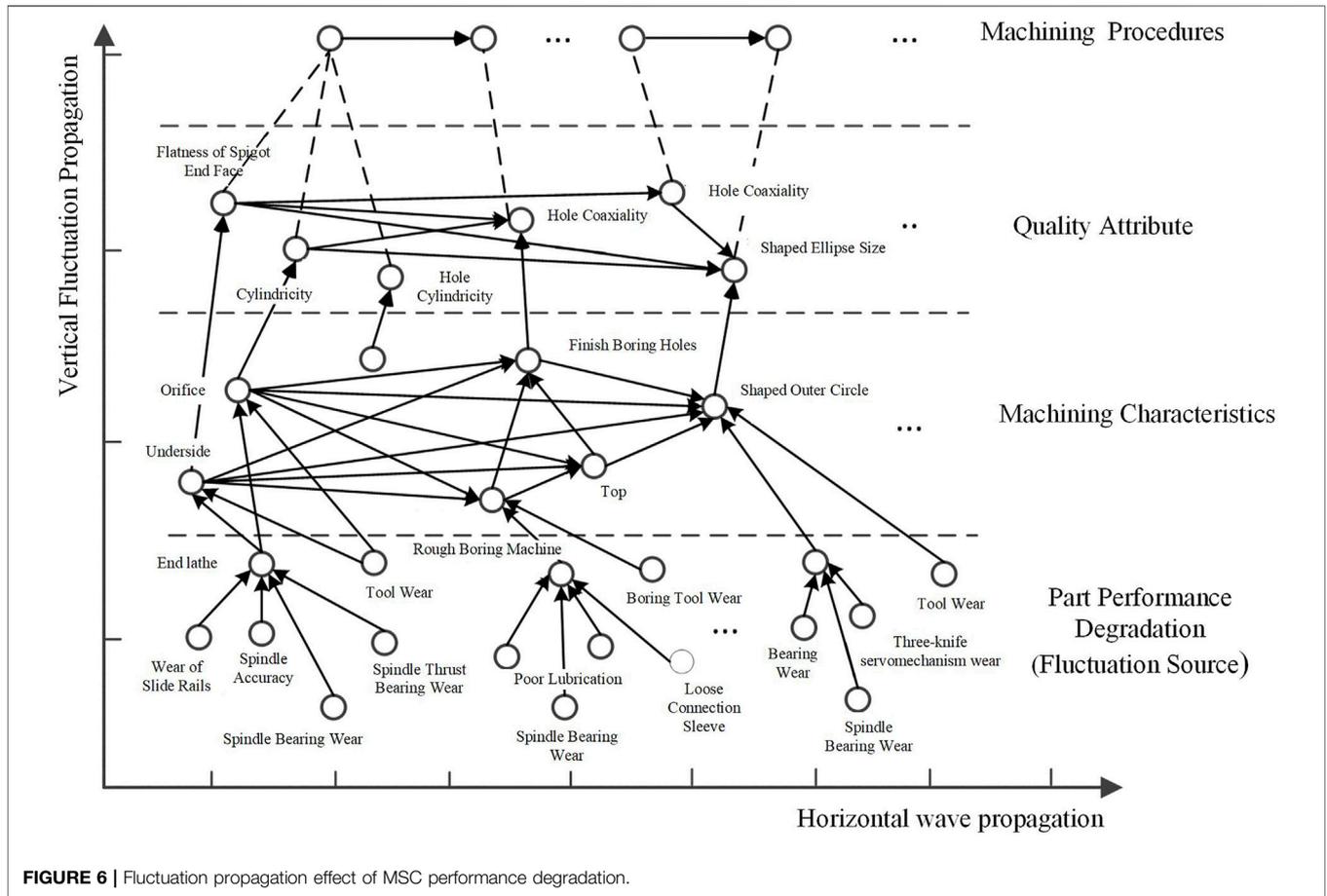
FIGURE 6 | Fluctuation propagation effect of MSC performance degradation.

4.2 Construction Steps of Fluctuation Transmission Network

According to the processing information, processing characteristics of workpiece and equipment accuracy factors that affect processing characteristics are abstracted into nodes, and the relationship between them is abstracted into edges. Then fluctuation transmission network of production line processing can be constructed. The specific modeling steps are as follows:

Step 1: Construct a network of machining feature nodes. Given production line workpiece set $P = \{P_1 \cdots P_i \cdots P_{NP}\}$, the machining feature set of workpiece P_i is $F_i = \{F_{i1}, F_{i2} \cdots F_{in_i}\}$ (n_i represents the number of P_i features, F_{in_i} represents the machining feature of the workpiece P_i). The machining feature is defined as network nodes, and the relationship between features (benchmark, etc.) is defined as edges. Then the feature nodes network for workpiece P_i processing can be described as:

$$G_{P_i} = \langle F_i, E_{F_i} \rangle \qquad (1)$$

Where E_{F_i} represents the edges set of machining feature nodes on P_i.

Step 2: Construct a network of accuracy reliability factors based on machining characteristics. Given machining feature

F_{ij} ($i = 1, 2, \cdots, NP$, $j = 1, 2, \cdots, n_i$), accuracy reliability factor nodes set $R_{ij} = \{r_{ij1}, r_{ij2}, \cdots, r_{ijs}\}$ (s represents the number of nodes of accuracy reliability factor of machining feature F_{ij}). Accuracy reliability factor is defined as network nodes, and the relationship between the accuracy reliability factor and machining feature is defined as edges. Then the network of machining feature F_{ij} and accuracy reliability factor R_{ij} can be described as:

$$G_{R_{ij}} = \langle \{F_{ij}, R_{ij}\}, E_{ij} \rangle \qquad (2)$$

where E_{ij} represents edges set of accuracy reliability factor and processing feature.

Step 3: Construct a network of quality attributes based on machining characteristics. Given machining feature F_{ij} ($i = 1, 2, \cdots, NP$, $j = 1, 2, \cdots, n_i$), quality attribute nodes set $D_{ij} = \{d_{ij1}, d_{ij2}, \cdots, d_{ijh}\}$ (h represents the number of quality attribute nodes of the processing feature F_{ij}). The quality attribute is defined as network nodes, and the corresponding relationship between quality attribute and processing feature is defined as edges. Then the network can be described as:

$$G_{D_{ij}} = \langle \{F_{ij}, D_{ij}\}, E_{D_{ij}} \rangle \qquad (3)$$

$E_{D_{ij}}$ represents edges set corresponding to fluctuation source and accuracy reliability factor.

Step 4: Combine workpiece processing feature network, accuracy reliability factor network, and quality attribute network. The edges G_{P_i}, $G_{R_{ij}}$ and $G_{D_{ij}}$ do not intersect each other but contain the same network nodes. By merging the same nodes of three networks, the fluctuation transmission network can be obtained:

$$G' = \langle \{F_i, R_{ij}, D_{ij}\}, E_{F_i} \cup E_{ij} \cup E_{D_{ij}} \rangle \qquad (4)$$

5 IMPORTANCE MEASURE *VIA* NETWORK CONTROLLABILITY

Some nodes seem to have a specific ability to control the network, and the fluctuation in their functions will cause a series of chain reactions, which will affect the state of other nodes. Therefore, based on network controllability analysis, this section quantifies the control ability of nodes on crucial quality attributes to measure its importance.

5.1 Network Controllability Analysis

Watts and Strogatz, (1998) reported the phenomenon of small-world networks in 1998. Barabasi and Albert (Barabasi et al., 1999) led the scale-free properties of complex networks in 1999. Liu et al. published an article on the controllability of complex networks in Nature in 2011. Combining complex networks with control theory opens new avenues for the controllability of networks (Diao et al., 2014). Since then, many scientific researchers have researched network controllability and have achieved fruitful results (Sorrentino et al., 2007; Pedro et al., 2014; Hou et al., 2015; Gao et al., 2016; Hou et al., 2016).

Nonlinear processes drive most real complex systems, but the literature (Ching-Tai, 1974; Liu et al., 2011) proved that its controllability is similar to linear systems in many aspects structurally. The elements of a complex system are abstracted as nodes, and the connection relationship between nodes is directed edges, which can construct a network model. According to control theory, given a linear steady system:

$$\frac{d\boldsymbol{X}(t)}{dt} = A\boldsymbol{X}(t) + B\boldsymbol{X}(t) \qquad (5)$$

$$\boldsymbol{Y}(t) = C\boldsymbol{X}(t) \qquad (6)$$

$\boldsymbol{X}(t) = (x_1(t), \cdots, x_N(t))^T$ represents the state of N nodes at time t. The system matrix A represents the connection relationship between nodes. If there is an interaction between node i and node j, then $a_{ij} \neq 0$, otherwise $a_{ij} = 0$. $B = (b_{ij})_{R \times N} (M \leq N)$ is the input matrix that nodes controlled by an external signal. $C = (c_{ij})_{R \times N}$ is the output matrix, and its output vector is $\boldsymbol{Y}(t) = (y_1(t), \cdots, y_R(t))^T$.

If given any initial state $x(t) = x_0$ and the desired state x_f, there is a control input $\boldsymbol{u}(t) = (u_1(t), \cdots, y_M(t))^T$ that can make $x(t) = x_f$ within a finite time t, then the system is said to be controllable. According to Kalman's controllability rank condition, the necessary and sufficient condition of system controllability is that its controllability matrix QC is full rank, namely:

$$rank(QC) = rank[B, AB, A^2B, \cdots, A^{N-1}B] = N \qquad (7)$$

Similarly, if for any given initial state $x(t) = x_0$, which can be determined in finite time t by measuring output vector. Then this system is completely observable. The necessary and sufficient condition of system observability is that its observability matrix QO is full rank, namely:

$$rank(QO) = rank\left[[C]^T, [CA]^T, [CA^2]^T, \cdots, [CA^{N-1}]^T\right] = N \qquad (8)$$

The simple network with four nodes shown in **Figure 7A**, matrix A can be expressed according to connection relationship between nodes:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{pmatrix} \qquad (9)$$

There are two input signals u_1 and u_2, and the input matrix B is expressed as:

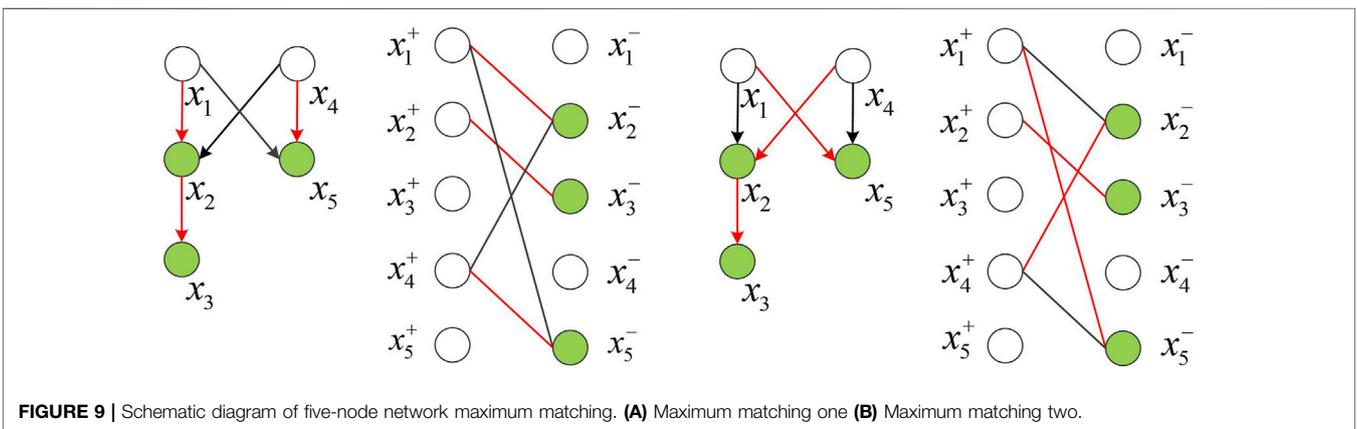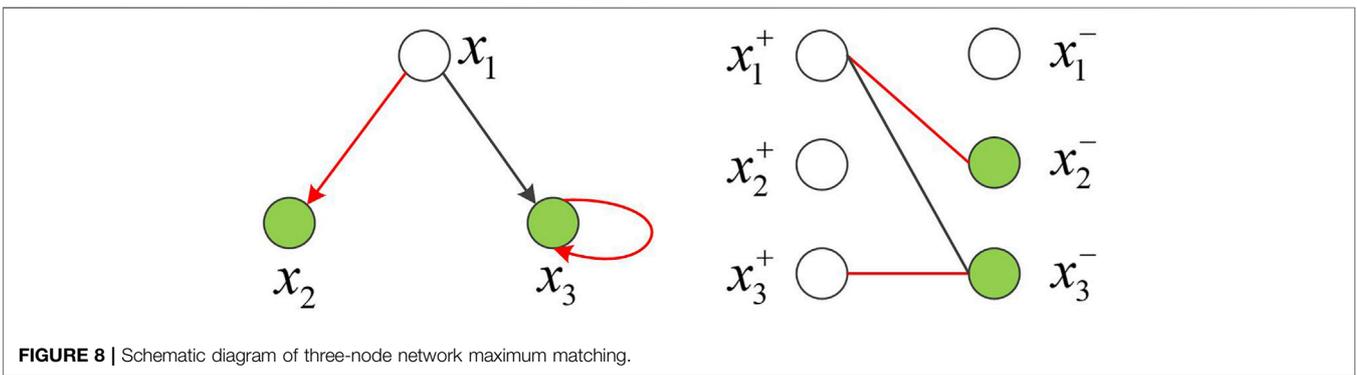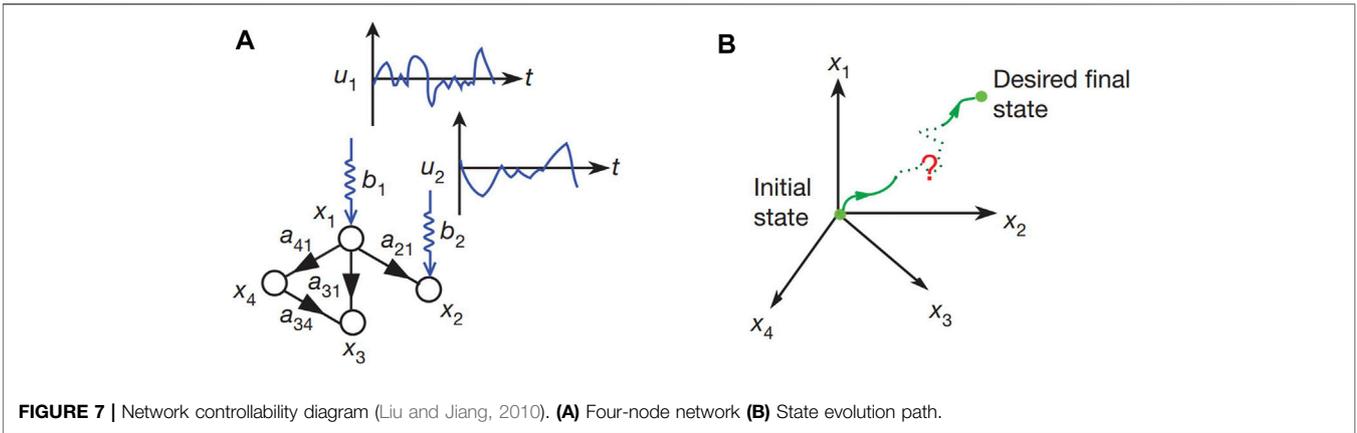$$B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad (10)$$

According to **Eq. 7**, QC is a representation of the controllability matrix:

$$QC = \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & a_{21}b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{31}b_1 & 0 & a_{34}b_1 & 0 & 0 & 0 \\ 0 & 0 & a_{41}b_1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (11)$$

The network is fully controllable, not only depending on the input of external signal but closely related to weight between directed edges, according to the new concept of structural controllability proposed by Lin in 1974 (Hopcroft and Karp, 1971), for different weight combinations of matrix A and matrix B, which are structurally controllable except for some extreme cases (such as all zeros). This structure controllability ignores weight value, which greatly reduces the difficulty of actual calculations and makes any networks controllability analysis based on Kalman ranks possible.

In **Eq. 11**, the elements of matrix QC can have different values according to weight values. Except for some extreme cases, it can be determined that $rank(QC) = N = 4$. Then the network can be controlled by controlling nodes x_1 and x_2, that is, the system can be transformed from the initial state to any desired state within a finite time through input signals u_1 and u_2 (as shown in **Figure 7**).

However, this is only one of many control input methods. At this time, each node has two choices of control and non-control. If a brute-force search is used to obtain all possible ranks of the controllability matrix, then a total of $2^n - 1$ different type is

FIGURE 7 | Network controllability diagram (Liu and Jiang, 2010). **(A)** Four-node network **(B)** State evolution path.

FIGURE 8 | Schematic diagram of three-node network maximum matching.

FIGURE 9 | Schematic diagram of five-node network maximum matching. **(A)** Maximum matching one **(B)** Maximum matching two.

needed. This is a difficult task as the number of network nodes increases.

To avoid the brute-force search for driver nodes, Liu et al. (Liu and Jiang, 2010) creatively combined network controllability analysis with graph theory analysis and proved that the number of minimum driver nodes could be obtained by a maximum matching algorithm, which solved the calculation problem of minimum driver nodes. For a directed network, if all directed edges in directed edges set M don't share the starting and ending nodes, then set M is a

matching of network. The end node of matched edge is called matched node. Matching nodes and matching edges can be found using a bipartite graph corresponding to the network. **Figure 8** includes a simple network with three nodes and its bipartite graph. The red edges are matched edges, green nodes are matched nodes, and white nodes are nonmatched nodes. It can be seen from the bipartite graph that the three edges have only one matching method so that these edges do not share an intersection point, that is, the maximum matching represented by the red edge. **Figure 9** shows a simple
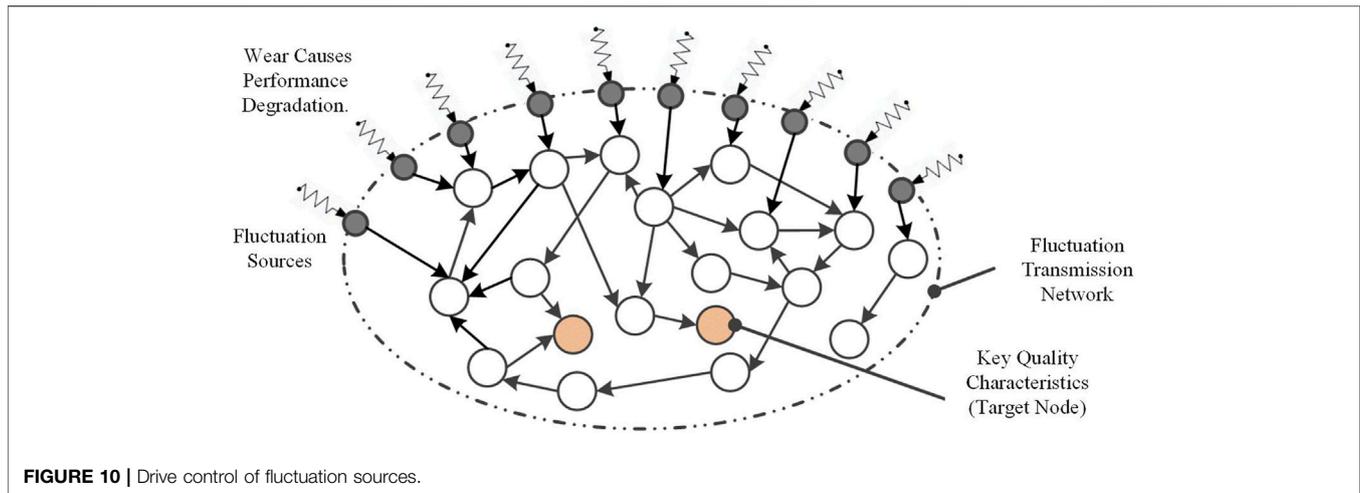
FIGURE 10 | Drive control of fluctuation sources.

network with five nodes and its bipartite graph. It can be known that it has two maximum matchings.

Liu et al.(Liu and Jiang, 2010) pointed out that if want to control nodes state fully, the minimum number of Driver Nodes required as the following formula:

$$N_D = max\{N - |M|, 1\} \tag{12}$$

As nodes and edges increase, the number of maximum matchings will increase so that the minimum driver nodes control methods will also increase. For complex networks, there are algorithms such as Hopcroft-Karp that can solve the maximum matching of the bipartite graph within $O(n^3)$ time complexity (Hopcroft and Karp, 1971), which makes it possible to solve the control of complex networks.

We can construct a cactus structure similar to graph theory through the maximum matching algorithm and select the root of the independent disjoint cactus structure as the input node to completely control the entire network.

5.2 Driving Control Ways of Key Quality Characteristics

The structure of a fluctuation transmission network makes many possible ways to cause quality deviation, as shown in **Figure 5**. Although the deterministic relationship (weight) of the mutual influence between network nodes is unknown, the directed edges can be constructed to indicate a mutual relationship between them. The external signals of u_1 and u_2 in **Figure 6A** can be understood as performance degradation of nodes x_1 and x_2. And the weights b_1 and b_2 can be understood as the degree of degradation. The node can be used as a driver node if an external input signal directly controls it. The in-degree of fluctuation source is zero. According to network controllability, if you want to control network fluctuation, the fluctuation source must be the driver node (Liu and Jiang, 2010). Controllability analysis of network can measure control mode and control ability of node whose in-degree is zero, so it can be applied to evaluate the importance of fluctuation sources.

The combination of different fluctuation sources determines the state space of the system. That is, the degraded state of different equipment components will ultimately be represented at a level of accuracy reliability. As shown in **Figure 10**, by applying external control signals (different degradation states of equipment) to these driver nodes, the key quality attribute nodes (also referred to as target nodes in this section) can be controlled.

Assuming that network has H fluctuating sources, it can correspond to 2^H different fluctuating sources with different combinations. At the same time, there are multiple maximum matchings for each type of fluctuating source combination, resulting in a wide variety of network drive control ways. This is also consistent with the diversity of error transmission paths that lead to deviations in the final quality attributes. Take the five-node network in **Figure 9**. There are two matchings for network structure. The driver node corresponding to x_3 is x_1.And the driver node corresponding to x_5 is x_4 in **Figure 7A**; in **Figure 7B**, the driver node corresponding to x_3 is x_4.And the driver node corresponding to x_5 is x_1. External input of driver nodes x_1 and x_4 will dominate state changes of x_3 and x_5. If x_1 and x_3 correspond to performance degradation factors of fluctuation source, x_3 and x_5 correspond to the critical quality characteristics to be controlled, then control capabilities of x_1 and x_3 on the entire network are equal.

5.3 Accuracy-Reliability Oriented Importance Measure

The network controllability analysis provides a way to quantify the control ability of fault source nodes on key quality characteristics nodes (target nodes) and then measure the importance that affects the accuracy reliability of the production line.

The focus of this section is not to use a maximum matching algorithm to find the least set of driver nodes, but to identify the control ability of fluctuation source on target nodes, thereby identifying its importance. Several key quality characteristics often determine the quality of the workpiece. In the
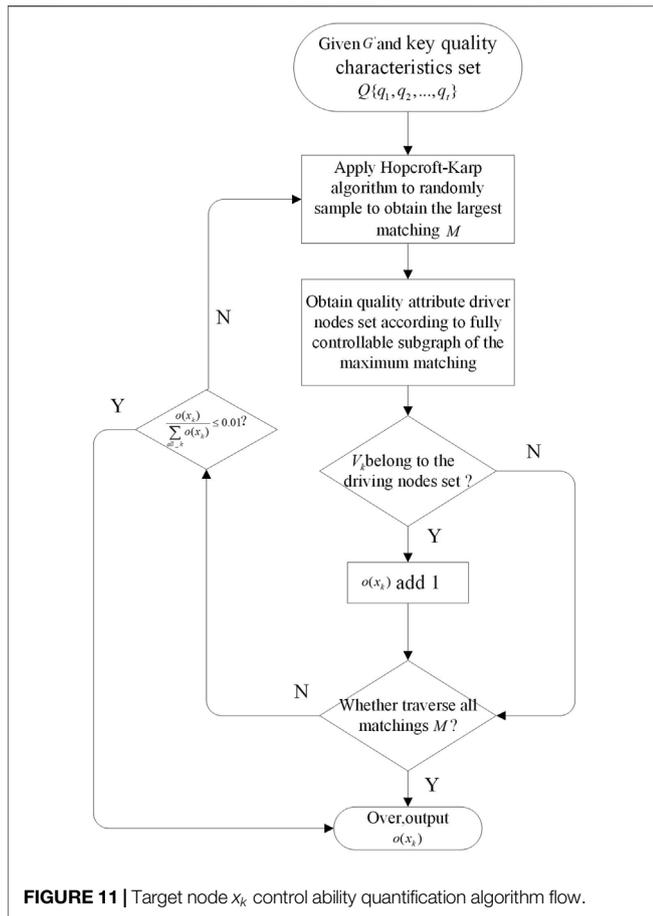
FIGURE 11 | Target node x_k control ability quantification algorithm flow.

fluctuation transmission network, some nodes can always appear as the root of the cactus structure, controlling key quality characteristics nodes. And the other nodes have never been able to control key quality characteristics nodes, and most nodes are in between. Given this, the importance of the fluctuation source node can be measured by the frequency of occurrence of cactus root in the maximum matching.

Some scholars have proposed measures of the importance of node Control capacity index and Control centrality. The control ability is defined as the probability of appearing in a set of all minimum driver nodes. In the network structure of this section, the fluctuation source node that represents a degrading factor of equipment always acts as driver nodes, so this method cannot be used to distinguish importance. In addition, control centrality measures a node's control ability over an entire network, but it fails to measure its control ability over target nodes.

In summary, this section proposes the concept of Target Control Capacity (TCC). In **Figure 11**, the algorithm for control ability of fluctuation source node x_k to target node can be described as follows:

Step 1: Given fluctuation transmission network G', determine key quality characteristics node set $Q\{q_1, q_2, \cdots, q_t\}$, and initialize = 0;

Step 2: Apply the Hopcroft-Karp algorithm to randomly sample to obtain the largest matching and determine a fully controllable subgraph of each matching plan. Find the root of the cactus structure of key quality characteristics node (target node), and determine the set of roots D, if $x_k \in D$, then add 1, otherwise do not add.

Step 3: Calculate the frequency of fluctuation source node x_k in fully controllable subgraph to which the key quality characteristics node belongs during the sampling process.

Step 4: When frequency change stabilizes (less than or equal to 0.001), or the algorithm has traversed all matchings, obtain the importance of x_k according to **Eq. 13**, and determine weaknesses that affect accuracy reliability.

The importance of fluctuation source node x_k can be calculated as follows:

$$I^{x_k} = [1 - R(x_k)]\frac{o(x_k)}{\sum_{all\_k} o(x_k)} \tag{13}$$

Among them, $1 - R(x_k)$ is the probability of accuracy failure of the fluctuation source node x_k, $o(x_k)$ is the frequency of occurrence of x_k in the fully controllable subgraph of crucial quality characteristics node.

6 CASE STUDIES

In this section, we give two examples to illustrate the developed methodology. The results show that network reliability can effectively measure the importance of fluctuation sources, thereby identifying the weak links that affect accuracy and reliability.

6.1 Case Study of a Launcher Part Machining Line

In order to verify the effectiveness of weakness identification, this section takes a launcher part machining line (Liu and Jiang, 2007) as a case and compares the method in this paper with it.

The coaxiality of hole A of the launcher part is a key quality characteristic, and the specification value of the shaft diameter of hole A is $2 - \phi 250_0^{+0.072}$. The coaxiality does not exceed 0.02 mm. Three processes process it (rough boring, semi-finish boring, and finish boring). **Figure 12** shows the fluctuation transmission network of these three processes. The bottom surface, process holes E1 and E2, left side surface, and hole D are used as the positioning datum. There is a benchmark relationship with hole A. Hole A has a precision evolution relationship between features with different precisions formed in different processes.

In **Figure 12**, the physical meaning of MSC and accuracy reliability factor nodes are shown in **Table 1**.

Therefore, this article modifies the probability of **Eq. 14** as follows:

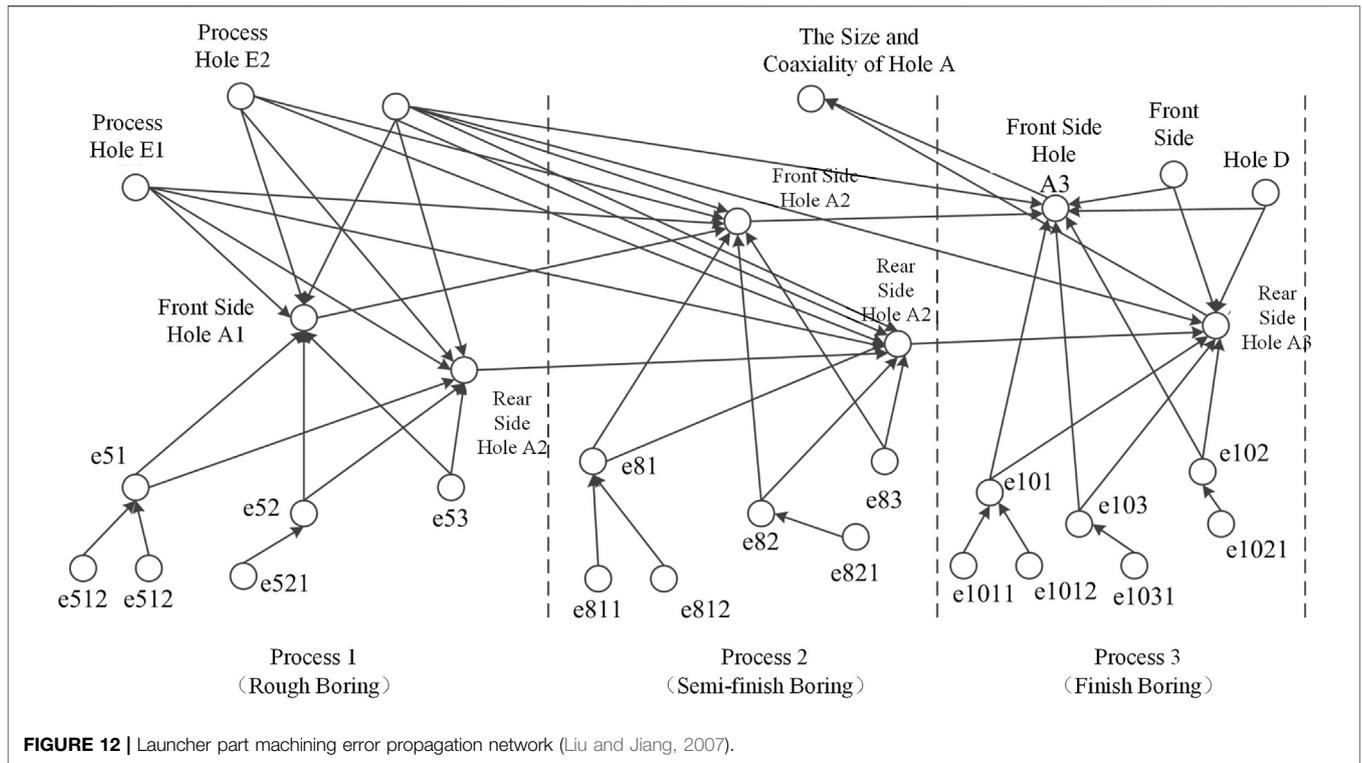$$I_B^{x_k} = V_k \frac{o(x_k)}{\sum_{all_k} o(x_k)} \tag{14}$$

FIGURE 12 | Launcher part machining error propagation network (Liu and Jiang, 2007).

TABLE 1 | The MSC and accuracy reliability factors of hole a

MSC	Accuracy reliability
horizontal boring machine (e51)	Spindle vibration (e511)
	back engagement (e512)
Boring tool (e52)	Cutting hot distortion (e521)
Fixture (e53)	—
horizontal boring machine (e81)	Spindle rotation accuracy (e811)
	worktable rotation accuracy (e812)
Boring tool (e82)	Tool wear (e821)
Fixture (e83)	—
horizontal boring machine (e101)	worktable rotation accuracy (e1011)
	Spindle rotation accuracy (e1012)
Fixture (e102)	fixture positioning components Wear (e1021)
Finish Boring Tool (e103)	Tool wear (e1031)

Because the literature (Liu and Jiang, 2007) doesn't give the probability of each accuracy reliability factor but gives grading grade V_k of its fluctuation level.

The literature (Liu and Jiang, 2007) determines important nodes by measuring the fluctuation transmission effect of nodes on key quality characteristics. The conclusion is that the finish boring process e101 and e102 have a greater impact on coaxiality. Using the algorithm in this article, measure the ability of each MSC node to control key quality characteristics of coaxiality on hole A, and the results obtained are shown in **Table 2**. It can be seen from **Table 2** that it is determined that e103, e101, and e102 have a greater impact on quality characteristics, and the wear of fine boring tools has the most significant impact. Both methods show that when improving coaxiality quality characteristics of hole A, the e101 and e102 of MSC need to be adjusted. That is, to

TABLE 2 | Algorithm result of nodes control ability.

MSC	V_k	$I_B^{x_k}$	
		Value	Sequence
e101	6	0.582	2
e102	5	0.485	3
e103	8	0.776	1
e51	6	0.390	6
e52	5	0.325	7
e81	6	0.450	4
e82	6	0.450	4

improve rotation accuracy of table and spindle of the horizontal boring machine to improve system accuracy reliability.

This case shows that the method in this paper can effectively obtain the importance of fluctuation sources, thereby identifying the weakness that affects accuracy and reliability.

6.2 Case Study of a Piston Production Line

Figure 13 shows the structural model of the fluctuation transmission network of a piston production line. The network has six characteristic processing nodes, two key quality characteristic nodes, and 32 precision reliability factor nodes.

The in-degree of accuracy reliability factor node (the fluctuation source node in **Figure 13**) representing performance degradation of machine tool components is 0, which must be the driver nodes of the network. Over time,
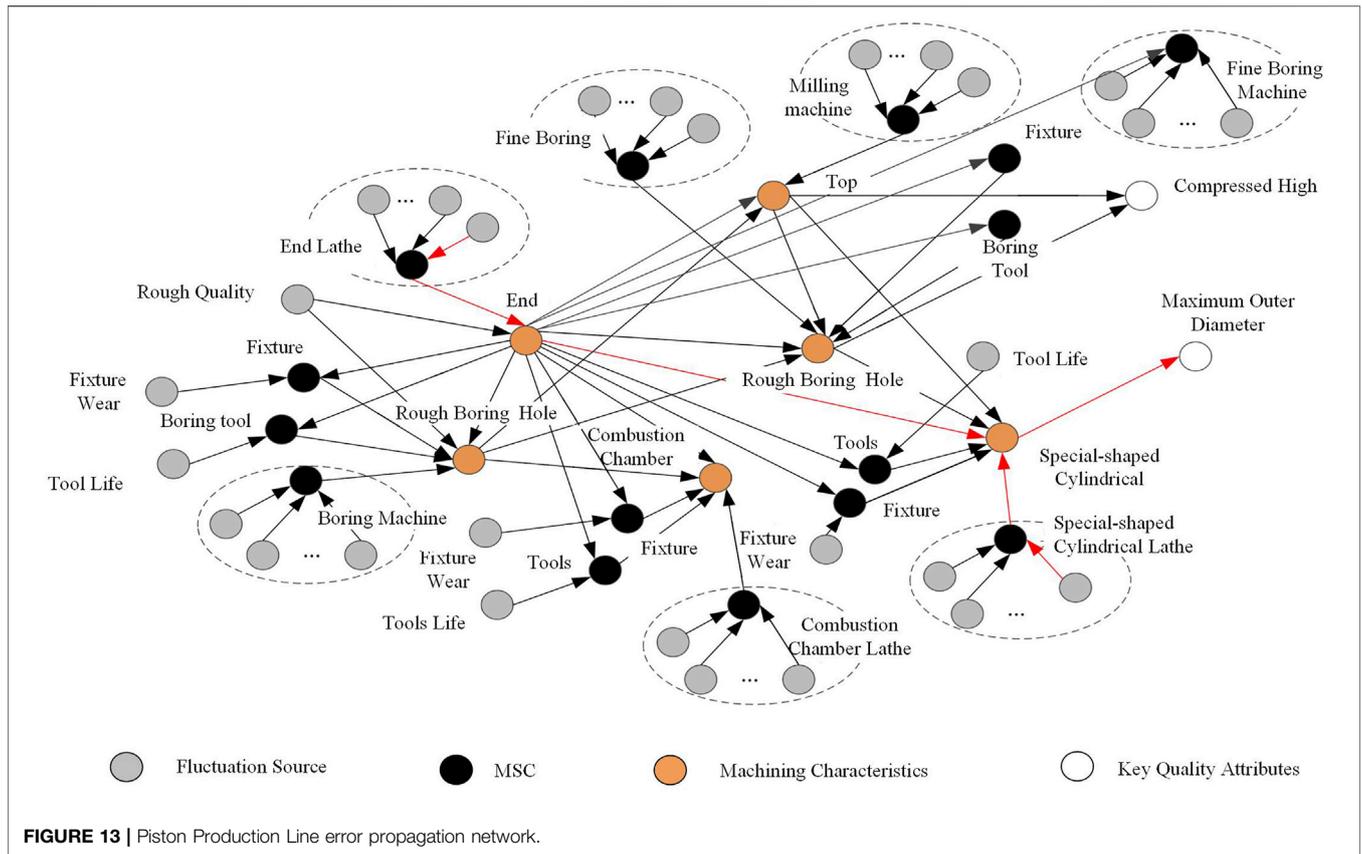
FIGURE 13 | Piston Production Line error propagation network.

TABLE 3 | Accuracy Reliability Oriented system weakness identification.

MSC	Accuracy reliability factors	o(x_k)	1 − R(x_k)	Importance
End lathe	slide rails Wear	652	1.12×10^{-4}	4.40×10^{-6}
	Spindle manufacturing accuracy failure	652	1.05×10^{-4}	4.12×10^{-6}
	Spindle bearing wear	652	2.02×10^{-4}	7.93×10^{-6}
	Thrust bearing wear	652	2.65×10^{-4}	1.04×10^{-5}
End lathe tool	Tool wear	652	5.52×10^{-3}	2.17×10^{-4}
End lathe fixture	Fixture wear	652	2.13×10^{-3}	0.84×10^{-4}
Rough boring machine	Bad gear meshing	458	1.94×10^{-4}	5.35×10^{-6}
	Spindle bearing wear	458	1.52×10^{-4}	4.19×10^{-6}
	Loose connection sleeve	458	2.36×10^{-4}	6.51×10^{-6}
Rough boring tool	Tool wear	458	3.25×10^{-3}	8.97×10^{-5}
Rough boring fixture	Fixture wear	458	2.13×10^{-3}	5.88×10^{-5}
Combustion chamber lathe	slide rails Wear	0	3.12×10^{-4}	0
	Spindle manufacturing accuracy	0	5.05×10^{-4}	0
	Spindle bearing wear	0	2.25×10^{-4}	0
	Manufacturing thrust bearing wear	0	2.90×10^{-4}	0
Combustion chamber lathe tool	Tool wear	0	4.51×10^{-3}	0
Combustion chamber lathe fixture	Fixture wear	0	2.03×10^{-3}	0
Milling machine	Rail wear	389	5.25×10^{-4}	1.23×10^{-5}
	Increased screw backlash	389	2.26×10^{-4}	5.30×10^{-6}
	Spindle bearing wear	389	2.85×10^{-4}	6.68×10^{-6}
Milling machine tool	Tool wear	389	2.81×10^{-3}	6.58×10^{-5}
Milling machine fixture	Fixture wear	389	2.03×10^{-3}	4.76×10^{-5}
Finish boring machine	Bad gear meshing	871	3.02×10^{-4}	1.58×10^{-5}
	Spindle bearing wear	871	3.25×10^{-4}	1.71×10^{-5}
	Loose connection sleeve	871	2.98×10^{-4}	1.56×10^{-5}
Finish boring tool	Tool wear	871	4.36×10^{-3}	2.29×10^{-4}
Finish boring fixture	Fixture wear	871	3.03×10^{-3}	1.59×10^{-4}
Special-shaped cylindrical lathe	Three-knife servomechanism wear	820	6.85×10^{-4}	3.38×10^{-5}
	Worn screw connection thrust bearing	820	3.60×10^{-4}	1.78×10^{-5}
	Spindle bearing wear	820	1.90×10^{-4}	0.94×10^{-5}
Special-shaped cylindrical tool	Tool wear	820	2.51×10^{-3}	1.23×10^{-4}
Special-shaped cylindrical fixture	Fixture wear	820	2.09×10^{-3}	1.03×10^{-4}

these nodes will experience varying degrees of performance degradation during their life cycle. This performance degradation can be understood as the network is controlled by "external input signals" and the driving state of the network can fully control the accuracy of the production line. The combination of different fluctuation source nodes corresponds to different precision states of the production line, and each state has a variety of network drive control methods. The maximum matching algorithm can be used to search for control methods to identify system weaknesses. The piston's compression height and the maximum outer diameter are two key quality characteristics, as shown in **Figure 13**. Different drive control methods can be used to control these two target nodes through the maximum matching algorithm.

According to the algorithm in **Figure 11**, the accuracy reliability factor nodes of equipment can be determined to control compression height and maximum outer diameter. Through the maximum matching of a network, control methods of these nodes can be found. Among many control methods, finding a driver node that is reachable to the node of compression height and maximum outer diameter and traversing all maximum matchings will quantify the control ability of the fluctuation source. The results obtained are shown in **Table 3**.

It can be seen from **Table 3** that the node with the greatest control capability for two key quality characteristics is the source of fluctuations for the boring finish machines. These fluctuation source nodes have the same value $o(x_k)$ of 871, which indicates that among all control methods, these nodes act as the roots of a fully controllable subgraph more frequently. Due to the wear of tools and fixtures of the production line, failure is higher. So its importance ranking is generally higher than the fluctuation source of machine tool functional parts. The higher-ranking fluctuation sources should be focused on accuracy, reliability, and maintenance measures.

7 CONCLUSION

The accuracy of the production line puts forward higher requirements from the perspective of the workpiece processing process. This paper proposes a method for importance measure

in production line system based on network controllability analysis. The performance degradation of the production line equipment will spread along the multi-stage manufacturing process. When the network scale is large, it is difficult to establish a complex coupling relationship among many nodes by using quantitative analysis methods such as FT and BN. The evaluation results are not ideal due to simulation accuracy or lack of training data. In a directed network, applying external control signals can effectively drive the state evolution of the entire network. Inspired by this, the use of network controllability analysis to quantify the control ability of networks can achieve the purpose of identifying a weakness. This paper provides a new idea for reliability analysis of machining systems and has certain reference significance for quality-reliability analysis and importance measure of a complex system.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

DZ, YP, and QL contributed to conception and design of the study.

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