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Forecasting crude oil futures volatility with extreme-value information and dynamic jumps

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In this paper, we propose the realized EGARCH model with jumps (hereafter REGARCH-Jump model) to model and forecast the crude oil futures volatility. A key feature of the proposed REGARCH-Jump model is its ability to account for the extreme-value information as well as time-varying jump intensity. We apply the REGARCH-Jump model to the Brent crude oil futures price data. Our empirical results provide evidence of the presence of time-varying jumps in the crude oil futures market. More importantly, we show that our proposed REGARCH-Jump model outperforms the GARCH, EGARCH, HAR, and REGARCH models in terms of both empirical return fit and out-of-sample volatility forecast. Moreover, the superior forecast performance of the REGARCH-Jump model is robust to alternative out-of-sample forecast windows. Finally, a Value at Risk (VaR) analysis demonstrates the economic value of the improved volatility forecasts from the REGARCH-Jump model. In summary, our findings highlight the importance of accommodating the extreme-value information and jump dynamics in forecasting the volatility of crude oil futures prices.

KEYWORDS

volatility forecasting, crude oil futures, extreme-value information, jump dynamics, realized EGARCH model

1 Introduction

As an indispensable energy resource in a country's development, the crude oil plays a vital role in industrial production and transportation. In recent years, major events, such as the outbreak of the COVID-19 pandemic at the beginning of 2020 and the Russia-Ukraine conflict in 2022, have lead to significant impact on the crude oil futures markets, resulting in large fluctuations in crude oil futures price. Notably, this would have adverse effects on economic activities. As a consequence, accurately modeling and forecasting the volatility of crude oil futures prices has become a crucial concern for market participants and policy makers. In fact, crude oil futures volatility plays an important role in asset allocation, risk management and derivative pricing. And there is now a large body of literature on forecasting the volatility of crude oil futures prices (Zhang et al., 2019, 2023; Kazemzadeh et al., 2022; Li et al., 2022; Zhang and Zhang, 2023a,b; Xu et al., 2024).

Given the importance of accurately forecasting the crude oil futures volatility, this paper aims to develop a new volatility model, namely the realized EGARCH model with jumps (hereafter REGARCH-Jump model), to model and forecast the crude oil futures volatility. Our proposed model has the capacity to account for the extreme-value information and the time-varying jumps in the crude oil futures prices. Moreover, the model is able to capture the complex volatility characteristics, such as the time-varying volatility and volatility asymmetry.

This paper contributes to the literature on crude oil futures volatility forecasting in several aspects. Firstly, we extend the REGARCH model to incorporate the dynamic jumps, and propose the REGARCH-Jump model to model and forecast the crude oil futures volatility. Our proposed model can capture the extreme-value information and the time-varying jumps in the crude oil futures prices simultaneously, which has the potential to improve the crude oil futures volatility forecasts.

Secondly, we apply the REGARCH-Jump model to the Brent crude oil futures price data. Our empirical results provide evidence of the presence of time-varying jumps in the crude oil futures market. More importantly, we show that our proposed REGARCH-Jump model outperforms the GARCH, EGARCH, HAR and REGARCH models in terms of both empirical return fit and out-of-sample volatility forecast. Moreover, the superior forecast performance of the REGARCH-Jump model is robust to alternative out-of-sample forecast windows.

Finally, a Value at Risk (VaR) analysis is conducted to demonstrate the economic value of the improved volatility forecasts from the REGARCH-Jump model. We confirm that the REGARCH-Jump model can produce reasonable VaR forecasts.

To facilitate quick reference for readers, Table 1 summarizes the model names and their abbreviations used in this paper. The remainder of the paper is organized as follows. Section 2 reviews relevant literature on crude oil volatility forecasting. In Section 3, we describe the REGARCH-Jump model and the maximum likelihood method for parameter estimation. In Section 4, we introduce the methods used for out-of-sample evaluation. The empirical results are presented in Section 5, and Section 6 concludes the paper.

2 Literature review

Accurately forecasting the crude oil futures volatility is important for market investors, risk managers and policy markers, since it has an important influence on investors' financial strategies and policymakers' decisions (Agnolucci, 2009; Wei et al., 2017; Wen et al., 2019; Lyu et al., 2021a,b; Huang et al., 2023). It has been well documented in the literature that financial Volatility exhibits complex characteristics, such as volatility clustering, leverage effects and mean-reverting properties, which poses great challenges in volatility measuring and forecasting. Traditionally, volatility estimator is derived from closing prices, and a variety of volatility models have been proposed to describe its dynamics in the last three decades. Since the seminal work by Engle (1982) and Bollerslev (1986), who propose the generalized autoregressive conditional heteroskedasticity (GARCH)-type models, numerous studies have been devoted to investigating the crude oil volatility modeling and forecasting by using GARCH-type models (see, e.g., Iglesias and Rivera-Alonso, 2022; Hong et al., 2022; Wang et al., 2021; Lin et al., 2020; Pan et al., 2017; Kang and Yoon, 2013). An alternative to the GARCH volatility model is the stochastic volatility (SV) models of Taylor (1986) and Heston (1993). Tsay (2005) presents a review of the two strands of the literature. Lyu et al. (2021a,b) analyse the time-varying effects of global economic uncertainty shocks on the volatilities of Brent and WTI crude oil prices through a time-varying parameter structural vector autoregressive model with stochastic volatility. Essentially, both the GARCH and SV models are return-based models, which are constructed based on daily closing prices, neglecting all intraday price movement, which reduces the predictive power of the models (Pu et al., 2016; Gong and Lin, 2018).

With the increasing availability of intraday high-frequency data, many authors have introduced the realized measures to measure the financial volatility (Andersen et al., 2001; Barndorff-Nielsen and Shephard, 2002, 2004; Kazemzadeh et al., 2023a,b, 2022). Corsi (2009) develop the HAR model based on realized volatility, which effectively captures the long-memory characteristics of volatility. Thanks to its ease of extensibility, the model has gained wide recognition and application. Wen et al. (2016) analyze the impact of stock market uncertainty on the volatility of the crude oil futures market by constructing an HAR model. Furthermore, Zhang et al. (2023) compare the performance of HAR models with different structural changes in forecasting the volatility of the crude oil futures market. Notably, the realized measure contains more information about the current level of volatility, which can provide more accurate volatility estimates (Andersen et al., 2001; Lyócsa et al., 2021). However, the realized measure based on intraday high-frequency data is sensitive to market microstructure noise, which leads to a biased volatility estimate.

As an alternative, the price range computed from the intraday high and low prices has been proposed to measure volatility. The idea of using price range in finance can be found in Mandelbrot (1971), who employs it to test the existence of long-term dependence in asset prices. The price range incorporates more (extreme-value) information on intra-period trajectory of the prices than the return-based volatility measure that only includes single measurement of the closing prices each period (Wu and Hou, 2020). Numerous studies have shown that the intraday price range is a more efficient measure of financial volatility relative to the commonly used return-based measure, such as the absolute (or squared) return or even the realized volatility from intraday returns. In fact, by employing the extreme-value theory and some well-known properties of range, Parkinson (1980) provide evidence of the superiority of using range as a volatility estimator. Alizadeh et al. (2002) show theoretically, numerically and empirically that range-based volatility estimator is not only highly efficient, but also approximately Gaussian and robust to microstructure noise. Brandt and Jones (2006) find that the range-based EGARCH model has better volatility forecast performance compared to the return-based

TABLE 1 Table of abbreviations.

Abbreviations	Meaning
GARCH	Generalized AutoRegressive conditional heteroskedasticity
EGARCH	Exponential generalized AutoRegressive conditional heteroskedasticity
HAR	Heterogeneous autoregressive
REGARCH	Realized exponential generalized autoregressive conditional heteroskedasticity

EGARCH model. More recently, Degiannakis and Livada (2013) show that the price range volatility estimator is more accurate than the realized volatility estimator based on five, or less, equidistance points in time.

To explore the intraday (extreme-value) information for modeling volatility, motivated by the insights of the realized SV model (Shirota et al., 2014; Asai and McAleer, 2022), Hansen et al. (2012) develop a joint model of return and realized measure, namely the realized GARCH (RGARCH) model. The RGARCH model can capture the leverage effect of volatility, and is well suited for situations where volatility changes rapidly to a new level. Using the RGARCH model, Hansen et al. (2012) demonstrate that the inclusion of the realized measures can improve the model's ability to forecast volatility. Further, Hansen and Huang (2016) extend the RGARCH model to incorporate an additional leverage function to capture leverage effect more flexibly. The resulting model is referred to as the REGARCH model. Importantly, the R(E)GARCH model has a simple structure, which can be estimated and filtered easily. In addition, the model can automatically adjust the bias in the realized measures caused by non-trading hours and market microstructure noise. Subsequently, the R(E)GARCH model has attracted a great deal of attention in the literature. For example, Huang et al. (2017) and Tong and Huang (2021) apply the R(E)GARCH model to option pricing, and find that the R(E)GARCH model provide better option pricing performance than the traditional GARCH models. Chen and Watanabe (2019) and Chen et al. (2022) apply the R(E)GARCH model to risk measurement.

Despite the empirical success of the R(E)GARCH model, it does not take into account the presence of jumps in asset prices, which have been well recognized in the literature (see, e.g., Arouri et al., 2019; Pan et al., 2020; Qiao et al., 2020; Guo et al., 2023; Wu et al., 2024; Zhang et al., 2024). The huge changes (i.e., jumps) in asset prices usually cannot be explained by the current level of volatility. In recent years, numerous studies have shown that the occurrence of jumps is time-varying (see, e.g., Chernov et al., 2018; Zhou et al., 2019; Dutta et al., 2021). Most of these studies capture the risk of market crashes by assuming that the intensity of jumps is a function of the variance of asset returns. Although this modeling approach is intuitive and simple, it can not capture the jumps of asset price adequately.

Table 2 summarizes the relevant studies in the literature, while Table 3 provides an overview of the associated econometric models. Motivated by the above insights, in this paper we use price range as a proxy of realized measure, and propose the REGARCH-Jump model to modeling and forecasting the crude oil futures volatility. Notably, our proposed model can capture the extreme-value information as well as the dynamic jumps through assuming the jump intensity is governed by a autoregressive conditional jump intensity process. It is worth pointing out that although significant contributions have been made in the literature on crude oil futures volatility forecasting, few studies have taken into account both the extreme-value information and dynamic jumps in the crude oil futures prices for predicting the crude oil futures volatility.

TABLE 2 Relevant literature review.

Theme	Year	References
Crude oil futures volatility	2009	Agnolucci, 2009
	2017	Wei et al., 2017
	2019	Zhang et al., 2019
		Wen et al., 2019
	2021	Lyu et al., 2021a,b
	2022	Li et al., 2022
	2023	Huang et al., 2023
		Zhang and Zhang, 2023a,b
	2024	Xu et al., 2024
High-frequency information	1971	Mandelbrot, 1971
	2001	Andersen et al., 2001
	2002	Barndorff-Nielsen and Shephard, 2002
		Alizadeh et al., 2002
	2004	Barndorff-Nielsen and Shephard, 2004
	2006	Brandt and Jones, 2006
	2013	Degiannakis and Livada, 2013
	2020	Wu and Hou, 2020
	2021	
		Lyócsa et al., 2021
	2022	Kazemzadeh et al., 2022
Jump dynamics information	2018	Chernov et al., 2018
	2019	Zhou et al., 2019
	2020	Pan et al., 2020
		Qiao et al., 2020
	2021	Dutta et al., 2021
	2006	Guo et al., 2023
	2024	Wu et al., 2024
	2024	Zhang et al., 2024

3 The model

In this section, we first provide a brief review of the GARCH, the EGARCH, the HAR and the REGARCH models. Then we introduce the extension of the REGARCH model, namely the REGARCH-Jump model, which simultaneously accommodates the extreme-value information and time-varying jump intensity. Finally, we describe the maximum likelihood method for estimation of the parameters of the proposed model.

3.1 GARCH model

In financial econometric literature, the GARCH model proposed by Bollerslev (1986) is a popular approach for measuring

TABLE 3 Relevant models review.

Theme	Year	References
GARCH-type models	1982	Engle, 1982
	1986	Bollerslev, 1986
	2013	Kang and Yoon, 2013
	2017	Pan et al., 2017
	2019	Zhang et al., 2019
		Wen et al., 2019
	2020	Lin et al., 2020
	2021	Wang et al., 2021
		Lyu et al., 2021a,b
	2022	Hong et al., 2022
		Iglesias and Rivera-Alonso, 2022
		Li et al., 2022
	2023	Huang et al., 2023
		Zhang and Zhang, 2023a,b
	2024	Xu et al., 2024
SV-type models	1986	Taylor, 1986
	1993	Heston, 1993
	2005	Tsay, 2005
	2016	Pu et al., 2016
	2017	Pan et al., 2017
	2018	Gong and Lin, 2018
	2021	Lyu et al., 2021a,b
Realized volatility models	2009	Corsi, 2009
	2012	Hansen et al., 2012
	2014	Shirota et al., 2014
	2016	Hansen and Huang, 2016
		Wen et al., 2016
	2017	Huang et al., 2017
	2019	Chen and Watanabe, 2019
	2021	Tong and Huang, 2021
	2022	Chen et al., 2022
		Asai and McAleer, 2022
	2023	Zhang et al., 2023

and forecasting financial volatility. The GARCH model can be written as

$$r_t = \mu + z_t \tag{1}$$
$$z_t = \sqrt{h_t}\epsilon_t, \quad \epsilon_t \sim i.i.d.N(0,1) \tag{2}$$
$$h_t = \omega + \alpha z_{t-1}^2 + \beta h_{t-1} \tag{3}$$

where $r_t = \log(P_t/P_{t-1})$ is the log-return on day t, where P_t is the closing price on day t; μ is the conditional mean of r_t; h_t is the conditional variance of r_t; z_t is the return innovation, and ϵ_t is the standardized return innovation.

3.2 EGARCH model

The EGARCH model was proposed by Nelson (1991), which has the capacity to capture the leverage effect, which has been found to be important for volatility forecasting. The EGARCH model is given by

$$r_t = \mu + z_t \tag{4}$$
$$z_t = \sqrt{h_t}\epsilon_t, \quad \epsilon_t \sim i.i.d.N(0,1) \tag{5}$$
$$\log(h_t) = \omega + \beta\log(h_{t-1}) + \gamma\epsilon_{t-1} + \alpha(|\epsilon_{t-1}| - E(|\epsilon_{t-1}|)) \tag{6}$$

where the coefficient γ captures the leverage effect when $\gamma < 0$.

3.3 HAR model

Based on the Heterogeneous Market Hypothesis, the HAR model proposed by Corsi (2009) aims to capture the volatility dynamics in financial markets, which can be written as

$$x_{t+1} = c_t + \beta_d x_t + \beta_w x_t^w + \beta_m x_t^m + \omega_{t+1} \tag{7}$$
$$\omega_t = \sigma_d\epsilon_t \quad \epsilon_t \sim i.i.d.N(0,1) \tag{8}$$
$$x_t^w = \frac{x_t^d + x_{t-1}^d + \cdots + x_{t-4}^d}{5} \tag{9}$$
$$x_t^m = \frac{x_t^d + x_{t-1}^d + \cdots + x_{t-21}^d}{22} \tag{10}$$

where x_t is the realized measure of volatility on day t; x_t^w and x_t^m represent the average realized volatility for the past weekly and monthly periods, respectively. (corresponding to lags of 5 days, and 22 days).

3.4 REGARCH model

Considering the fact that the RGARCH model cannot capture the leverage effect (asymmetric response of volatility to positive and negative shocks) adequately, Hansen and Huang (2016) extend the RGARCH to the REGARCH model, which can be written as

$$r_t = \mu + z_t \tag{11}$$
$$z_t = \sqrt{h_t}\epsilon_t, \quad \epsilon_t \sim i.i.d.N(0,1) \tag{12}$$
$$\log(h_t) = \omega + \beta\log(h_{t-1}) + d(\epsilon_{t-1}) + \alpha u_{t-1} \tag{13}$$
$$\log(x_t) = \xi + \varphi\log(h_t) + \upsilon(\epsilon_t) + u_t, u_t \sim \text{ i.i.d.}N\left(0, \sigma_u^2\right) \tag{14}$$
$$d(\epsilon_t) = d_1\epsilon_t + d_2\left(\epsilon_t^2 - 1\right) \tag{15}$$
$$\upsilon(\epsilon_t) = \upsilon_1\epsilon_t + \upsilon_2\left(\epsilon_t^2 - 1\right) \tag{16}$$

where u_t is the realized measure innovation, which is independent of the return innovation ϵ_t; $d(\epsilon_t)$ and $\upsilon(\epsilon_t)$ are the leverage functions, which are used to capture the leverage effect, satisfying $E_{t-1}\left[d(\epsilon_t)\right] = E_{t-1}\left[\nu(\epsilon_t)\right] = 0$. If $d_1 < 0$, $\upsilon_1 < 0$, it means that there exists leverage effect.

In the REGARCH model, Equations 11, 13 and 14 are referred to as the return equation, the variance (GARCH) equation and the measurement equation, respectively. The measurement equation relates the ex-post realized measure to the ex-ante conditional variance, with bias-correction coefficients ξ and φ used to correct the bias in the realized volatility measure caused by non-trading hours and market microstructure noise. Consistent with Takahashi et al. (2009), Koopman and Scharth (2012) and Wu et al. (2020), we assume that $\varphi = 1$ to facilitate model estimation and to improve out-of-sample performance.

3.5 REGARCH-jump model

The REGARCH model fall short in capturing the jumps (huge changes) in the asset prices. In light of this, this paper proposes the REGARCH-Jump model that extends the REGARCH model of Hansen and Huang (2016) to incorporate the dynamic jump intensity to model and forecast the volatility of crude oil futures markets. The specification of the REAGRCH-Jump model is as follows

$$r_t = \mu + z_t + y_t \tag{17}$$

$$z_t = \sqrt{h_{z,t}}\epsilon_t, \quad \epsilon_t \sim i.i.d.N(0,1) \tag{18}$$

$$y_t = \sum_{j=0}^{n_t} Y_t^j, \quad j = 0,1,2,\ldots,n_t \tag{19}$$

$$\log(h_{z,t}) = \omega + \beta\log(h_{z,t-1}) + d(\epsilon_{t-1}) + \alpha u_{t-1} \tag{20}$$

$$\log(x_t) = \xi + \varphi\log(h_t) + \upsilon(\epsilon_t) + u_t, u_t \sim i.i.d.N(0,\sigma_u^2) \tag{21}$$

where z_t and y_t denote the normal component and the jump component, respectively, which are assumed to be independent. The normal component z_t is assumed to be distributed as $N(0,h_{z,t})$, where $h_{z,t}$ is the conditional variance of the normal component, which is governed by the REGARCH dynamic. The jump component y_t follows a compound Poisson process with jump intensity $h_{y,t}$ and jump size Y_t^j, where Y_t^j is independently drawn from a normal distribution with mean θ and variance δ^2, that is, $Y_t^j \sim i.i.d.N(\theta,\delta^2)$. The mean and variance of the jump component, y_t, are given by $\theta h_{y,t}$ and $\left(\theta^2+\delta^2\right)h_{y,t}$, respectively. n_t is the number of jumps arriving between $t-1$ and t, which follows a Poisson counting process with intensity $h_{y,t}$, that is, $n_t \sim Poisson\left(h_{y,t}\right)$. The conditional probability of n_t can be written as

$$\Pr_{t-1}(n_t = j) = \frac{\exp(-h_{y,t})h_{y,t}^j}{j!}, \quad j = 0,1,2,\ldots \tag{22}$$

Therefore, the conditional expectation of the number of jumps arriving over the time interval $(t-1,t)$ equals the jump intensity, that is, $E_{t-1}[n_t] = h_{y,t}$. To describe the dynamics of jump intensity process $h_{y,t}$, we follow Maheu and McCurdy (2004) and assume that it follows the autoregressive conditional jump intensity model:

$$h_{y,t} = \rho + \kappa h_{y,t-1} + \psi\zeta_{t-1} \tag{23}$$

where $\rho > 0, \kappa > 0, \psi > 0$; ζ_{t-1} denotes the jump intensity residual, which can be written as

$$\begin{aligned}\zeta_{t-1} &= E_{t-1}(n_{t-1}) - h_{y,t-1}\\ &= \sum_{j=0}^{\infty} j\Pr_{t-1}\left(n_{t-1}=j\right) - h_{y,t-1}\end{aligned} \tag{24}$$

It is clear that $E(\zeta_{t-1}) = 0$. Thus, κ describes the persistence of the jump intensity process. If $h_{y,t}$ is stationary, we have $0 < \kappa < 1$. Then the unconditional jump intensity is given by $E(h_{y,t}) = \frac{\rho}{1-\kappa}$.

In the measurement Equation 21, h_t denotes the total unconditional variance of return, which can be written as

$$h_t = h_{z,t} + \left(\theta^2+\delta^2\right)h_{y,t} \tag{25}$$

It has been well documented in the literature that the use of intraday high and low prices leads to more accurate estimate of volatility than daily returns (Molnár, 2011; Chou and Liu, 2010). Therefore, in the paper we utilize the intraday price range calculated from the intraday high and low prices as a proxy for the realized measure in the HAR, REGARCH specifications, and their extensions. The intraday price range of Parkinson (1980) is defined as

$$RNG_t = \frac{\log(H_t) - \log(L_t)}{\sqrt{4\log(2)}} \tag{26}$$

where H_t and L_t are the high and low prices observed at day t, respectively. The intraday range given by Equation 26 is not only a highly efficient volatility proxy, capturing information regarding the entire intraday trajectory of the price, but also robust to microstructure noise (Alizadeh et al., 2002; Brandt and Jones, 2006). In this paper, we set $x_t = RNG_t^2$ the HAR, REGARCH specifications, and their extensions.

3.6 Maximum likelihood estimation

The proposed REGARCH-Jump model is intuitive and easy to implement. We can use the classical maximum likelihood method to estimate the parameters of the model. To be specific, the log-likelihood function of the REGARCH-Jump model can be written as

$$\ell(r,x;\Theta) = \ell(r;\Theta) + \ell(x|r;\Theta) \tag{27}$$

$$\ell(r;\Theta) \propto \sum_{t=1}^{T}\ln(f_{t-1}(r_t)) \tag{28}$$

$$\ell(x|r;\Theta) \propto \sum_{t=1}^{T}\ln(f_{t-1}(x_t|r_t)) \tag{29}$$

where $\Theta = \left(\mu,\omega,\alpha,\beta,d_1,d_2,\xi,\varphi,\sigma_u,\upsilon_1,\upsilon_2,\rho,\kappa,\psi,\theta,\delta\right)'$ is the vector of model parameters, $\ell(r;\Theta)$ and $\ell(x|r;\Theta)$ are the partial log-likelihood functions for the returns and realized measure, respectively. $f_{t-1}(r_t)$ and $f_{t-1}(x_t|r_t)$ denote the conditional probability density functions of r_t and x_t, respectively, which can be written as

$$f_{t-1}(r_t) = \sum_{j=0}^{\infty} f_{t-1}\left(r_t \mid n_t = j\right)\Pr_{t-1}\left(n_t=j\right) \tag{30}$$

$$f_{t-1}\left(x_t \mid r_t\right) = \frac{1}{\sqrt{2\pi\sigma_u^2}} \exp\left(-\frac{\left(\log\left(x_t\right) - \xi - \varphi \log\left(h_t\right) - \nu\left(\epsilon_t\right)\right)^2}{2\sigma_u^2}\right) \quad (31)$$

where $\Pr_{t-1}\left(n_t = j\right)$ is the conditional probability of the number of jumps, which is given in Equation 22. It should be noted that the summation in Equation 30 must be truncated when implementing the maximum likelihood estimation. We truncate the summation at 50 jumps per day. In fact, since the tail probability of Poisson distribution at $n_t \geq 50$ is sufficiently small and negligible, summing to $n_t = 50$ can ensures the accuracy of the estimation process. $f_{t-1}\left(r_t \mid n_t = j\right)$ in Equation 30 can be written as

$$f_{t-1}\left(r_t \mid n_t = j\right) = \frac{1}{\sqrt{2\pi\left(h_{z,t} + j\delta^2\right)}} \exp\left(-\frac{\left(r_t - \mu - j\theta\right)^2}{2\left(h_{z,t} + j\delta^2\right)}\right) \quad (32)$$

In order to calculate the likelihoods (Equations 30–32), it is necessary to determine the normal innovation z_t and the number of jumps n_t, and further filter the conditional variance $h_{z,t+1}$ and the conditional jump intensity $h_{y,t+1}$. This process can be easily implemented by using analytic filtering. Applying Bayes' rule, the filtering density is given by

$$\begin{aligned} \Pr_t\left(n_t = j\right) &\equiv \Pr_{t-1}\left(n_t = j \mid r_t\right) \\ &= \frac{f_{t-1}\left(r_t \mid n_t = j\right) \Pr_{t-1}\left(n_t = j\right)}{f_{t-1}\left(r_t\right)} \end{aligned} \quad (33)$$

where the expressions on the right-hand side of the Equation 33 are given by Equations 22, 30 and 32. $\Pr_t\left(n_t = j\right)$ represents the ex-post inference on n_t, or the probability that j jumps have arrived between time $t-1$ and t conditional on the information available at time t. The filtered number of jumps is then given by

$$\tilde{n}_t = E_t\left(n_t\right) = \sum_{j=0}^{\infty} j \Pr_t\left(n_t = j\right) \quad (34)$$

The actual number of jumps, n_t, can be larger than one but is restricted to be less than fifty as mentioned above. According to Christoffersen et al. (2012), the filtering of the normal innovation z_t can be written as

$$\tilde{z}_t = E_t\left(z_t\right) = \sum_{j=0}^{\infty} \frac{\tilde{h}_{z,t}}{\tilde{h}_{z,t} + j\delta^2}\left(r_t - \mu - j\theta\right) \Pr_t\left(n_t = j\right) \quad (35)$$

Once $\tilde{n}_t$ and $\tilde{z}_t$ are known, we can infer the filtered $\tilde{h}_{z,t+1}$ and $\tilde{h}_{y,t+1}$ easily through Equations 20 and 23.

Finally, the parameters of the REGARCH-Jump model can be estimated via maximum likelihood method by solving

$$\hat{\Theta} = \arg\max_{\Theta} \ell(r, x; \Theta) \quad (36)$$

4 Volatility forecast evaluation

4.1 Loss function

The loss function is commonly used to evaluate the forecasting accuracy of the competing models. In the paper, we employ four popular loss functions, including the mean absolute error (MAE), mean absolute percentage error (MAPE), mean squared error (MSE) and Quasi-likelihood (QLIKE). The four evaluation criteria are defined as

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|h_{t+1} - \hat{h}_{t+1}(m)\right| \quad (37)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{h_{t+1} - \hat{h}_{t+1}(m)}{h_{t+1}}\right| \quad (38)$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(h_{t+1} - \hat{h}_{t+1}(m)\right)^2 \quad (39)$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{h_{t+1}}{\hat{h}_{t+1}(m)} - log\left(\frac{h_{t+1}}{\hat{h}_{t+1}(m)}\right) - 1\right) \quad (40)$$

where T is the number of out-of-sample forecasts, h_{t+1} and $\hat{h}_{t+1}(m)$ denote the measured (true) volatility and forecasted volatility, respectively, and m stands for the GARCH, EGARCH, HAR, REGARCH or REGARCH-Jump models. It is worth noting that MSE and QLIKE are robust loss functions, which could provide a consistent ranking of the volatility models with a conditionally unbiased volatility proxy (Patton, 2011).

Since the true volatility is unobservable, the evaluation and comparison of volatility forecasting models requires the proxy of true volatility. In the paper, we employ the scaled realized volatility (RV) as a proxy of the true volatility. The scaled RV is defined as $RV_t^{scale} = c \times RV_t$, where RV_t is computed based 5-min intraday returns on day t and $c = \sum_{t=1}^{T} r_t^2 / \sum_{t=1}^{T} RV_t$. This adjustment is used to account for the non-trading hours.

4.2 MCS test

For robustness, we adopt the model confidence set (MCS) test proposed by Hansen et al. (2011) to examine whether the difference in forecasting performance between the competing models is statistically significant. The MCS method tests a given set of competing models and identifies a set of optimal predictive models or MCS with a certain level of confidence. Specifically, the MCS test relies on an equivalence test $\delta_{\mathcal{M}}$ and an elimination rule $e_{\mathcal{M}}$. Let $\mathcal{M}^0$ be the initial set of all competing models. Set $\mathcal{M} = \mathcal{M}^0$, and use the equivalence test $\delta_{\mathcal{M}}$ to test the null hypothesis that the competing models have the same expected loss (equal forecasting performance), i.e.,

$$H_{0,\mathcal{M}}: E\left[d_{uv,t}\right] = 0, \quad \forall u, v \in \mathcal{M} \quad (41)$$

where $d_{uv,t} = Loss_t(u) - Loss_t(v)$ is the loss difference between the models u and v. Hansen et al. (2011) propose the following statistics to test the null hypothesis $H_{0,\mathcal{M}}$:

$$T_{\mathcal{M}} = \max_{u,v \in \mathcal{M}} |t_{uv}|, \quad t_{uv} = \frac{\bar{d}_{uv}}{\sqrt{\widehat{\text{var}}(\bar{d}_{uv})}} \quad (42)$$

where $\bar{d}_{uv}$ is the average loss difference, and $\widehat{\text{var}}(\bar{d}_{uv})$ is the bootstrapped estimate of the variance of $\bar{d}_{uv}$, i.e., $\text{var}(\bar{d}_{uv})$. For a

TABLE 4 Descriptive statistics of Brent crude oil futures.

	Mean	Min	Max	SD	Skew	Kurt	J-B	$Q(20)$
r_t	0.0001	−0.2798	0.1908	0.0233	−0.6860	16.0821	33,811.4909	53.8946
RNG_t	0.0184	0.0016	0.2489	0.0127	4.8252	56.7418	582,597.2715	22,881.7334

SD, standard deviation; J–B, Jarque–Bera statistics; Q(20), Ljung-Box Q-statistic for autocorrelation up to 20 lags.

given significance level α, if the null hypothesis $H_{0,\mathcal{M}}$ is accepted, define $\mathcal{M}^*_{1-\alpha} = \mathcal{M}$. Otherwise, we use the elimination rule $e_{\mathcal{M}} = \arg\max_{u\in\mathcal{M}} \sup_{v\in\mathcal{M}} t_{uv}$ to exclude the model that has poor forecasting performance. This process is repeated until no model can be excluded. Finally, the set of surviving models $\mathcal{M}^*_{1-\alpha} = \mathcal{M}$ is referred to as the MCS, i.e. the set of optimal predictive models at a given confidence level of $1-\alpha$.

Since the asymptotic distribution of the test statistics $T_{\mathcal{M}}$ is non-standard, this paper uses a block bootstrap of 10^5 replications for approximate calculation. In addition, we set the significance level as $\alpha = 10\%$.

5 Empirical application

5.1 Data and descriptive statistics

For our empirical analysis, we use data on the daily open, high, low and close prices for the Brent crude oil futures. We focus on Brent crude oil futures market due to the fact that Brent crude oil is currently considered to be the global oil pricing benchmark (Dowling et al., 2016; Zavadska et al., 2020). We employ the squared price range (RNG_t^2) as the realized volatility measure (i.e., x_t). The data is obtained from Wind Database of China, and the sample period is from January 4, 2005 to February 28, 2023.

Table 4 presents the descriptive statistics of the daily returns and price ranges of Brent crude oil futures. As can be seen from the table, the return distribution for the Brent crude oil futures is negatively skewed and leptokurtic, while the price range distribution is positively skewed and leptokurtic. The Jarque-Bera statistics indicate that both the return and price range distributions are non-Gaussian. The Ljung-Box Q-statistic for autocorrelation up to 20 lags shows that the price range series is highly autocorrelated, suggesting high persistence of Brent crude oil futures volatility.

Figure 1 presents the time series plots of the daily Brent crude oil futures returns and price ranges. It can be seen from the figure that the well-known behavior of volatility clustering is apparent. In addition, the Brent crude oil futures experience large fluctuations (jumps) during the periods of 2008–2009 global financial crisis (GFC), 2020 COVID-19 pandemic and 2022 Russo-Ukrainian conflict.

5.2 In-sample parameter estimation

In this subsection, we estimate the five competitor models (GARCH, EGARCH, HAR, REGARCH and REGARCH-Jump) using the maximum likelihood method based on the in-sample data covering the period from January 4, 2005 to December 31, 2020. Table 5 reports the maximum likelihood estimates for the five models. As can be seen from the table, the daily coefficient of x_t in the HAR model is significantly positive, while the weekly coefficient is significantly negative. This indicates that daily information positively affects crude oil futures volatility, whereas weekly information has a negative impact. Additionally, the estimates of volatility persistence in the GARCH, EGARCH, REGARCH and REGARCH-Jump models are larger than 0.98, suggesting the stylized fact of high volatility persistence. Regarding the leverage parameters, γ, d_1, d_2, υ_1 and υ_2, they are all significantly different from zero. In particular, γ, d_1 and υ_1 are all negative, suggesting the presence of the leverage effect in the Brent crude oil futures market.

Moreover, the jump intensity parameters (ρ, κ, ψ) are all positive and statistically significant, suggesting the presence of time-varying jump intensity and that the REGARCH-jump model is correctly specified. In particular, the parameter κ is estimated to be 0.9578, which provides evidence of high persistence of the conditional jump intensity process. The estimated mean jump size θ in the REGARCH-Jump model is significantly negative, while the estimated jump volatility δ is significantly positive. The unconditional mean of dynamic jump intensity is $E(h_{y,t}) = \frac{\rho}{1-\kappa} = 0.1493$, implying that jumps arrive at a frequency of 37.6 jumps per year. In addition, the contribution of return jumps to the total return variance is $\frac{(\theta^2+\delta^2)h_{y,t}}{(\theta^2+\delta^2)h_{y,t}+h_{z,t}} = 12.73\%$, which is close to the results reported in Christoffersen et al. (2012) ($12\% \sim 15\%$), Andersen et al. (2007) (14.6%) and Pan et al. (2020) (11%), suggesting that our estimation results of jump intensity is reasonable.

Finally, we can observe that the REGARCH-Jump model improves the empirical return fit relative to all other models (GARCH, EGARCH and REGARCH) in terms of the partial log-likelihood for the returns $\ell(r)$ and the full log-likelihood $\ell(r, x)$.

Figure 2 presents the time series plots of the jump intensity and the number of jumps. It is clear that the jumps have a time-varying feature, and the number of jumps related to market uncertainty events have increased during the periods of 2008–2009 global financial crisis, 2015 global oil price decline and 2020 COVID-19 pandemic. Note also that, the number of jumps in most cases is less than one jump per day.

5.3 Out-of-sample forecast results

In the in-sample analysis, we document that the jump intensity and the number of jumps has time-varying characteristic. Moreover, we show that incorporating the extreme-value information and jump dynamics into the REGARCH model could improve the model's ability to fit the Brent crude oil futures

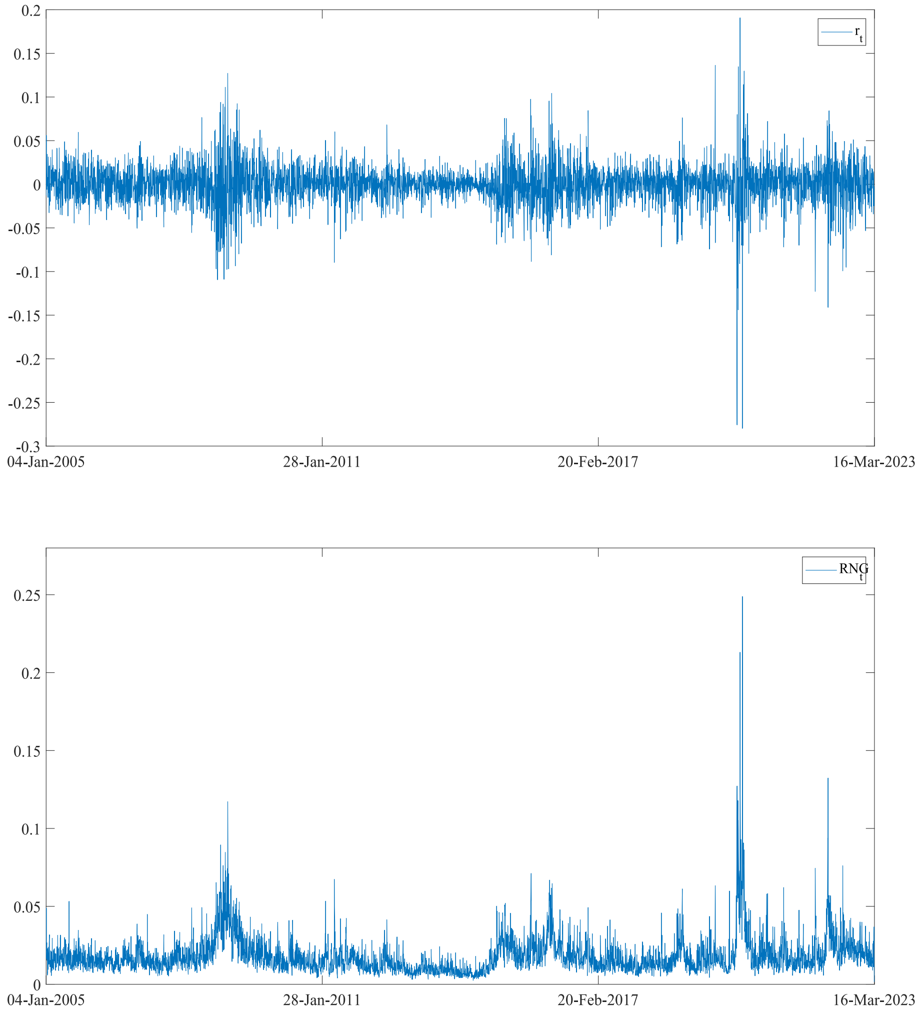

FIGURE 1
Time series plots of the daily Brent crude oil futures returns and price ranges.

returns. In this subsection, we investigate the importance of accounting for the extreme-value information and jump dynamics for forecasting the crude oil futures volatility relying on our proposed REGARCH-Jump model. Importantly, we compare the out-of-sample forecasting performance of the REGARCH-Jump model with that of the GARCH, EGARCH, HAR and REGARCH models.

The out-of-sample forecast exercise is performed relying on a rolling window scheme. The out-of-sample period is from January 4, 2021 to February 28, 2023. Figure 3 presents the out-of-sample volatility forecasts for the competing models. Table 6 reports the out-of-sample forecast evaluation results based on the four loss functions. As can be seen from the table, the REGARCH model offers smaller loss values (MAE, MAPE, MSE, QLIKE) than the GARCH and EGARCH models. In particular, the REGARCH-Jump model yields the smallest loss values in all cases. Our findings indicate that incorporating the extreme-value information and jump dynamics into volatility model is important for improving the out-of-sample volatility forecasts.

Further, Table 7 presents the MCS test results for the out-of-sample forecasts of competing models. It is clear that the REGARCH-Jump model is always included in the MCS, and always has the highest MCS p-value ($p = 1$), confirming the superior performance of the REGARCH-Jump model over all other models in forecasting the crude oil futures volatility.

5.4 Robustness check

For robustness, we perform the out-of-sample forecast exercise for different out-of-sample windows. To be specific, we consider two alternative out-of-sample windows: 200 and 400. The out-of-sample forecast evaluation results for the alternative out-of-sample windows are reported in Table 8.

In line with the results reported in Tables 6, 7, the REGARCH-Jump model that accounts for the extreme-value information

TABLE 5 Parameter estimation results for the Brent crude oil futures.

	GARCH	EGARCH	HAR	REGARCH	REGARCH-Jump
μ	0.0005 (0.0003)	0.0002 (0.0002)			–0.0000 (0.0002)
c			0.0001 (0.0000)	–0.0000 (0.0002)	
ω	0.0000 (0.0000)	0.0001 (0.0010)		–0.1154 (0.0012)	–0.1034 (0.0012)
α	0.0866 (0.0039)	0.1290 (0.0034)		0.1311 (0.0021)	0.1286 (0.0030)
β	0.9053 (0.0046)	0.9994 (0.0002)		0.9855 (0.0002)	0.9863 (0.0002)
β_d	0.9053 (0.0046)	0.9994 (0.0002)	0.3898 (0.0031)	0.9855 (0.0002)	0.9863 (0.0002)
β_w	0.9053 (0.0046)	0.9994 (0.0002)	–0.0205 (0.0103)	0.9855 (0.0002)	0.9863 (0.0002)
β_m	0.9053 (0.0046)	0.9994 (0.0002)	–0.4674 (0.0132)	0.9855 (0.0002)	0.9863 (0.0002)
γ		–0.0670 (0.0025)			
d_1				–0.0847 (0.0022)	–0.0732 (0.0025)
d_2				0.0423 (0.0012)	0.0455 (0.0023)
ξ				-0.4010 (0.1240)	–0.3737 (0.0031)
σ_u				0.6103 (0.0060)	0.5600 (0.0049)
σ_d			0.0013 (0.0000)		
υ_1				–0.0272 (0.0058)	–0.0704 (0.0055)
υ_2				0.2191 (0.0024)	0.4149 (0.0047)
ρ					0.0063 (0.0005)
κ					0.9578 (0.0034)
ψ					0.7475 (0.0062)
θ					–0.0040 (0.0006)
δ					0.0217 (0.0006)
$\ell(r)$	10,464.1643	10,487.7063	21,492.5989	10,552.1596	10,602.4880
$\ell(r,x)$				6,737.8636	7,141.7314

The number in parenthesis is the standard error; $\ell(r)$ denotes the partial log-likelihood for the returns, $\ell(r,x)$ denotes the full log-likelihood.

and jump dynamics yields the most accurate volatility forecasts, dominating all other models. This result confirms that the superior performance of the REGARCH-Jump model in forecasting the crude oil futures volatility is robust to alternative out-of-sample forecast windows.

5.5 Economic value analysis

To illustrate the economic value of the improved volatility forecasts from the REGARCH-Jump model, we perform a VaR analysis in this subsection. Accurate measurement of financial market risk is of great significance to the investors, policy makers and regulators who are trying to manage the risk of portfolio as well as to maintain the functioning and the stability of financial markets. The standard tool for measuring market risk is VaR, which is intuitive, simple and easy to compute. It is used by financial institutions and financial regulators worldwide for market risk mointoring and management.

5.5.1 VaR forecast

The one-day-ahead forecast of VaR for a given probability (significance level) α satisfies:

$$\Pr_{t-1}(r_t < VaR_t(\alpha)) = \alpha \tag{43}$$

According to the definition of VaR in Equation 39, the VaR under the REGARCH-Jump model can be formulated as

$$VaR_t(\alpha) = \mu + \tilde{\epsilon}_\alpha \sqrt{h_{z,t} + (\theta^2 + \delta^2) h_{y,t}} \tag{44}$$

where $\tilde{\epsilon_\alpha}$ can be written as follows (Chiu et al., 2006):

$$\tilde{\epsilon}_\alpha = \epsilon_\alpha + \frac{1}{6}(\epsilon_\alpha - 1) \times Sk(r_t) \tag{45}$$

$$Sk(r_t) = \frac{h_{y,t}(\theta^3 + 3\theta\delta^2)}{\left(h_{z,t} + h_{y,t}\theta^2 + h_{y,t}\delta^2\right)^{3/2}} \tag{46}$$

where ϵ_α is the left quantile at level α for standard normal distribution, and $Sk(r_t)$ is the conditional skewness of returns.

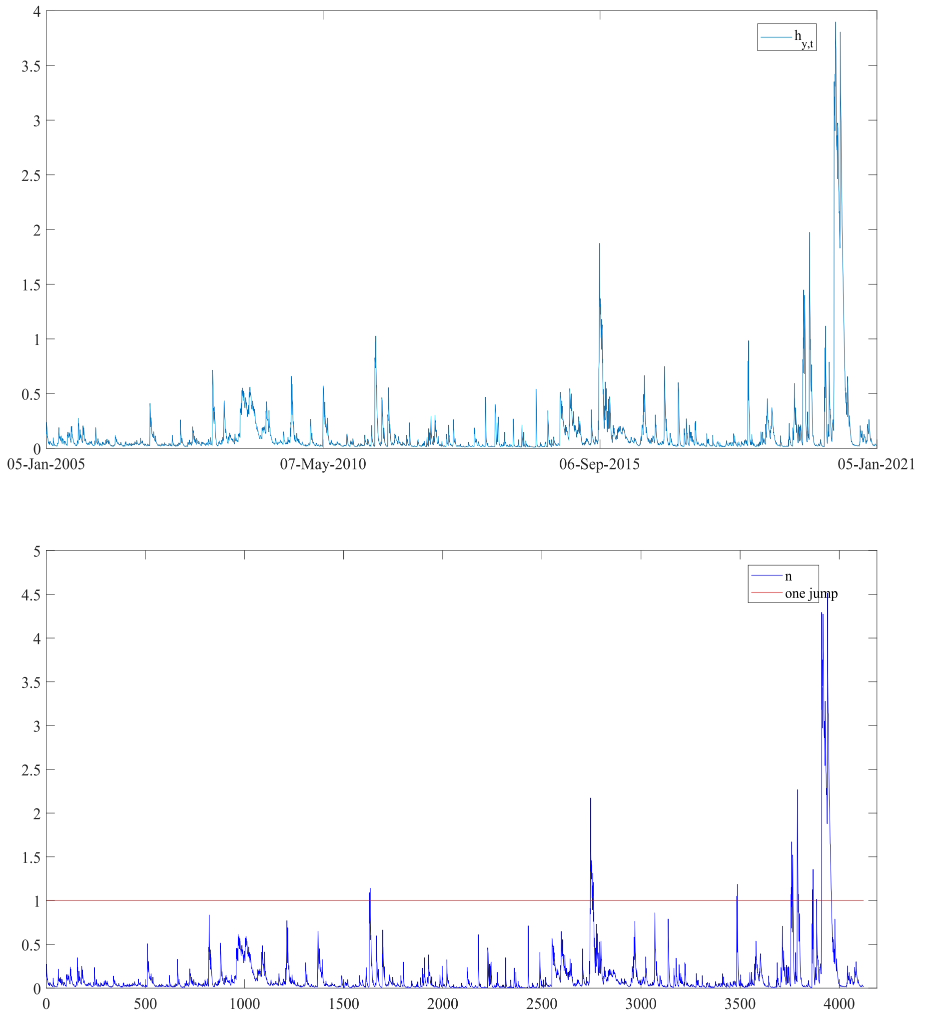

FIGURE 2
Time series plots of the jump intensify $h_{y,t}$ and the number of jumps n_t for the Brent crude oil futures.

5.5.2 Backtesting

To examine the accuracy of VaR forecast, we perform the backtesting relying on the failure rate test, the likelihood ratio test of unconsitional coverage (Kupiec, 1995) and the likelihood ratio test of consitional coverage (Christoffersen, 1998).

The likelihood ratio test statistic of unconsitional coverage can be written as

$$LR_{uc} = -2\ln\left[\alpha^{T_1}(1-\alpha)^{T_1-T_0}FR^{-T_0}(1-FR)^{T_1-T_0}\right] \sim \chi^2(1) \quad (47)$$

where FR is failure rate, that is, $FR = T_0/T_1$, T_0 is the number of times the VaR is violated, T_1 is the total number of VaR forecasts.

The likelihood ratio test statistic of unconsitional coverage (LR_{uc}) can not examine the independence of VaR exceptions. In light of this, Christoffersen (1998) propose the conditional coverage test that can examine the independence of VaR exceptions, which can be written as

$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2) \quad (48)$$

where

$$LR_{ind} = -2\ln\left[\frac{(1-FR_2)^{T_{00}+T_{10}}FR_2^{T_{00}+T_{11}}}{(1-FR_{01})^{T_{00}}FR_{01}^{T_{10}}FR_{11}^{T_{11}}}\right] \sim \chi^2(1) \quad (49)$$

and T_{ij} is the number of observations with value i followed by j, $FR_{01} = \frac{T_{01}}{T_{00}+T_{01}}$, $FR_{11} = \frac{T_{11}}{T_{10}+T_{11}}$, $FR_2 = \frac{T_{01}+T_{11}}{T_{00}+T_{01}+T_{10}+T_{11}}$. It is clear that the conditional coverage test LR_{cc} builds on the LR_{uc} and LR_{ind} tests.

Figure 4 presents the VaR forecasts for the competing models. Table 9 reports the results of VaR backtesting including the failure rates (FR) test, the likelihood ratio test of unconsitional coverage (LR_{uc}) and the likelihood ratio test of consitional coverage (LR_{cc}) for the five competing models. As can be seen from Table 9, all models have passed the likelihood ratio test at the 10% and 5% significance levels, indicating that they all perform well in measuring the crude oil futures market risk in the cases. Importantly, it is interesting to note that the FR of the VaR forecasts

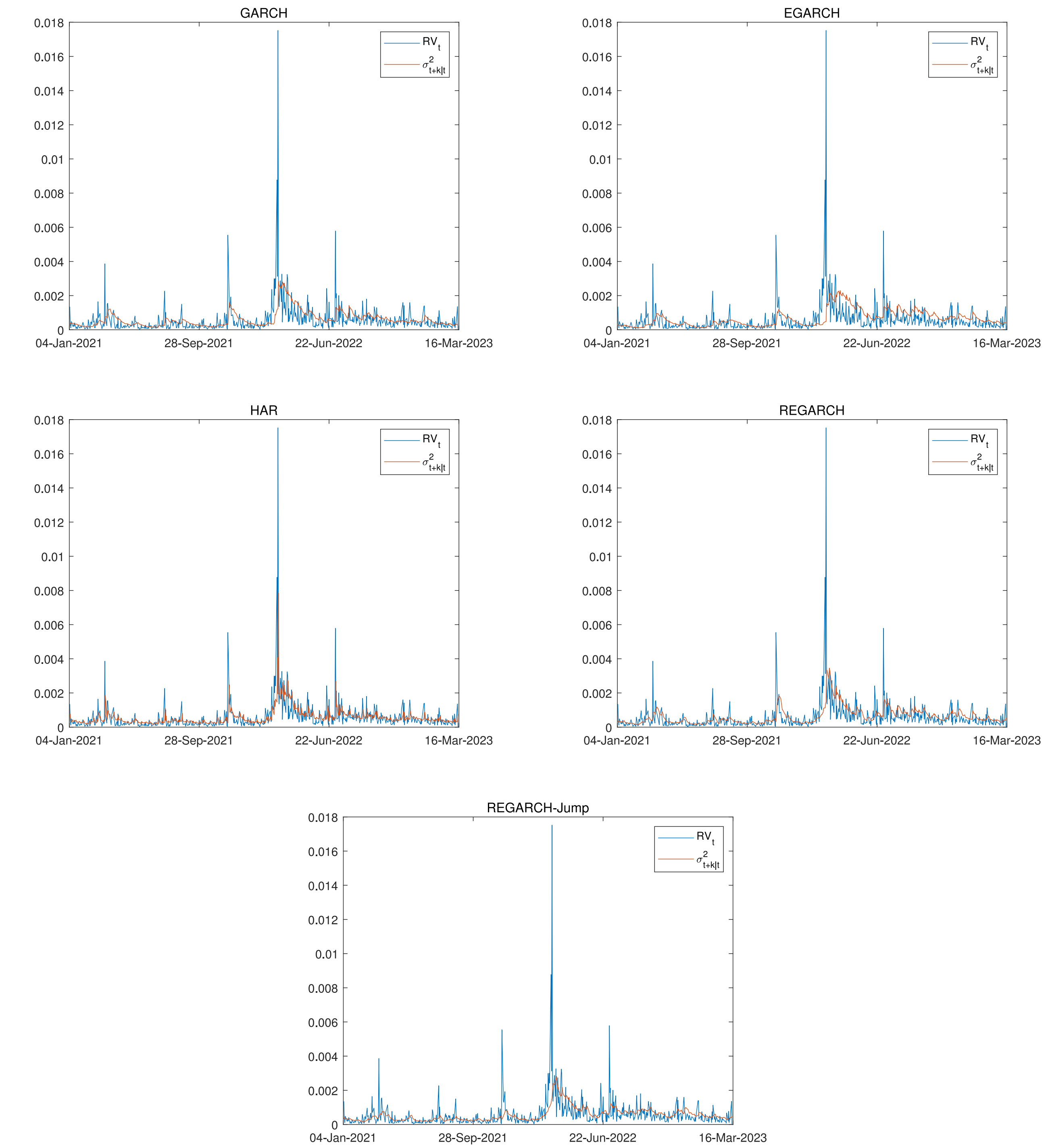

FIGURE 3
Out-of-sample volatility forecasts.

for the REGARCH-Jump model are closer to the corresponding theoretical values (α) under the 95% and 90% confidence levels than the other models.

Regarding the confidence level of 99%, the FR of VaR forecasts of the EGARCH model significantly deviates from the theoretical value ($\alpha = 0.01$), and all of the models can not forecast VaR accurately in terms of the unconditionally covered likelihood ratio test. However, in terms of the conditionally covered likelihood ratio test, all the models except for the EGARCH model performs well.

TABLE 6 Out-of-sample forecast evaluation results.

	GARCH	EGARCH	HAR	REGARCH	REGARCH-Jump
MAE	3.9727e-04	4.5305e-04	3.7919e-04	3.9187e-04	**3.7141e-04**
MAPE	1.3601e+00	1.5519e+00	1.5427e+00	1.4622e+00	**1.3364e+00**
MSE	9.2982e-07	1.0537e-06	8.9767e-07	9.0246e-07	**8.6799e-07**
QLIKE	4.0616e-01	5.3022e-01	3.5707e-01	3.4907e-01	**3.4152e-01**

The bold numbers in the table indicate that the model yields the lowest loss value (in each row).

TABLE 7 MCS test results.

	GARCH	EGARCH	HAR	REGARCH	REGARCH-Jump
MAE	0.0114	0.0000	0.4581	0.0159	**1.0000**
MAPE	0.8253	0.0008	0.3805	0.0181	**1.0000**
MSE	0.0231	0.0231	0.4992	0.0231	**1.0000**
QLIKE	0.0013	0.0013	0.3546	0.3546	**1.0000**

Shaded entries indicate the model is included in the MCS at a significance level of 10%. The numbers in the table are the *p*-values of the MCS test. The bolded numbers represent the model with the best predictive ability relative to the other competing models.

TABLE 8 Out-of-sample forecast evaluation results for alternative out-of-sample windows.

	GARCH	EGARCH	HAR	REGARCH	REGARCH-Jump
Out-of-sample window: 200					
MAE	3.3741e-04	4.2706e-04	3.7267e-04	3.3204e-04	**3.2531e-04**
MAPE	**1.4942e+00**	2.2955e+00	2.0547e+00	2.0871e+00	1.7982e+00
MSE	3.2299e-07	3.6970e-07	3.1801e-07	3.2705e-07	**2.9816e-07**
QLIKE	2.9271e-01	3.0689e-01	2.9223e-01	2.7860e-01	**2.6224e-01**
Out-of-sample window: 400					
MAE	3.9398e-04	4.4089e-04	4.3243e-04	3.8255e-04	**3.5174e-04**
MAPE	1.4095e+00	1.6105e+00	1.6713e+00	1.4633e+00	**1.3110e+00**
MSE	8.1292e-07	9.0956e-07	**1.1947e-06**	7.7427e-07	7.1005e-07
QLIKE	4.1551e-01	5.1729e-01	3.4917e-01	3.5386e-01	**3.4150e-01**

The bold numbers of the table indicate that the model yields the lowest loss value (in each row). Shaded entries indicate the model is included in the MCS at a significance level of 10%.

Finally, we can observe that, based on the 95% and 90% confidence levels, the VaR forecasts obtained by the GARCH, EGARCH, HAR and REGARCH models are more conservative than that obtained by the REGARCH-Jump model, which may underestimate the risk. In summary, our results provide support for combining the extreme-value information and dynamic jumps for improving the accuracy of VaR forecasts. Note also that there is still room to improve the accuracy of VaR forecasts in extreme risk scenarios.

6 Conclusion

In this paper, we propose the REGARCH-Jump model, which incorporates the extreme-value information and jump dynamics, to model and forecast the crude oil futures volatility. An empirical analysis based on the daily Brent crude oil futures prices data shows the presence of time-varying jump intensity with high persistence. In addition, we observe that the REGARCH-Jump model outperforms the GARCH, EGARCH, HAR and REGARCH models in terms of both empirical return fit and out-of-sample volatility forecast. Moreover, we confirm that the superior forecasting performance of the REGARCH-Jump model is robust to alternative out-of-sample forecast windows. Finally, a VaR analysis is conducted to demonstrate the economic value of the improved volatility forecasts from the REGARCH-Jump model. We confirm that the REGARCH-Jump model can produce reasonable VaR forecasts. In summary, our findings highlight the importance of accommodating the extreme-value information as well as the jump dynamics in forecasting the volatility of the crude oil futures market.

Our work offers theoretical and methodological insights into modeling and forecasting the crude oil futures volatility, with great significance related to both academic researchers and

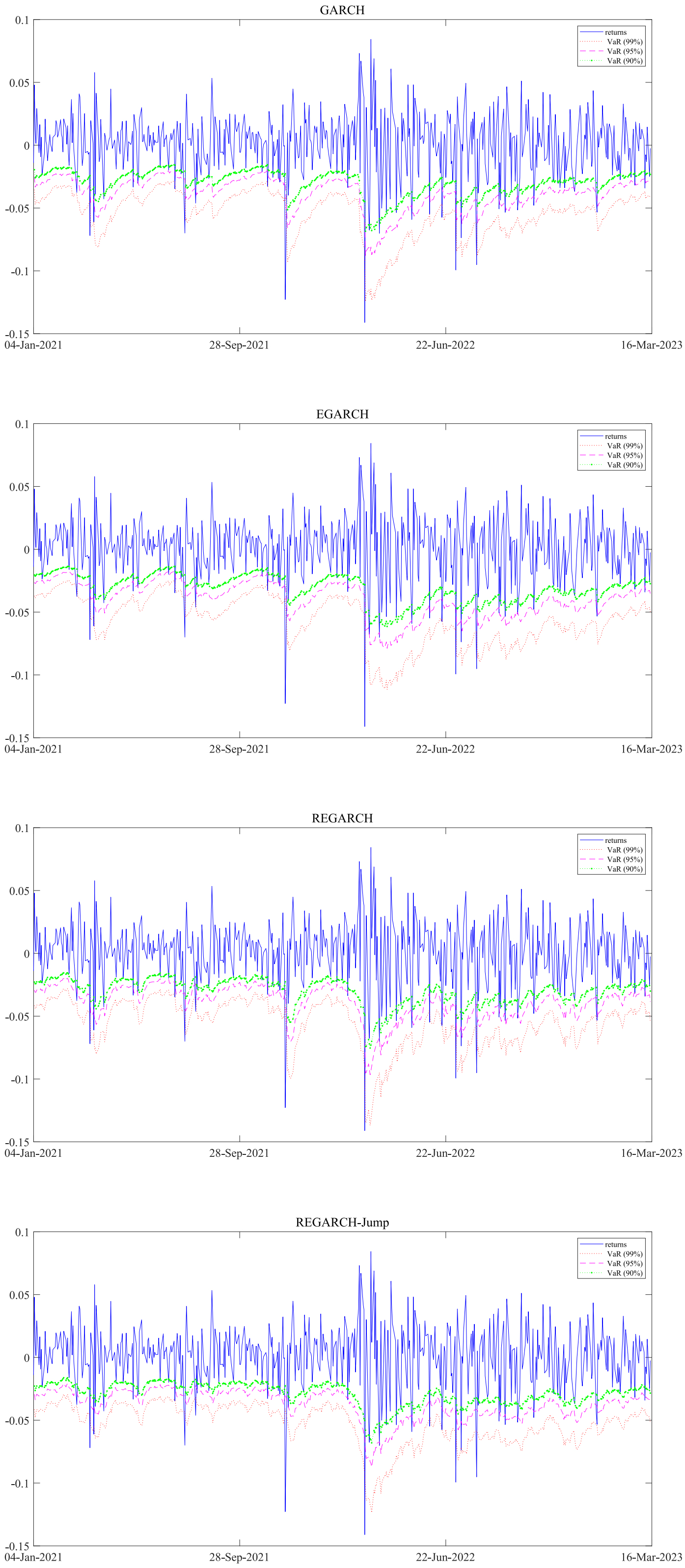

FIGURE 4
VaR forecasts.

TABLE 9 Backtesting analysis of VaR forecasts.

1-α	Model	FR	LR_{uc}	P_{uc}	LR_{cc}	P_{cc}
Panel A: Subsample, 2005–2020						
99%	GARCH	0.0218	8.6546**	0.0033	9.3538**	0.0093
	EGARCH	0.0206	7.1331**	0.0076	7.9852**	0.0185
	HAR	0.0290	19.9843**	0.0000	20.1080**	0.0000
	REGARCH	0.0218	8.6546**	0.0033	9.3538**	0.0093
	REGARCH-Jump	0.0180	2.6126*	0.1060	2.6126	0.2708
95%	GARCH	0.0556	0.5319	0.4658	0.6121	0.7364
	EGARCH	0.0520	0.0684	0.7936	0.0977	0.9523
	HAR	0.0580	1.0728	0.3003	1.3491	0.5094
	REGARCH	0.0508	0.0107	0.9176	0.0204	0.9898
	REGARCH-Jump	0.0510	0.0141	0.9054	2.0618	0.3567
90%	GARCH	0.0979	0.0391	0.8433	0.6668	0.7165
	EGARCH	0.0955	0.1864	0.6659	0.6022	0.7400
	HAR	0.1088	0.6980	0.4034	2.7933	0.2474
	REGARCH	0.0967	0.0989	0.7531	1.4328	0.4885
	REGARCH-Jump	0.1014	0.0154	0.9011	0.4816	0.7860
Panel B: Subsample, 2005–2021						
99%	GARCH	0.0176	2.7214*	0.0990	2.7214	0.2565
	EGARCH	0.0212	5.4049**	0.0201	5.4049*	0.0670
	HAR	0.0247	8.7725**	0.0031	8.7725**	0.0124
	REGARCH	0.0176	2.7214*	0.0990	2.7214	0.2565
	REGARCH-Jump	0.0194	3.9703**	0.0463	3.9703	0.1374
95%	GARCH	0.0547	0.2534	0.6147	0.3106	0.8561
	EGARCH	0.0529	0.0993	0.7527	0.3769	0.8282
	HAR	0.0529	0.0993	0.7527	0.3769	0.8282
	REGARCH	0.0459	0.2107	0.6463	0.2473	0.8837
	REGARCH-Jump	0.0511	0.0156	0.9007	0.2128	0.8990
90%	GARCH	0.0899	0.6567	0.4177	2.8193	0.2442
	EGARCH	0.0864	1.2121	0.2709	1.6892	0.4297
	HAR	0.1058	0.2098	0.6469	2.8593	0.2394
	REGARCH	0.0864	1.2121	0.2709	2.9202	0.2322
	REGARCH-Jump	0.0988	0.0096	0.9218	1.6757	0.4326

1-α is the confidence level, FR is the failure rate, and LR_{uc} and LR_{cc} denote the unconditionally covered likelihood ratio test statistic and the conditionally covered likelihood ratio statistic, respectively. * and ** indicate significant at the 10% and 5% levels of significance, respectively, indicating that the model is rejected.

practitioners. Further extensions and applications of the proposed model are encouraged. For example, incorporating intraday high-frequency data into the realized volatility measure presents a promising area for future research. In addition, future studies could involve applying the model to derivative pricing and asset allocation.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

WS: Methodology, Software, Supervision, Writing – review & editing. HL: Conceptualization, Data curation, Formal analysis, Writing – original draft.

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Conflict of interest

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