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On-line strength assessment of distribution systems with distributed energy resources

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To enable the online strength assessment of distribution systems integrated with Distributed Energy Resources (DERs), a novel hybrid model and data-driven approach is proposed. Based on the IEC-60909 standard, a new short-circuit calculation method is developed, allowing inverter-based DERs (IBDERs) to be represented as either voltage or current sources with controllable internal impedance. This method also accounts for the impact of distant generators by introducing a site-dependent Short Circuit Ratio (SCR) index to evaluate system strength. An adaptive sampling strategy is employed to generate synthetic data for real-time assessment. To predict the strength of distribution systems under various conditions, a rectified linear unit (ReLU) neural network is trained and further reformulated as a mixed-integer linear programming (MILP) problem to verify its robustness and input stability. The proposed method is validated through case studies on modified IEEE-33 and IEEE-69 bus systems, demonstrating its effectiveness regarding the varying operating conditions within the system.

KEYWORDS

system strength, short circuit ratio, distribution systems, input stability verification, online forecasting

1 Introduction

1.1 Motivation

Circuit calculation is a fundamental tool for distribution system protection and control (Boutsika and Papathanassiou, 2008). With the increasing integration of inverter-based distributed energy resources (IBDERs), such as photovoltaic generators (PVs) and battery energy storage systems (BESSs), in distribution systems, short-circuit levels are rising, pushing the systems closer to their static voltage stability limits (Wu et al., 2017) and reducing overall system strength (Qays et al., 2023). System strength is widely evaluated using the Short Circuit Ratio (SCR), which depends nonlinearly on factors such as net power injection and the control strategies of IBDERs (Qays et al., 2023). Traditional SCR quantification methods rely on short-circuit calculations and impedance estimation under fault conditions (He et al., 2023), but these offline approaches are not suitable for real-time estimation under time-varying operating conditions induced by fluctuating loads and DERs.

To address these limitations, data-driven approaches for short-circuit calculations have emerged, leveraging deep learning techniques to achieve faster and more adaptable predictions. Various studies have explored the use of artificial neural networks (ANNs) to estimate short-circuit characteristics under diverse operating scenarios (Aljarrah et al., 2023), detect faults and disturbances (Guillen et al., 2020), and improve prediction accuracy by incorporating network topology information (Ruikai et al., 2024). However, while these data-driven models show promise, they often lack interpretability and robustness, making them unsuitable for safety-critical applications without further validation. To enhance the robustness and interpretability of deep learning models, several techniques—such as gradient-based visualization (Zhang and Zhu, 2018), mixed-integer linear programming (MILP)-based robustness verification (Anderson et al., 2020), and inverse optimization methods (Genzel et al., 2022)—have been proposed. Despite these advancements, the robustness and interpretability of data-driven models for short-circuit calculation and SCR estimation have not yet been thoroughly explored. This gap motivates the need for a comprehensive framework to ensure reliable and interpretable online strength assessment of distribution systems with IBDERs.

1.2 Literature review

The strength of distribution systems is a key characteristic that describes the extent of voltage changes in response to faults or disturbances (Gu et al., 2019). It is commonly quantified using the Short Circuit Ratio (SCR)[1], which can take various forms, such as composite, weighted, multi-infeed effective, interaction factors, inverter interaction, and site-dependent SCRs (Qays et al., 2023). Impedance is often used to represent system strength, as the SCR can approximate this impedance (Gavrilovic, 1991). In Wu et al. (2017), a site-dependent SCR (SDSCR) metric is proposed for transmission systems with high penetration of renewable energy sources, providing insights into the impact of these sources on voltage stability through the relationship between voltage stability and the Jacobian matrix. To address complex inter-inverter interactions, a hierarchical-infeed interactive effective SCR is introduced in Xiao et al. (2022). Additionally, a novel grid strength impedance metric is proposed in Henderson et al. (2024) to quantify AC system strength across a wide range of frequencies. However, the relationship between short-circuit current and SCR, specifically in terms of short-circuit-based SCR calculation, has not been fully explored.

Various approaches have been developed to calculate the short-circuit currents in distribution systems with DERs. For AC distribution systems, DERs can be represented as either voltage or current sources according to the IEC-60909 standard (Thurner and Braun, 2018). The influence of different load models on short-circuit behavior in distribution systems has been examined in Mathur et al. (2015). To facilitate the integration of three-phase IBDERs into existing short-circuit calculation procedures, a general Δ short-circuit model for DERs is introduced in Strezoski et al. (2017). Additionally, a fast short-circuit calculation method tailored for unbalanced three-phase distribution systems with IBDERs is proposed in He et al. (2023), which incorporates fault ride-through control and converter current limiting. For assessing topology and line impedance parameters, a numerical approach based on a specialized Newton-Raphson iteration and power flow equations is proposed in Zhang et al. (2020). The post-fault temporary over voltage has been incorporated into the SCR calculation in Xin et al. (2024). While these methods provide accurate results, they are typically model-based, making them computationally intensive and time-consuming, thereby limiting their suitability for real-time or online applications.

With the advancement of deep learning techniques, data-driven methods for short-circuit calculations have gained prominence in recent years. In Guillen et al. (2020), a fault detection and location method is developed using graph theory representation and microsynchrophasors, accounting for the uncertain operating conditions and intermittency of IBDERs. Similarly Aljarrah et al. (2023), employs an artificial neural network (ANN) to estimate short-circuit current characteristics—such as sub-transient, transient, and peak currents—under varying scenarios driven by high renewable integration. A supervised learning approach for internal short-circuit detection in Li-ion batteries is presented in Naha et al. (2020). Meanwhile Gholami et al. (2019), introduces a short-circuit fault location method using current and voltage synchrophasors from PMUs along with pre-fault state estimation results as input features. In Ruikai et al. (2024), the superposition theorem is utilized to improve data-driven short-circuit calculations by incorporating the effects of network topology. While these data-driven models demonstrate effectiveness, especially neural networks, they often suffer from a “black-box” nature, lack interpretability, and cannot ensure robustness in the generated results.

Recent studies have proposed using machine learning techniques to forecast the SCR. A multilayer perceptron neural network is trained on data collected from sensors in inverter systems (e.g., voltage and current), with its hyperparameters optimized via an evolutionary algorithm Priyadarshini et al. (2024). A multi-objective machine learning algorithm has been proposed to forecast the SCR for the next day or week, utilizing ground truth data from both experimental and simulated cases Qays et al. (2025). To assess site-dependent SCR under varying operating conditions, an artificial neural network is trained to predict site conditions under varying cloud distributions Javadi et al. (2018).

To enhance the robustness and interpretability of deep networks, several techniques have been proposed in recent years. A stable training method was introduced in Zheng et al. (2016) to improve the robustness of neural networks. This robustness—defined as the resistance of the model to small perturbations in input samples without causing significant performance degradation—is further analyzed from a geometrical perspective in Fawzi et al. (2017). Gradient-based localization techniques have been employed to visualize and interpret the hidden layers of deep networks, thereby offering insight into the network’s decision-making process (Zhang and Zhu, 2018). Additionally, in Anderson et al. (2020), trained neural networks are reformulated as mixed-integer linear programming (MILP) problems to verify their robustness over

1 Apart from the SCR and its variations, the impedance equivalent is another approach (Henderson et al., 2024), which is not suitable for higher voltage or multiple RESs interaction.

specified input regions, where robust optimization methods are used to determine the maximum and minimum network outputs. The robustness of neural networks has also been examined using inverse optimization techniques, as demonstrated in Genzel et al. (2022), to validate their performance over a given dataset (Amini and Ghaemmaghami, 2020). However, to the best of the author's knowledge, the robustness of trained short-circuit calculation or SCR data-driven models has yet to be thoroughly investigated.

1.3 Contributions

In this work, a novel online system strength assessment method for distribution systems with high penetration of IBDERs is proposed. To accurately capture the influence of DERs on system strength, a new SCR calculation method is developed based on the IEC-60909 standard, where the short-circuit current contribution from IBDERs can be limited. The SCR is formulated as a parametric function of renewable energy outputs and load levels, which is further approximated using a neural network with adaptive sampling. The robustness of the trained neural network is validated using a MILP approach. The key contributions of this study are as follows:

- A new SCR calculation method is proposed based on the IEC-60909 standard, which effectively incorporates the impact of DERs on network impedance. This approach enables more accurate assessment of system strength in distribution networks with high DER penetration.
- A novel online SCR forecasting method is proposed, utilizing robust optimization techniques to verify the performance of the neural network. This approach enables real-time and reliable evaluation of system strength, addressing the dynamic and uncertain nature of DERs within the distribution system.

1.4 Outline

This paper is organized as follows: Section 2 introduces the SCR calculation method for distribution systems with high penetration of IBDERs. In Section 3, a ReLU neural network is trained using adaptive sampling to generate synthetic data for SCR approximation under uncertain operating conditions. Section 4 verifies the robustness of the trained ReLU network. In Section 5, comprehensive case studies are presented to validate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 6.

2 Short circuit ratio assessment for distribution systems with inverter-based distributed energy resources

In this section, the SDSCR of distribution systems with IBDERs is derived to quantify the strength of distribution systems under given conditions. The short circuit under a three-phase fault is used to calculate the short circuit impedance following the IEC-60909 standard, where one IBDER can be integrated as either a voltage source or a current source.

2.1 Network topology

A distribution system with IBDERs is defined as a connected graph, i.e., $\mathcal{G} \coloneqq \{\mathcal{N}, \mathcal{E}, \mathcal{D}, \mathcal{G}, \mathcal{R}, \mathcal{W}\}$ with a set of branches, where

- $i \in \mathcal{N}$ is the set of AC buses
- $ij \in \mathcal{E}$ is the set of AC branches[2]
- $d \in \mathcal{D}$ is the set of AC loads
- $g \in \mathcal{G}$ is the set of conventional generators
- $r \in \mathcal{R} \coloneqq \mathcal{R}' \cup \mathcal{R}''$ is the set of IBDERs
- $w \in \mathcal{W}$ is the set of DERs as current sources

For distribution systems, the network topology is assumed to be radial. Following the revision of IEC-60909, the IBDERs with full converters can be modeled as constant current sources (Thurner et al., 2018). $\mathcal{R}'$ is the set of grid-forming IBDERs, and $\mathcal{R}''$ is the set of grid-following IBDERs. The $g \in \mathcal{G}$ and $r \in \mathcal{R}'$ are treated as voltage sources. The $w \in \mathcal{W}$ and $r \in \mathcal{R}^{prime\prime}$ are treated as current sources.

2.2 Short circuit ratio calculation

The short circuit includes two components, i.e., the short circuit calculation contribution from voltage sources and current sources (Thurner and Braun, 2018).

2.2.1 Voltage source current contribution

At the fault location i, according to the theorem of Thevenin, the equivalent voltage $U_{\mathrm{Q},i}$ after there phase short circuit fault is given as follows:

$$U_{\mathrm{Q},i} = \frac{cU_{\mathrm{R},i}}{\sqrt{3}}, \forall i \in \mathcal{G} \cup \mathcal{R}', \tag{1}$$

where $U_{\mathrm{R},i}$ is the nominal voltage of bus i. c is the voltage correction factor, which depends on the voltage levels (Thurner and Braun, 2018).

By neglecting all current source elements, The short circuit current contributed by the voltage source at bus i, i.e., I''_{kIi}, can be derived by the following network equations (Thurner et al., 2018):

$$\begin{bmatrix} Y_{11} & \cdots & Y_{n1} \\ \vdots & \ddots & \vdots \\ Y_{1n} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ \vdots \\ U_{Qi} \\ \vdots \\ U_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ I''_{kIi} \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

To solve Equation 2, the inverse of admittance matrix, i.e., impedance matrix, is introduced as follows:

2 The transformers are embedded in the branch set.

$$\begin{bmatrix} 0 \\ \vdots \\ I''_{kIi} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{n1} \\ \vdots & \ddots & \vdots \\ Z_{1n} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ \vdots \\ U_{Qi} \\ \vdots \\ U_n \end{bmatrix} \tag{3}$$

The short circuit at bus i is given as follows:

$$I''_{kIi} = \frac{U_{Qi}}{Z_{ii}}, \forall i \in \mathcal{G} \cup \mathcal{R}' \tag{4}$$

Remark 1: $\mathbf{Y} = \begin{bmatrix} Y_{11} & \cdots & Y_{n1} \\ \vdots & \ddots & \vdots \\ Y_{1n} & \cdots & Y_{nn} \end{bmatrix}$ is the admittance matrix, which is typically sparse for power networks. It should be noted that, different integration method will affect the the diagonal element, Y_{ii} by the impedance of $\mathcal{R}'$ and $\mathcal{R}''$.

Remark 2: If the fault impedance at location i is given, the short circuit current should be modified $I''_{kIi} = \frac{U_{Qi}}{(Z_{ii}+Z_{\text{fault}})}, \forall i \in \mathcal{G} \cup \mathcal{R}'$, where Z_{fault} is the fault impedance.

2.2.2 Current source current contribution

For the current source injection at current source i, its current injection under fault, i.e., I''_{kCi}, is given as follows Thurner and Braun (2018):

$$I''_{kCi} = -j\left(kI_{\text{R},i}\right), \tag{5}$$

where k is the ratio of short circuit to rated current $I_{\text{R},i}$, which is given by the manufacturer.

By short-circuiting all voltage resources, the bus current injection at bus i, can be derived as follows:

$$\begin{bmatrix} U_1 \\ \vdots \\ 0 \\ \vdots \\ U_n \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & & Z_{n1} \\ \vdots & \ddots & & \vdots \\ & & Z_{ii} & \\ \vdots & & & \vdots \\ Z_{1n} & \cdots & & Z_{nn} \end{bmatrix} \begin{bmatrix} -I''_{kC1} \\ \vdots \\ I''_{kIIi} - I''_{kCi} \\ \vdots \\ -I''_{kCn} \end{bmatrix} \tag{6}$$

As shown in Equation 6, the voltage at the location i is 0, the fault current is given as follows:

$$I''_{kIIi} = \frac{1}{Z_{ii}} \cdot \sum_{m=1}^{n} Z_{im} \cdot I''_{kC,m} \tag{7}$$

Based on the Equations 4, 7, the initial short circuit at location i can be derived as follows:

$$I''_{ki} = \frac{U_{Qi}}{Z_{ii}} + \frac{1}{Z_{ii}} \cdot \sum_{m=1}^{n} Z_{im} \cdot I''_{kC,m} \tag{8}$$

With dc and ac correction factors, i.e., m and n, the thermal short circuit current is defined as follows:

$$I_{\text{th},i} = \sqrt{m_i + n_i} I''_{ki} \tag{9}$$

Remark 3: Correction factors m_i and n_i are site dependent. For distribution systems with synchronous generators, if the fault location i is far from the generator, some empirical functions can be found in Bolgaryn et al. (2022).

2.2.3 Short circuit ratio

Considering the impacts of renewable energy sources on the SDSCR (Wu et al., 2017), the following index is proposed to quantify the SDSCR:

$$SCR_i = \frac{U_{\text{R},i} I_{\text{th},i}}{P_{r,i} + \sum_{j \in \mathcal{N}, j \neq i} P_{r,j} \omega_{ij}}, \tag{10}$$

where $P_{r,i}$ is the output of IBDER at bus i, ω_{ij} is the coefficient to quantify the impacts of remote IBDERs,

$$\omega_{ij} = \left| \frac{Z_{ij}}{Z_{ii}} \left(\frac{U_i}{U_j} \right)^* \right| \tag{11}$$

Remark 4: As shown in Equation 10, ω_{ij} plays an important role in the SDSCR (Qays et al., 2023). In this work, ω_{ij} considers the impacts of voltage variation within the distribution systems. It should be noted that, ω_{ij} is taken as the magnitude of ω_{ij} defined in Wu et al. (2017), as the voltage angle is close to each other within the same distribution network.

The system strength of distribution systems with IBDERs can be further defined as the following parametric function depending on the operating conditions $\boldsymbol{\xi} \coloneqq \{P_r, C_r, P_d, \forall r, d\}$[3]:

$$f_{\text{SS}}(\boldsymbol{\xi}) = \min_{i \in \mathcal{N}} SCR_i(\boldsymbol{\xi}) \tag{12}$$

Based on the derivation within this section, the system strength is highly nonlinear.

3 ReLU neural networks for online short circuit ratio assessment

Considering the varying operating condition $\boldsymbol{\xi}$ within the distribution systems and nonlinear nature of $f_{\text{SS}}(\boldsymbol{\xi})$, function Equation 12 can not always be solved efficiently for online applications. In this section, it is reformulated as a data-driven application problem via active sampling over $\xi \in \mathcal{U}$ (Bamdad et al., 2020).

3.1 ReLU neural work based short circuit ratio approximation

For a given set of system strength samples, i.e., $\{(\xi_\omega, f_{\text{SS}}(\xi_\omega)), \forall \omega \in \Omega\}$[4], a ReLU neural network is designed and trained to capture the relation between ξ_s and $f_{\text{SS}}(\xi_s)$.

A ReLU neural network is shown in Figure1, where the activation function is a ReLU function, as follows:

$$\text{ReLU}(x) = \max(0, x) \tag{13}$$

3 C_r is the control mode of the r-th IBDER, i.e., voltage source or current source.

4 The Ω is defined Subsection 3.2.

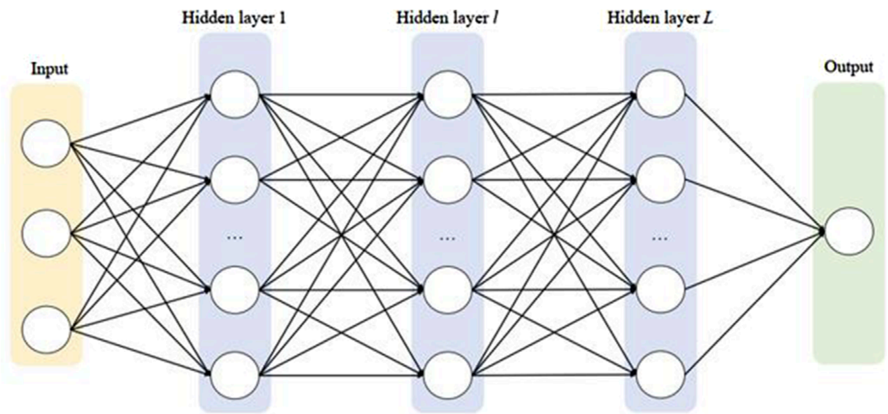

FIGURE 1
An illustrative neural network with ReLU activation function.

The propagation of the ReLU neural network is defined as follows:

$$\begin{aligned}
&\mathbf{a}^{(0)} = \mathbf{x},\\
&\mathbf{z}^{(l)} = \mathbf{W}^{(l)}\mathbf{a}^{(l-1)} + \mathbf{b}^{(l)}, \forall l \in \{1,\ldots,L\},\\
&\mathbf{a}^{(l)} = \mathbf{ReLU}\left(\mathbf{z}^{(l)}\right) = \max\left(0, \mathbf{z}^{(l)}\right), \forall l,\\
&f'_{SS} = \mathbf{W}^{(L+1)}\mathbf{a}^{(L)} + \mathbf{b}^{(L+1)},
\end{aligned} \tag{14}$$

where $\Theta := \{\mathbf{W}^l, \mathbf{b}^{(l)}, \forall l \in \{1,\ldots,L+1\}\}$ are the weight factors and bias for the neuron on each layer l, which should be optimized to minimize the following mean square error (MSE) function:

$$\min_{\Theta}\mathbf{MSE}(\Theta) = \frac{1}{|\Omega|}\sum_{\omega\in\Omega}\left(f_{SS}(\xi_\omega) - f'_{SS}(\xi_\omega;\Theta)\right)^2 \tag{15}$$

For the training process, the gradient approaches are widely adopted, e.g., adaptive moment estimation (ADAM) (Zhang, 2018).

Remark 5: It can be observed that, the training set $\Omega \subseteq \mathcal{U}$.

Input: System strength assessment function $f_{SS}(\xi)$; parameter space $\mathcal{U}$; initial sample size N; maximum iterations K; error tolerance ϵ
Output: ReLU neural network model $\hat{f}(\xi;\Theta)$ approximating $f_{SS}(\xi)$

```
1  Initialization
2  Uniformly sample an set of samples {ξ_i}_{i=1}^N from U as Ω^0.
3  Derive the system strength f_SS(ξ_i) for each sample ξ_i.
4  Build an initial model f̂_0(ξ; Θ^0) using the samples Ω^0, loss function (15), and the ADAM
     algorithm.
5  for k = 1 to K do
6      Error Estimation:
7      Evaluate the approximation error of the trained model f̂_{k−1}(ξ; Θ^{k−1}) over U.
8      Identify Sampling Candidates:
9      Determine regions in U where the error is maximized.
10     Sample Selection:
11     Select new sample points {ξ_{new,j}}_{j=1}^N from the identified regions.
12     Function Evaluation at New Samples:
13     Derive the system strength f_SS(ξ_{new,j}) for each new sample ξ_{new,j} by solving (12).
14     Update Training Set:
15     Augment the dataset with the new samples and their function values, i.e.,
        Ω^k = Ω^{k−1} ∪ {{(ξ_{new,j}, f_SS(ξ_{new,j}))}_{j=1}^N}.
16     Retrain the f̂_0(ξ; Θ^{k−1}) to obtain Θ^k.
17     Convergence Check:
18     if maximum estimated error or uncertainty across U is below ϵ then
19         Terminate the loop.
20     end
21 end
22 Return the final model f̂_SS(ξ; Θ) = f̂_k(ξ; Θ^k).
```

Algorithm 1. Active Sampling for Approximating System Strength Function $f_{SS}(\xi)$.

3.2 Active sampling for synthetic system strength assessment data set generation

As shown in Equation 15, the Ω will affect the performance of the trained neural network, from the input perspectives. The following algorithm adopts an iterative procedure to generate the training dataset Ω, i.e., an active sampling algorithm. In Line 9 of Algorithm 1, the cross-validation method is used to identify the region with maximum error.

Remark 6: In Algorithm 1, a finite large scalar should be assigned to K, to meet the stopping criterion in Line 18.

4 Mixed-integer linear programming based input stability verification

In this section, the trained ReLU neural network is further reformulated as a MILP problem to verify the robustness of the derived neural network. The linear transformation and activation function are formulated as equal and unequal constraints to reformulate the neural network. The main merit of reformulating the trained neural network as a mixed-integer linear programming problem is to derive the adversarial samples within local area. This sample can be further used to check the local stability of the trained neural network.

4.1 Linear transformation equations

For each neuron j in layer l, the relation between this neuron and the output from neurons in layer $l-1$ can be depicted as follows:

$$y_j^{(l)} = \sum_{i=1}^{n^{(l-1)}} W_{ji}^{(l)} z_i^{(l-1)} + b_j^{(l)}, \quad \text{for } l = 1, 2, \ldots, L, \tag{16}$$

where $y_j^{(l)}$ and $z_i^{(l-1)}$ are the input and output respectively; $W_{ji}^{(l)}$ and $b_j^{(l)}$ are the weight factor and bias, respectively.For the input layer, the following constraint is added to map the input to the first hidden layer linearly:

$$z_i^{(0)} = x_i, \quad \text{for } i = 1, 2, \ldots, |\xi|,$$

where x_i is treated as the decision variable as well, where $x_i \in [\xi_{i,\min}, \xi)i, \max]$.

4.2 ReLU activation constraints

To reformulate the ReLU activation function Equation 13, a binary variable $\delta_j^{(l)} \in \{0,1\}$ is introduced to realize the exactly reformulated. For each neuron j in layer l:

$$z_j^{(l)} \geq 0, \tag{17}$$

$$z_j^{(l)} \geq y_j^{(l)}, \tag{18}$$

$$z_j^{(l)} \leq y_j^{(l)} - y_{\min,j}^{(l)}\left(1 - \delta_j^{(l)}\right), \tag{19}$$

$$z_j^{(l)} \leq y_{\max,j}^{(l)}\delta_j^{(l)}, \tag{20}$$

$$\delta_j^{(l)} \in \{0,1\}, \tag{21}$$

where $y_{\min,j}^{(l)}$ and $y_{\max,j}^{(l)}$ are the minimum and maximum boundary value of neuron j in layer l.

Theorem 1: Equations 17–21 are the exact reformulation of ReLU activation function Equation 13.

The proof Theorem 1 is a direct result by enumerating $\delta_j^{(l)}$ as either 0 or 1. The $y_{\min,j}^{(l)}$ and $y_{\max,j}^{(l)}$ play an important role in the reformulation Equations 17–21, regarding the solution space and feasibility. The boundary-tightening technique is widely adopted to formulate a compact model (Liu et al., 2024).

4.3 Variable bounds

For each neuron j in layer l, the following constraints are placed on the input and output of each neuron:

$$y_{\min,j}^{(l)} \leq y_j^{(l)} \leq y_{\max,j}^{(l)} \tag{22}$$

$$0 \leq z_j^{(l)} \leq z_{\max,j}^{(l)} = \max\left(0, y_{\max,j}^{(l)}\right) \tag{23}$$

4.4 Objective function

When it comes to the objective function, the objective function is set to minimize or maximize the output, as follows:

$$\min or \max \quad z^L \tag{24}$$

As shown in Equations 16–24, the reformulated problem, i.e., one MILP problem, can be solved by the off-the-shelf commercial solver.

Remark 7: For some neural network embedded optimization techniques, the objective function Equation 24 is set to 0.

Remark 8: For boundary-tightening, the following two problems are solved for each neuron to derive the $y_{\min,j}^{(l)}$ and $y_{\max,j}^{(l)}$:

$$\begin{aligned} y_{\min,j}^{(l)} = \min \quad & y_j^{(l)} \\ \text{s.t.} \quad & (16)-(23) \end{aligned} \tag{25}$$

$$\begin{aligned} y_{\max,j}^{(l)} = \max \quad & y_j^{(l)} \\ \text{s.t.} \quad & (16)-(23) \end{aligned} \tag{26}$$

In Equations 25, 26, the boundaries of the input and output are set to a sufficient big scalar.

5 Case studies

In this section, the performance of the proposed system strength assessment method is assessed using the numerical results conducted on the modified IEEE-33 bus systems with a set of IBDERs.

5.1 Case description

As shown in Figure 2, the modified IEEE-33 bus systems have 33 buses, 32 AC branches, 32 loads, and 8 IBDERs. The base load level is 2.9 MW. The installation capacity of IBDERs is 2.835 MVA. All IBDERs are assumed to be controlled as either voltage or current sources. The k parameter is set to 1.2 for each IBDER. The transient dynamic parameters are set to 1 and 0.5 p.u. regarding the resistance and reactance.

For the ReLU neural networks, there are 6 layers, and there are 64, 128, 256, 128, 64, and 1 neuron in each layer. There are 49 key features for the system strength assessment. The Latin hypercube sampling approach is adopted to generate the initial training set with 1,000 samples. In Algorithm 1, after evaluating the error distribution, the first 10 samples with the highest errors are used to generate new samples, whereas simple random sampling is used to generate additional 5 samples. K is set to 100. The frequency distribution of the system strength within set Ω^0 is shown in Figure3.

The formulated problem Equation 16–24 is solved by Gurobi.To verify the claimed contributions, the following cases are conducted:

I. The base case with either voltage source or current source IBDERs.
II. The load levels are changing.
III. A initial neural network is trained with Ω^0
IV. Algorithm 1 is adopted to train the neural network.

5.2 Result analysis

The system strength, i.e., SCR, in Cases I are reported in Table 1, where C stands for grid-following and V stands for grid-forming.

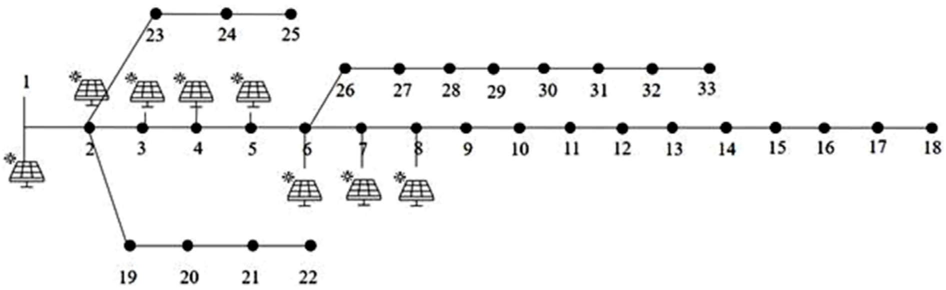

FIGURE 2
Modified IEEE-33 bus system with IBDERs.

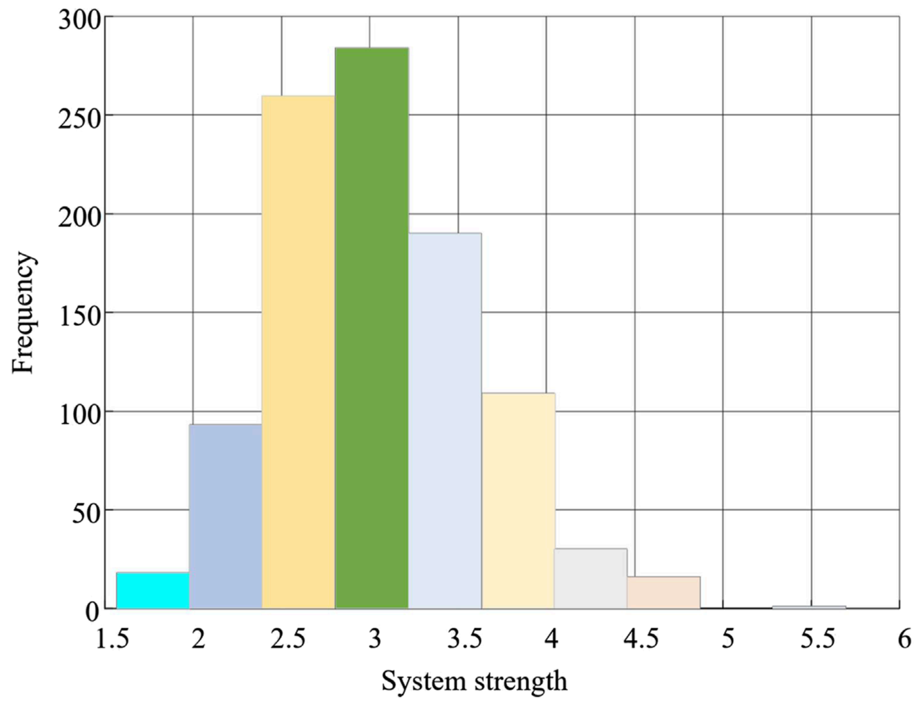

FIGURE 3
Histogram of system strength across Ω^0.

TABLE 1 System strength under different control modes and location.

	IBDER1	IBDER2	IBDER3	IBDER4	IBDER5	IBDER6	IBDER7	IBDER8	SS
S1	C	C	C	C	C	C	C	C	1.99
S2	V	C	C	C	C	C	C	C	2.42
S3	C	C	C	C	V	C	C	C	2.35
S4	C	C	C	C	C	V	C	C	2.32

As it can be observed, the voltage control of IBDERs can always increase the system strength. When the voltage-controlled IBDER is close to the load center, e.g., bus 2 and bus 6, this contribution is higher. These results indicate the proposed system's strength assessment method is Section 2 can effectively identify the location for IBDER sitting regarding SCR.

The system strength under different control modes and load levels are illustrated in Figure4. It can be observed that the increase

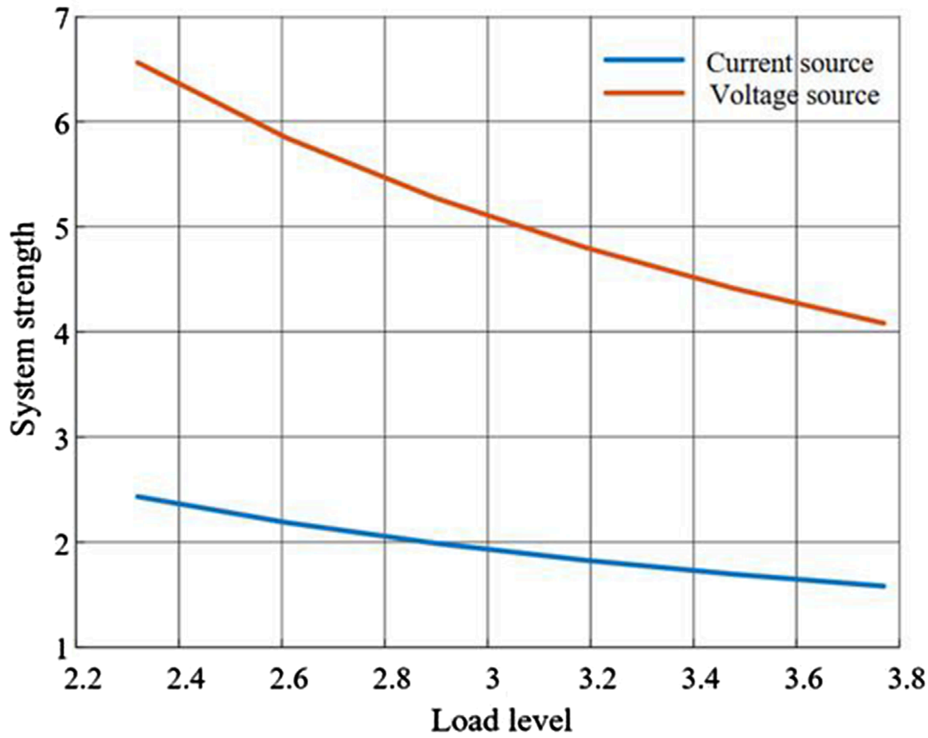

FIGURE 4
System strength under different control modes and load levels.

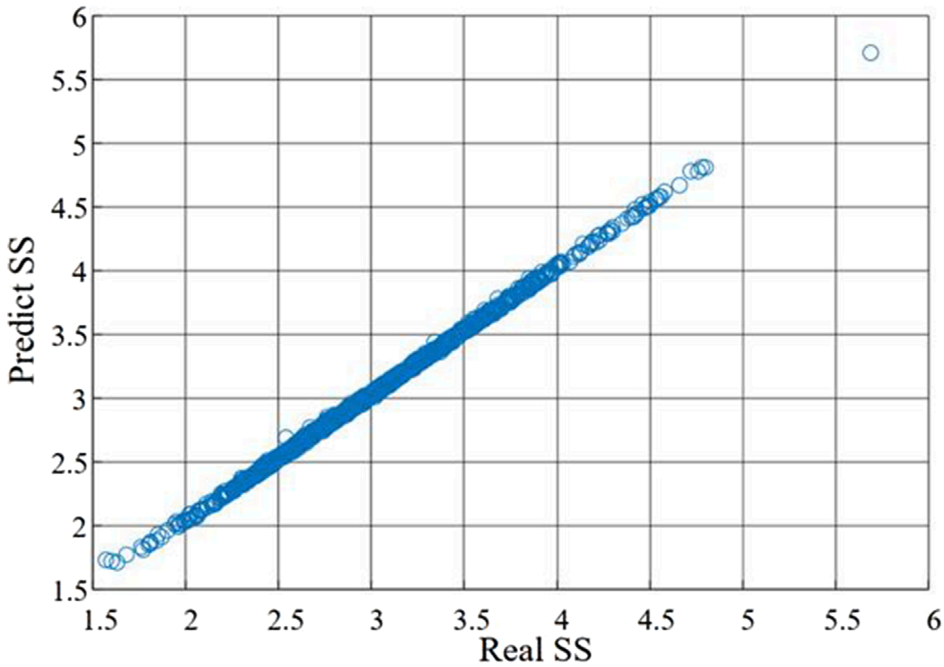

FIGURE 5
Forecasting and real system strength results in Case III.

of load leads to the rise of system strength directly, and this trend is not affected by the control mode. Along with the increase in load level, the system strength is always higher, when all IBDERs are under voltage control mode. Moreover, when the load arrives at 3.7 MW, the SCR is close to 1.5, indicating the system is close to the voltage stability margin. These results indicate that the formulated

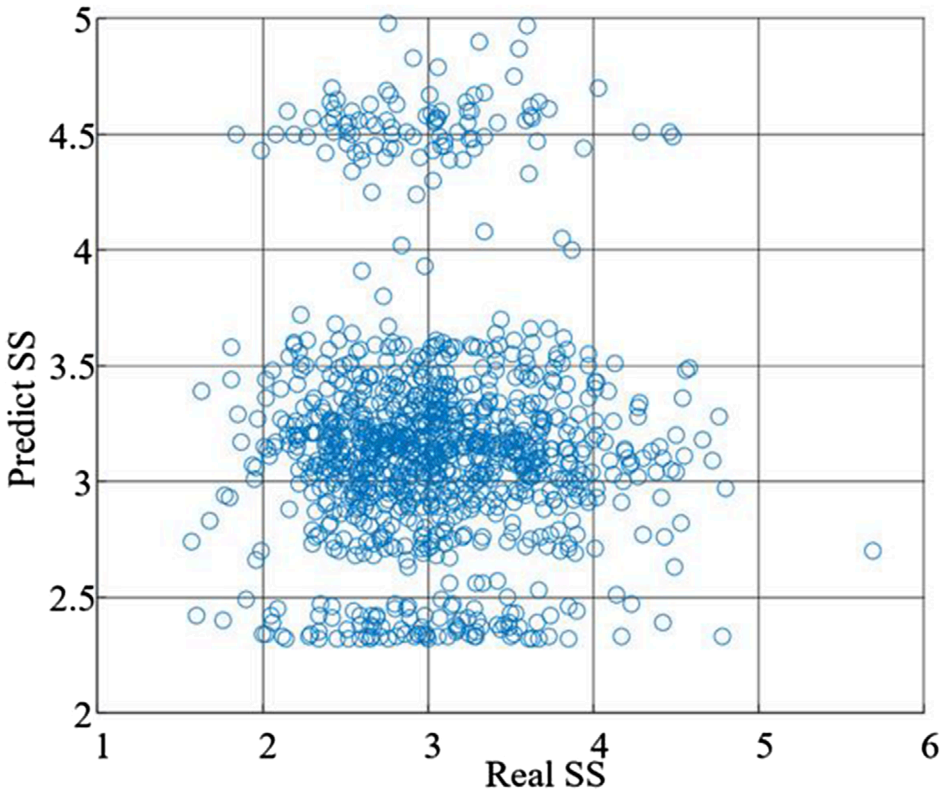

FIGURE 6
Forecasting and real system strength results on the test set in Case III.

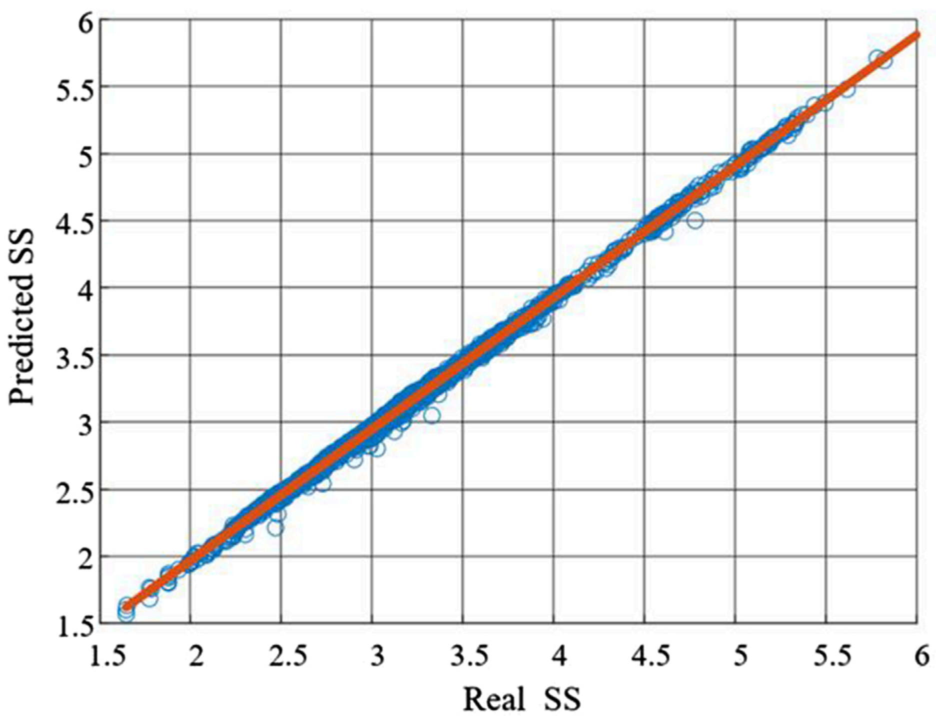

FIGURE 7
Forecasting and real system strength results in Case IV.

SS assessment method can identify the weak points in the operating conditions.

The forecasting result and ground truth SS information in Case III are shown in Figures 5, 6, respectively. It can be observed that the training performance, i.e., mean squared error (MSE), of the ReLU network is 0.6585. However, when it comes to the extended training set, the MSE has been increased to 0.7500. It indicates the trained ReLU network obtained from Ω^0 is unstable on the complete event space.

The forecasting result and ground truth SS information of Case IV are shown in Figure 7. As it can be observed, the forecasting and real SS are close to each other, i.e., the R^2 score is 0.99848. With the help of extended training samples, the performance of the trained ReLU neural network is acceptable. What is more, the minimum SS derived by Section IV is given as 1.3577, almost 10% smaller than the training sample performance over the whole sample space.

Regarding the computational feasibility, the proposed method demonstrates significant efficiency improvements over traditional simulation-based approaches. The offline SCR calculation requires an average processing time of 1.7 s under varying operating conditions, while the online forecasting scheme achieves a processing time of just 0.003 s. This demonstrates that the proposed method is approximately 560 times faster, making it well-suited for real-time applications in distribution systems. Such efficiency ensures timely system strength assessment, even in dynamic operating environments, and enhances its practical applicability for modern distribution systems with high DER penetration.

6 Conclusion

In this paper, an innovative online forecasting scheme for distribution systems with inverter-based distributed energy resources is presented. A new system strength metric, derived from the site-dependent short circuit ratio and based on the IEC-60909 standard, quantifies the impacts of inverter control modes on system strength. A ReLU neural network-based forecasting technique with adaptive sampling and embedded cross-validation is proposed, enhancing prediction accuracy and robustness. The trained neural network is reformulated as a mixed-integer linear programming problem to verify its input robustness.

Numerical results on the IEEE-33 bus system demonstrate that the proposed system strength metric effectively captures the influence of voltage control on short circuit behavior. Adaptive sampling improves training data quality, leading to a more robust neural network. The reformulated MILP approach provides a quantitative measure of the neural network's robustness, confirming the feasibility of the proposed framework for practical applications.

While the proposed system strength metric and online forecasting method show promise, their scalability to larger and more complex networks may require further refinement. Additionally, the computational burden of the MILP-based robustness verification could pose challenges for real-time applications in highly dynamic systems. Future work will focus on improving scalability and computational efficiency while validating the approach in more diverse and complex scenarios.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

JL: Formal Analysis, Project administration, Supervision, Writing–original draft. SR: Conceptualization, Investigation, Software, Writing–review and editing. TY: Methodology, Project administration, Resources, Validation, Writing–review and editing. TL: Formal Analysis, Methodology, Project administration, Software, Writing–original draft. HQ: Investigation, Methodology, Writing–review and editing. YC: Data curation, Methodology, Supervision, Validation, Writing–original draft.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Generative AI statement

The author(s) declare that no Generative AI was used in the creation of this manuscript.

Publisher's note

Supplementary material

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