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Collaborative source-grid-load frequency regulation strategy for DC sending-end power grid considering electrolytic aluminum participation

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As renewable energy sources are increasingly being integrated into power systems, traditional frequency regulation methods have faced challenges, such as reduced system inertia and diminished regulatory capacity. We present a source-grid-load collaborative control strategy for the participation of electrolytic aluminum in the frequency regulation of the DC sending-end power grid. First, the frequency response characteristics of the ultrahigh-voltage DC (UHVDC) sending-end system are analyzed, and an electrolytic aluminum load model is established. Then, a hierarchical source-grid-load control strategy is proposed. The upper-layer control assigns the frequency support tasks to synchronous generators, electrolytic aluminum stations, and UHVDC systems based on the frequency dead zones. The lower-layer control aims to minimize the cost of controlling the electrolytic aluminum loads by distributing power adjustment commands to each electrolytic aluminum series within the power station. Simulations were conducted, and the results validate the effectiveness and economic benefits of the proposed strategy in reducing the control costs while maintaining system stability.

KEYWORDS

frequency regulation, ultrahigh-voltage DC system, electrolytic aluminum load, source-grid-load control strategy, hierarchical control

1 Introduction

Comprehensive measures have been implemented in China to address carbon peaking and carbon neutrality, with particular focus on the strategic initiative of "accelerating the construction of a new energy system." The aim of this initiative is to simultaneously ensure secure and stable energy supply while promoting green and low-carbon development efforts (Wang et al., 2023; Yang et al., 2023a). Traditionally, frequency regulation in a power system relies on the balancing control mode, where the generating units track the active power demand of the loads. However, given the increasing proportion of new energy generation on the supply side, challenges such as declining system inertia, reduced regulation capacity, and heightened instability have emerged (Han et al., 2021; Han et al., 2023; Han and Chen, 2024). These challenges render the traditional regulation methods inadequate for new operational scenarios, thereby necessitating the introduction of new regulation resources to alleviate the pressure on the active power reserves (Wang et al., 2023).



The ultrahigh-voltage direct current (UHVDC) system offers significant advantages, such as strong controllability, fast response time, and high degree of control flexibility. It enhances the ability of the system to integrate renewable energy by maintaining frequency stability at the sending-end and maximizing the new energy output. A multi-time-scale coordinated control strategy has been proposed to actively support the frequency stability of the sending-end system by optimizing the use of frequency regulation resources while improving the system stability (Xin et al., 2023). Similarly, measures to enhance renewable energy accommodation in the sending-end grids of UHVDC systems that address both frequency and voltage concerns have been validated through a planning example of the Qinghai power grid in 2025 (Zhang et al., 2022). Additionally, a control strategy has been developed for the double-terminal converter stations of UHVDC systems based on virtual synchronous generator technology. Under load mutation conditions, this strategy reduces the amplitude, speed, and overshoot of the frequency fluctuations, thereby suppressing voltage fluctuations and ensuring frequency regulation without deviations (Guo et al., 2022). The frequency limit controller (FLC) in the UHVDC transmission system also offers unique advantages for maintaining frequency stability in an islanded system. The causes of ultralow-frequency oscillations in a multi-DC sending-end system after asynchronous networking as well as the mechanism of the DC FLC in suppressing these oscillations were analyzed to propose a suppression scheme that effectively mitigates the ultralow-frequency oscillations of the transmitter grid (Li et al., 2019). A multiobjective double-layer optimization method was also introduced for the design of the DC FLC parameters to improve its performance and reduce its impact on the frequency of the receiving-end grid (Wang et al., 2022). Furthermore, in response to the operational safety requirements of the multiterminal DC systems of the Wudongde DC project, stability control measures and FLC settings were proposed to resolve the system instabilities caused by AC line faults near the Wudongde power station, thus enhancing the adaptability of the high-frequency generator tripping scheme in the Yunnan power grid (You et al., 2018).



However, adjusting the control strategy at the sending end of a UHVDC transmission line will inevitably impact the receivingend power grid. This effect is particularly pronounced in scenarios with high proportions of wind and photovoltaic power sources as the volatility of renewable energy significantly influences the power system. The demand for real-time active power balancing in such systems is increasing; hence, relying solely on DC lines to compensate for power shortages can lead to large fluctuations





Overall control structure of the sending-end ultrahigh-voltage direct current (UHVDC) system.



in the receiving-end grid and may even exacerbate the system disturbances. Therefore, it is crucial to explore new regulation resources to alleviate the pressure of frequency regulation.

To improve the system regulation capacity, distributed resource (DR) and flexible loads are often used in frequency modulation auxiliary services to improve the overall performance of the system



TABLE 1 Parameters of the designed simulation experiment.

Item	Value
Rated AC voltage (kV)	500
Maximum regulating power of the synchronous generation units (MVA)	2,700
Capacity of the new energy grid connection (MVA)	2,700
Rated capacity of the converter station (MVA)	1,500
Maximum reserve capacity for frequency modulation (MW)	270
Primary frequency modulation deviation coefficient	0.05
Rated power of the electrolytic aluminum load (MW)	1,300

TABLE 2 Parameters of the electrolytic aluminum series.

Name	ASL1	ASL2	ASL3	ASL4	ASL5
Voltage (V)	312	624	1,024	840	840
Current (kA)	160	300	300	400	400
Electrolytic power (MW)	49.92	187.2	307.2	336	336
Current	160	300	300	400	400
Electrolytic power	49.92	187.2	307.2	336	336

(Mo et al., 2023; Yang et al., 2021). When traditional generation units lack sufficient frequency modulation capacity, flexible loads can reduce their operating power or temporarily shut down electrical equipment without significantly impacting user experience. This alleviates the pressure on the load side of the power system and enhances the grid frequency stability (Yang et al., 2023b; Wang et al., 2024a). For instance, Wang et al. (2024b) proposed a stable control method incorporating the control parameters of an analogous virtual synchronous generator along with flexible load regulation; by utilizing resources such as virtual damping, virtual inertia, and flexible loads, this approach ensures that the initial steady-state point is included within the largest estimated domain of attraction of the target steady-state point. This method effectively eliminates the risk of instability due to oscillations and significantly improves the frequency stability of system operation.

Among the various types of flexible loads, electrolytic aluminum as a high-energy-consuming industrial load offers significant potential for frequency regulation owing to its power intensity and large thermal inertia. As a large-scale industrial load with a constant and substantial power demand, it plays a crucial role in energy-intensive industries. Despite its continuous power consumption, the aluminum electrolysis process allows shortterm power modulation without disrupting production, making it an ideal candidate for demand-side frequency regulation. In regions with high renewable energy penetration, integrating such controllable loads becomes increasingly important. In particular, in high-voltage DC (HVDC) systems at the sending end where renewable power fluctuations are common, electrolytic aluminum can rapidly adjust its power consumption to provide essential flexibility while stabilizing the frequency and mitigating renewable generation variability. Thus, incorporating electrolytic aluminum not only enhances the reliability of the HVDC system but also addresses the operational challenges of modern grids with high renewable energy integration.

Considering the response of the electrolytic aluminum load, a source-grid-load coordinated active power and frequency control strategy is proposed herein by taking an industrial power grid including electrolytic aluminum loads as the sending-end system to provide power support to the receiving-end system via a flexible direct current transmission system (Bao, 2021; Nie et al., 2024). Additionally, the participation of high-energy-consuming electrolytic aluminum loads in frequency regulation and auxiliary services has been studied, and their fast dynamic regulation capabilities have been verified (Bao et al., 2020). A coordinated frequency modulation strategy for wind farms and electrolytic aluminum has also been proposed, although it lacks consideration of the entire power system (Luo et al., 2023). Another work proposed a secondary frequency control strategy using a virtual



FIGURE 7

Simulation results under Condition 1 showing (A) a comparison of the effects of different frequency modulation methods and (B) the power consumed by the electrolytic aluminum station or delivered by the UHVDC grid.



power plant with aluminum smelter loads for optimized power support via a voltage source converter (VSC)-HVDC link by focusing on secondary regulation and demand response under renewable energy uncertainty (Bao, 2023). A hierarchical control strategy for aluminum smelters was introduced to provide costefficient primary frequency support by utilizing a novel DC control scheme (Bao et al., 2022). These studies focus on the frequency regulation requirements of the power grid. In contrast, our proposed strategy emphasizes primary frequency regulation with comprehensive source-grid–load coordination by integrating synchronous generators, HVDC systems, and electrolytic aluminum loads. Furthermore, our approach addresses the physical constraints and economic impacts during the regulation process to provide a more detailed control structure at the electrolytic cell level, which has been insufficiently investigated in previous studies.

In response to the aforementioned issues, we propose a source-grid-load coordinated control strategy for the participation of electrolytic aluminum in the UHVDC sending-end system. The proposed strategy entails construction of power-frequency response models for both the UHVDC transmission system and electrolytic aluminum load along with analysis of the frequency modulation potential of the source-grid-load at the sending end. Additionally, a hierarchical control architecture was developed for the participation of electrolytic aluminum loads in primary frequency modulation. The upper layer of the control scheme establishes a coordinated frequency modulation scheme for source-grid-load interactions based on fixed-value coordination, whereas the lower-layer control scheme focuses on the economic aspects of electrolytic aluminum to formulate a decentralized frequency response strategy. The effectiveness and economic



TABLE 3 Control costs (×10³ \$/h) under different strategies.

ΔP_{ASL}	Proposed strategy			Traditional strategy		
	C_{PW}	C _{PD}	C_R^{sum}	C_{PW}	C _{PD}	C_R^{sum}
100 MW	0.000	1.148	1.148	0.000	1.239	1.239
150 MW	0.000	1.276	1.276	0.984	2.774	3.758
200 MW	0.716	2.159	2.875	4.607	8.280	12.887
250 MW	16.430	27.744	44.174	27.274	35.113	62.387

viability of the proposed control strategy are finally validated through simulations.

2 Frequency response model of the UHVDC sending-end system with electrolytic aluminum load

2.1 Frequency response characteristics of the UHVDC sending-end system

The topological structure of the sending-end system for largescale wind power transmission via line-commutated converter (LCC)-HVDC is shown in Figure 1. Here, the traditional units are represented by synchronous units, wind turbines are represented by doubly fed induction generators, and the traditional LCC-HVDC system is adopted for the HVDC transmission line.

As shown in Figure 1, during stable operation, the active power of the sending-end system maintains equilibrium that can be expressed as Equation 1

$$P_G = P_m + P_w = P_{ASL} + P_{LD1} + P_{dc} = P_L.$$
 (1)

Considering the primary frequency regulation of the system, the system frequency deviation Δf can be calculated by Equation 2:

$$\Delta f = \frac{1}{sH_{sys} + D} \left(\Delta P_G - \Delta P_L \right). \tag{2}$$

It is seen from Equation 2 that when there is a disturbance in the system, the system frequency deviation is closely related to the system inertia. However, for the system in which largescale new energy bases are transmitted through UHVDC links, the inertia of the sending-end system is low, and a small power disturbance will cause a large fluctuation in the system frequency. When the frequency deviation caused by the system load disturbance is within the primary frequency regulation range of the synchronous units, the system operates stably. However, once the system load disturbance exceeds the limit of the primary frequency regulation adjustment of the synchronous units, it will be difficult for the synchronous units to suppress the frequency fluctuations caused by the unbalanced power; thus, the system frequency will decrease rapidly and seriously threaten the system security (Li et al., 2024). At this time, relying only on the synchronous units to achieve frequency regulation no longer meets the demand for frequency stability. Hence, it is necessary to further introduce the sending-end electrolytic aluminum load and UHVDC transmission system for coordinated frequency regulation.

2.2 Model of the electrolytic aluminum load

The grid connection diagram of the rectification equipment for electrolytic aluminum is shown in Figure 2. By utilizing the rapid adjustment characteristics of thyristors, electrolytic aluminum can quickly regulate the active power of the electrolytic aluminum series according to the frequency regulation requirements, achieving a reduction of 0%–40% of the rated load and an increase of 0%–10% of the rated load within approximately 80 ms (Xu et al., 2014; Du et al., 2023).

The active power consumption of the *i*-th electrolysis cell P_i^{ASL} is calculated as Equation 3

$$P_i^{ASL} = U_i^{ASL} I_i^{ASL}.$$
(3)

The structure of the rectifier controller in the electrolytic aluminum load is shown in Figure 3. To eliminate the influences of external disturbances on I_i^{ASL} , the electrolytic aluminum load controller takes the difference between the reference value $I_{ref,i}^{ASL}$ of the DC of the electrolytic aluminum load and actual value I_i^{ASL} as the input signal. Using the proportional-integral (PI) controller, a pulse-width modulation (PWM) signal is generated to change the duty cycle of the input switching circuit and change the output power of the electrolytic cell to achieve frequency support.

3 Frequency control strategy for the UHVDC sending-end system with electrolytic aluminum load

3.1 Overall control structure

The frequency control structure of the UHVDC sending-end system with electrolytic aluminum load is divided into the upper and lower layers. The overall control structure is shown in Figure 4.

3.1.1 Upper-layer control

The upper layer is mainly focused on coordinating the synchronous machine, electrolytic aluminum station, and UHVDC system. Specifically, it determines the frequency regulation power allocation for the synchronous machines, electrolytic aluminum loads, and UHVDC system based on the magnitude of the frequency deviation. When $|\Delta f| < |\Delta f_1|$, the reserve capacity of the synchronous machine is sufficient to absorb the unbalanced power, and the frequency regulation task is handled exclusively by the synchronous machine to maintain system stability without engaging the electrolytic aluminum station. When $|\Delta f_1| \le |\Delta f| < |\Delta f_2|$, both the synchronous machine and electrolytic aluminum station share the burden of frequency regulation as the power disturbance exceeds the capabilities of the synchronous machine. When $|\Delta f| \ge |\Delta f_2|$, the frequency regulation task is jointly undertaken by the synchronous machine, electrolytic aluminum station, and UHVDC system. Specifically, under the circumstances of $|\Delta f_1| \le |\Delta f| < |\Delta f_2|$ and $|\Delta f| \ge |\Delta f_2|$, the frequency regulation power of the electrolytic aluminum station is transmitted to the lower-layer control.

3.1.2 Lower-layer control

The lower layer is mainly focused on coordinating the electrolytic aluminum cell series within the electrolytic aluminum station. Once the electrolytic aluminum station receives the command from the upper-layer control, the lower-layer control distributes the required regulation power to the individual electrolytic aluminum cell series by solving an optimization problem. This optimization considers the production state and control costs, including the production cost from output reduction and electrical energy cost during the heat preservation state when production is suspended. The lower-layer control must also meet the total regulated power constraint while ensuring that the combined regulation power of all cell series remains within their allowable limits. It also fine-tunes the self-saturable reactor to adjust the series current, maintaining it within the reactor's voltage regulation capacity and minimum current limit.

3.2 Upper-layer source-grid-load coordinated control strategy

Frequent participation of the electrolytic aluminum load in system frequency regulation over a long period of time has a certain impact on the production efficiency of the high-energy-consuming load. Similarly, the power regulation of the UHVDC system also directly affects the power balance of the receiving-end power grid. Therefore, it is necessary to limit the ranges of participation of the electrolytic aluminum load and UHVDC system in frequency regulation to reduce the impacts of power disturbances on the loads and receiving-end power grid. Considering that different degrees of disturbance can lead to different amounts of unbalanced power, we divided the frequency support requirements into three scenarios based on the maximum reserve capacity and unbalanced power of the primary frequency regulation of the sending-end AC system.

Case 1: The power disturbance is small, and the absolute frequency deviation $|\Delta f|$ is less than the dead zone value of frequency regulation $|\Delta f_1|$, as shown in Equation 4. At this time, the reserve capacity of the synchronous machine completely absorbs the unbalanced power, and the frequency regulation task of the sendingend system is only undertaken by the synchronous machine, thus avoiding the influence of frequent participation of the electrolytic aluminum load in the system frequency on the stability of the equipment.

$$\left|\Delta f\right| < \left|\Delta f_1\right| \tag{4}$$

Case 2: The absolute frequency deviation $|\Delta f|$ is between the dead zone values $|\Delta f_1|$ and $|\Delta f_2|$ of frequency regulation, as shown in Equation 5. At this time, the synchronous machine of the sending-end AC system cannot completely regulate the unbalanced power, and the frequency regulation task of the sending-end system is jointly undertaken by the synchronous machine and electrolytic aluminum station.

$$\left|\Delta f_{1}\right| \leq \left|\Delta f\right| < \left|\Delta f_{2}\right| \tag{5}$$

Case 3: The absolute frequency deviation $|\Delta f|$ is greater than or equal to the dead zone value of frequency regulation $|\Delta f_2|$, as shown in Equation 6. At this time, the frequency regulation task of the sending-end system is jointly undertaken by the synchronous machine, electrolytic aluminum station, and UHVDC system.

$$\left|\Delta f\right| \ge \left|\Delta f_2\right| \tag{6}$$

It can be seen that the dead zone values $|\Delta f_1|$ and $|\Delta f_2|$ of frequency regulation determine the unbalanced power undertaken by the electrolytic aluminum load and UHVDC system in the process of frequency coordinated control.

3.2.1 Structure and parameter settings of the electrolytic aluminum load participating in primary frequency control

When the absolute frequency deviation $|\Delta f|$ is less than the dead zone value of frequency regulation $|\Delta f_1|$, the electrolytic aluminum load and UHVDC system do not participate in system frequency regulation, and the frequency is borne only by the synchronous generation units.

Here, the frequency regulation of the dead zone value $|\Delta f_1|$ is Equation 7

$$\left|\Delta f_{1}\right| = \Delta P_{Gmax}\sigma_{p}.$$
(7)

When the absolute frequency deviation $|\Delta f|$ is between the dead zone values $|\Delta f_1|$ and $|\Delta f_2|$ of frequency regulation, the synchronous generation units and electrolytic aluminum load

participate in frequency control. In this paper, a frequency droop control structure is introduced in the electrolytic aluminum station, as shown in Figure 5. The droop control coefficient of the electrolytic aluminum load is mainly related to the frequency regulation capacities of the synchronous generation units and electrolytic aluminum. The primary regulation amount of the synchronous generation units and system frequency change can be expressed as

$$\Delta f = \sigma_p \Delta P_G \frac{f_N}{P_{GN}}.$$
(8)

Then, the relationship between the regulation amount of the electrolytic aluminum load and system frequency deviation can be expressed as

$$\Delta f = K_{f,ASL} \Delta P_{ASL} \frac{f_N}{P_{ASLN}}.$$
(9)

The power regulation undertaken by the electrolytic aluminum load is expressed as

$$\Delta P_{ASL} = \Delta P_{unb} - \Delta P_G. \tag{10}$$

By substituting Equations 8, 9 into Equation 10, the droop control coefficient of the electrolytic aluminum load can be obtained as Equation 11

$$K_{f,ASL} = \frac{\sigma_p \Delta P_{Gmax} P_{ASLN}}{(\Delta P_{unb} - \Delta P_{Gmax}) P_{GN}}.$$
 (11)

3.2.2 Parameter settings of the frequency regulation dead zone for the UHVDC system

When the absolute frequency deviation $|\Delta f|$ is greater than or equal to the dead zone value of frequency regulation $|\Delta f_2|$, the synchronous generation units, electrolytic aluminum loads, and UHVDC system participate in frequency control. The dead zone values of frequency regulation of the UHVDC system depend mainly on the upward and downward regulating capacities of the electrolytic aluminum load. The capacity of the electrolytic aluminum load depends on the depth of voltage regulation of the self-saturated reactors equipped to each electrolytic series and current power level of the electrolytic series.

The electrolytic aluminum station monitors the voltage, current, and power levels of the electrolytic series in real time. Based on the depth of voltage regulation of the self-saturated reactor equipped in the electrolytic series and current power level of the electrolytic series, the available active capacity of the *i*-th electrolytic series can be expressed as Equations 12–14

$$\Delta P_{max,i}^{ASL} = U_{max,i}^{ASL} I_{max,i}^{ASL} - U_{d,i}^{ASL} I_{d,i}^{ASL},$$
(12)

$$I_{max,i}^{ASL} = \frac{U_{max,i}^{ASL} - E_{d,i}^{ASL}}{R_{d,i}^{ASL}},$$
(13)

$$\Delta P_{max}^{ASL} = \sum i \Delta P_{max,i}^{ASL}.$$
 (14)

Combined with the reserve capacity of the primary frequency regulation of the synchronous generation unit, the dead zone parameter of the frequency regulation action of the UHVDC system is obtained as Equation 15

$$|\Delta f_2| = \left(\Delta P_{max}^{ASL} + \Delta P_{Gmax}\right) \sigma_p. \tag{15}$$

When the frequency deviation is greater than the dead zone value of frequency regulation $|\Delta f_2|$, the UHVDC sending-end system adopts DC frequency limit control to participate in frequency regulation, and its overall control structure is shown in Figure 6.

When the frequency change is greater than the dead zone value of frequency regulation $|\Delta f_2|$, the formula for the power modulation amount ΔP_{dc} is Equation 16

$$\Delta P_{dc} = K_p \Delta f + \int K_i \Delta f dt. \tag{16}$$

3.3 Decentralized frequency response strategy of the electrolysis cell

Once the electrolytic aluminum station receives the upper-layer control command ΔP_{ASL} , it decomposes the value among all the electrolysis aluminum cell series. Next, we simulate the control cost and power regulation command distribution architecture for the electrolytic aluminum load separately.

3.3.1 Modeling the control cost of the electrolytic aluminum load

The direct current series current value I_i^{ASL} of the electrolytic cell is a key parameter in the electrolysis process. Under normal production conditions, the series current I_i^{ASL} is maintained near its rated value $I_{i,0}^{ASL}$, and the electrolytic aluminum load is in normal production. However, when the electrolytic aluminum load is in the reduced load state, based on the magnitude of I_i^{ASL} , the production state of the electrolytic aluminum load can be divided into three types as production reduction, heat preservation, and cooling.

Under production reduction, the series current I_i^{ASL} is 90%–100% of the rated value; in this state, the electrolytic aluminum load can still maintain production, but the output will be reduced. Under heat preservation, the series current I_i^{ASL} is 70%–90% of the rated value; the electrolytic cell is energized for heat preservation to maintain the molten electrolyte from cooling; in this state, the production of electrolytic aluminum cannot be maintained but this will not cause equipment damage. Under cooling, the series current I_i^{ASL} is 70% of the rated value; in this state, the electrolytic cell has difficulty maintaining the heat preservation state so that the molten electrolyte will cool gradually, and maintaining this state for a long time will cause equipment damage.

If the power consumed by the electrolytic aluminum load changes in response to frequency deviation, the production state will inevitably be affected. Compared to the normal production state of the electrolytic aluminum load, the economic loss caused by regulation due to the frequency response is the control cost and will be quantitatively analyzed.

Considering that a lengthy period of load reduction causes electrolyte cooling (8 h in summer and 4 h in winter), the duration of primary frequency regulation is relatively short. Therefore, the cost of damage to the electrolytic cell from maintaining electrolyte cooling for a long time can be ignored. Based on the different production states of the electrolytic aluminum load, the source of the control cost C_R of the electrolytic aluminum load in this paper is divided into two parts as production and electrical energy costs.

Production cost C_{PD} is the loss from production suspension or production reduction caused by the participation of the electrolytic

aluminum load in regulation. This loss component is obtained by multiplying the difference between the original aluminum output under normal production and the original aluminum output after participating in regulation with the profit of the unit output of the original aluminum.

Electrical energy cost C_{PW} refers to the large amount of electrical energy consumed in the absence of production of original aluminum in the heat preservation state under production suspension of the electrolytic aluminum load. This loss component is obtained by multiplying the electrical energy consumed under the production suspension state with the unit electricity cost.

Thus, $C_{R,i}$ can be expressed as

$$C_{PD,i} = C_{PD,i} + C_{PW,i}$$

$$C_{PD,i} = \begin{cases} (M_{0,i} - M_{R,i})c_{prft} & I_i^{ASL} \ge 0.9I_{i,0}^{ASL} \\ M_{0,i}c_{prft} & I_i^{ASL} < 0.9I_{i,0}^{ASL} \end{cases}$$

$$C_{PW,i} = \begin{cases} 0 & I_i^{ASL} \ge 0.9I_{i,0}^{ASL} \\ (I_i^{ASL}R_{d,i} + E_{d,i})I_i^{ASL}c_{free} & I_i^{ASL} < 0.9I_{i,0}^{ASL}. \end{cases}$$
(17)

The relationship between the primary aluminum output and DC series current value can be expressed as Equation 18

$$M_{R,i} = \frac{0.3356 \cdot \eta_{0,i}}{I_{i,0}^{ASL}} (I_i^{ASL})^2.$$
(18)

To facilitate analysis of the piecewise function in Equation 17, the flag bit r_i is introduced as follows:

$$r_{i} = \begin{cases} 0 & I_{i}^{ASL} < 0.9 I_{i,0}^{ASL}, \\ 1 & I_{i}^{ASL} \ge 0.9 I_{i,0}^{ASL}. \end{cases}$$
(19)

By substituting Equation 19 in Equation 17, we obtain

$$C_{PD,i} = r_i \left(M_{0,i} - \frac{0.3356\eta_{0,i}}{I_{i,0}^{ASL}} \left(I_i^{ASL} \right)^2 \right) c_{prft} + (1 - r_i) M_{0,i} c_{prft}, \quad (20)$$

$$C_{PW,i} = 0 \cdot r_i + (1 - r_i) \left(I_i^{ASL} R_{d,i} + E_{d,i} \right) I_i^{ASL} c_{free}.$$
 (21)

By substituting Equations 20, 21 in Equation 17, the control load cost of the electrolytic aluminum series can be obtained as

$$C_{R,i} = r_i \left(M_{0,i} c_{prft} - \frac{0.3356 \eta_{0,i} c_{prft}}{I_{d0,i}} I_i^{ASL2} \right) + (1 - r_i) \left[M_{0,i} c_{prft} + \left(I_i^{ASL} R_{d,i} + E_{d,i} \right) I_i^{ASL} c_{free} \right].$$
(22)

The control cost of the entire electrolytic aluminum station is the sum of the control costs of the *n* electrolytic series:

$$C_R^{sum} = \sum_{i=1}^n C_{R,i}.$$
 (23)

3.3.2 Power distribution control strategy for the electrolytic aluminum series

Considering the form of $C_{R,i}$ in Equation 22, Equation 23 can be written in the following compact form as shown in Equations 24–28:

$$\min C_R^{sum} = \sum_{i=1}^n \left[r_i \Big(\varepsilon_{i,1} \Big(I_{d,i}^{ASL} \Big)^2 + \nu_i \Big) + (1 - r_i) \Big(\varepsilon_{i,2} \Big(I_{d,i}^{ASL} \Big)^2 + \nu_i I_{d,i}^{ASL} + \nu_i \Big) \right],$$
(24)

where

$$\varepsilon_{i,1} = -\frac{0.3356\eta_{0,i}c_{prft}}{I_{d0,i}},$$
(25)

$$\varepsilon_{i,2} = c_{free} R_{d,i},\tag{26}$$

$$\gamma_i = c_{free} E_{d,i},\tag{27}$$

$$v_i = M_{0,i} c_{prft}.$$
 (28)

The expression of r_i is then constrained as Equation 29

$$\frac{I_{d,i}}{0.9I_{d0,i}} - 1 < r_i \le \frac{I_{d,i}}{0.9I_{d0,i}}.$$
(29)

The model also needs to satisfy the total regulated power constraint, which can be expressed as Equations 30, 31

$$\Delta P_{ASL} = \sum i \Delta P_i^{ASL}, \tag{30}$$

$$\sum i \Delta P_i^{ASL} \le \sum_{i=1}^n U_{max,i}^{ASL} I_{max,i}^{ASL} - U_{d,i}^{ASL} I_{d,i}^{ASL}.$$
(31)

Steady current control of the electrolytic series is achieved by fine-tuning the self-saturable reactor. Owing to the limited depth of voltage regulation of the self-saturable reactor, there are minimum values for both the voltage and series current of the electrolytic aluminum series. Therefore, the set series current should not exceed the voltage regulation capacity of the self-saturable reactor as shown in Equation 32:

$$I_{min,i}^{ASL} \le I_{d,i}^{ASL} \le I_{max,i}^{ASL}.$$
(32)

The above model is a mixed-integer quadratic programming problem. In each dispatch control cycle, the power adjustment sequence for each electrolytic aluminum series is calculated through optimization and distributed to the lower-layer electrolytic series 1 to *n*. The lower electrolytic series respond automatically to system frequency deviation control by using the current command value as the input to the PI control link.

4 Case study

4.1 Simulation design

To verify the effectiveness of the abovementioned frequency regulation strategy, the UHVDC transmission system model with electrolytic aluminum load shown in Figure 1 was built and simulated in MATLAB/Simulink. The relevant simulation parameters are shown in Table 1. Among these, the load at the electrolytic aluminum station contains five electrolytic series ASL1–ASL5, all of which have the same type of self-saturable reactor with a voltage regulation depth of 70 V. The equipment parameters are shown in Table 2.

4.2 Source–grid–load coordinated frequency control experiment

In the simulations, two working conditions corresponding to frequency rise and frequency drop were set, and the dynamic response curves of the system frequency under participation or non-participation of the electrolytic aluminum load in the source-grid-load coordinated regulation were compared and analyzed to highlight the effectiveness and advantages of the proposed control strategy.

- Condition 1: A wind farm failure occurs, resulting in a 450-MW power shortage in the system that causes the frequency of the sending-end power grid to drop.
- Condition 2: A monopolar blocking fault occurs in the DC line, resulting in a 400-MW power surplus in the system that causes the frequency of the sending-end power grid to rise.

4.2.1 Simulation results under Condition 1

A 450-MW wind turbine generator was disconnected at t = 5s, resulting in a 450-MW power shortage in the system. Since this shortage exceeds the primary frequency regulation reserve capacity of the synchronous units, the power shortage of the system should be compensated by the electrolytic aluminum station or UHVDC system after disturbance occurs. Under this working condition, we simulated the frequency and power variation curves for three control modes: 1) neither the electrolytic aluminum load nor the UHVDC system is involved in frequency regulation; 2) the electrolytic aluminum load does not participate in frequency regulation, but the UHVDC system participates in frequency regulation; 3) the proposed source–grid–load coordinated control strategy with the electrolytic aluminum load participates in frequency regulation. The results under these conditions are shown in Figure 7.

As shown in Figure 7A, when only the synchronous units participate in frequency regulation without the electrolytic aluminum load or UHVDC system, the primary frequency regulation capacity of the synchronous generator units is maximum but still insufficient to compensate the unbalanced power of the system, eventually leading to rapid decline of the system frequency and to system collapse. As shown in Figure 7A, when the synchronous units and UHVDC system participate in frequency regulation, the final frequency will be stabilized near the frequency dead zone value of the UHVDC system. As seen in Figure 7B, during the adjustment process, there was a large power fluctuation in the receiving-end power grid; after reaching the steady state, the UHVDC system had a power transmission reduction of 180 MW that accounted for nearly 20% of the transmission capacity of the DC system and greatly reduced its utilization rate. In addition, although this scheme can suppress the frequency drop of the system, the frequency difference at steady state is large and there is much room for improvement.

As seen in Figure 7A, when the electrolytic aluminum load participates in grid frequency support through the source–grid–load fixed-value coordination strategy proposed in this paper, the power shortage reaches the action threshold of the electrolytic aluminum load that then participates immediately in the system frequency regulation control strategy. As seen in Figure 7B, under the proposed

control strategy, the electrolytic aluminum station responds quickly to the frequency change and reduces the power of the entire station to that required by the dispatch control center within 2 s. Compared with the frequency regulation achieved with the control schemes of the participating units and UHVDC system, the proposed method increases the lowest point of frequency drop by 0.7 Hz. At the same time, compared with the UHVDC frequency regulation scheme, the frequency adjustment time of this strategy is significantly shorter, which has substantial advantages.

4.2.2 Simulation results under Condition 2

At t = 5s, a monopolar blocking occurs in the UHVDC line, resulting in 400 MW of surplus power in the system that exceeds the primary frequency regulation capacity of the synchronous generator units. Once the disturbance occurs, the electrolytic aluminum load immediately responds to the unbalanced power of the system, and the corresponding simulation results are shown in Figure 8. As shown in Figure 8A, when only the synchronous units are used for frequency modulation, their electromagnetic power reaches the upper limit of regulation when the surplus power exceeds the maximum frequency modulation reserve capacity of the sendingend AC system owing to the limited frequency modulation resources of the system. At this time, if the electrolytic aluminum load does not participate in regulation, the frequency continues to increase, and it is ultimately difficult to maintain the frequency stability of the system.

As shown in Figure 8B, when the electrolytic aluminum load participates in frequency regulation, the system power shortage reaches the action threshold of the electrolytic aluminum load that then immediately participates in system frequency modulation control and achieves rapid suppression of the frequency fluctuations within 20 s. At this time, the UHVDC line will have not reached the frequency action threshold yet and the transmission power remains unchanged, ensuring safe operation of the DC line. In addition, as shown in Figure 8A, the adjustment time, overshoot, and steady-state frequency difference of the proposed strategy have significant advantages compared to the frequency modulation scheme of the DC grid alone.

To further explore the influence of the participation degree of the electrolytic aluminum station in frequency modulation on the dynamic performance indicators of the system and operating process of the DC line, the processes causing frequency variations under different participation degrees of the electrolytic aluminum load in frequency modulation (100%, 90%, and 80%) were compared; these results are shown in Figure 9. As the participation degree of the electrolytic aluminum load in frequency modulation decreases, the system response speed gradually decreases, maximum point of frequency drop gradually decreases, and steady-state frequency difference gradually increases.

By comparing the frequency variation curves of the DC grid participating in frequency modulation in Figures 8A, 9, it is understood that higher degrees of participation of the electrolytic aluminum load in frequency modulation result in lower adjustment power from the DC grid; moreover, both the dynamic and steady-state system performances are improved, fully demonstrating the superiority of the proposed method. With increase in the coordination degree of the electrolytic aluminum load, the system frequency can be quickly adjusted to the steady-state frequency with a small overshoot, thereby avoiding changes in the transmission power during the operation of the UHVDC line and effectively avoiding overload in the UHVDC grid while greatly improving the safety and reliability of the system.

4.3 Control cost analysis of the electrolytic aluminum station

To verify the effectiveness of the proposed optimal control strategy in reducing the control cost of the electrolytic aluminum load, a contrastive method of proportional dispatch is designed. This scheme dispatches the required frequency regulation power proportionally among all operating generators based on their available regulation power, which can be expressed as Equation 33

$$P_{i}^{ASL} = \frac{P_{\text{avail},i}^{ASL}}{\sum_{i=1}^{n} P_{\text{avail},i}^{ASL}} \Delta P_{ASL}.$$
(33)

The profit of each ton of primary aluminum is set at 71.4 \$/MWh here, and the electricity fee is 57.1 \$/MWh. By setting ΔP_{ASL} to 100, 150, 200, and 250 MW, we analyzed the production cost C_{PD} , electricity cost C_{PW} , and total cost C_{R}^{sum} under the two methods, which are shown in Table 3.

According to the results in Table 3, the proposed method shows superior performance in reducing the control cost of the electrolytic aluminum load. When ΔP_{ASL} is 100 MW, the total costs of the proposed and comparable methods are 1.148×10^3 \$/h and 1.239 $\times 10^3$ \$/h, respectively, and the total cost of the proposed method is slightly lower than that of the comparable method. As ΔP_{ASL} increases, the cost advantage of the proposed method becomes more obvious. When ΔP_{ASL} is 150 MW, the total cost of the proposed method is 1.276×10^3 \$/h while that of the comparable method is 3.758×10^3 \$/h, and the total cost with the proposed method is only about one-third of that of the comparable method. When ΔP_{ASL} is 200 MW, the total cost of the proposed method is 2.875 $\times 10^3$ \$/h while that of the comparable method is as high as 12.887×10^3 \$/h, which is a significant difference. Finally, when ΔP_{ASL} is 250MW, the total cost of the proposed method is 44.174 \times 10³ \$/h while that of the comparable method is 62.387 \times 10³ \$/h; it can be seen that the total cost with the proposed method is always lower than that with the comparable method for all values of ΔP_{ASL} and especially at higher values of ΔP_{ASL} . These data verify the significant superiority of the proposed optimal control strategy in reducing the control cost of the electrolytic aluminum load and prove its potential economic benefits in practical applications.

5 Conclusion

This paper presents a source-grid-load coordinated control strategy for the participation of electrolytic aluminum in

frequency regulation at the DC sending-end grid. The proposed method effectively enhances system stability and reduces control costs. Simulation results confirm that this method significantly improves both the dynamic and steady-state performances of the power system. By leveraging the rapid response capabilities of electrolytic aluminum loads and the UHVDC system, the proposed strategy ensures efficient frequency regulation, minimizes power fluctuations, and prevents overload conditions. Additionally, the proposed method consistently demonstrates lower control costs compared to traditional proportional dispatch methods, particularly at higher regulation power levels. Thus, the proposed strategy offers substantial economic and operational benefits, underscoring its viability for practical applications in power system frequency regulation.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material; further inquiries can be directed to the corresponding author.

Author contributions

CX: conceptualization, methodology, and writing-original draft. XX: conceptualization, formal analysis, and writing-original draft. XH: formal analysis, investigation, validation, and writing-review and editing. CD: formal analysis, supervision, and writing-review and editing. MZ: resources, visualization, and writing-review and editing.

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Conflict of interest

Authors CX, XX, XH, CD, and MZ were employed by the Electric Power Research Institute of Yunnan Power Grid Co., Ltd.

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Nomenclature

Abbreviations

UHVDC	ultrahigh-voltage direct current
FLC	frequency limit controller
DC	direct current
AC	alternating current
DR	distributed resources
VSC-HVDC	voltage source converter high-voltage direct current
LCC-HVDC	line-commutated converter high-voltage direct current
LCC	line-commutated converter
ASL	aluminum smelting line
Ы	proportional-integral
PWM	pulse-width modulation

Frequency response model

LCC ₁	rectifier station connecting the sending-end system including the wind farm, synchronous units, and electrolytic aluminum load	ΔP_{max}^{ASL}	
Bus ₁	bus of the alternating current system on the rectifier side	K _p	
R_{d1}	resistance of the direct current line		
LCC ₂	inverter station connecting the receiving-end alternating current power grid	ΔP_{dc0}	
Bus ₂	bus of the alternating current system on the inverter side	۸D	
P _m	output of the synchronous unit	∆1 dc	
P_w	output of the wind turbine unit	I_i^{ASL}	
P_{LD1}	load power of the sending-end system	$I_{i,0}^{ASL}$	
P _{ASL}	power of the electrolytic aluminum load	C_R	
P _{dc}	transmission power of the high-voltage direct current line	C_{PD}	
P_{G}	total generated output of the sending-end system	C_{PW}	
P_L	total active load demand of the sending-end system	C_{RI}	
H_{sys}	equivalent inertia of the sending-end power grid	Mor	
ΔP_G	active power of the primary frequency regulation response of	0,1	
	the sending-end power grid	$M_{\mathrm{R},I}$	
ΔP_L	variation in the active load	c _{prft}	
D	damping coefficient of the sending-end power grid	c _{free}	
Δf	frequency deviation of the sending-end power grid	ŋ _{0,I}	
f	actual frequency of the sending-end power grid	r _i	
f_0	rated frequency (50 Hz)	ε,,γ,,ν,	
P_i^{ASL}	active power consumption of the <i>i</i> th electrolysis cell	P_i^{ASL}	
U_i^{ASL}	voltage of the <i>i</i> th electrolysis cell	P ASL	
I_i^{ASL}	current of the <i>i</i> th electrolysis cell	ava11, 1	
I _{ref, i} ^{ASL}	reference value of the current of the <i>i</i> th electrolysis cell	sum R	

Frequency control strategy

$ \Delta f $	absolute frequency deviation		
$ \Delta f_1 , \Delta f_2 $	dead zone values of the frequency regulation		
σ_p	droop coefficient of the primary frequency regulation of the synchronous generation units		
P _{Gmax}	maximum regulating power of the synchronous generation units		
a	initial control angle of the converter		
a _{ord}	command control angle of the converter		
P _{GN}	rated power of the generation unit		
P _{ASLN}	rated power of the electrolysis cell		
$K_{f,ASL}$	frequency droop coefficient of the electrolytic aluminum load		
ΔP_{ASL}	power deviation of the electrolytic aluminum station		
ΔP_{unb}	power disturbance generated by the system		
$\Delta P_{max,i}^{ASL}$	available active capacity of the <i>i</i> th electrolytic aluminum series		
$U_{max, i}^{ASL}$	maximum voltage of the <i>i</i> th electrolytic aluminum series		
I _{max, i} ^{ASL}	maximum current of the <i>i</i> th electrolytic aluminum series		
ΔP_{max}^{ASL}	available active capacity of the entire electrolytic aluminum station		
K _p	coefficients of the proportional links		
K _i	coefficients of the integral links		
ΔP_{max}	upper limit of the power adjustment amount		
ΔP_{dc0}	reference value of the transmission power of the high-voltage direct current line		
ΔP_{dc}	transmission power value of the high-voltage direct current line after adjustment		
I_i^{ASL}	direct current series current value		
I _{i,0} ^{ASL}	rated value of the direct current series current		
C_R	source of the control cost		
C_{PD}	production cost		
C_{PW}	electrical energy cost		
$C_{R,I}$	control cost of the <i>i</i> th electrolytic series		
<i>M</i> _{0,<i>I</i>}	output of the primary aluminum under normal production conditions		
$M_{\mathrm{R},I}$	output of the primary aluminum after load reduction or increase		
c _{prft}	profit of the primary aluminum per unit output		
c _{free}	unit electricity cost		
ŋ _{0,I}	rated current efficiency in the <i>i</i> th electrolytic series		
r _i	flag bit		
$\varepsilon_i, \gamma_i, \nu_i$	coefficients of the piecewise quadratic function		
P_i^{ASL}	active power that the <i>i</i> th electrolytic series must generate		
$P_{avail, i}^{ASL}$	available reactive power of the <i>i</i> th electrolytic aluminum series at a specific moment		
sum R	total cost		