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EDITED BY

Kaiqi Sun,
Shandong University, China

REVIEWED BY

Bing Sun,
Tianjin University, China
Lv Chaoxian,
China University of Mining and Technology,
China
Yongliang Liang,
Shandong University, China

*CORRESPONDENCE

Tiankai Yang,
✉ Tiankai900@dlmu.edu.cn

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Distributionally robust chance-constrained operation of distribution grids considering voltage constraints

Chao Wang¹, Junjie Sun¹, Xinwei Li¹, Tiankai Yang^{2*} and
Wansong Liu¹

¹Electric Power Research Institute of State Grid Liaoning Electric Power Co., Ltd, Shenyang, China,

²College of Marine Electrical Engineering, Dalian Maritime University, Dalian, China

The distribution grid experiences node voltage fluctuations due to the growing uncertainty of large-scale renewable energy sources. A practical solution is establishing a chance-constrained optimal model to deal with the uncertainties. However, using this method needs to know the accurate probability distribution of node power injections, which has limitations in application. Therefore, this paper proposes a distributionally robust chance-constrained optimization method for power grid operation based on the ambiguity set of probability distributions. Firstly, considering voltage security constraints, this paper establishes a chance-constrained model to minimize the cost of active power regulation. Besides, based on the Wasserstein ambiguity set, a linearized method is proposed to convexify the objective function. Moreover, the conditional risk value (CVaR) is applied to convert the uncertain model into a deterministic model. The effectiveness of the proposed method is validated through optimization results obtained for the modified PG&E69-bus distribution grid.

KEYWORDS

uncertainty of renewable energy sources, voltage security constraints, distributionally robust chance constraints, Wasserstein distance, conditional risk value, linearized method

1 Introduction

The global energy crisis and environmental issues have prompted continuous energy-resource structure transformation and upgrading. The gradual proliferation of renewable energy sources (RESs) such as photovoltaic and wind power on a global scale has become an inevitable trend. However, large-scale RESs have intense uncertainty, which brings enormous challenges to the dispatch and operation of active distribution grids (Alismail et al., 2017). For instance, in optimizing real-time operations using active and reactive regulation devices, the traditional dispatch approach overlooks power output uncertainties, leading to scheduling plans based solely on expected node power injections. As a result, there is a risk of violating the secure constraints (Hu et al., 2022). Therefore, considering the uncertainties of various RESs in the power grid and selecting appropriate optimal dispatch strategies have become research hotspots. In other words, it is necessary to propose a new dispatch method to integrate active power regulation devices, deal with the uncertainty of various distributed energy sources, and make the distribution grid operate securely and economically.

Currently, there are three kinds of methods to describe the uncertainty of RESs: stochastic optimization-based method, robust optimization-based method, and distributionally robust chance-constrained (DRCC) optimization-based method. When using the robust optimization-based method, the dispatchers do not need all probability distribution of RES outputs and only consider the worst-case scenario during the optimization process, which often leads to overly conservative results (Chen et al., 2020; Zhai et al., 2020; Chen et al., 2021). Han et al. (2020) considered the security constraints and established a robust optimization model for distribution systems. The constructed probability distribution fuzzy set is complex, resulting in plodding optimization speed. Ding et al. (2015) proposed a robust two-stage operation model. Although the proposed method considers the uncertainties within the power grid, it leads to high economic scheduling costs. Xia et al. (2019) considered the impact of wind power integration into the grid and proposed a robust optimization-based operation model. However, this method sacrifices significant economic costs to ensure secure operation.

According to Zhai et al. (2022), Wang et al. (2012) and Zhao et al. (2014), the stochastic optimization-based method mainly includes the Monte Carlo method, scenario generation method, and chance constraint method. This method needs to assume that the uncertainty of each power injection follows a specific probability distribution. Yang and Yu (2018) used the chance constraint method to convert uncertainty problems into deterministic problems and estimate the probability distribution of node voltages. Guggilam et al. (2016) established a two-stage energy management model for microgrids based on the chance-constrained method. Zhao et al. (2018) applied chance-constrained programming dealing with power flow, voltage, and reserve constraints. Tang et al. (2023) proposed a chance-constrained optimization scheduling method for distribution grids considering photovoltaic carrying capacity. Zhong et al. (2023) proposed a distributed optimization method for integrated transmission and distribution grids with chance constraints. They established a restoration operation model for the transmission and distribution grid to minimize restoration costs. Roald et al. (2013) introduced the fuzzy theory and credibility theory to handle wind power uncertainty, converting deterministic constraints into fuzzy chance constraints. The stochastic reactive power optimization model is established to minimize the entire grid loss and voltage offset, and it is solved by the improved particle swarm optimization algorithm. Lubin et al. (2015) and Dai et al. (2017) proposed the approximate derivation method for chance constraints based on the analytical method and established the two-step optimal scheduling for the power system. Zhao and Jiang, (2017) introduced the chance-constrained programming method to minimize the energy storage configuration in the distribution grid and used the analytical method to establish the deterministic optimal model. Ciftci et al. (2019) proposed a data-driven chance-constrained optimization model for microgrid energy management, constructing confidence sets for uncertainties based on historical data. Since the established optimal model of the stochastic optimization-based method has probability constraints, it is nonlinear and usually solved by intelligent algorithms (Li et al., 2013; Huang, 2017; Xu and Li, 2019). However, the obtained results will likely fall into the local

optimum, which means the accuracy is far from the existing engineering requirements.

In contrast to the stochastic optimization-based method, the DRCC optimization-based approach does not necessitate precise probability distribution information for uncertainties; instead, it relies on ambiguity sets, which exhibit enhanced robustness (Cao et al., 2021; Du et al., 2022). Proposed as a hybrid of stochastic and robust optimization, the DRCC optimization-based method addresses uncertain challenges. It operates under the assumption that the probability distribution falls within a predefined ambiguity set and employs the worst-case scenario from this set as the optimization reference. By utilizing the worst-case probability distribution in an ambiguity set, the optimization outcomes can be adjusted, thus mitigating the risk of over-conservatism compared to robust optimization-based methods. He et al. (2019) utilized the moment of uncertainty is to distribute and robustly solve wind power uncertainty and energy optimization problems. Ao et al., 2020 used the DRCC for multiple discrete scenarios to deal with the load uncertainties in distribution grids. However, the above two methods for constructing DRCC ambiguity sets fail to utilize historical data fully and remain highly conservative (Esfahani and Kuhn, 2018). Unlike the above methods, Wasserstein distance is data-driven and uses historical data of distributed power generation outputs to construct an ambiguity set. As the number of samples increases, the ambiguity set gradually converges to the actual situation (Duan et al., 2018). Based on this principle, the ambiguity set based on Wasserstein distance is proposed to depict the degree of difference between actual and empirical distributions (Zhu et al., 2019; Poolla et al., 2020). The conservatism and economic benefits of the flexible control optimization scheme and the established model are easy to solve and have strong applicability. Based on the above, this paper applies DRCC to the optimal operation in the distribution grid.

In addition, the existing nonlinear voltage chance constraints significantly increase the number of variables processed in the operation model. The second-order cone method is used to deal with the voltage chance constraints, but the accuracy of the optimal result is low (Cai et al., 2022). The relaxation variables and interval estimation method were introduced to relax the nonlinear chance constraints (Xiao et al., 2021). However, the approximate model introduces too many variables, resulting in low solving efficiency. For the nonconvex voltage chance-constrained forms, Liu et al. (2020) and Xie W. (2021) used the conditional risk values to transform the nonlinear chance constraints into convex approximation. Chen et al. (2024) applied the risk value and duality theory to transform the chance-constrained form of nonconvex voltages into convex expressions. Although the above-simplified method deals with fewer variables in the operation process than the traditional nonlinear optimization methods, there is still a time-consuming problem. Therefore, it is necessary to propose a fast solution method.

In summary, this paper proposes a distributionally robust chance-constrained dispatch for distribution grids that considers voltage secure constraints. The specific structure is as follows. Section 2 establishes a chance-constrained model considering voltage constraints. In Section 3, based on the Wasserstein ambiguity set, a linearized method is proposed for the objective function. Section 4 uses the conditional risk value to convert the uncertain model into the deterministic model. Simulations

validating the effectiveness of the proposed method are conducted in Section 5. Conclusions are shown in Section 6.

2 The chance-constrained model considering voltage constraints

Assume the node set is N (where the dimension is n) and the output of renewable energy sources set is W (where the dimension is w) in the distribution grid. $\mathbf{x}^0 = (P_1^0, \dots, P_n^0, Q_1^0, \dots, Q_n^0)^T$ is the current operating point. To minimize the regulation cost of distribution grids, the objective function F is as follows:

$$F = \min E \left(\sum_{i=1}^n (c_i P_i + c_i^U P_i^U + c_i^D P_i^D) \right) \quad (1)$$

where $E(\bullet)$ is the expectation of \bullet ; P_i and c_i are the active power injection and regulation cost at node $i \in N$, respectively; P_i^U and P_i^D are the reserve capacity corresponding to the upper and lower power limits at node $i \in N$, respectively; c_i^U and c_i^D are the corresponding costs of P_i^D and P_i^U , respectively.

Considering the uncertainty of the power injection, we have:

$$P_i = \bar{P}_i - P_i^L + \Delta P_i(\xi) + \xi_i, \forall i \in N \quad (2)$$

where \bar{P}_i is the power adjustment of node i ; $\Delta P_i(\xi)$ is the adjustment error caused by the uncertainty of the power injection at node i ; $\Delta P(\xi) = [\Delta P_1(\xi)^T, \Delta P_2(\xi)^T, \dots, \Delta P_n(\xi)^T]^T$; P_i^L is the load of node i . ξ_i is the renewable energy output at node i .

According to the linearization method proposed by Ordoudis et al. (2021), $\Delta P(\xi)$ can be transformed into the following:

$$\Delta P_i(\xi) = Y_i \xi, Y_i \in \mathbb{R}^{1 \times n} \quad (3)$$

where $Y_i = [y_{i,1}, y_{i,2}, \dots, y_{i,n}]$ is the adjustment coefficient vector to be solved.

Combining (1-3), we have (4):

$$F = \min_{\bar{P}_i, P_i^U, P_i^D, Y} \sum_{i=1}^n (c_i \bar{P}_i - c_i P_i^L + c_i^U P_i^U + c_i^D P_i^D) + E(c^T Y \xi) \quad (4)$$

where $c = [c_1, c_2, \dots, c_n]$, $Y = [Y_1^T, Y_2^T, \dots, Y_n^T]^T$.

Additionally, the operational constraints include:

$$\bar{P}_i - P_i^D \geq P_i^m \quad (5)$$

$$\bar{P}_i + P_i^U \leq P_i^M \quad (6)$$

$$0 \leq P_i^U \leq (P_i^M - P_i^m) \quad (7)$$

$$0 \leq P_i^D \leq (P_i^M - P_i^m) \quad (8)$$

$$\psi(V_i \leq V_i^M) \geq 1 - \varepsilon_V \quad (9)$$

$$\psi(V_i \geq V_i^m) \geq 1 - \varepsilon_V \quad (10)$$

where (5) and (6) are the constraints for the outputs of power regulation devices; (7) and (8) are the constraints for reserve capacities; (9) and (10) are the chance constraints for node voltage magnitudes; V_i^M represents the upper limit, while V_i^m denotes the lower limit of the node voltage magnitude; ε_V is the confidence level.

For (9, 10), this paper linearizes the node voltage constraints based on current constraints. According to Yang and Yu. (2018), the voltage security constraints of nodes $k \in N$ can be written as (11) and (12):

$$\sum_{j=1}^n (\alpha_{k,j}^M P_j + \beta_{k,j}^M Q_j) \leq 1 \quad (11)$$

$$\sum_{j=1}^n (\alpha_{k,j}^m P_j + \beta_{k,j}^m Q_j) \leq 1 \quad (12)$$

where $\alpha_{k,j}^M$, $\beta_{k,j}^M$, $\alpha_{k,j}^m$, and $\beta_{k,j}^m$ can be represented as follows:

$$\left\{ \begin{array}{l} \alpha_{k,j}^M = \frac{1}{V_k^M H_k^M} \frac{\partial V_k}{\partial P_j} \\ \beta_{k,j}^M = \frac{1}{V_k^M H_k^M} \frac{\partial V_k}{\partial Q_j} \\ \alpha_{k,j}^m = \frac{1}{V_k^m H_k^m} \frac{\partial V_k}{\partial P_j} \\ \beta_{k,j}^m = \frac{1}{V_k^m H_k^m} \frac{\partial V_k}{\partial Q_j} \end{array} \right. \quad (13)$$

where $H_k^M = V_k^M - V_0^0 + \sum_{\forall j \in N} (\frac{\partial V_k}{\partial P_j} P_j^0 + \frac{\partial V_k}{\partial Q_j} Q_j^0)$, $H_k^m = V_k^m - V_0^0 + \sum_{\forall j \in N} (\frac{\partial V_k}{\partial P_j} P_j^0 + \frac{\partial V_k}{\partial Q_j} Q_j^0)$.

Combining (2, 3), and (12), we can get the explicit expression of the chance constraints for voltage magnitude as (14, 15):

$$\psi \left[\sum_{j=1}^n \alpha_{k,j}^M Y_j \xi + \sum_{j=1}^n \alpha_{k,j}^M \xi_j + \sum_{j=1}^n (\alpha_{k,j}^M \bar{P}_j + \beta_{k,j}^M Q_j - \alpha_{k,j}^M P_j^L) \leq 1 \right] \geq 1 - \varepsilon_V \quad (14)$$

$$\psi \left[\sum_{j=1}^n \alpha_{k,j}^m Y_j \xi + \sum_{j=1}^n \alpha_{k,j}^m \xi_j + \sum_{j=1}^n (\alpha_{k,j}^m \bar{P}_j + \beta_{k,j}^m Q_j - \alpha_{k,j}^m P_j^L) \leq 1 \right] \geq 1 - \varepsilon_V \quad (15)$$

3 Linearization of objective based on distributionally robust theory

For the renewable energy sources output ξ , its historical statistical data is $\hat{\xi}$, and the empirical probability distribution is in (16) $\hat{\mathbb{P}}$:

$$\hat{\mathbb{P}} = \frac{1}{m} \sum_{i=1}^m \delta_i \quad (16)$$

where m is the number of historical data vectors used; δ_i is the Dirac measure of the i th historical statistical data $\hat{\xi}_i$.

Considering that the actual probability distribution \mathbb{P} of the distributed power generation vector ξ is unknown, this paper constructs a Wasserstein ambiguity set \mathbb{D} to describe the uncertainty of ξ as (17):

$$\mathbb{D} = \left\{ \mathbb{P} \in M(\Xi) \mid \mathbb{W}(\mathbb{P}, \hat{\mathbb{P}}) \leq \rho \right\} \quad (17)$$

where $\rho > 0$ is the radius of the ambiguity set; $M(\Xi)$ is the space set to which the probability distribution \mathbb{P} belongs. $\mathbb{W}(\mathbb{P}, \hat{\mathbb{P}})$ is the Wasserstein distance between the actual and empirical probability distribution of the output of renewable energy sources, which can be expressed as (18):

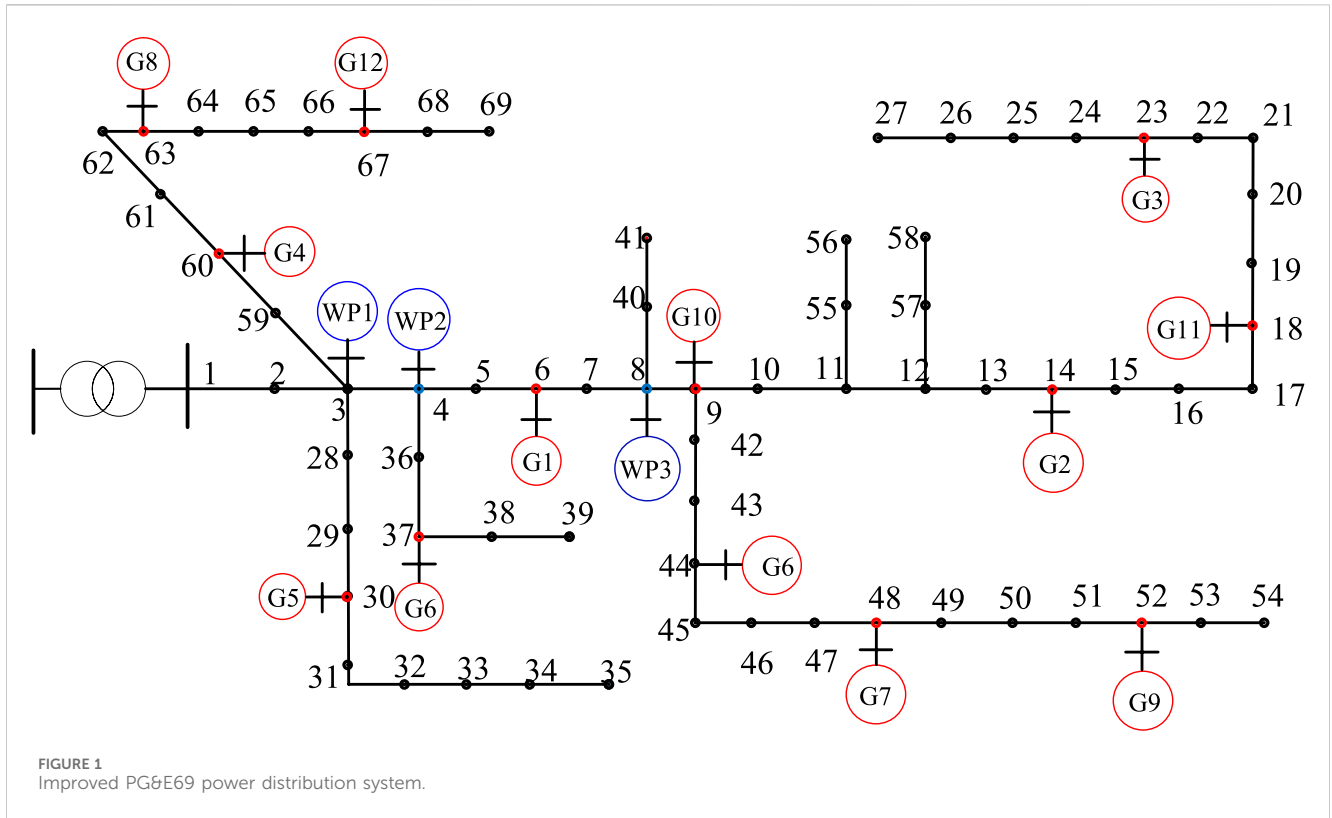


FIGURE 1 Improved PG&E69 power distribution system.

TABLE 1 Comparison of optimization results of various methods.

Method	Adjustment cost/\$
Robust optimization	3.10×10^4
The proposed method	2.77×10^4
Monte Carlo method	2.74×10^4

$$\begin{aligned} \mathbb{W}(\mathbb{P}, \hat{\mathbb{P}}) &= \inf_{\Pi} \left\{ \int_{\Xi^2} \|\hat{\xi} - \xi\| \Pi(d\hat{\xi}, d\xi) \right\} \\ &= \inf_{Q_i} \left\{ \frac{1}{m} \sum_{i=1}^m \int_{\Xi} \|\hat{\xi}_i - \xi\| Q_i(d\xi) \right\} \end{aligned} \quad (18)$$

where Π is the joint distribution set of $\hat{\xi}$ and ξ ; Q_i is the conditional probability distribution of Π when $\hat{\xi} = \hat{\xi}_i$. we have (19):

$$\Pi(d\hat{\xi}_i, d\xi) = \frac{1}{m} Q_i(d\xi) \quad (19)$$

Then, based on the robust optimization theory, the objective function is relaxed as (20, 21):

$$\min_{P_i, P_i^U, P_i^D, Y} \sum_{i=1}^n (c_i \bar{P}_i - c_i P_i^L + c_i^U P_i^U + c_i^D P_i^D) + \sup_{P \in \mathbb{D}} E(c^T Y \xi) \quad (20)$$

Let

$$l(\xi) = \sup_{P \in \mathbb{D}} E(c^T Y \xi) = \sup_{\hat{\mathbb{P}}} \int_{\Xi} c^T Y \xi \hat{\mathbb{P}}(d\xi) \quad (21)$$

Considering the total probability equation:

$$\hat{\mathbb{P}}(d\xi) = \frac{1}{m} \sum_{i=1}^m Q_i(d\xi) \quad (22)$$

Based on the Lagrange function, we have (23):

$$\begin{aligned} &\sup_{Q_i} \frac{1}{m} \sum_{i=1}^m \int_{\Xi} c^T Y \xi Q_i(d\xi) + \lambda \left(\rho - \frac{1}{m} \sum_{i=1}^m \int_{\Xi} \|\hat{\xi}_i - \xi\| Q_i(d\xi) \right) \\ &= \lambda \rho + \frac{1}{m} \sum_{i=1}^m \sup_{\xi} (l(\xi) - \lambda \|\hat{\xi}_i - \xi\|) \end{aligned} \quad (23)$$

where $\lambda \geq 0$ is the Lagrange multiplier under the condition $\mathbb{W}(\mathbb{P}, \hat{\mathbb{P}}) \leq \rho$.

By introducing the auxiliary variable s_i , the dual problem of (22) can be expressed as (24, 25):

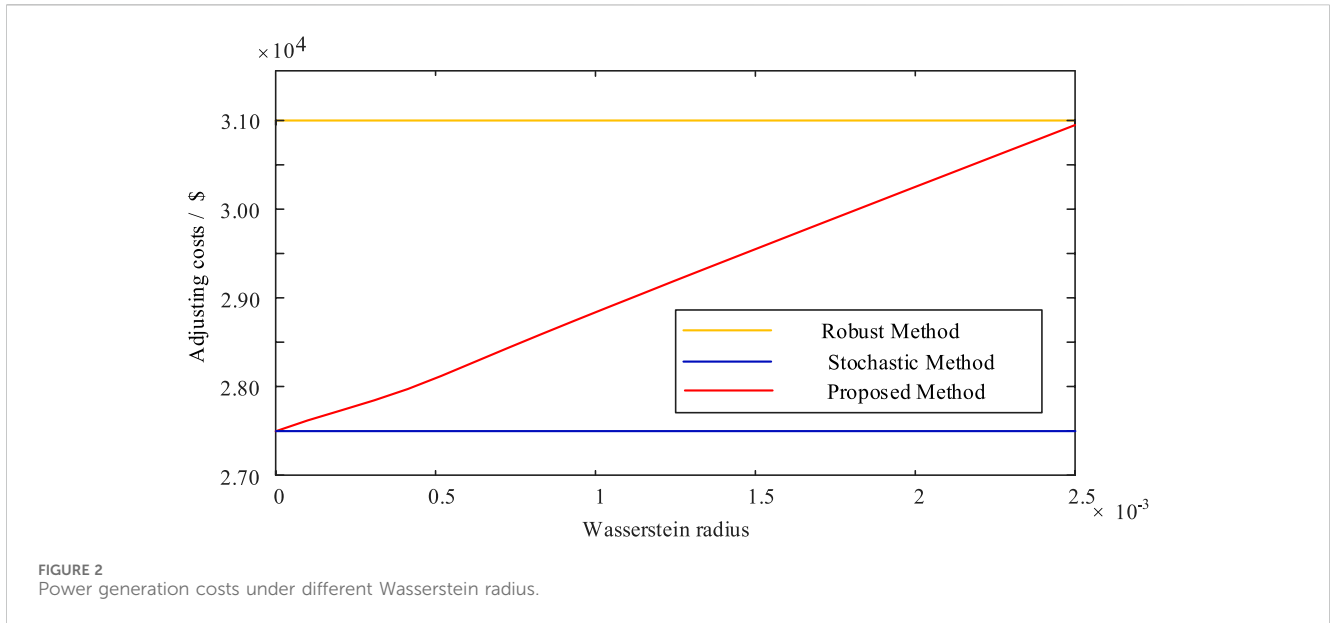
$$\lambda \rho + \inf_{s_i} \frac{1}{m} \sum_{i=1}^m s_i \quad (24)$$

$$\text{s.t.} \quad \sup_{\xi} l(\xi) - \lambda \|\hat{\xi}_i - \xi\| \leq s_i, \forall i \in \{1, 2, \dots, m\} \quad (25)$$

Besides, by introducing the intermediate variable z_i ($\|z_i\|_{\infty} \leq \lambda$), (24) can be equivalent as (26):

$$\begin{aligned} \sup_{\xi \in \Xi} l(\xi) - z_i^T (\hat{\xi}_i - \xi) &= -z_i^T \hat{\xi}_i + \sup_{Q_i} \int_{\Xi} (c^T Y \xi + z_i^T \xi) Q_i(d\xi) \\ &= -z_i^T \hat{\xi}_i + \sup_{Q_i} \int_{\Xi} (z_i + Y^T c)^T \xi Q_i(d\xi) \leq s_i \end{aligned} \quad (26)$$

According to the dual theory, we have (27):



$$z_i = -Y^T c \tag{27}$$

Then, the optimal objective can be expressed as (28, 29):

$$\min_{\bar{P}_i, P_i^U, P_i^D, Y, \lambda, s_i, y} \sum_{i=1}^n (c_i \bar{P}_i - c_i P_i^L + c_i^U P_i^U + c_i^D P_i^D) + \lambda \rho + \frac{1}{m} \sum_{i=1}^m s_i \tag{28}$$

$$\text{s.t.} \begin{cases} -z_i^T \hat{\xi}_i = c^T Y \hat{\xi}_i \leq s_i \\ \|z_i\|_\infty = \|-Y^T c\|_\infty \leq \lambda \end{cases} \tag{29}$$

$$\begin{aligned} & \sup_{\mathbb{P} \in \mathbb{D}} \inf_{\tau_k^M} \tau_k^M + \frac{1}{\varepsilon_V} E[a_k^M \xi - b_k^M - \tau_k^M]_+ \\ & = \inf_{\tau_k^M} \tau_k^M + \frac{1}{\varepsilon_V} \sup_{\mathbb{P} \in \mathbb{D}} E[a_k^M \xi - b_k^M - \tau_k^M]_+ \leq 0 \end{aligned} \tag{32}$$

where τ_k^M is the dual variable; $E[\bullet]$ is the expectations of \bullet ; $[\bullet]_+ = \max(\bullet, 0)$. By (31), the original nonconvex chance constraint form can be rewritten as a distributionally robust form
For (31), we have

4 Convex approximation of chance constrained combining conditional risk value

Given the nonconvex nature of chance constraints, solving the model becomes challenging. To address this issue, the paper proposes utilizing a conditional risk value, as suggested by Ben-Tal et al. (2009), to formulate a convex approximation of these constraints in the worst-case scenario. For the convenience of expression, this paper takes the upper limit of node voltage as an example to deduce, and (13) is recorded as (30):

$$\psi[a_k^M \xi - b_k^M \leq 0] \geq 1 - \varepsilon_V \tag{30}$$

where $\alpha_k^M = \alpha_k^M (Y + I)$; $\alpha_k^M = [\alpha_{k,1}^M, \alpha_{k,2}^M, \dots, \alpha_{k,n}^M]$; $b_k^M = 1 - \sum_{j=1}^n (\alpha_{k,j}^M \bar{P}_j + \beta_{k,j}^M Q_j - \alpha_{k,j}^M P_j^L)$; $I \in \mathbb{R}^{n \times n}$ is the unit diagonal matrix.

This paper uses the conditional risk value to approximate the chance constraint of node voltage upper limit in the worst case, shown as follows:

$$\sup_{\mathbb{P} \in \mathbb{D}} \psi[a_k^M \xi - b_k^M \leq 0] \geq 1 - \varepsilon_V \tag{31}$$

And the following constraints can be established by conditional risk value (Rockafellar and Uryasev, 2000) in (32):

$$\sup_{\mathbb{P} \in \mathbb{D}} E([a_k^M \xi - b_k^M - \tau_k^M]_+) = \sup_{\mathbb{Q}_i} \frac{1}{m} \sum_{i=1}^m \int_{\Xi} [a_k^M \xi - b_k^M - \tau_k^M]_+ \mathbb{Q}_i(d\xi) \tag{33}$$

Then, based on the Lagrange function, we have (34):

$$\begin{aligned} & \sup_{\mathbb{Q}_i} \frac{1}{m} \sum_{i=1}^m \int_{\Xi} [a_k^M \xi - b_k^M - \tau_k^M]_+ \mathbb{Q}_i(d\xi) + \lambda_k^M \left(\rho - \frac{1}{m} \sum_{i=1}^m \int_{\Xi} \|\hat{\xi}_i - \xi\| \mathbb{Q}_i(d\xi) \right) \\ & = \lambda_k^M \rho + \frac{1}{m} \sum_{i=1}^m \sup_{\xi} \left(E([a_k^M \xi - b_k^M - \tau_k^M]_+) - \lambda_k^M \|\hat{\xi}_i - \xi\| \right) \end{aligned} \tag{34}$$

where $\lambda_k^M \geq 0$ is the Lagrange coefficient corresponding to corresponding to the constraint $\mathbb{W}(\mathbb{P}, \hat{\mathbb{P}}) \leq \rho$.

Then, by introducing auxiliary variables $s_{k,i}^M$, the dual problem of (33) can be expressed as (35, 36):

$$\lambda_k^M \rho + \inf_{s_{k,i}^M} \frac{1}{m} \sum_{i=1}^m s_{k,i}^M \tag{35}$$

$$\text{s.t.} \begin{cases} \sup_{\xi} \left(E([a_k^M \xi - b_k^M - \tau_k^M]_+) - \lambda_k^M \|\hat{\xi}_i - \xi\| \right) \leq s_{k,i}^M \\ \sup_{\xi} \left(0 - \lambda_k^M \|\hat{\xi}_i - \xi\| \right) \leq s_{k,i}^M \end{cases} \tag{36}$$

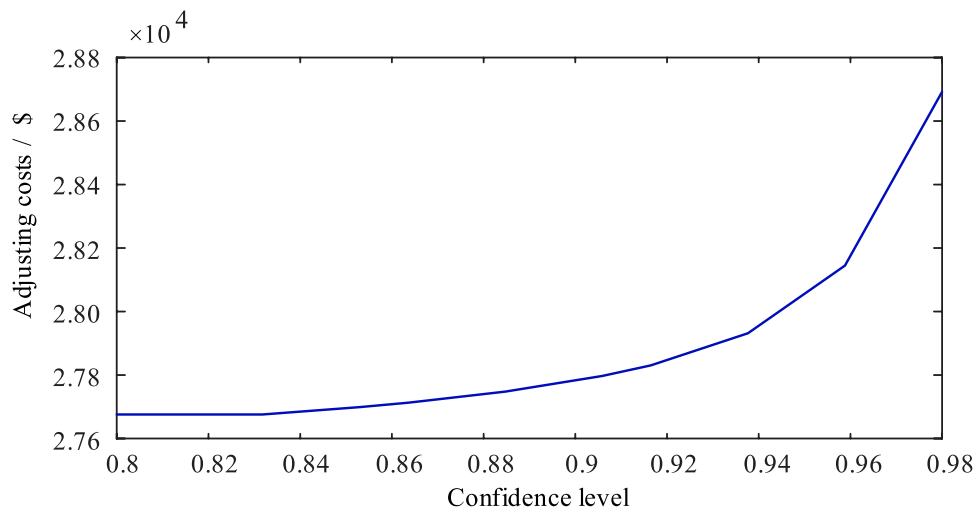


FIGURE 3 Adjustment cost curve with the confidence level.

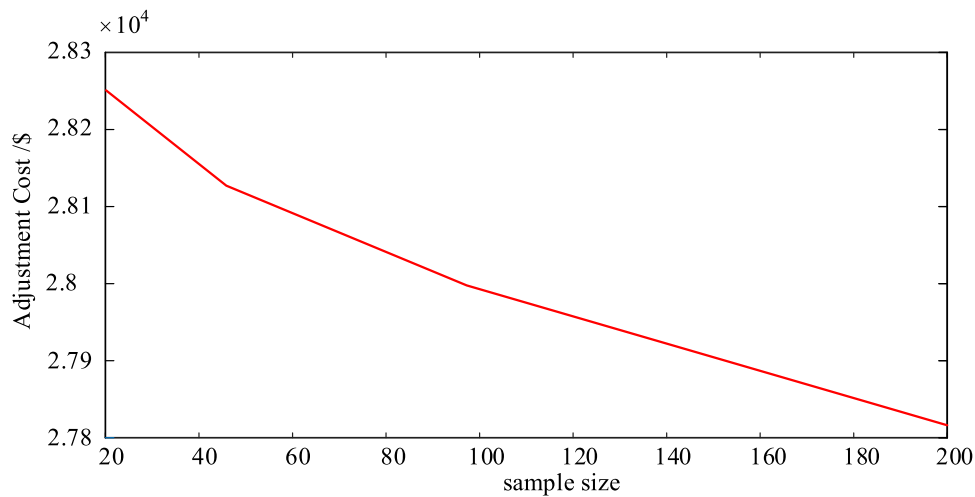


FIGURE 4 Adjustment cost changes with sample size curve.

Because $s_{k,i}^M \geq 0$, $-\sup_{\xi} (\lambda_k^M \|\hat{\xi}_i - \xi\|) \leq 0 \leq s_{k,i}^M$ should be established.

Let

$$l_k^M(\xi) = \sup_{P \in \mathbb{D}} E([a_k^M \xi - b_k^M - \tau_k^M]) \quad (37)$$

Introduce variables $z_{k,i}^M, \forall i \in \{1, 2, \dots, m\}$, where $\|z_{k,i}^M\|_{\infty} \leq \lambda_k^M$, and (35) can be equivalent to (38):

$$\begin{aligned} & \sup_{\xi \in \Xi} l_k^M(\xi) - \sup_{\|z_{k,i}^M\|_{\infty} \leq \lambda_k^M} (z_{k,i}^M)^T (\hat{\xi}_i - \xi) \\ &= -(z_{k,i}^M)^T \hat{\xi}_i - b_k^M - \tau_k^M + \sup_{Q_i} \int_{\Xi} (z_{k,i}^M + (a_k^M)^T)^T \xi Q_i(d\xi) \leq s_{k,i}^M \end{aligned} \quad (38)$$

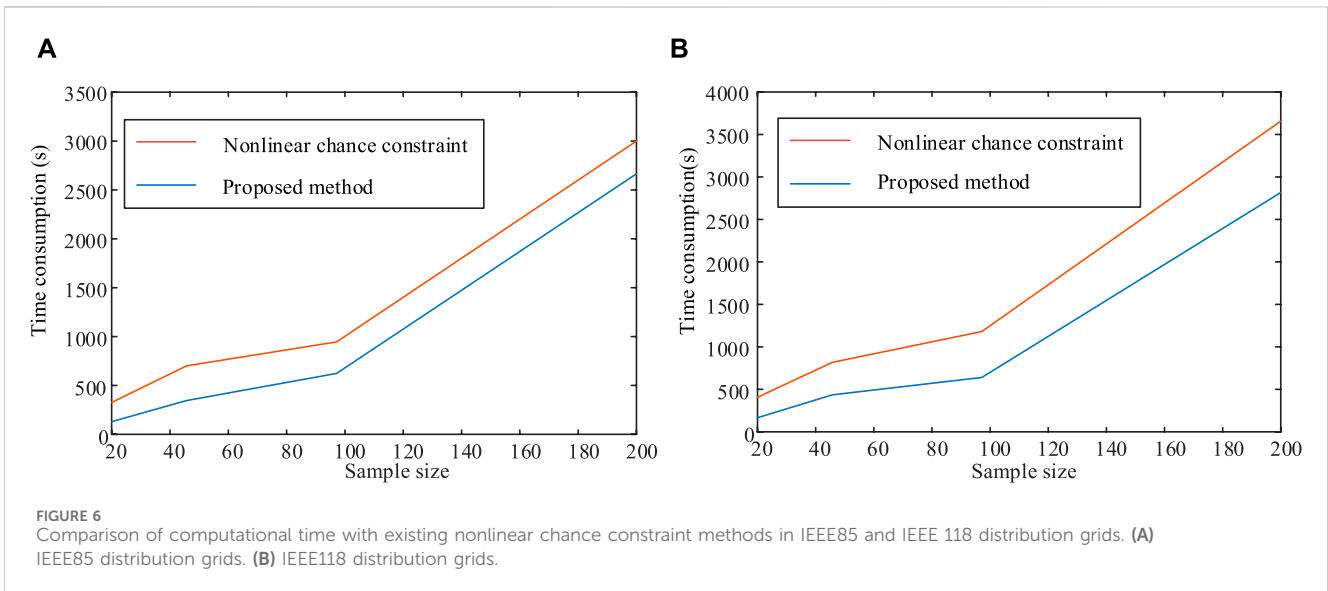
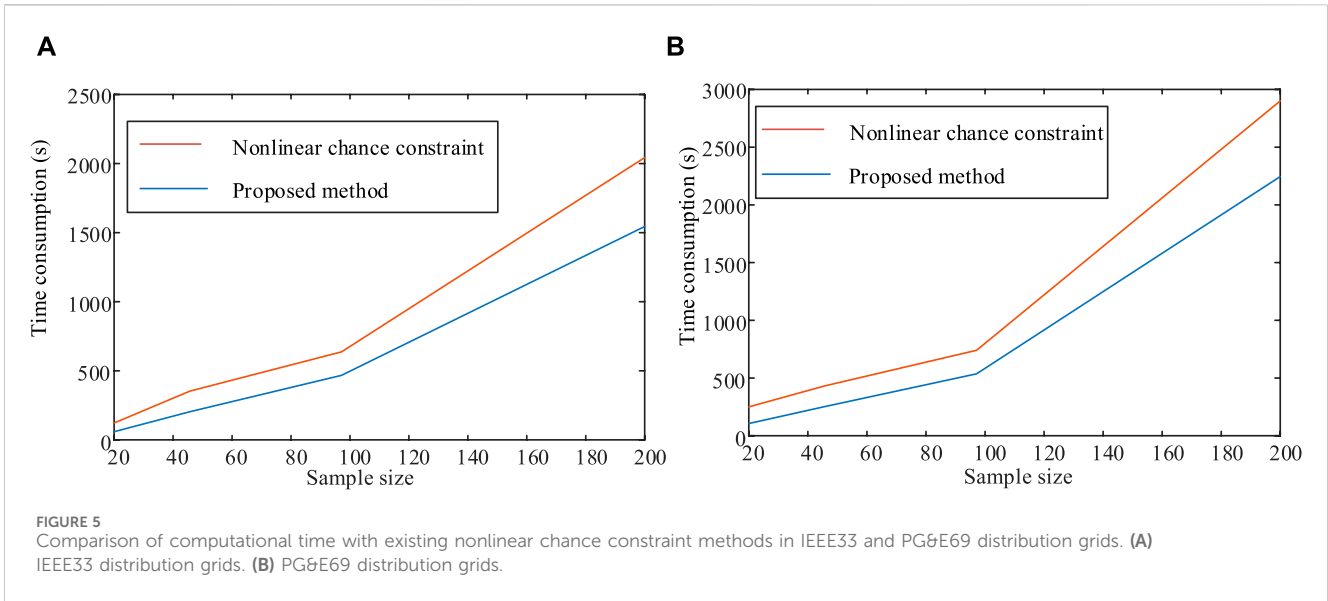
According to the duality theory, we have (39):

$$z_{k,i}^M = -(a_k^M)^T \quad (39)$$

The conditional risk value constraints can be depicted as (40):

$$\begin{cases} -(z_{k,i}^M)^T \hat{\xi}_i - b_k^M - \tau_k^M = a_k^M \hat{\xi}_i - b_k^M - \tau_k^M \leq s_{k,i}^M \\ \|z_{k,i}^M\|_{\infty} = \| -a_k^M \|_{\infty} \leq \lambda_k^M \\ \tau_k^M + \frac{1}{\varepsilon_V} \left(\lambda_k^M \rho + \frac{1}{m} \sum_{i=1}^m s_{k,i}^M \right) \leq 0 \end{cases} \quad (40)$$

In the same way, the node voltage lower limit chance constraint can be expressed as (41):



$$\begin{cases} a_k^m \hat{\xi}_i - b_k^m - \tau_k^m \leq s_{k,i}^m \\ \| -a_k^m \|_\infty \leq \lambda_k^m \\ \tau_k^m + \frac{1}{\varepsilon_V} \left(\lambda_k^m \rho + \frac{1}{m} \sum_{i=1}^m s_{k,i}^m \right) \leq 0 \end{cases} \quad (41)$$

where $a_k^m = \alpha_k^m (Y + I)$; $\alpha_k^m = [\alpha_{k,1}^m, \alpha_{k,2}^m, \dots, \alpha_{k,n}^m]$;
 $b_k^m = 1 - \sum_{j=1}^n (\alpha_{k,j}^m \bar{P}_j + \beta_{k,j}^m Q_j - \alpha_{k,j}^m P_j^L)$

In summary, the distributionally robust chance-constrained proposed in this paper is as (42, 43):

$$\min_{P_i, r^+, r^-, Y, \lambda, s_i, \gamma} \sum_{i=1}^n (c_i \bar{P}_i - c_i P_i^L + c_i^U P_i^U + c_i^D P_i^D) + \lambda \rho + \frac{1}{m} \sum_{i=1}^m s_i \quad (42)$$

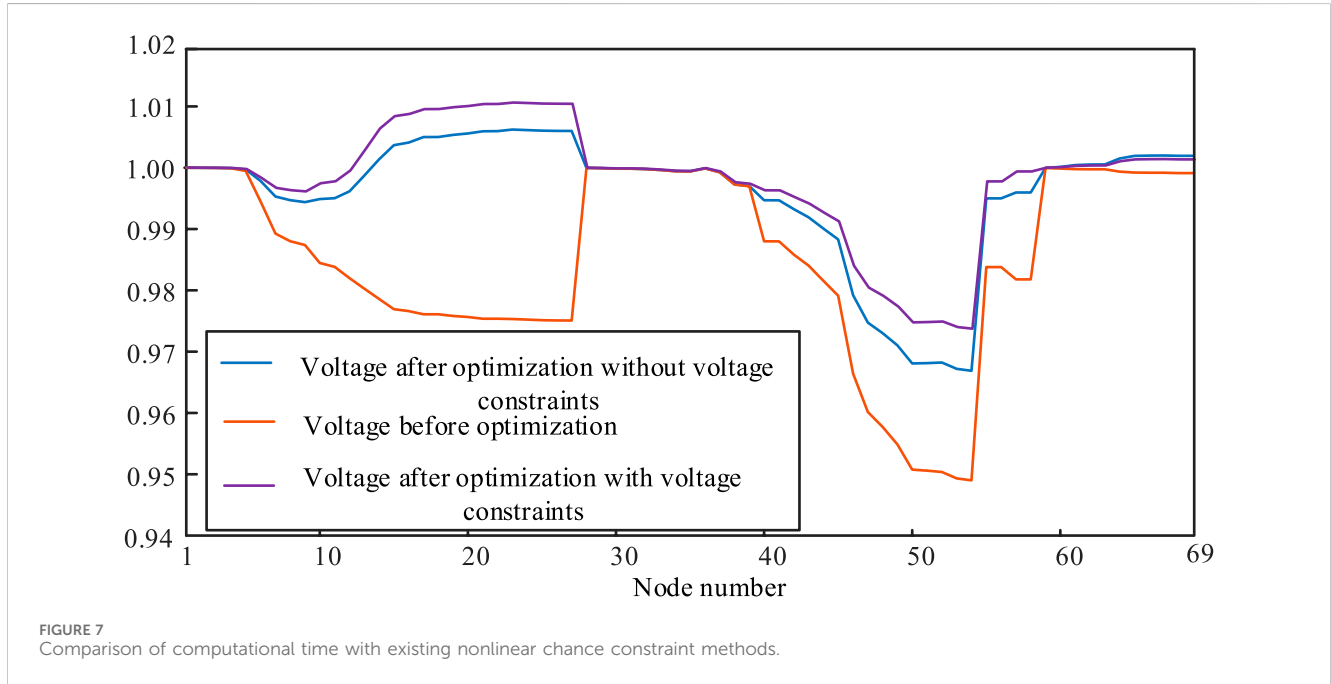
$$s.t. \begin{cases} c^T Y \hat{\xi}_i \leq s_i, \| -Y^T c \|_\infty \leq \lambda \\ a_k^M \hat{\xi}_i - b_k^M - \tau_k^M \leq s_{k,i}^M, \| -a_k^M \|_\infty \leq \lambda_k^M, \tau_k^M + \frac{1}{\varepsilon_V} \left(\lambda_k^M \rho + \frac{1}{m} \sum_{i=1}^m s_{k,i}^M \right) \leq 0 \\ a_k^m \hat{\xi}_i - b_k^m - \tau_k^m \leq s_{k,i}^m, \| -a_k^m \|_\infty \leq \lambda_k^m, \tau_k^m + \frac{1}{\varepsilon_V} \left(\lambda_k^m \rho + \frac{1}{m} \sum_{i=1}^m s_{k,i}^m \right) \leq 0 \end{cases} \quad (43)$$

In summary, it is not difficult to see that the operation model of the distribution grid proposed in this paper is linear and easy to solve. In addition, compared with the traditional method, the model proposed in this paper needs to deal with fewer variables, which can shorten the calculation time.

Let the error rate function as (44):

TABLE 2 Voltage deviation comparison.

	Maximum voltage deviation (%)	Correspondent node/\$
Before optimization	5.20	54
After optimization without voltage constraints	3.31	54
After optimization	2.80	54



$$\varepsilon = \frac{|F - F^{MC}|}{F^{MC}} \tag{44}$$

Where F^{MC} represents the result obtained by the Monte Carlo method, and F represents the result obtained by other methods. Besides, this paper defines the voltage deviation function as (45):

$$\Delta = \max \frac{|V_i - V_0|}{V_0} \times 100\%, i = 1, 2, 3, \dots, n \tag{45}$$

where $|\bullet|$ is the absolute value of \bullet .

5 Simulation results

This paper tests the modified PG&E69-bus distribution grids for simulations. The rated output power of the distributed generation device in the example is 150 kW. The power grid in southeastern Australia from 2012 to 2013 was used to construct the Wasserstein ambiguity set and distributed power generation output data. The rated active power of each active power regulating equipment is 333kW, 267kW, 300kW, 300kW, 333kW, 267kW, 300kW, 300kW, 333kW, 267kW, 300kW, 300kW, respectively. The specific grid structure is shown in Figure 1. In addition, the upper and lower limits of the voltage magnitude are 1.03 p.u. and 0.97 p.u. The sample capacity of the Wasserstein ambiguity set is 2000; the Wasserstein radius is 0.5×10^{-3} .

Compared with robust optimization and the Monte Carlo method, the comparison of the optimization results between the method proposed in this paper and the two is shown in Table 1:

Combined with the optimization results described in Table 1, we can see that the optimization results obtained by the proposed method have an error rate of only 1% compared to the Monte Carlo method, while the error rate of the robust optimization method is 13.1% compared to the Monte Carlo method. Therefore, the method proposed in this paper can fully meet practical engineering requirements in terms of calculation accuracy.

Moreover, compared with the robust and stochastic optimization methods, the adjustment cost of the optimization method proposed in this paper varies with the Wasserstein radius, as shown in Figure 2. It is observable from our analysis that the optimization outcomes yielded by our method gradually diverge from those of risk-neutral stochastic optimization and incline towards robust optimization outcomes as the Wasserstein radius increases. When leaning towards stochastic optimization, it indicates that the decision-maker believes that the probability distribution of uncertain variables in constructing the ambiguity set is accurate enough or there is sufficient historical data to describe the uncertainty of distributed generation, and the decision tends to be economic. Conversely, the shift towards robust optimization indicates the decision-

maker's anticipation of significant future risks, prioritizing decision reliability over economic concerns. In summary, the proposed method can reflect the subjective risk preferences of decision-makers by adjusting the Wasserstein radius parameter, thereby achieving a reasonable choice between scheduling economy and robustness. In other words, this method can realize the flexible choice of optimization decision in economy and robustness, but whether the optimization result tends to be robust or economical is determined by the risk preference of decision makers.

For the optimization case where the sample size is 2000, and the Wasserstein radius is 0.5×10^{-3} , the impact of different confidence levels on the adjustment cost is shown in Figure 3. We can see that the adjustment cost will also increase as the confidence level increases. This is in line with the general law. When the confidence level is improved, the tolerance of the grid to the voltage limit is reduced, so the output of each active regulation unit will be raised, which will further increase the cost. In general, with the increase of confidence level, it is necessary to sacrifice certain economy to meet the requirements.

In addition, when the confidence level is 95%, and the Wasserstein radius is 0.5×10^{-3} , changes in sample size will also lead to adjustment costs, as shown in Figure 4. We can see that the corresponding optimization cost will decrease as the sample size increases. This is because as the sample size increases, the probability information required by the Wasserstein fuzzy set is more comprehensive. Therefore, the Wasserstein fuzzy set describes the uncertainty of distributed generation output more accurately, reducing the decision-making conservatism. As reflected in the optimization results, the optimization cost will be reduced.

However, it should be noted that the selection of sample size is closely related to the calculation speed. Although the increase in sample size will reduce the optimization cost, the calculation efficiency will also decrease. Therefore, this paper gives the optimization time under different sample methods as shown in Figures 5, 6.

We can see that, compared to the nonlinear chance constraint form, the method proposed in this paper is more time-saving. The calculation time will increase as the number of nodes in the system increases, or the sample size increases. However, the method proposed in this paper takes less time than the existing nonlinear chance-constrained method. Especially when the number of nodes is large and the sample size is enormous, the method proposed in this paper is more time-saving and has more engineering application value.

The maximum voltage deviation before and after optimization and its correspondent node are shown in Table 2.

In addition, the voltage distribution of nodes before and after optimization is shown in Figure 7. Based on Figure 7 and Table 2, it is not difficult to see that the optimized voltage deviation of the proposed method in this paper is significantly smaller than in the other two cases. In addition, there are voltage violation situations before optimization and situations without considering voltage constraints. Still, there is no voltage violation after considering voltage constraints, which is sufficient to demonstrate the effectiveness of the proposed method in ensuring system voltage security.

6 Conclusion

This paper proposes a distributionally robust chance-constrained optimization dispatch model considering voltage security constraints. First, the comparison between robust optimization and stochastic optimization shows that the method proposed in this paper can reflect the subjective risk preference of distribution grid decision-makers by adjusting the Wasserstein radius parameter, thereby achieving a reasonable choice between dispatch economy and robustness. Second, compared to existing calculation methods, this paper uses the linear power flow method and conditional risk value to convert nonconvex chance constraints into linear constraints, reduce computational complexity, and shorten calculation time, especially for the system with multiple nodes and a high sample size. Finally, as mentioned in the example, the optimized voltage deviation is minor, and no voltage exceeds the limit, indicating the effectiveness of the method proposed in this article in ensuring the security of the distribution grids.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

CW: Conceptualization, Investigation, Methodology, Project administration, Writing–review and editing. JS: Conceptualization, Funding acquisition, Resources, Software, Writing–review and editing. XL: Conceptualization, Data curation, Methodology, Writing–review and editing. TY: Conceptualization, Formal Analysis, Visualization, Writing–review and editing. WL: Conceptualization, Supervision, Validation, Writing–original draft.

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Conflict of interest

Authors CW, JS, XL, and WL were employed by Electric Power Research Institute of State Grid Liaoning Electric Power Co., Ltd.

The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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