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EDITED BY
Harish Puppala,
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Vivek Tiwari,
Indian Institute of Technology Kanpur, India

*CORRESPONDENCE
Shreekant Varshney,
skvarshney91@gmail.com

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Optimal management strategies of renewable energy systems with hyperexponential service provisioning: an economic investigation

Shreekant Varshney[1]*, Chandra Shekhar[2], A. V. Dhanunjaya Reddy[3], K. S. Pritam[1], M. V. V. Prasad Kantipudi[4], Hossam Kotb[5], Kareem AboRas[5] and Mohammed Alqarni[6]

[1]Department of Mathematics, Pandit Deendayal Energy University, Gandhinagar, India, [2]Department of Mathematics, Birla Institute of Technology and Science, Pilani, Rajasthan, India, [3]Department of Mechanical Engineering, Srinivasa Ramanujan Institute of Technology, Andhra Pradesh, India, [4]Department of Electronics and Tele-Communication Engineering, Symbiosis Institute of Technology, Symbiosis International University, Pune, Maharashtra, India, [5]Department of Electrical Power and Machines, Faculty of Engineering, Alexandria University, Alexandria, Egypt, [6]Electrical Engineering Department, University of Business and Technology, Jeddah, Saudi Arabia

The current research proposes optimal management strategies for queueing modeling-based renewable energy systems with hyper-exponentially distributed maintenance/repair under the assumption of an admission control policy. Using the concept and steps of the matrix-analytical method, the steady-state probability distribution associated with energy systems is explicitly presented. A relatively straightforward computation that can help with modeling wind energy generation, investigating wind farm performance, optimizing energy based on system storage, reliability inspection, service maintenance planning, and numerous other purposes can be employed to mathematically derive several system performance indicators. The investigation findings are validated via quantitative outcomes, illustrative possesses, and a step-by-step recursive methodology for efficient management of the renewable energy system. Additionally, considering multiple governing parameter values, the nature-inspired optimization technique, Cuckoo Search (CS), is employed to demonstrate the optimum anticipated cost of renewable energy system. A comparison with other metaheuristics and semi-classical approaches is also presented to establish the best convergence results. In order to help system designers, policymakers, engineers, and researchers, several numerical examples are also provided to construct more practical strategies based on the production of energy, storage, and system management. The economic, parametric, and performance investigation findings are highlighted, and the opportunities and recommendations for further research are provided. In a nutshell, the outcomes of the present analysis can be adopted to formulate the most effective economic strategies and regulate decision-making processes in the energy sectors.

KEYWORDS

renewable energy systems, admission control policy, hyperexponential distribution, cost optimization, cuckoo search algorithm, particle swarm optimization, Quasi-Newton method

1 Introduction

The management and controlling the inflow of energy from renewable sources is a process used by system designers, engineers, and policymakers to enhance better service strategies and provide a high grade of service to individuals based on energy demand fluctuations in the energy systems. This type of queueing scenario specifies the demand management of energy or energy storage and buffering in any industry or organization. Energy generation assets can be available for the better service in real-time scenario as per service distribution and can be changed as per requirement. In this era of modernization, management and control of energy flow from renewable sources in the energy systems becomes a primary concern. Therefore, the optimal management strategies to control energy dispatch and distribution in energy systems is a challenge for decision/policymakers, scholars, and researchers from an economic point of view. In the queuing literature, it has been observed that controllable queuing systems, in general, have no buffer for long-term waiting demands. Therefore, these systems are also known as a loss system. Stidham Jr and Weber (1993) provided a survey based on optimal control of the broad network of queues and emphasized the models based on Markov decision theory. Furthermore, over the recent past, many studies related to optimal management and control of queues have been provided that can help researchers and investigators to enumerate various system parameters for better service strategies (*cf.* Aghighi et al. (2021); Jain and Sanga (2020); Wu and Yang (2021); Yang (2022)).

In the congestion situations that occur in our day-to-day life and technical/managerial circumstances, it has been found that phase-type services within queueing-based systems are essential in outlining and examining Markovian/Non-Markovian queueing problems. In conventional queueing problems, each newly joined individual requires the primary service to be accomplished in a single phase. Though, in many realistic circumstances, it has been examined that service provisioning may be performed in many different stages. The concept of phase-type service was first proposed by Jackson (1954) utilizing the queueing-theoretic approach in literature. Generally, the mean time of inclusion of a continuous stochastic process for a single system state can be characterized by the distribution that follows a phase-type service (*cf.* Latouche and Ramaswami (1999)). The concept of phase-type service patterns has been extensively used in various models involving stochastic applications in diverse fields, namely, telecommunications, finance and portfolio management, reliability engineering, queuing modeling, biotechnology and biostatistics, supply chain, inventory control, etc. The phase-type service discipline has gained considerable popularity among researchers and systems analysts in the queuing literature because it constitutes a uniform and versatile class of distributions described by nonnegative real numbers that confer dominance over algorithmically controllable queuing models. In the past, several illustrations have been performed based on numerous queueing nomenclatures along with phase-type service patterns (*cf.* Dudin and Dudin (2016); Dudin et al. (2016); Dudin and Dudina (2019)).

In recent times, the growing interest in maximizing energy utility and productivity from renewable sources, energy distribution and storage has contributed significantly to research on queuing-based renewable energy systems (*cf.* Kocaman et al. (2016); Talari et al. (2018); Noorollahi et al. (2020); Samain et al. (2021); Baik and Ko (2023); Momenitabar et al. (2023); Nagababu et al. (2023); Patel et al. (2023)). To the best of our collective understanding, no literature-based investigation has yet been presented on the economic and performance evaluations of renewable energy systems with admission control and hyper-exponentially distributed maintenance/repair. Our motivation for the current study stems from the literature's lack of qualitative research opportunities. Besides, inspired by the adaptive capabilities of the nature-inspired CS and PSO algorithms, optimal system design parameters and the total anticipated cost associated with the renewable system have been calculated utilizing these evolutionary algorithms. The present research aims to assist practitioners, system analysts, and policymakers in constructing and analyzing the queueing-theoretic stochastic modeling of renewable energy systems. The novelty of the current research is to achieve the proper characterization based on convergence results and statistical inference incorporating different evolutionary algorithms. In algorithms. In addition to the parametric analysis, a comparative investigation among CS, PSO algorithms, QN, and DS methods is also provided to verify the optimum combinations of design variables and the superiority of the metaheuristics. The primary contribution of the present investigation is the execution of the MATLAB codes along with associated algorithms for an empirical comparison of the research findings of the CS algorithm, PSO algorithm, QN method, and DS method with respect to computational time, statistical parameters, and standard operating procedures, etc.

In order to propose the best energy management approaches, the primary objectives of the present investigation are to analyze and estimate the system performance indicators to demonstrate the efficiency, utility, and quality performance of the energy systems. The entire research can be categorized into categories as follows.

(i) To categorize a novel and challenging stochastic scenario based on congestion situations that corresponds to a practical renewable energy system in a random environment;
(ii) To establish an adequate mathematical approach to interpret the numerical results according to equilibrium conditions;
(iii) Demonstration, analysis, and classification of numerous energy systems' efficiency metrics depending on fluctuations in energy consumption, transmission of energy, cost-effective generation, and strategies used for controlling and managing the energy generated by renewable energy sources;
(iv) To identify the essential energy system characteristics required for maintaining the service system's higher efficiency;
(v) Utilization of evolutionary algorithms, such as PSO and CS algorithms, for the economic investigation of constructed queueing-based renewable energy system;
(vi) To validate the convergence results produced by the cost optimization problem by comparing the PSO, CS, QN, and DS algorithms.

The current study addresses a variety of stochastic problems pertaining to queue-based energy systems that originated and evolved in accordance with numerous industrial, managerial, and economic problems. The following points highlight how the research findings analyzed for the developed model in the present investigation differ from the outcomes of previous research.

(i) To demonstrate the equilibrium queueing distribution employing the matrix-analytical approach for the energy system having finite buffer storage with admission control policy and hyper-exponential service pattern;
(ii) To establish a specific optimization problem and demonstrate which optimization approaches are most effective in minimizing the energy system's overall anticipated expenditure;
(iii) To construct the converging results in terms of optimum cumulative values of discrete and continuous system design parameters F, μ_1, and μ_2, simultaneously by implementing semi-classical optimizers: QN and DS methods and nature-inspired algorithms: CS and PSO algorithms.

2 Literature review

Examining the subsequent research investigations drawn from the queueing literature could suggest essential perspectives on how stochastic/Markovian modeling can be utilized in the context of renewable energy systems. A more comprehensive understanding of the theoretical foundations and efficient strategies for enhancing the reliability and effectiveness of renewable energy systems may be acquired through the investigation.

2.1 Control policies

The quality-of-service (QoS) and efficiency of service systems can be characterized in terms of the variable number of active units, the number of servers available/service rate of the service provider, throughput, total anticipated cost of the service system, etc. In our daily life, there is a requirement to control the queues as it causes resource consumption, high operational costs, additional energy expenditures, and contrary environmental impacts of these consumptions. Thus, analysis of queue discipline, queue-size distribution, and control policies to minimize the waiting period can lead to more efficient service systems with positive significance in reaching viable development goals. The control of queues in service systems can be classified in two ways, first includes the arriving control policy (F-policy), introduced by Gupta (1995). The other are service control policies, namely, the T-policy, N-policy, and D-policy, which are proposed by Heyman (1977), Yadin and Naor (1963), and Balachandran (1973), respectively. Apart from that, several research problems can be found in the queueing literature on the controllable queues (*cf.* Chang and Ke (2011); Efrosinin D. V. et al. (2018); Shekhar et al. (2017); Yang and Wang (2013); Yang et al. (2010; 2011); Yeh et al. (2017)), which put the controlling policy in the context of Markovian/Non-Markovian queueing system. Recently, Chen (2018) studied the finite population machining system and performed the sensitivity investigation for reliability and mean-time-to-first-failure (MTTFF). Efrosinin D. et al. (2018) studied the queueing situation incorporating the Markov arrival process (MAP) and analyzed the deterministic control strategies for the distribution of arrived customers among the available servers. Again, Efrosinin and Sztrik (2018) extended the work done in Efrosinin D. et al. (2018) for the unreliable heterogeneous servers and provided the algorithms for finding the MTTF, reliability function, and stationary reliability characteristics for the machining repair system. To stabilize a partially-controllable network, Liang and Modiano (2019) designed the optimal control algorithms. They investigated the system states where the controllable and uncontrollable nodes use the dynamics of queue-dependent service systems. Further, Shekhar et al. (2020b) investigated a controllable queue with vacation interruption of the service provider. They applied the matrix-analytical approach to determine the closed-vector form expressions of several system performance indicators. In the last 4 years, many scholars and researchers, including Yen et al. (2020), Yang et al. (2021), Safaei et al. (2022), and Wu et al. (2023), used different solution methodologies and optimization algorithms to study the admission control policy under different hypotheses via queueing-theoretic approach.

2.2 Service in phases

Numerous mathematicians and research analysts have studied optional phase-type services to improve queue-based service systems' workability, utility, and service quality (*cf.* Flatto (1985); Flatto and Hahn (1984); Sharma (2014); Wang et al. (1999; 2004)). Shang and Wolter (2016) addressed the impact of the probability of messages in terms of packets on the execution of open-flow networks using the queueing-theoretic approach. They provided the expressions based on the sojourn time distribution for switches and controllers. Tarasov and Bakhareva (2018) analyzed three different queueing models based on service in phases. They proposed an approach to calculate the expected values and variances of the time intervals using the general assumptions of the probability distribution between the system states. In recent years, Tarasov (2022), Khayyati and Tan (2022), and Kumar and Jain (2023) examined the queueing-based service systems and discussed different methods for the numerical approximation of queue-size distribution under diverse service regimes.

Queueing systems following Earlangian distributed service patterns have also been considered in different frameworks recently. One can easily extend the single server Markovian queueing models to the Earlang models utilizing a series of exponential and identical service phases using several theoretical aspects and phenomenons of queueing modeling. The Erlang-type service distribution produces much better flexibility in modeling real-time service patterns that does the exponential. Further studies on this topic were done by many researchers (*cf.* Baek et al. (2014); Griffiths et al. (2006); Yu et al. (2011); Yue et al. (2009); Zhang et al. (2016)). Besides, Liu and Fralix (2019) employed the lattice path counting techniques to examine the time-dependent and equilibrium behavior of continuous-time Markov chains (CTMC) and Markovian queueing systems. More recently, Pandey and Gangeshwer (2020) utilized the probability-generating function approach to derive the steady-state

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queue-size distribution by employing the ambulance service as a server.

2.3 Evolutionary algorithms

Due to the complex stochastic problems, the conventional linear and nonlinear optimization techniques, namely, LPP, and gradient-based algorithms, are usually insufficient to demonstrate solutions to such problems efficiently. Evolutionary algorithms such as heuristics and metaheuristics are among the best solution techniques used in various computational, engineering, and industrial problems because they provide efficient solutions to complex stochastic problems in reasonable time intervals. Most of these are motivated by wild animals' physical and biological behaviors and use Darwin's theory of sustainability of the fittest. For instance, taking inspiration from birds flocking and fish schooling, Kennedy and Eberhart (1995) introduced the particle swarm optimization (PSO) algorithm. In contrast, the clever algorithm, namely, the bees optimization algorithm, was established by taking motivation from the social grouping behavior of honey bees (*cf.* Pham et al. (2006)). New optimization algorithms, including cuckoo search (*cf.* Bulatović et al. (2013)) and an algorithm having flashing behavior of fireflies (*cf.* Yang (2009)) have also been recently evolved. During the past decade, the aforementioned techniques have been extensively implemented to solve several decision-making situations in supply-chain management, inventory problems, production systems, scheduling problems, etc (*cf.* Khajehzadeh et al. (2013); Kumar et al. (2022); Shekhar et al. (2021); Subbaiah and Kannayaram (2021); Zhang et al. (2011)).

Yang and Deb (2009) developed the cuckoo search (CS) algorithm in 2009 by motivating the adaptation to parasitism of cuckoos. Cuckoos lay their eggs in other species' communal nests for reproduction. In fact, for the CS algorithm to execute, host eggs in a host nest must always be distinguished from cuckoo eggs. After spotting cuckoo eggs, birds have two options: either abandon the eggs or permanently abandon the nest and construct a new one. Certain cuckoos have developed a specialization in imitating host bird eggs, which lowers their risk of being recognized and leaving the nest and boosts their chances of reproducing. CS performs significantly better than PSO and other algorithms. In addition, because of its assured global convergence quality, it is appropriate for multi-modal/multiple constraint-based optimization problems, such as queueing aspects, inventory management, mechanical design, computer graphics and computer vision, and many more. Moreover, the CS algorithm requires minimal algorithmic parameters than the PSO algorithm and several other population-based optimization methods. Because of its enhanced efficiency, the CS algorithm is best suited for a wide range of real-time optimization problems. Several computer scientists, engineers, decision-makers, and academicians have recently used various nature-inspired algorithms for multi-objective complex optimization problems. They emphasized various selection strategies of these algorithms (*cf.* Baskar (2023); Dwivedi et al. (2022); Gupta et al. (2023); Han et al. (2023a; b); Li et al. (2023a; b,c); Liu et al. (2022); Mareli and Twala (2018); Mitra and Acharyya (2022); Rani et al. (2023); Sahu et al. (2023); Shehab et al. (2017); Zhang et al. (2019)).

3 Model description

3.1 Practical justification of the model

In the context of the renewable and wind energy sectors, the proposed queueing framework can be viewed as a special type of queueing model incorporating a hyperexponential service time distribution with a Markovian arrival procedure and a single server. It can be implemented significantly in many scenarios, including modeling wind energy generation, performance investigation of wind farms, storage systems for energy optimization, reliability inspection, service planning, and maintenance planning. The designed queueing framework can be used to establish an efficient forecasting system for estimating wind energy production. This mathematical model might help to predict and forecast the outcomes of wind energy infrastructure, enabling enhanced management and planning of energy generation. It empowers the scenario by incorporating the variations of wind speeds and their impact on the generation of energy as a Markovian process. Additionally, by combining the variations in wind speed and the resulting energy production, this model can help to determine the effectiveness of wind farms. The suggested queueing model can provide perspectives on the efficient operation of energy conversion procedures within wind farms by analyzing the arrival and service times of wind energy production, however, enabling it to be easier to recognize feasible challenges and opportunities for optimization.

The proposed queueing model based on a hyperexponential service pattern can be used to enhance the performance of energy storage systems concerning the intermittent nature of wind energy. This model can help to identify the ideal dimensions, location, and functioning of energy storage units within the renewable energy framework. Thus, enhancing energy efficiency and the grid's stability, It accomplishes this situation by considering the stochastic nature of energy inflows and outflows. Further, the developed queueing model may aid with the development of optimal management strategies for planning services and maintenance scheduling for wind energy installations. This approach can help optimize the scheduling of maintenance operations/activities, minimize downtime, and maximize the overall operational efficiency of wind energy infrastructure by examining the arrival and service times of maintenance requests and considering the stochastic nature of the system breakdowns. The wind and renewable energy industries can discover considerable information concerning system performance, reliability, and operational effectiveness via the proposed queueing model. It will help researchers, system designers, and decision-makers to develop more effective approaches based on energy production, storage, and controlling energy systems.

3.2 Assumptions and notations

In this section, strategies established to monitor and regulate the proportion of energy generated by renewable sources with hyper exponentially distributed repair of energy infrastructure are discussed in detail. The concept of the consumer's arrival and customer service procedure in the queueing modeling can be associated with numerous factors related to the energy generation and distribution processes, namely, energy demand fluctuations, load balancing, grid integration, maintenance, and repairs, et cetera in context with the wind and renewable energy fields. Inspired by these actual situations, the following fundamental assumptions for the queueing modeling have been presented for analysis purposes.

Arrival Pattern.

- Customers enter the system and proceed in accordance with an independent Poisson process using λ as a parameter. If the server is already occupied, the subsequent arriving customer joins the waiting queue in the system; otherwise, he receives service immediately.
- Depending on the access in the system, the newly arrived customers arrange themselves into one particular waiting line.
- The system has two different categories of customers: type-1 and type-2. The likelihood that a new type-1 (or type-2) customer will enter the system is p (or $(1-p)$).
- The system prohibits any customer from joining the system until the size of the queue surpasses a prefix threshold value F after the system's capacity K is reached.

Service Pattern.

- For type 1 and type 2 customers, the service times follow an exponential distribution with corresponding parameters, μ_1 and μ_2, respectively.
- When customers are permitted to rejoin the system to begin the service, the service provider needs an exponentially distributed startup time with rate parameter γ.

All processes and occurrences happened repeatedly and independently of one another.In order to model the system states of the developed model at time t, the states are described as

$$N(t) = \text{Number of customers in the system at time } t$$

$$J(t) = \begin{cases} 0, & \text{Server is idle at time instant } t \\ 1, & \text{Server is busy and in} - \text{service customer is of type 1 at time instant } t \\ 2, & \text{Server is busy and in} - \text{service customer is of type 2 at time instant } t \end{cases}$$

$$Y(t) = \begin{cases} 0, & \text{the customers are allowed to enter in the system at time instant } t \\ 1, & \text{the customers are not allowed to enter in the system at time instant } t \end{cases}$$

Then, $X(t) = \{(N(t), J(t), Y(t));\ t \geq 0\}$ represents the continuous stochastic process with the state space

$$\Theta \equiv \{(0,0,l);\quad l = 0,1\} \cup \{(n,j,0);\ n = 1,2,\ldots,K-1,\ \&\, j = 1,2\} \cup \{(n,j,1);\ n = 1,2,\ldots,K\ \&\, j = 1,2\}$$

It signifies the irreducibility of the Markov chain $\{X(t);\ t \geq 0\}$. Due to the finite nature of the state space Θ, the Markov chain $\{X(t);\ t \geq 0\}$ is also positively recurrent. Furthermore, The state probabilities at the instant of time t are represented by the following annotations.

$P_{0,0,0}(t) \equiv$ Probability that service provider is in idlestate and that no customer is initially present inthe system.

$P_{n,1,0}(t) \equiv$ Probability that service provider is in busy state and new customers are allowed and the in – service customer is of type – 1.

$P_{n,2,0}(t) \equiv$ Probability that service provider is in busy state and new customers are allowed and the in – service customer is of type – 2.

$P_{n,1,1}(t) \equiv$ Probability that service provider is in busy state and new customers are not allowed and the in – service customer is of type – 1.

$P_{n,2,1}(t) \equiv$ Probability that service provider is in busy state and new customers are not allowed and the in – service customer is of type – 2.

Now, employing the appropriate birth-death axioms and preventing the transitions from inflow to outflow in equilibrium, the governing set of Kolmogorov differential equations is constructed to demonstrate the state probabilities linked with different system states as follows.

(i) Customers are permitted to join the system during idle state of the server.

$$\frac{dP_{0,0,0}(t)}{dt} = -\lambda P_{0,0,0} + \mu_1 P_{1,1,0} + \mu_2 P_{1,2,0} + \gamma P_{0,0,1} \tag{1}$$

(ii) Customers can join the service system while the service provider is active and the in-service customer is a type 1 customer.

$$\frac{dP_{1,1,0}(t)}{dt} = -(\lambda+\mu_1)P_{1,1,0} + \lambda p P_{0,0,0} + p\mu_1 P_{2,1,0} + p\mu_2 P_{2,2,0} + \gamma P_{1,1,1} \tag{2}$$

$$\frac{dP_{n,1,0}(t)}{dt} = -(\lambda+\mu_1)P_{n,1,0} + \lambda P_{n-1,1,0} + p\mu_1 P_{n+1,1,0} + p\mu_2 P_{n+1,2,0} + \gamma P_{n,1,1}; \qquad 2 \leq n \leq F \tag{3}$$

$$\frac{dP_{n,1,0}(t)}{dt} = -(\lambda+\mu_1)P_{n,1,0} + \lambda P_{n-1,1,0} + p\mu_1 P_{n+1,1,0} + p\mu_2 P_{n+1,2,0}; \qquad F+1 \leq n \leq K-2 \tag{4}$$

$$\frac{dP_{K-1,1,0}(t)}{dt} = -(\lambda+\mu_1)P_{K-1,1,0} + \lambda P_{K-2,1,0} \tag{5}$$

(iii) Customers can join the service system while the service provider is active and the in-service customer is a type 2 customer.

$$\frac{dP_{1,2,0}(t)}{dt} = -(\lambda+\mu_2)P_{1,2,0} + \lambda(1-p)P_{0,0,0} + (1-p)\mu_2 P_{2,2,0} + (1-p)\mu_1 P_{2,1,0} + \gamma P_{1,2,1} \tag{6}$$

$$\frac{dP_{n,2,0}(t)}{dt} = -(\lambda+\mu_2)P_{n,2,0} + \lambda P_{n-1,2,0} + (1-p)\mu_2 P_{n+1,2,0} + (1-p)\mu_1 P_{n+1,1,0} + \gamma P_{n,2,1};\ 2 \le n \le F \tag{7}$$

$$\frac{dP_{n,2,0}(t)}{dt} = -(\lambda+\mu_2)P_{n,2,0} + \lambda P_{n-1,2,0} + (1-p)\mu_2 P_{n+1,2,0} + (1-p)\mu_1 P_{n+1,1,0};\qquad F+1 \le n \le K-2 \tag{8}$$

$$\frac{dP_{K-1,2,0}(t)}{dt} = -(\lambda+\mu_2)P_{K-1,2,0} + \lambda P_{K-2,2,0} \tag{9}$$

(iv) Customers can not enter in the system during idle state of the server.

$$\frac{dP_{0,0,1}(t)}{dt} = -\gamma P_{0,0,1} + \mu_1 P_{1,1,1} + \mu_2 P_{1,2,1} \tag{10}$$

(v) Customers cannot join the system during servers' busy period; however, the in-service customer is of type 1.

$$\frac{dP_{1,1,1}(t)}{dt} = -(\mu_1+\gamma)P_{1,1,1} + p\mu_1 P_{2,1,1} + p\mu_2 P_{2,2,1} \tag{11}$$

$$\frac{dP_{n,1,1}(t)}{dt} = -(\mu_1+\gamma)P_{n,1,1} + p\mu_1 P_{n+1,1,1} + p\mu_2 P_{n+1,2,1};\ 2 \le n \le F \tag{12}$$

$$\frac{dP_{n,1,1}(t)}{dt} = -\mu_1 P_{n,1,1} + p\mu_1 P_{n+1,1,1} + p\mu_2 P_{n+1,2,1};\ F+1 \le n \le K-1 \tag{13}$$

$$\frac{dP_{K,1,1}(t)}{dt} = -\mu_1 P_{K,1,1} + \lambda P_{K-1,1,0} \tag{14}$$

(vi) Customers cannot join the system during servers' busy period; however, the in-service customer is of type 2.

$$\frac{dP_{1,2,1}(t)}{dt} = -(\mu_2+\gamma)P_{1,2,1} + (1-p)\mu_2 P_{2,2,1} + (1-p)\mu_1 P_{2,1,1} \tag{15}$$

$$\frac{dP_{n,2,1}(t)}{dt} = -(\mu_2+\gamma)P_{n,2,1} + (1-p)\mu_2 P_{n+1,2,1} + (1-p)\mu_1 P_{n+1,1,1};\ 2 \le n \le F \tag{16}$$

$$\frac{dP_{n,2,1}(t)}{dt} = -\mu_2 P_{n,2,1} + (1-p)\mu_2 P_{n+1,2,1} + (1-p)\mu_1 P_{n+1,1,1};\ F+1 \le n \le K-1 \tag{17}$$

$$\frac{dP_{K,2,1}(t)}{dt} - \mu_2 P_{K,2,1} + \lambda P_{K-1,2,0} \tag{18}$$

The state probabilities for the steady-state characterization in equilibrium (*i.e.*, as $t \to \infty$) are shown as follows

$$P_{0,0,0} = \lim_{t\to\infty} P_{0,0,0}(t),\ P_{0,0,1} = \lim_{t\to\infty} P_{0,0,1}(t),$$
$$P_{K,j,1} = \lim_{t\to\infty} P_{K,j,1}(t);\quad j = 1,2\ \&\quad l = 0,1$$

and

$$P_{n,j,l} = \lim_{t\to\infty} P_{n,j,l}(t);\quad n = 1,\ldots,K-1,\quad j = 1,2\ \&\quad l = 0,1$$

4 Model formulation

4.1 Matrix analytic method

In general, it is not straightforward to derive the equlibrium state probabilities from the governing differential equations. The reason is the existence of multiple variables, multiple equations, and numerous parameters in stochastic environments. Many researchers followed Neuts (1981) matrix solution technique to tackle such types of complex engineering and queueing situations. Therefore, the transition block matrix **Q** is represented in the subsequent manner to figure out the flow balance differential-difference equations for the probability distribution.

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{B}_0 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_1 & \mathbf{A}_1 & \mathbf{B}_1 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 & \mathbf{A}_1 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_1 & \mathbf{B}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_2 & \mathbf{A}_1 & \mathbf{B}_1 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_2 & \mathbf{A}_2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_2 & \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_2 & \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_3 & \mathbf{A}_3 \end{bmatrix}$$

where, the block-diagonal matrices $\mathbf{A}_0$ and $\mathbf{A}_3$ are the square matrices of order 2, while $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{C}_2$ are the square matrices of order 4. Similarly, block matrices $\mathbf{B}_0$ and $\mathbf{C}_3$ are the rectangular matrices of dimension (2×4) and $\mathbf{B}_2$ and $\mathbf{C}_1$ have the dimension (4×2), respectively. The block sub-matrices for the environmental process are $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$. In contrast, the super and sub-diagonal matrices for the Markov process are $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ having elements λ, μ_1, μ_2 and p, respectively. The following are the structures of each block sub-matrix of the rate matrix **Q**.

$$\mathbf{A}_0 = \begin{bmatrix} -\gamma & \gamma \\ 0 & -\lambda \end{bmatrix};\quad \mathbf{B}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & p\lambda & (1-p)\lambda \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} -(\gamma+\mu_2) & 0 & 0 & \gamma \\ 0 & -(\gamma+\mu_1) & \gamma & 0 \\ 0 & 0 & -(\lambda+\mu_1) & 0 \\ 0 & 0 & 0 & -(\lambda+\mu_2) \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} -\mu_2 & 0 & 0 & 0 \\ 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & -(\lambda+\mu_1) & 0 \\ 0 & 0 & 0 & -(\lambda+\mu_2) \end{bmatrix};$$

$$\mathbf{A}_3 = \begin{bmatrix} -\mu_2 & 0 \\ 0 & -\mu_1 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}; \quad \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \lambda \\ \lambda & 0 \end{bmatrix}$$

$$\mathbf{C}_1 = \begin{bmatrix} \mu_2 & 0 \\ \mu_1 & 0 \\ 0 & \mu_1 \\ 0 & \mu_2 \end{bmatrix}; \quad \mathbf{C}_2 = \begin{bmatrix} (1-p)\mu_2 & p\mu_2 & 0 & 0 \\ (1-p)\mu_1 & p\mu_1 & 0 & 0 \\ 0 & 0 & p\mu_1 & (1-p)\mu_1 \\ 0 & 0 & p\mu_2 & (1-p)\mu_2 \end{bmatrix}$$

$$\mathbf{C}_3 = \begin{bmatrix} (1-p)\mu_2 & p\mu_2 & 0 & 0 \\ (1-p)\mu_1 & p\mu_1 & 0 & 0 \end{bmatrix}$$

With the transition rate matrix $\mathbf{Q}$ partitioned in $\mathbf{\Pi}_0, \mathbf{\Pi}_1, \mathbf{\Pi}_2, \ldots, \mathbf{\Pi}_{K-1}, \mathbf{\Pi}_K$, such that $\mathbf{\Pi}_0 = [P_{0,0,0}, P_{0,0,1}]$ and $\mathbf{\Pi}_K = [P_{K,1,1}, P_{K,2,1}]$ are the sub-vectors of dimension 1×2 and.$\mathbf{\Pi}_n = [P_{n,1,0}, P_{n,2,0}, P_{n,1,1}, P_{n,2,1}]$; $n = 1, 2, \ldots, K-1$ are of dimension (1×4), let. $\mathbf{\Pi} = [\mathbf{\Pi}_0, \mathbf{\Pi}_1, \mathbf{\Pi}_2, \ldots, \mathbf{\Pi}_{K-1}, \mathbf{\Pi}_K]$ signifies the associated probability vector in equilibrium. The homogeneous system of linear equations $\mathbf{\Pi Q} = \mathbf{0}$ is now analyzed using the recursive algorithm and the normalizing condition $\mathbf{\Pi e}_1 = 1$. Further, $\mathbf{e}_1$ is the column vector of order 4 in this instance, and it has a single element as 1. The detailed iterative procedure of the recursive methodology is outlined in the subsequent procedure.

$$\mathbf{\Pi}_0\mathbf{A}_0 + \mathbf{\Pi}_1\mathbf{C}_1 = \mathbf{0} \tag{19}$$
$$\mathbf{\Pi}_0\mathbf{B}_0 + \mathbf{\Pi}_1\mathbf{A}_1 + \mathbf{\Pi}_2\mathbf{C}_2 = \mathbf{0} \tag{20}$$
$$\mathbf{\Pi}_{n-1}\mathbf{B}_1 + \mathbf{\Pi}_n\mathbf{A}_1 + \mathbf{\Pi}_{n+1}\mathbf{C}_2 = 0;\ 2 \le n \le F \tag{21}$$
$$\mathbf{\Pi}_{n-1}\mathbf{B}_1 + \mathbf{\Pi}_n\mathbf{A}_2 + \mathbf{\Pi}_{n+1}\mathbf{C}_2 = 0;\ F+1 \le n \le K-2 \tag{22}$$
$$\mathbf{\Pi}_{K-2}\mathbf{B}_1 + \mathbf{\Pi}_{K-1}\mathbf{A}_2 + \mathbf{\Pi}_K\mathbf{C}_3 = \mathbf{0} \tag{23}$$
$$\mathbf{\Pi}_{K-1}\mathbf{B}_2 + \mathbf{\Pi}_K\mathbf{A}_3 = \mathbf{0} \tag{24}$$

Now, using a recursive approach, state probability vectors are obtained after suitable substitution in the following form

$$\begin{aligned} \mathbf{\Pi}_0 &= \mathbf{\Pi}_1\mathbf{C}_1\left(-\mathbf{A}_0^{-1}\right) = \mathbf{\Pi}_1\mathbf{X}_0 \\ \mathbf{\Pi}_n &= \mathbf{\Pi}_{n+1}\left\{-\mathbf{C}_2\left(\mathbf{X}_{n-1}\mathbf{B}_1 + \mathbf{A}_1\right)^{-1}\right\} = \mathbf{\Pi}_{n+1}\mathbf{X}_n; \quad n = 1, 2, \ldots, F \\ \mathbf{\Pi}_n &= \mathbf{\Pi}_{n+1}\left\{-\mathbf{C}_2\left(\mathbf{X}_{n-1}\mathbf{B}_1 + \mathbf{A}_2\right)^{-1}\right\} = \mathbf{\Pi}_{n+1}\mathbf{X}_n;\ n = F+1, \\ &\qquad F+2, \ldots, K-2 \end{aligned}$$

and

$$\mathbf{\Pi}_{K-1} = \mathbf{\Pi}_K\left\{-\mathbf{C}_3(\mathbf{X}_{K-2}\mathbf{B}_1 + \mathbf{A}_2)^{-1}\right\} = \mathbf{\Pi}_K\mathbf{X}_{K-1}$$

Again, the equilibrium probability vector $\mathbf{\Pi}_n$ in the compact form of $\mathbf{X}_n$; $n = 0, 1, 2, \ldots, K-1$ can be efficiently interpreted as

$$\begin{aligned} \mathbf{\Pi}_n &= \mathbf{\Pi}_K\left[\mathbf{X}_{K-1}\mathbf{X}_{K-2}\mathbf{X}_{K-3}\cdots\mathbf{X}_{n+2}\mathbf{X}_{n+1}\mathbf{X}_n\right]; \quad n = 0, 1, 2, \ldots, K-1 \\ \mathbf{\Pi}_n &= \mathbf{\Pi}_K\prod_{i=K-1}^{n}\mathbf{X}_n = \mathbf{\Pi}_K\mathbf{\Phi}_n; \quad n = 0, 1, 2, \ldots, K-1 \end{aligned} \tag{25}$$

Let $\mathbf{e}_1$ and $\mathbf{e}_2$ are the column vectors having form $[1111]^T$, $[11]^T$, respectively. Hence, the normalization condition is re-expressed as

$$\begin{aligned} &\mathbf{\Pi}_0\mathbf{e}_2 + \sum_{n=1}^{K-1}\mathbf{\Pi}_n\mathbf{e}_1 + \mathbf{\Pi}_K\mathbf{e}_2 = 1 \\ &[\mathbf{\Pi}_1 + \mathbf{\Pi}_2 + \cdots + \mathbf{\Pi}_{K-2} + \mathbf{\Pi}_{K-1}]\,\mathbf{e}_1 + [\mathbf{\Pi}_0 + \mathbf{\Pi}_K]\,\mathbf{e}_2 = 1 \\ &[\mathbf{\Pi}_K\mathbf{\Phi}_1 + \mathbf{\Pi}_K\mathbf{\Phi}_2 + \mathbf{\Pi}_K\mathbf{\Phi}_3 + \cdots + \mathbf{\Pi}_K\mathbf{\Phi}_{K-2} + \mathbf{\Pi}_K\mathbf{\Phi}_{K-1}]\,\mathbf{e}_1 \\ &\qquad + [\mathbf{\Pi}_K\mathbf{\Phi}_0 + \mathbf{\Pi}_K]\,\mathbf{e}_2 = 1 \end{aligned}$$

Finally, the normalizing conditions closed-form expression takes the following form.

$$\mathbf{\Pi}_K\left[\sum_{n=1}^{K-1}\mathbf{\Phi}_n\mathbf{e}_1 + \mathbf{\Phi}_0\mathbf{e}_2 + \mathbf{e}_2\right] = 1 \tag{26}$$

Therefore, using the vector Eq 24 and the normalization condition (26), one can easily demonstrate the probability vector $\mathbf{\Pi}_K$. In addition, all the additional probability vectors of steady-state probabilities are easily determined by substituting $\mathbf{\Pi}_K$ in Eq 25. These probability vectors and steady-state probabilities are utilized to construct several systems' quality performance indicators and the anticipated cost function in the following sections to authenticate our modelling and approaches.

4.2 System performance measures

To estimate the service quality and efficacy of the established model, several performance indicators of the system are defined in the context of the equilibrium probabilities of various states and closed vector form representation. The system's key performance indicators are expressed as following.

- Average count of customers in the system

$$\begin{aligned} L_S &= \sum_{n=1}^{K-1}\sum_{j=1}^{2} nP_{n,j,0} + \sum_{n=1}^{K}\sum_{j=1}^{2} nP_{n,j,1} \\ &= \mathbf{\Pi}_1\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \mathbf{\Pi}_2\begin{bmatrix}2\\2\\2\\2\end{bmatrix} + \cdots + \mathbf{\Pi}_{K-1}\begin{bmatrix}K-1\\K-1\\K-1\\K-1\end{bmatrix} + \mathbf{\Pi}_K\begin{bmatrix}K\\K\end{bmatrix} \\ &= [1\mathbf{\Pi}_1\mathbf{e}_1 + 2\mathbf{\Pi}_2\mathbf{e}_1 + \cdots + (K-1)\,\mathbf{\Pi}_{K-1}\mathbf{e}_1 + K\mathbf{\Pi}_K\mathbf{e}_2] \\ &= \sum_{n=1}^{K-1} n\mathbf{\Pi}_n\mathbf{e}_1 + K\mathbf{\Pi}_K\mathbf{e}_2 \end{aligned} \tag{27}$$

- Average number of waiting customers

$$
\begin{aligned}
L_Q &= \sum_{n=1}^{K-1}\sum_{j=1}^{2}(n-1)P_{n,j,0} + \sum_{n=1}^{K}\sum_{j=1}^{2}(n-1)P_{n,j,1} \\
&= \mathbf{\Pi}_1\begin{bmatrix}0\\0\\0\\0\end{bmatrix} + \mathbf{\Pi}_2\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \cdots + \mathbf{\Pi}_{K-1}\begin{bmatrix}K-2\\K-2\\K-2\\K-2\end{bmatrix} + \mathbf{\Pi}_K\begin{bmatrix}K-1\\K-1\end{bmatrix} \\
&= [0\mathbf{\Pi}_1\mathbf{e}_1 + 1\mathbf{\Pi}_2\mathbf{e}_1 + \cdots + (K-2)\mathbf{\Pi}_{K-1}\mathbf{e}_1 + (K-1)\mathbf{\Pi}_K\mathbf{e}_2] \\
&= \sum_{n=1}^{K-1}(n-1)\mathbf{\Pi}_n\mathbf{e}_1 + (K-1)\mathbf{\Pi}_K\mathbf{e}_2
\end{aligned} \tag{28}
$$

- Probability associated to the busy state of the server

$$
\begin{aligned}
P_B &= \sum_{n=1}^{K-1}\sum_{j=1}^{2}P_{n,j,0} + \sum_{n=1}^{K}\sum_{j=1}^{2}P_{n,j,1} \\
&= \mathbf{\Pi}_1\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \mathbf{\Pi}_2\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \cdots + \mathbf{\Pi}_{K-1}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \mathbf{\Pi}_K\begin{bmatrix}1\\1\end{bmatrix} \\
&= [\mathbf{\Pi}_1\mathbf{e}_1 + \mathbf{\Pi}_2\mathbf{e}_1 + \cdots + \mathbf{\Pi}_{K-1}\mathbf{e}_1 + \mathbf{\Pi}_K\mathbf{e}_2] \\
&= \sum_{n=1}^{K-1}\mathbf{\Pi}_n\mathbf{e}_1 + \mathbf{\Pi}_K\mathbf{e}_2
\end{aligned} \tag{29}
$$

- Probability associated to the idle state of the server

$$
P_I = \sum_{l=0}^{1}P_{0,0,l} = \mathbf{\Pi}_0\begin{bmatrix}1\\1\end{bmatrix} = \mathbf{\Pi}_0\mathbf{e}_2 \tag{30}
$$

- Throughput of the system

$$
\begin{aligned}
\tau_P &= \sum_{n=1}^{K-1}\mu_1P_{n,1,0} + \sum_{n=1}^{K-1}\mu_2P_{n,2,0} + \sum_{n=1}^{K}\mu_1P_{n,1,1} + \sum_{n=1}^{K}\mu_2P_{n,2,1} \\
&= \mu_1\mathbf{\Pi}_1\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + \mu_1\mathbf{\Pi}_2\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + \cdots + \mu_1\mathbf{\Pi}_{K-1}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + \mu_2\mathbf{\Pi}_1\begin{bmatrix}0\\1\\0\\1\end{bmatrix} \\
&\quad + \mu_2\mathbf{\Pi}_2\begin{bmatrix}0\\1\\0\\1\end{bmatrix} + \cdots + \mu_2\mathbf{\Pi}_{K-1}\begin{bmatrix}0\\1\\0\\1\end{bmatrix} + \mu_1\mathbf{\Pi}_K\begin{bmatrix}1\\0\end{bmatrix} + \mu_2\mathbf{\Pi}_K\begin{bmatrix}0\\1\end{bmatrix} \\
&= [\mu_1\mathbf{\Pi}_1\mathbf{u}_1 + \mu_1\mathbf{\Pi}_2\mathbf{u}_1 + \cdots + \mu_1\mathbf{\Pi}_{K-1}\mathbf{u}_1 + \mu_2\mathbf{\Pi}_1\mathbf{v}_1 \\
&\quad + \mu_2\mathbf{\Pi}_2\mathbf{v}_1 + \cdots + \mu_2\mathbf{\Pi}_{K-1}\mathbf{v}_1 + \mu_1\mathbf{\Pi}_K\mathbf{u}_2 + +\mu_2\mathbf{\Pi}_K\mathbf{v}_2] \\
&= \sum_{n=1}^{K-1}\mu_1\mathbf{\Pi}_n\mathbf{u}_1 + \sum_{n=1}^{K-1}\mu_2\mathbf{\Pi}_n\mathbf{v}_1 + \mu_1\mathbf{\Pi}_K\mathbf{u}_2 + \mu_2\mathbf{\Pi}_K\mathbf{v}_2
\end{aligned} \tag{31}
$$

- The probability that customers are re-allowed to enter the queueing system

$$
\begin{aligned}
P_A &= P_{0,0,1} + \sum_{n=1}^{F}\sum_{j=1}^{2}P_{n,j,1} \\
&= \mathbf{\Pi}_0\begin{bmatrix}0\\1\end{bmatrix} + \mathbf{\Pi}_1\begin{bmatrix}0\\0\\1\\1\end{bmatrix} + \mathbf{\Pi}_2\begin{bmatrix}0\\0\\1\\1\end{bmatrix} + \cdots + \mathbf{\Pi}_F\begin{bmatrix}0\\0\\1\\1\end{bmatrix} \\
&= \mathbf{\Pi}_0\mathbf{w}_1 + \sum_{n=1}^{F}\mathbf{\Pi}_n\mathbf{w}_2
\end{aligned} \tag{32}
$$

- The probability that customers are blocked

$$
\begin{aligned}
P_D &= P_{0,0,1} + \sum_{n=1}^{K}\sum_{j=1}^{2}P_{n,j,1} \\
&= \mathbf{\Pi}_0\begin{bmatrix}0\\1\end{bmatrix} + \mathbf{\Pi}_1\begin{bmatrix}0\\0\\1\\1\end{bmatrix} + \mathbf{\Pi}_2\begin{bmatrix}0\\0\\1\\1\end{bmatrix} + \cdots + \mathbf{\Pi}_{K-1}\begin{bmatrix}0\\0\\1\\1\end{bmatrix} \\
&\quad + \mathbf{\Pi}_K\begin{bmatrix}1\\1\end{bmatrix} \\
&= \mathbf{\Pi}_0\mathbf{w}_1 + \sum_{n=1}^{K-1}\mathbf{\Pi}_n\mathbf{w}_2 + \mathbf{\Pi}_K\mathbf{e}_1
\end{aligned} \tag{33}
$$

- The effective arrival rate of the customers

$$
\begin{aligned}
\lambda_{\text{eff}} &= \lambda P_{0,0,0} + \sum_{n=1}^{K-1}\sum_{j=1}^{2}\lambda P_{n,j,0} \\
&= \lambda\mathbf{\Pi}_0\begin{bmatrix}0\\1\end{bmatrix} + \lambda\mathbf{\Pi}_1\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + \lambda\mathbf{\Pi}_2\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + \cdots + \lambda\mathbf{\Pi}_{K-1}\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \\
&= [\lambda\mathbf{\Pi}_0\mathbf{w}_3 + \lambda\mathbf{\Pi}_1\mathbf{w}_4 + \lambda\mathbf{\Pi}_2\mathbf{w}_4 + \cdots + +\lambda\mathbf{\Pi}_{K-1}\mathbf{w}_4] \\
&= \lambda\mathbf{\Pi}_0\mathbf{w}_3 + \sum_{n=1}^{K-1}\lambda\mathbf{\Pi}_n\mathbf{w}_4
\end{aligned} \tag{34}
$$

- Total anticipated waiting period of customers in the system

$$
W_S = \frac{L_S}{\lambda_{\text{eff}}} \tag{35}
$$

4.3 Cost function

The optimization function of the governing model is developed in this section, which benefits system analysts, decision-makers, and system engineers in decision-making by finding the appropriate operating policies and service and maintenance costs. The proposed model considers three decision variables: F, μ_1, and μ_2. Our intuition's fundamental goal is to present the optimum threshold

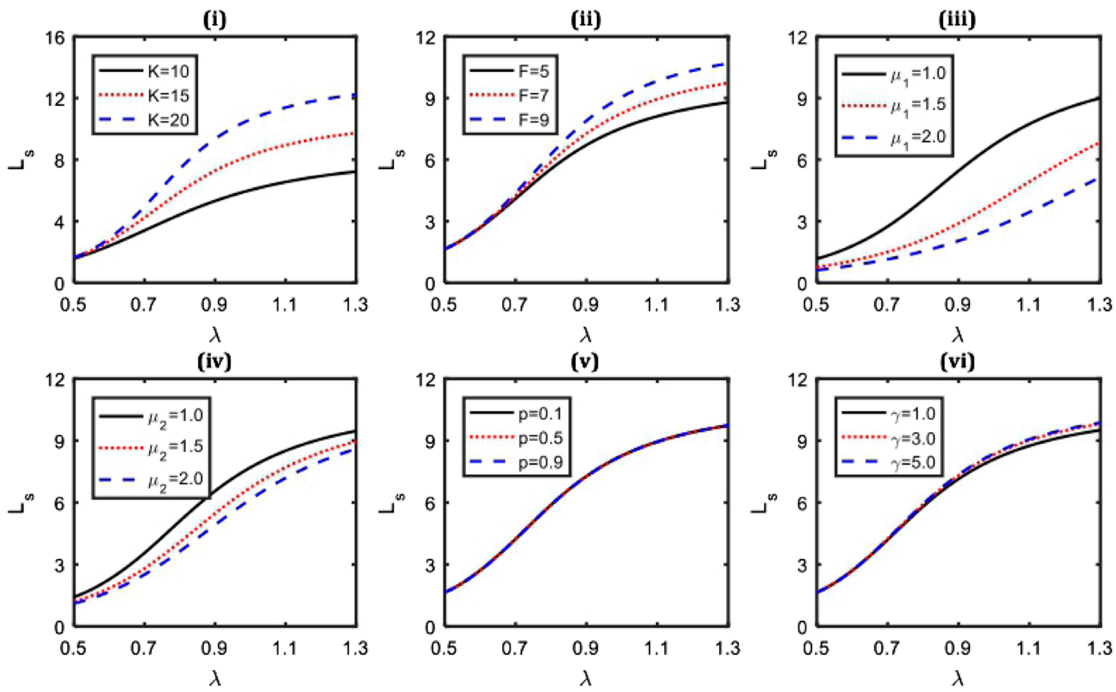

FIGURE 1
Anticipated customers in the system (L_S) with respect to λ for different system parameter values.

value F^* and the preferred service rates μ_1^* & μ_2^*, respectively. The system engineers, researchers and professionals would execute the most effective operating strategy to streamline the anticipated total cost. The subsequent cost components correlated with various quality performance indicators are taken into consideration and characterized in the following manner.

$C_h \equiv$ Holding cost associated with each customer in the system

$C_b \equiv$ Cost associated with the busy state of the server

$C_d \equiv$ Fixed cost associated with each lost customer

$C_a \equiv$ Unit cost to allow the customers to enter in the system

$C_K \equiv$ Fixed cost associated with the systems' capacity (K)

$C_1 \equiv$ Associated cost for providing the service with rate μ_1

$C_2 \equiv$ Associated cost for providing the service with rate μ_2

The cost function is characterized as follows using the queueing-theoretic framework and the fundamental principle enabling the utilization of the aforementioned cost elements.

$$TC(F,\mu_1,\mu_2) = C_h L_S + C_b P_b + \lambda C_d P_d + C_a P_a + C_k K + C_1\mu_1 + C_2\mu_2 \tag{36}$$

The discussed model's cost minimizing problem can be mathematically represented as an unconstrained problem in the following manner

$$TC(F^*, \mu_1^*, \mu_2^*) = \min_{(F,\mu_1,\mu_2)} TC(F, \mu_1, \mu_2) \tag{37}$$

5 Economic analysis

Economic investigation is a comprehensive approach to understanding and evaluating diverse financial decisions, policies, and circumstances. It is usually associated with examining the advantages and disadvantages of a statistical procedure or methodology. It is usually associated with examining the advantages and disadvantages of a statistical procedure or methodology. Economic researchers typically use economic and statistical methods to balance both advantages and disadvantages to establish whether a decision is feasible from a financial perspective. Furthermore, economic investigation involves analyzing how different strategies affect stakeholders and determining whether the approach meets its primary objectives. Statistical and economic analysis makes logical decisions possible based on mathematical and economic principles and enhances rational decision-making by considering marginal analysis, opportunity costs, and trade-offs. Therefore, from an economic perspective, one can demonstrate that resource allocation optimization, cost minimization, service quality improvement, and customer satisfaction depend on the economic analysis of queuing models. In fact, a thorough understanding of system dynamics, customer behavior, and trade-offs between various costs and benefits is essential to make better economic decisions that support corporate objectives.

The majority of real-time stochastic optimization problems are typically nonlinear and have a high degree of complexity in nature due to the presence of multiple combinatorial constraints, which makes it very challenging to compute the solution analytically. In addition, other associated cost elements further increase the complexity of these optimization problems. In order to

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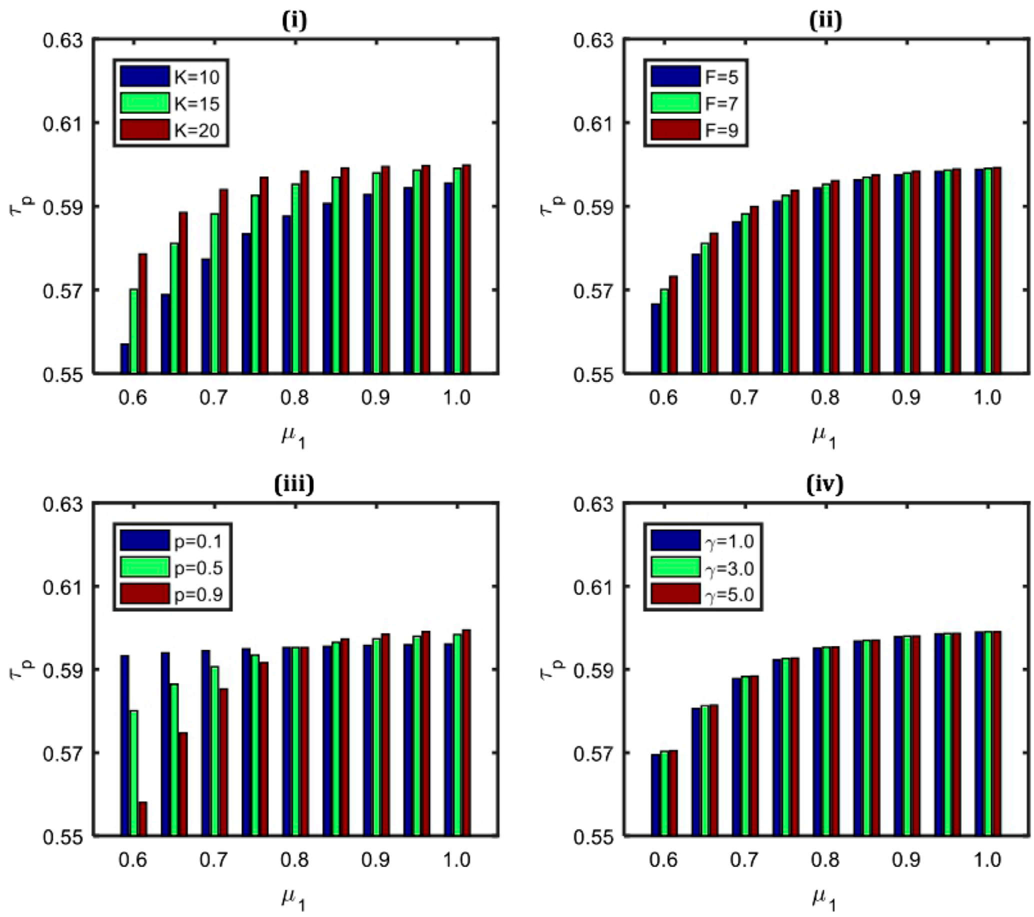

FIGURE 2
System throughput (τ_P) measured in terms of μ_1 for various system parameter values.

overcome this restriction, effective alternative solutions are needed, including global optimizers like metaheuristics and gradient-based optimization algorithms, which are particularly effective in solving such kinds of problems numerically. Using the conceptual framework of the CS algorithm, an optimization approach that depends on natural circumstances is carried out for the economic investigation purpose in the current research. Under the same constraints and service system-based decision-making processes, the outcomes are compared to the research findings of the well-known evolutionary optimization method, PSO technique, and heuristics like DS and QN approaches. For the detailed study and utilization of these algorithms in the context of queueing-based service systems, refer to (*cf.* Shekhar et al. (2020b); Shekhar et al. (2020a; c)) and references therein. Furthermore, the subsequent subsections provide a more detailed description of the procedures involved in these algorithms in order to visualize the necessary successive iterations.

5.1 Cuckoo Search Algorithm

One of the most prominent nature-inspired optimization computational techniques, cuckoo search is widely applied to tackle a variety of challenging and realistic optimization concerns in different engineering fields, namely, queueing systems, inventory systems, manufacturing systems, and many more. It is much more convenient than the other heuristic techniques for solving stochastic global optimal. This is because it can use the switching parameter to maintain a balance between the local and global random walks. The CS algorithm takes its source of inspiration from the behavior of brood parasites, a natural occurrence that may be transformed into an optimization problem as follows.

- When acting like a search agent, each cuckoo only lays one egg at a time in a nest that has been randomly searched. The term randomly selected nest corresponds to the problem's solution.
- A certain number of eggs are passed down from generation to generation. The fitness functions of all agents at the current solution point are evaluated to identify the most suitable solutions.
- The host bird has a chance with probability q_a of identifying the cuckoo's egg inside its nest. It is commonly referred to as switching probability for optimization algorithms and is used to conduct exploitation and exploration.
- Nest positions are modified in the neighborhood such that the eggs in each nest compete to be the best. It is described as a CS algorithm exploitation process.

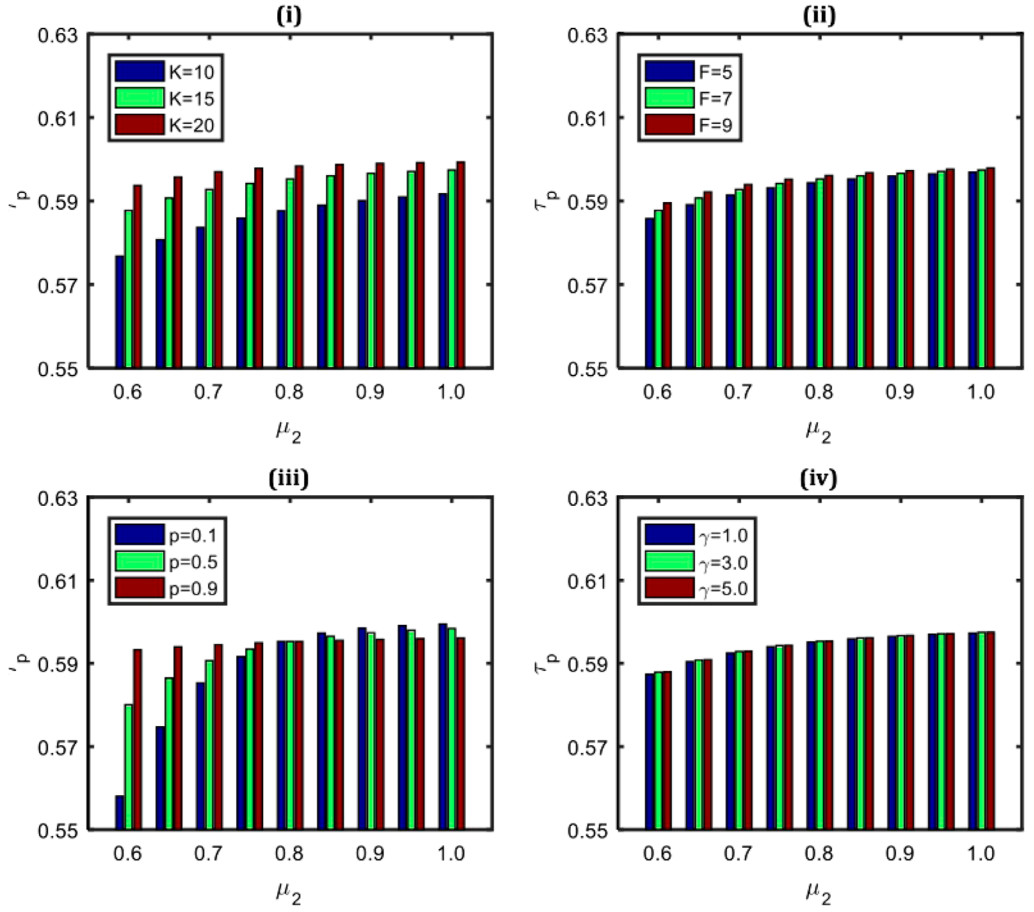

FIGURE 3
System throughput (τ_p) measured in terms of μ_2 for various system parameter values.

- Most likely, the parent birds will evacuate the nest and arbitrarily construct a new one. When certain agents are unable to locate a better solution in their immediate region, exploration for a new solution is conducted in this type of situation.

The behavior of local and global random walks is one of this algorithm's primary concerns. This algorithm's switching parameter q_a balances the usage of a local random walk and a global exploratory random walk. The local random walk can be defined as

$$Y_i^{t+1} = Y_i^t + \omega^1 \beta \otimes H(q_a - \bar{\psi}) \otimes (Y_j^t - Y_k^t) \tag{38}$$

where β is the step size, Y_j^t and Y_k^t are two independent solutions that are opted by stochastic recombination, and $\bar{\psi}$ is a arbitrary number identified from a uniform distribution. The entry-wise product of two vectors is referred to here as ⊗.

Global convergence, as mentioned earlier, is responsible for its effectiveness and the broad range of applications. Levy flights regulate the CS algorithm's global random walks. The exponentially decreasing tails of the distribution function in isotropic random walks make significant step sizes less likely than in population-based approaches. Animals, birds, and insects use Ĺevy flights to generate extremely rational random walks for a variety of survival methods, including searching for food. Due to its power characteristics, the Ĺevy distribution (40) has a strong tail and permits substantially larger step sizes than the normal distribution.

$$Y_i^{t+1} = Y_i^t + \omega^1 L_f(\beta, \psi_1) \tag{39}$$

where

$$L_f(\beta, \psi_1) = \frac{\psi_1 \Gamma(\psi_1) \sin\left(\frac{\pi\psi_1}{2}\right)}{\beta^{(1+\psi_1)}\pi}; \ \beta \gg \beta_0 > 0 \tag{40}$$

In the present scenario, the scaling factor for step size is $\omega^1 > 0$. It should be chosen on the basis of the problem's characteristics. Eq. 38 uses the formula $\omega^1 = O\left(\frac{\zeta}{10}\right)$, where ζ is the problem of interest's characteristic scale. These phases in global optimization would guarantee that the solution would not become stuck in a particular local optimum.

The step-size β in Mantegna's algorithm may be computed by using the concept proposed by Yang (2014).

$$\beta = \frac{u}{|\Xi|^{\frac{1}{\psi_1}}}$$

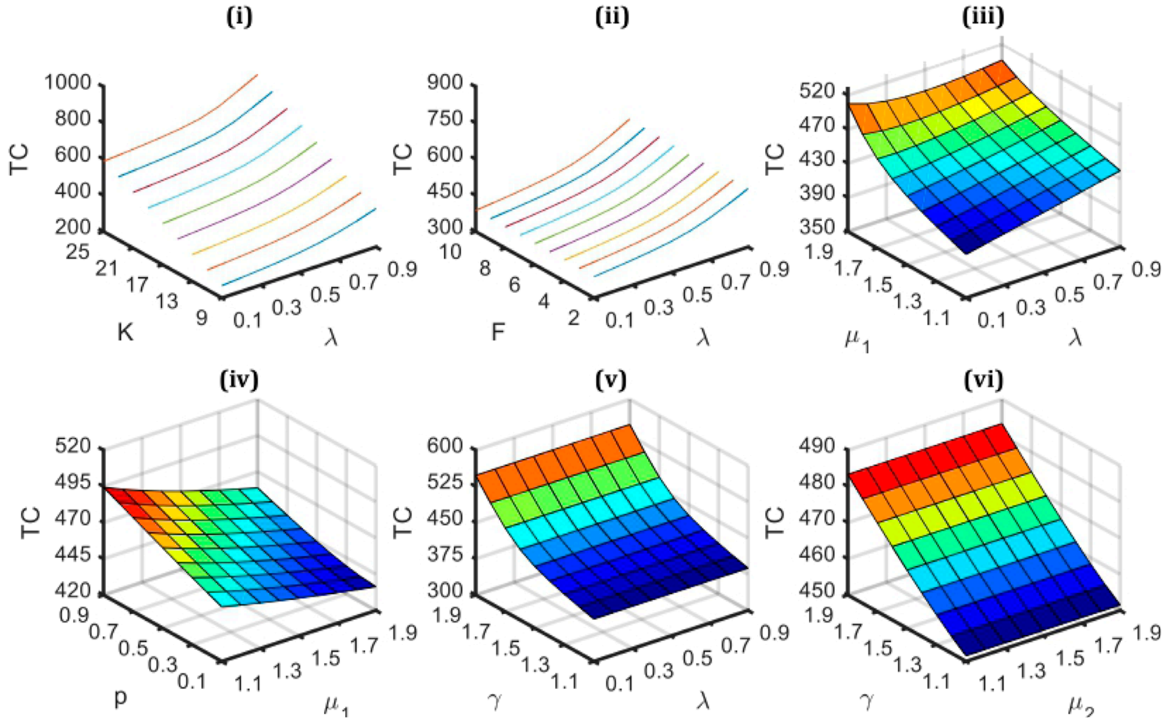

FIGURE 4
The system's anticipated cost (*TC*) concerning different combinations of design parameters.

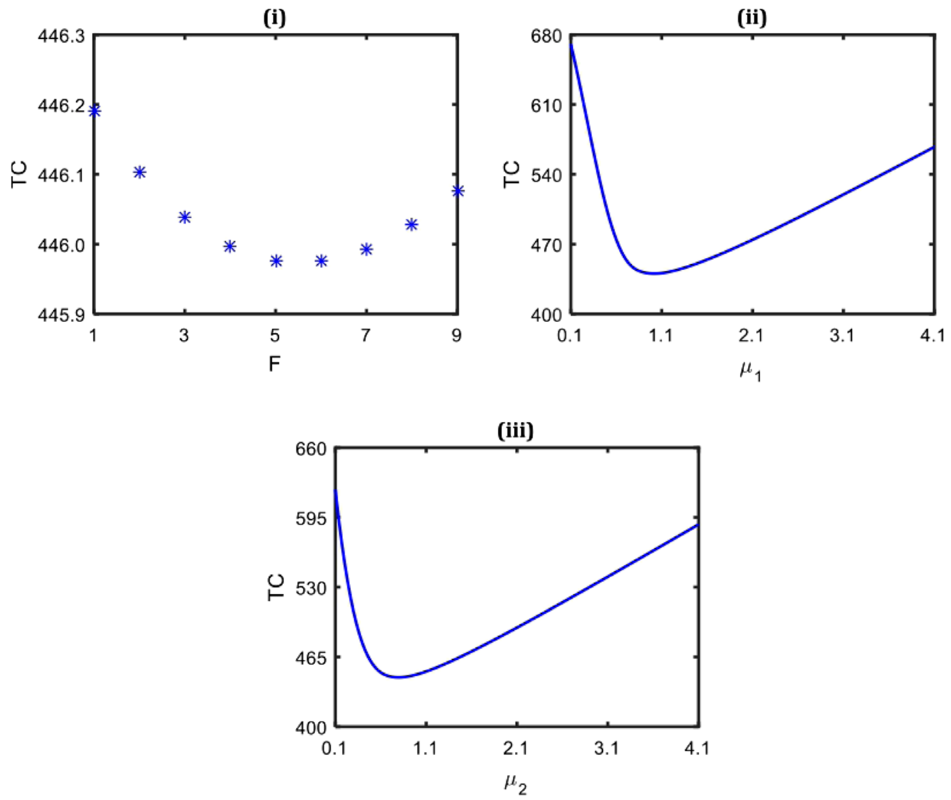

FIGURE 5
The minimum systems' cost along with the best system design parameters value.

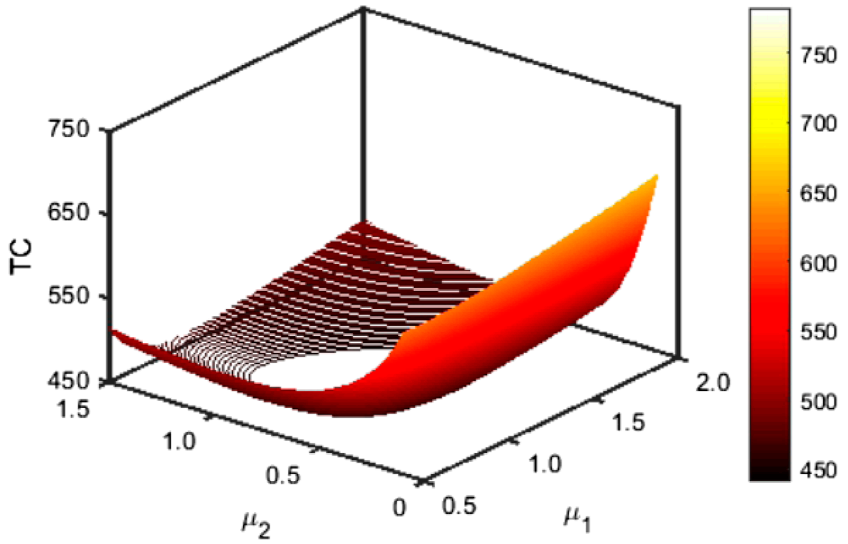

FIGURE 6
Three dimensional contour plot for the optimal combination of (μ_1,μ_2) wrt optimal cost of the system.

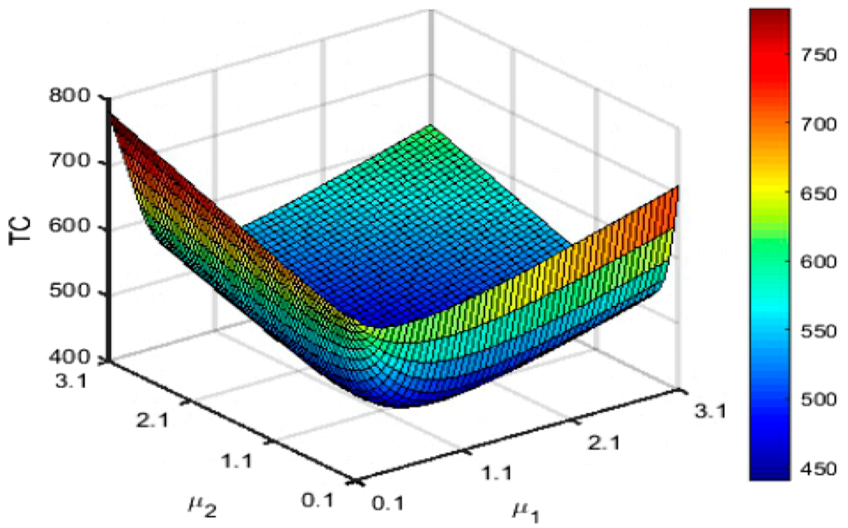

FIGURE 7
Surface plot for the optimal combination of (μ_1,μ_2) wrt optimal cost of the system.

where u and Ξ are opted from the Gaussian distribution, *i.e.*, $u \sim N(0,\sigma_u^2)$ and $\Xi \sim N(0,\sigma_v^2)$ s.t.

$$\sigma_u = \left\{ \frac{\Gamma(1+\psi_1)\sin\left(\frac{\pi\psi_1}{2}\right)}{\Gamma\left[\frac{(1+\psi_1)}{2}\right]\psi_1\, 2^{\left\{(\psi_1-1)/_2\right\}}} \right\}^{\frac{1}{\psi_1}}, \qquad \sigma_v = 1$$

When $|\beta| \geq |\beta_0|$, where β_0 is the least step exists, the distribution of β follows the predicted Lévy distribution. Although in practice, β_0 may be considered a reasonable amount, which is $\beta_0 = 0.1$ to 1. The cuckoo search with Lévy flights has the following pseudo-code.

5.2 Particle swarm optimization

An optimization technique named PSO is extensively utilized to tackle a diverse set of situations. The PSO algorithm was first suggested by Kennedy and Eberhart (1995). The grouping behavior of a community of fish or birds that schools together served as the model for such a stochastic optimization technique. The PSO method uses a particle population that roams randomly through the search space. Based on every particle's best position (p-best) and common/global best positions (g-best) in the search area, its intrinsic velocity as well as position are updated. Each particle's velocity component is updated according to a predetermined

10.3389/fenrg.2023.1329899

```
Input: Size of the population, shifting
       parameter, and system parameters (all
       having  constant values)
Output: Determine the cost function's value
        TC corresponding to the approximated
        solution (F,μ1,μ2)
 Phase 1: Initialization of host nest population
          through objective function
          TC(Y); Y = [Y1,Y2,...,Yd]^T
 while (t < MaxGeneration) or (Stoping framework)
 Phase 2: Get the cuckoo at random
 Phase 3: Develop a solution employing Lévy
          flights
 Phase 4: Examine the solution's effectiveness or
          objectivity
 Phase 5: Choose one nest at random from n nests
          if(TCi < TCj)
          Substitute j by the updated outcome i
          end
 Phase 6: Abandon worse nests with fraction (qa)
 Phase 7: Create new perspectives while
          preserving the best options (or nests
          using better  alternatives
 Phase 8: Determine the actual best by ranking
          the solutions
 Phase 9: Keep updating t ← t+1
 end while
Outcomes from post-processing and visualizations
```

Algorithm 1. Pseudo-code.

formula. The updated velocity is determined from the particle's position, current velocity, and the swarm's local best and common best positions. The modified velocity of each particle is subsequently utilized to determine its new position. This procedure continues until the swarm identifies the best solution.

Assume that $\mathbf{Y}_i$ and $\mathbf{U}_i$ are the particles' location and velocity vectors of the i^{th} polpulation agent, resp ectively. The velocity formula may be further organised using the procedure given below

$$\mathbf{U}_i^{t+1} = \mathbf{U}_i^t + \beta^{(1)}\boldsymbol{\xi}_1\left(\mathbf{G}^* - \mathbf{Y}_i^t\right) + \beta^{(2)}\boldsymbol{\xi}_2\left(\mathbf{Y}_i^*(t) - \mathbf{Y}_i^t\right) \quad (41)$$

where $\beta^{(1)}$ and $\beta^{(2)}$ are learning variables with numerical value as 2. $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are two randomly generated vectors having components ranging from 0 to 1. Hence, the i^{th} particle's updated position formula is expressed as

$$\mathbf{Y}_i^{t+1} = \mathbf{Y}_i^t + \mathbf{U}_i^{t+1} \quad (42)$$

To control the exploitation and exploration between each particle, an inertia function $\psi_2(t)$ is used further by Shi and Eberhart (1998) in the PSO approach. So the reframed equation of velocity can be depicted as

$$\mathbf{U}_i^{t+1} = \psi_2\mathbf{U}_i^t + \beta^{(1)}\boldsymbol{\xi}_1\left(\mathbf{G}^* - \mathbf{Y}_i^t\right) + \beta^{(2)}\boldsymbol{\xi}_2\left(\mathbf{Y}_i^*(t) - \mathbf{Y}_i^t\right) \quad (43)$$

Further, the range of 0.5–0.9 is chosen as the expected value of the inertia function $\psi_2(t)$.

```
Input: Initial parameters' values, density of the
       population, and learning variables.
Output: Estimate a basic solution (F*,μ1*,μ2*) and
        quantify the cost function's value.
        TC(F*,μ1*,μ2*).
 Phase 1: Initialization: locate
          n particles locations Yn.
 Phase 2: Demonstrate G* from
          {TC(Y1),TC(Y2),...,TC(Yn)}.
 Phase 3: while(t < MaximumGeneration) or
          (stoping framework)
          for (n agentsanddimensiond).
 Phase 4: Explore updated speed of the i^th moving
          agent Ui^(t+1).
 Phase 5: Locate current location for the i^th
          agent Yi^(t+1) = Yi^t + Ui^(t+1).
 Phase 6: Investigate the cost value at updated
          locations Yi^(t+1).
 Phase 7: Discover the p-best of each agent Yi*.
          end for
 Phase 8: Keep Updating g-best G*.
          t → t+1
          end while
 Phase 9: Updated research findings Yi* and G*.
Phase 10: Final outcomes of cost optimization
          problem: TC* at G*.
```

Algorithm 2. PSO technique: Pseudo-code.

Besides this, PSO has been utilized to resolve a numerous of global optimization issues, including difficulties concerning cost and economic efficiency. It is a preferred feature for resolving challenging computational optimization problems because of its flexibility to simultaneously exploiting and exploring the region of search interest. In conclusion, PSO is a dynamic optimization algorithm that uses a swarm of particles to explore and take advantage of the search space. It has become a popular alternative for resolving challenging combinatorial optimization problems because of its capacity to explore and exploit the solution area continuously.

The following is a representation of the PSO technique's pseudo-code.

5.3 Quasi-Newton method

Executing the convergence analysis of the cost function (36) analytically is challenging because of the high degree of convexity and significant non-stationary behavior of the derived cost optimization problem. In the queueing literature, many researchers used traditional optimization approaches to overcome this restriction for numerical simulation. As a result, the Quasi-Newton (QN) methodology is used to illustrate the optimal combinations of the system design variables μ_1 and μ_2 together with the optimum anticipated cost. The expected cost function's optimal

Input: Initialization of the vector $\mathbf{\Phi}_{(0)} = [\mu_1^0, \mu_2^0]^T$, input variables, tolerance ϵ.
Output: Calculate the appropriate cost $TC(\mu_1^*, \mu_2^*)$ as a result of estimating the feasible solution $[\mu_1^*, \mu_2^*]^T$.
Phase 1: Calculate $TC(\mathbf{\Phi}_{(0)})$ by interpolating the actual trial solution $\mathbf{\Phi}_{(0)}$.
Phase 2: **while** $\left|\frac{\partial TC}{\partial \mu_1}\right| > \epsilon$ or $\left|\frac{\partial TC}{\partial \mu_2}\right| > \epsilon$, **do** steps 3–4.
Phase 3: Compute the gradient of cost function $\vec{\nabla} TC(\mathbf{\Phi}) = \left[\frac{\partial TC}{\partial \mu_1}, \frac{\partial TC}{\partial \mu_2}\right]^T$. Also, compute the Hessian matrix $\mathbf{H}(\mathbf{\Phi}) = \begin{bmatrix} \frac{\partial^2 TC}{\partial \mu_1^2} & \frac{\partial^2 TC}{\partial \mu_1 \partial \mu_2} \\ \frac{\partial^2 TC}{\partial \mu_2 \partial \mu_1} & \frac{\partial^2 TC}{\partial \mu_2^2} \end{bmatrix}$ at point $\vec{\mathbf{\Phi}}_i$.
Phase 4: Keep updating the test solution $\mathbf{\Phi}_{(i+1)} = \mathbf{\Phi}_{(i)} - [\mathbf{H}(\mathbf{\Phi}_{(i)})]^{-1} \vec{\nabla} TC(\mathbf{\Phi}_{(i)})$.
end
Phase 5: Final outcome

Algorithm 3. Quasi-Newton method: Pseudo-code.

value (36) is achieved by iteratively searching the values of μ_1 and μ_2 in the QN methodology. For this purpose, the vector representation $\mathbf{\Omega}_0$ first initializes the design parameters μ_1 and μ_2. Next, the cost optimization problem's gradients are mathematically estimated in the subsequent steps.

5.4 Direct-Search method

The Direct-Search (DS) technique is employed to achieve the appropriate value F^*, resulting in the anticipated function having its least value, say TC^*. The least cost problem can now be analytically represented as follows

$$TC(F^*, \mu_1^*, \mu_2^*) = \min_{F\epsilon(1,2,\ldots,K-1)} TC(F, \mu_1^*, \mu_2^*) \tag{44}$$

The following is a representation of the DS method's pseudo-code.

Input: K and all other system parameters.
Output: Initialization of (F^*, μ_1^*, μ_2^*) and computation of anticipated cost $TC(F^*, \mu_1^*, \mu_2^*)$.
Phase 1: **for** F=1 to $K-1$
Phase 2: Fix a initial iteration (μ_1, μ_2)
Phase 3: Utilize the QN approach to calculate the pair (μ_1^*, μ_2^*) and $TC(F, \mu_1^*, \mu_2^*)$
Phase 4: **if** diverging solution obtained, return to step 2 **end if**
Phase 5: **if** $TC(F, \mu_1^*, \mu_2^*) < TC^*$
Phase 6: $TC^* = TC(F, \mu_1^*, \mu_2^*)$
Phase 7: **end if**
Phase 8: **end**
Phase 9: Outcomes $TC^* = TC(F^*, \mu_1^*, \mu_2^*)$

Algorithm 4. Direct-Search method: Pseudo-code.

6 Numerical results

The present investigation examines the Markovian circumstances' significant and quantitative outcomes based on the hyper-exponential service approach. In this context, many different formulae for quality performance measurements in closed vector form are presented for straightforward computation. In order to validate our hypothesis and methodology, this section performs numerical simulations using a variety of numerical experiments. To commence the quantitative simulations, the system variables' default values are taken as $K = 15$; $F = 7$; $\lambda = 0.6$; $\mu_1 = 0.8$; $\mu_2 = 0.8$; $p = 0.7$ and $\gamma = 2.0$. In addition, the distribution of probability in an equilibrium state is mathematically illustrated opting the MATLAB (2018b) software and the matrix solution method.

Based on various considerable adjustments to the system parameter values, the sensitivity investigation for the anticipated customers in the service system (L_S) is carried out for the parametric analysis, as shown in Figure 1. The variability of L_S with mean arrival rate (λ) for varied values of the system's capacity (K) is shown in Figure 1I. It makes intuitive sense that the length of the queue would increase with higher values of λ. More demonstration of the variable nature of K is provided because for a fixed extent of λ and incremental variation of K, the value of L_S improved in a consistence manner. In other words, at higher values of K, a relatively large value of L_S is observed. The value of L_S seems to increase for increasing values of λ in Figure 1iii, iv; however, for a fixed value of λ, the opposite behavior is observed for increasing values of μ_1 and μ_2, which is pretty apparent. Similar trends of L_S are perceived for increasing pattern of the other system parameters in the other sub-figures.

The consequences of the parameters μ_1, and μ_2, together with the various combinations of the system parameters K, F, p, and γ, on the throughput of the service system (τ_P), are illustrated in Figures 2, 3. The system's throughput (τ_P) improves as the value of the service rate μ_1 appears to increase with the higher values of K, F, and γ, as shown in Figures 2I, ii,iv. However, it can be observed in Figure 2ii, iv that the rate of incremental change in the τ_p is slightly slower for higher values of γ. The higher values of p in Figure 2iii exhibit the reverse tendency, indicating that the lower value of p in such service systems is best suited to increase the number of consumers served in the system. Particularly in comparison to this, it is possible to recognize that the research outcomes correspond with the perception from Figure 3.

Now, some numerical examples are carried out by taking into account the following cost aspects to show that the suggested model and methodology are convenient for the system designers, policymakers, and practitioners in system modeling and decision-making. For analysis purposes, the numerical values of numerous associated cost variables are choosen as follows: $C_h = 10$; $C_b = 50$; $C_d = 350$; $C_a = 100$; $C_k = 20$; $C_1 = 50$ and $C_2 = 50$, and for all other default system parameters is taken as used in Figure 1. Figure 4 highlights the consequences of the various values for the system parameters on the anticipated system cost. To examine this effect, numerous values of the combinations (K,λ), (F,λ), (μ_1,λ), (p,μ_1), (γ,λ), and (γ,μ_2) up to a substantial level are presented. The

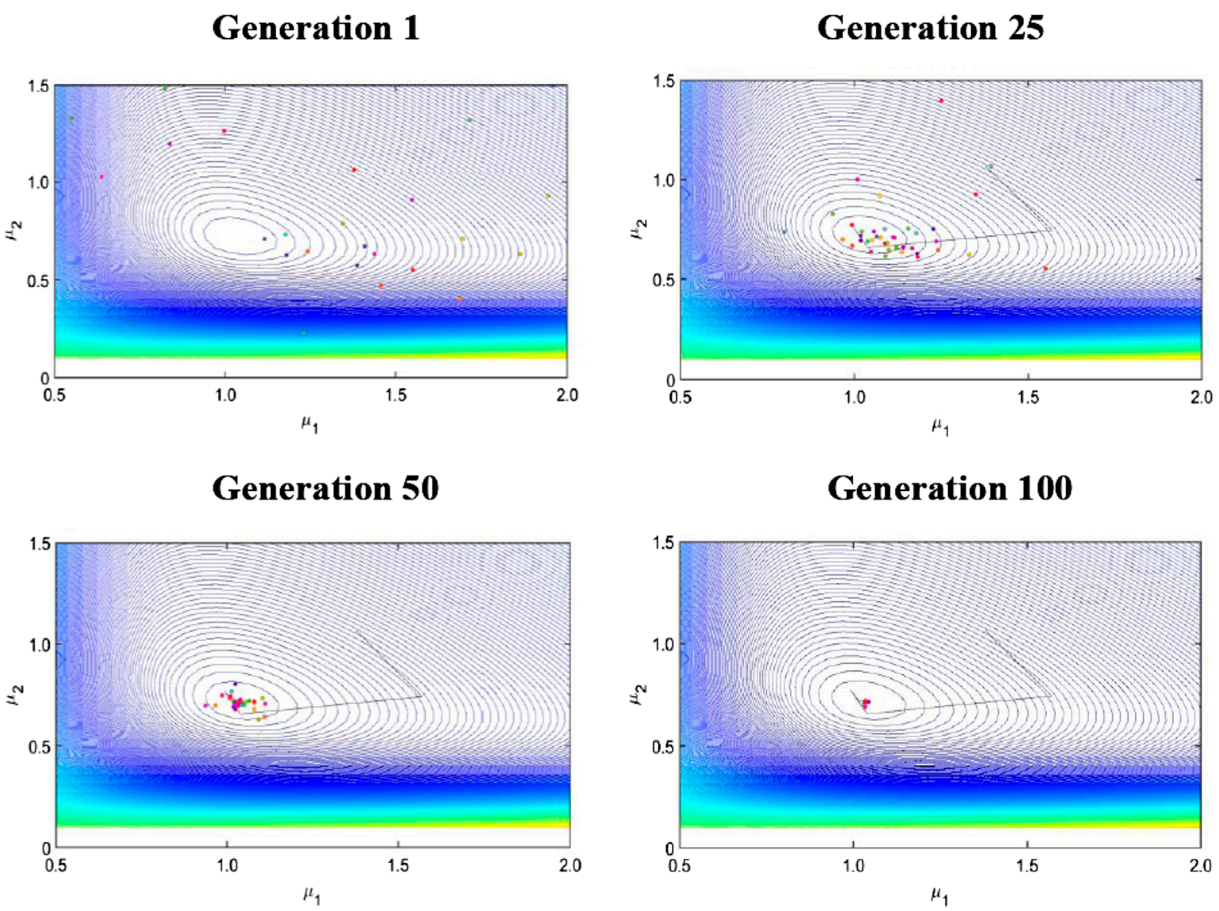

FIGURE 8
Classification of CS optimization technique.

sub-Figure 4i-iii, v demonstrate that the estimated cost of the system keeps rising as λ grows significantly. It is genuine because the system's associated cost clearly grows significantly in proportion to the higher number of customers. The validation of our formulation can be observed in Figure 4iv, which shows that the lower value of p and higher service rate μ_1 minimizes the associated cost (TC) of the system. Similarly, from Figure 4iii, vi, it is clear that anticipated cost of the system become higher with higher service rates μ_1 and μ_2, respectively. These outcomes demonstrate that an additional service facility might not always be considered into account to enhance system performance.

To illustrate the most appropriate strategies and the optimum anticipated cost for the suggested model, three system design parameters F, μ_1, and μ_2 are recommended. Therefore, we first verify that the anticipated cost (TC) is convex and analogous to the decision variables F, μ_1, and μ_2. Our intuition indicates that the convex behavior of the desired cost function is guaranteed by the graphic combinations of these parameters' variable values shown in Figure 5. It should be noticed that F, μ_1, and μ_2 have optimal values that are nearly identical to 5, 1.0, and 0.8, respectively. Since it is really very difficult task to generate the solution analytically, the nature-inspired optimization technique, CS algorithm, is implemented to determine the optimum system design combinations simultaneously. We fix the range of the decision parameter F as [1 14] and [0.5 3.0] for μ_1 and μ_2, respectively. Moreover, the combined optimal variations of μ_1 and μ_2 via the three-dimensional contour plot and surface plot are provided in Figures 6, 7 for a better understanding of research findings. As used in Figure 5, the default system parameter values and associated unit cost elements were taken. The cost function is clearly highly convex for the combined design parameter values μ_1 and μ_2, which is apparent in Figures 6, 7. So, it implies that engineers and system designers would be benefited more from using queueing modeling while making decisions.Now, some specific generations of the CS algorithm (*cf.* Figure 8) are specified in the feasible domain for illustration purposes by defining the minimal and higher ranges of the design variables μ_1 and μ_2 as [0.5 3.0] with $F^* = 9$ as the default value. With the aid of these generations, we present the efficient expected system cost as well as the most effective combinations of decision parameters. As the CS technique is evolutionary in nature, it is straightforward to verify that all search agents in the first generation are distributed randomly throughout the feasible region. Subsequently, they approach closer to the convergence outcomes by investigating unexplored regions, including each subsequent generation. This suggests that the CS

TABLE 1 Optimal choices for (F^*, μ_1^*, μ_2^*) and minimal system cost TC^* incorporating CS algorithm.

(K, λ, p, γ)	F^*	μ_1^*	μ_2^*	TC^*	Mean $\left\{\frac{TC}{TC^*}\right\}$	Max $\left\{\frac{TC}{TC^*}\right\}$	CPU time
(15, 0.6, 0.7, 2.0)	9	1.036013	0.714905	439.860911	1.0000006936	1.0000010443	730.7813
(20, 0.6, 0.7, 2.0)	8	1.040046	0.719131	539.936933	1.0000002721	1.0000008080	840.4874
(25, 0.6, 0.7, 2.0)	8	1.042306	0.719935	639.968386	1.0000002538	1.0000004125	960.2196
(15, 0.4, 0.7, 2.0)	7	0.769406	0.529343	409.136949	1.0000000808	1.0000001460	740.3206
(15, 0.8, 0.7, 2.0)	11	1.295901	0.894984	468.243607	1.0000019037	1.0000049741	735.7208
(15, 0.6, 0.5, 2.0)	8	0.899836	0.899809	443.189543	1.0000001074	1.0000002017	735.6285
(15, 0.6, 0.9, 2.0)	11	1.121175	0.435621	426.668703	1.0000010943	1.0000029453	727.2163
(15, 0.6, 0.7, 1.5)	9	1.035993	0.714638	439.865751	1.0000001421	1.0000003996	700.6391
(15, 0.6, 0.7, 2.5)	9	1.035598	0.714991	439.858109	1.0000003510	1.0000009364	693.9718

TABLE 2 Optimal choices for (F^*, μ_1^*, μ_2^*) and minimal system cost TC^* incorporating PSO algorithm.

(K, λ, p, γ)	F^*	μ_1^*	μ_2^*	TC^*	Mean $\left\{\frac{TC}{TC^*}\right\}$	Max $\left\{\frac{TC}{TC^*}\right\}$	CPU time
(15, 0.6, 0.7, 2.0)	9	1.035894	0.715072	439.860912	1.0000001105	1.0000003226	370.4333
(20, 0.6, 0.7, 2.0)	8	1.040021	0.718163	539.936935	1.0000072347	1.0000160244	468.3631
(25, 0.6, 0.7, 2.0)	9	1.042236	0.720193	639.968393	1.0000004749	1.0000012235	704.2960
(15, 0.4, 0.7, 2.0)	7	0.769324	0.529415	409.136949	1.0000158244	1.0000400613	479.9172
(15, 0.8, 0.7, 2.0)	11	1.295001	0.895602	468.243635	1.0000295733	1.0000443848	475.4190
(15, 0.6, 0.5, 2.0)	9	0.900871	0.900034	443.189549	1.0000496516	1.0000744922	416.5908
(15, 0.6, 0.9, 2.0)	11	1.121242	0.435747	426.668704	1.0000022885	1.0000068605	357.1831
(15, 0.6, 0.7, 1.5)	9	1.035541	0.714986	439.865754	1.0000051400	1.0000153885	353.2034
(15, 0.6, 0.7, 2.5)	9	1.035756	0.715193	439.858109	1.0000209068	1.0000626951	355.9429

approach is efficient for all underlying conceptual experiments. It also demonstrates the capability of the CS algorithm to converge on optimal outcomes within an appropriate time span. From the research findings, as a concluded remark, one can easily observe that the best agent's coordinates are $[\mu_1^*, \mu_2^*] = [1.036013, 0.714905]$ and the system's associated minimum cost is $TC^* = 439.860911$ for the CS algorithm.For an in-depth investigation of the most desirable outcomes, a numerical simulation covering a wide range of values for other system parameters is conducted in Table 1, which provides an overview of the converging research outcomes. We independently take 50 search particles, 100 iterations, and 20 total cycles for every test to outline each of these numerical evaluations for the CS technique. The concept of the combination of statistical variables is used in all iterations of the CS algorithm, particularly the average and maximal ratios of the optimized cost, in order to confirm the algorithm's adaptive capability. The average and maximum values of $\left[\frac{TC}{TC^*}\right]$, displayed in Table 1, ranging from 1.0000000808 to 1.0000019037 and 1.0000001460 to 1.0000049741, respectively. It indicates that the CS algorithm's performance to move towards the best position is significantly improved.

Finally, a comparison between the CS algorithm, the swarm intelligence-based optimization approach, the PSO algorithm, and the traditional optimization techniques, the DS and QN method, is conducted to highlight the validity of the obtained converging results of the CS algorithm. As we know, the optimal outcomes, the statistical parameters, namely, mean ratio and maximum ratio in all independent runs, and computation time (CPU time) are some of the crucial and fundamental aspects to consider when comparing any algorithm's utility and efficacy. As a result of this fact, some numerical experiments are shown in Tables 1–5. The minimum anticipated cost generated by the PSO, DS, QN, and CS algorithms can be visualized to three to four decimal places. From all of the numerical experiments, it can be observed that the CS methodology's adequate search quality to reach optimality is significantly superior to the PSO approach. It can also be strongly recommended based on the optimal strategies obtained

TABLE 3 Ideal variations of (μ_1^*, μ_2^*) with corresponding minimum cost *TC** for different *F*.

F	Initial value	μ_1^*	μ_2^*	TC*	Iterations
F = 1	[1.5, 1.5]	1.037576	0.717363	439.992064	09
F = 2	[1.5, 1.5]	1.036834	0.716572	439.960164	08
F = 3	[1.5, 1.5]	1.036251	0.715923	439.932623	08
F = 4	[1.5, 1.5]	1.035831	0.715420	439.909608	08
F = 5	[1.5, 1.5]	1.035573	0.715063	439.891159	08
F = 6	[1.5, 1.5]	1.035471	0.714848	439.877229	09
F = 7	[1.5, 1.5]	1.035514	0.714766	439.867692	08
F = 8	[1.5, 1.5]	1.035690	0.714808	439.862343	09
F = 9	[1.5, 1.5]	**1.035713**	**0.714957**	**439.860916**	**07**
F = 10	[1.5, 1.5]	1.036378	0.715213	439.863063	07
F = 11	[1.5, 1.5]	1.036856	0.715549	439.868447	07
F = 12	[1.5, 1.5]	1.037402	0.715957	439.876783	07
F = 13	[1.5, 1.5]	1.038016	0.716436	439.888177	07

The bold combination of values indicates the optimal operating strategies along with the minimal anticipated cost of the governing model.

TABLE 4 An illustration of the Direct-Search method's iterative process using the values *K* = 15, *λ* = 0.6, *p* = 0.7, and *γ* = 2.0, and initial combination (*F*, μ_1, μ_2) = (9, 1.5, 1.5).

Iterations	0	1	2	3	4	5	6	7
μ_1	1.5	1.094767	1.017981	1.054008	1.034769	1.036283	1.036159	**1.035713**
μ_2	1.5	1.000000	0.701614	0.723721	0.717739	0.715029	0.715132	**0.714957**
TC	476.666965	445.855240	439.918502	439.900798	439.861663	439.860919	439.860917	**439.860916**

The bold combination of values indicates the optimal operating strategies along with the minimal anticipated cost of the governing model.

TABLE 5 The best combinations of (F^*,μ_1^*,μ_2^*) and the optimum anticipated cost *TC** for various combinations of *λ*, *p*, and *γ*.

(λ, μ_1, μ_2)	(0.6,0.7,2.0)	(0.4,0.7,2.0)	(0.8,0.7,2.0)	(0.6,0.5,2.0)	(0.6,0.9,2.0)	(0.6,0.7,1.5)	(0.6,0.7,2.5)
F*	**9**	7	11	9	11	9	9
(μ_1^0,μ_2^0)	[1.5, 1.5]	[1.5, 1.5]	[1.5, 1.5]	[1.5, 1.5]	[1.5, 1.5]	[1.5, 1.5]	[1.5, 1.5]
Total Iteration	**7**	7	11	9	11	9	9
μ_1^*	**1.035713**	0.769351	1.296033	0.900453	1.121456	1.036650	1.035996
μ_2^*	**0.714957**	0.529391	0.895235	0.900462	0.435633	0.715061	0.715195
$TC(F^*,\mu_1^*,\mu_2^*)$	**439.860916**	409.136949	468.243608	443.189548	426.668705	439.865759	439.858110

The bold combination of values indicates the optimal operating strategies along with the minimal anticipated cost of the governing model.

by the CS algorithm compared to PSO. This demonstrates the robustness and better economic significance of the CS algorithm since the PSO requires numerous computations to update the p-best and g-best outcomes. CS algorithm naturally maintains population diversity due to its random search and cuckoo behavior, which can be beneficial for escaping local optima. While PSO can struggle with maintaining diversity as the particles converge towards a single solution, potentially getting trapped in local optima. The CS algorithm outperforms the semi-classical optimization methodology, QN, and DS methods in all testing scenarios. The QN approach requires a numerical estimation of the gradient or direction of optimality due to the optimization function's high nonlinearity and high degree of complexity. Hence, the substantial degree of approximations connected to the internal iterations of the QN approach negatively impacts the QN method's efficiency and searching quality.

As a concluding remark, we can emphasize that improved system efficiency, resource utilization, and customer satisfaction are generally considered to be the foundation of the theoretical implications of the queueing models. For instance, in this section, we have shown that the incremental changes in the arrival rate of customers significantly increase the queue length, and higher service rates reduce average customer waiting times and improve overall system efficiency. Further, several illustrative simulations based on the governing model are provided to validate these implications quantitatively and outcomes from these simulations are then compared with the model's predictions. Thus, the quantitative results demonstrate a decrease in average waiting times when the service provider's service rate increases, supporting the theoretical inference of enhanced system performance. This research confirmation supports the theoretical understanding that optimizing certain parameters in the queueing model can effectively enhance system performance and meet the model's predictions.

7 Managerial insights

The study's numerical findings in Section 6 can benefit service system managers and decision-makers dealing with complex and non-linear optimization problems. They can optimize the system's design parameters and minimize the overall anticipated cost of renewable energy systems by implementing the suggested CS algorithm. It may lower anticipated costs, enhance energy demand fluctuations, and improve energy storage management, system reliability, and repair and maintenance policies within renewable energy infrastructures. To avoid maintenance interruptions, system managers may employ these insights in developing preventive service strategies, such as regular maintenance inspections, risk identification and mitigation, and outages.

System managers implementing the suggested approach into practice have to take care a variety of aspects into account. For instance, they must ensure that they have the required data to optimize the system design parameters. Additionally, they should consider the impact of the model parameters on their existing repair and maintenance processes, resources, and costs. System managers and decision-makers should also carefully consider the mathematical validity of the model, particularly when forecasting future service requirements. To make sure that just-in-time operational efficiency is maintained, they should also take economic limitations and the accessibility of quick maintenance services into account.

In a nutshell, researchers, policymakers, and system managers of renewable energy systems can use the study's mathematical outcomes to optimize demand for energy fluctuations, lower expected costs, and optimize system design parameters. They can ensure efficient service provisioning throughout each stage of the energy service systems and improve the quality of their repair and maintenance services by implementing the suggested recommendations.

8 Conclusion and future perspectives

The proposed model based on renewable energy systems with hyperexponentially distributed service regimes and finite buffer storage has several unique findings and contributions. Firstly, the model considers the impact of admission control policy, which is an essential aspect in any energy sector but is often overlooked in traditional models. Secondly, the model considers a finite buffer storage, a more realistic assumption than infinite buffer in many practical scenarios. Thirdly, the closed-vector form expressions for different renewable system performance measures have been derived using steady-state probability distributions, which can aid in efficiently evaluating the energy system's performance. Finally, the cost optimization problem is constructed to design the appropriate threshold F^*, reasonable service parameters μ_1 and μ_2, and the minimal anticipated cost associated to the energy system. The current investigation has significant implications for researchers and practitioners seeking to optimize several energy parameters and improve the individual experience.

The economic analysis performed in this study also contributes to its uniqueness. The analysis shows how the established model can support system managers and decision-makers in decreasing the expense of service or maintenance, an extremely desirable component of renewable energy sector. The provision of nature-inspired metaheuristics, such as the CS and PSO techniques, further enhances the practical importance of the model. The validation of the findings using the comparative investigation with the PSO algorithm, QN method, and DS method also adds credibility to the results. Further, the current research introduces novel parametric and optimal analyses for a renewable energy system with an admission control policy and a two-stage hyperexponentially distributed service pattern. Our study illustrates the dynamic behavior of the developed model using several hypotheses based on the queueing-theoretic approach. Specifically, the current research employs the essential assumptions of transition between consecutive transition states to demonstrate the differential equations using the fundamental assumptions of stochastic process. The use of the queueing-theoretic approach enables us to provide a rigorous and analytical framework to analyze the controllable energy system. This framework offers insights into the optimal control policies that can be employed to minimize system costs and improve the individual experience. Our study also provides a detailed analysis of the system's robust behavior, allowing us to check the impact of distinct design variables on the operational characteristics of the energy system.

Despite its contributions, the proposed model has some limitations that need to be considered. One limitation is the assumption of hyperexponential distribution for service times and startup times. While this assumption may hold in some cases, it may not be applicable in all scenarios. Additionally, the model assumes a limited buffer storage, which may not be accurate in situations where buffer storage is not constrained. Moreover, the model assumes a fixed number of service channels, which may not be realistic in scenarios where the number of active service providers can change over the time.The following potential hypotheses can be introduced to expand the scope of the present investigation.

Extension to other service distributions: Extending the suggested model to other customer service distributions, such as Erlangian, Weibull, and geometric distributions, is one possible direction for future investigation. This will facilitate an evaluation of the performance of the renewable energy systems under multiple service distribution scenarios, offering a more thorough comprehension.

Dynamic server allocation: An additional possibility for future study could involve examining how the suggested model addresses dynamic service distribution. This could result in improved performance, higher efficiency, and reduced expenses associated with of system performance indicators by enabling a more adaptable and effective management of resources in the energy system.

Application to real-world scenarios: The current investigation can further be employed to numerous realistic scenarios such as healthcare systems, computer communication systems, and transportation systems. The viability and applicability of the suggested model in such circumstances may be further examined in future research.

Machine learning-based admission control policies: Using machine-learning-based admission control policies in the proposed model could be another potential research direction. This will allow individuals optimal connectivity and control over energy systems, potentially leading to improved performance and lower expenditure.

Robustness analysis: A more in-depth analysis of the convergence and robustness of the research findings may also be a future research direction. This may involve examining the effects of various types of breakdowns or interruptions on system performance and identifying strategies to mitigate those effects.

Analysis of real-time data: Real-time data analysis via the model having hyper-exponential distribution can facilitate dynamical parameter adjustments, improving forecast accuracy and enabling real-time decision-making.

In Implementation of Modern Technologies: The queueing-based hyper-exponentially service model can find applications in developing domains like cloud computing, edge computing, and the Internet-of-Things (IoT) over time as technology keeps growing. Understanding and improving these modern technologies' waiting times and service patterns can help with more effective resource management and enhanced customer satisfaction.

Data availability statement

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Author contributions

SV: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Supervision, Validation, Visualization, Writing–original draft, Writing–review and editing. CS: Investigation, Supervision, Validation, Writing–review and editing. AD: Formal Analysis, Validation, Writing–review and editing. KP: Formal Analysis, Writing–review and editing. MK: Formal Analysis, Writing–review and editing. HK: Formal Analysis, Writing–review and editing. KA: Formal Analysis, Writing–review and editing. MA: Formal Analysis, Writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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