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Optimal scheduling of pumped storage hydropower plants with multi-type of units in day-ahead electricity market considering water head effects

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Pumped-storage hydropower plant (PSHP) is a type of valuable energy storage system and a flexible resource to the modern power system with increasing renewable energy integration. As independent market participants, a PSHP can participate in both the energy market and frequency regulation market to maximize its revenue and contribution to the secure and economic operation of the power system. In some PSHPs, both fixed-speed and variable-speed units are installed to improve the flexibility, especially when operating in the pumping mode. However, it's difficult to deal with the nonlinear relationships among power, flow, and water head in pumping and generating modes. This paper proposes iterative solution methods for scheduling the PSHP by considering the relationship between power and flow at different water heads for different types of units. The scheduling problem is established as a scenario-based optimization formulation by considering PSHP's participation in both the energy market and frequency regulation market. In each iteration, the optimal dispatch model is formulated as a Mixed Integer Linear Programming (MILP) problem. Case studies are performed and simulation results validate the effectiveness of the model and the iterative solution methods.

KEYWORDS

pumped-storage hydropower plant, water head effects, scenario-based optimization, fixed-speed units, variable-speed units

1 Introduction

The large-scale development of renewable energy, such as wind and solar, is very important to achieve the targets of carbon neutrality (Outlook, 2020; Stančin et al., 2020). However, the volatile and intermittent characteristic of renewable energy generation brings great challenges to power balance of power systems (Cai et al., 2019). Energy storage systems, especially pumped storage hydropower plants (PSHP), will play a key role in the future power systems with very high penetration of renewable power generation.

Currently, pumped-storage hydropower units can be divided into fixed-speed units and variable-speed units. Compared to variable-speed units, fixed-speed units are mature and used in most pumped storage hydropower plants worldwide (Yang and Yang, 2019; Vasudevan et al., 2021). Meanwhile, with the development of AC excitation technology, power can be adjusted for variable speed units in pumping mode. In addition, variable-speed units can generate a lower percentage of the rated power in comparison to fixed-speed units, which means that variable-speed units have a stronger power regulation capability to cope with future large-scale renewable energy output fluctuations and renewable energy consumption.

In the US and some regions in Europe, PSHPs can participate in the local electricity market as independent players. The PSHPs can pump water to store energy when the electricity tariff is low, while generating to get profits during periods with high electricity price (Kim et al., 2021; Liu et al., 2021). Therefore, the PSHP declares its generating and pumping curve in the day-ahead market according to the forecast electricity price and receives the revenue as a price-taker. In Emmanuel and Denholm (2022), a modified price-taker model with a market feedback function is established to simulate the impact of increased storage including PSHP. A market participation strategy and schedule for a PSHP with hydraulic short-circuit technology are proposed using a price-taker model presented in Kwon et al. (2021). However, with the development of pumped storage unit technology and the improvement of the electricity market, pumped storage power plants can participate not only in the energy market, but also in the regulation and reserve market (Chazarra et al., 2017). In Chazarra et al. (2017), a scheduling model is established for a closed-loop PSHP participating in both the energy and secondary regulation market. Besides, using variable-speed units also makes it possible to get profits from the regulation market, when units operate in pumping mode (Rayati et al., 2022). By establishing an accurate model for PSHPs, PSHPs can be dispatched effectively, contributing to improving the overall benefits of PSHPs.

In the previous research, studies related to the optimal scheduling of PSHPs were mostly based on the constant head model, which means PSHPs were considered to maintain a constant head in operation. With this assumption, power shows a linear relationship with the flow in pumping and generating modes. In Bruninx et al. (2015); Moradi et al. (2017); Li et al. (2018); Abdelshafy et al. (2020); He et al. (2020), different coefficients are used to describe the linear relationship between the power and flow in pumping mode and generating mode. The pumping and generating constraints with corresponding coefficients are included in the optimization model. In fact, many PSHPs are head-sensitive plants, meaning the actual pumping and generating power of the pumped storage units is influenced by the combination of the head and flow. In order to dispatch PSHPs more effectively, it is necessary to establish an optimal dispatch model for PSHPs reflecting the nonlinear relationship of "power-flow-water head" for both fixed-speed and variable-speed units in pumping and generating modes.

Some researchers proposed linearized methods to accurately demonstrate the relationship between the flow and power under different hydraulic heads (Su et al., 2019; Toubeau et al., 2019; Wang et al., 2021; Yuan et al., 2021). In Toubeau et al. (2019), nonlinear pump/turbine head-dependent curves are modeled and linearized to decrease the endogenous uncertainties in the models. In Su et al. (2019); Wang et al. (2021), the MILP formulation is used to model the hydropower plants, which can give an approximate model to demonstrate the model of plants with nonlinear and non-convex features. However, it takes a long time or cannot get the optimal solution, when the PSHP contains multiple units or too many linearized constraints with a large number of binary variables included in the model to give a better approximation. Besides, the different operating characteristics of variable-speed and fixed-speed units make it difficult to optimize the scheduling of pumped storage power plants with multi-type of units.

Two major contributions have been made in this paper. One of the contributions is to propose an optimal day-ahead dispatch method for a PSHP containing multi-type of units considering the effect of hydraulic heads. The optimization objective is to maximize the revenue of PSHPs in the energy market and the regulation market. Nonlinear characteristics among the "power-flow-water head" of fixed-speed and variable-speed units are considered to establish a more realistic model for the PSHP. The other contribution is to propose an iterative method for solving the optimal scheduling model of the PSHP considering the influence of hydraulic effects. A heuristic-based acceleration method is proposed to improve the effectiveness of the computational results. The proposed method significantly shortens the solution time compared to directly solving complicated linearized models and gives a better approximation solution for the model. Therefore, it is more practical for the proposed algorithm to be used in engineering.

The remaining of this paper is organized as follows. Section 2 establishes the day-ahead dispatch model to maximize the revenues for the PSHP in the energy market and regulation market. Section 3 introduces the mathematical formulation of the fixed-speed and variable-speed units in both pumping and generating states. Section 4 describes the iteration strategy for optimal scheduling. Several cases are conducted in Section 5 to validate the correctness and effectiveness of the method of the proposed methods. Finally, conclusions are drawn in Section 6.

2 Basic day-ahead dispatch model for a PSHP containing multi-type units

In contrast to a fixed-speed unit, the pumping power of a variable-speed unit can be changed in the pumping mode. Therefore, a PSHP containing multi-type of units can participate in energy and frequency regulation with higher flexibility. By optimizing the generating and pumping power of units, it is possible for the PSHP's operator to get more revenue by fully making use of the multiple units. Considering the participation of PSHP in both energy and frequency regulation markets, we construct the basic day-ahead self-scheduling model for a PSHP to maximize the expected total revenue in this section.

In the model, the PSHP is regarded as a price-taker of the electricity markets, and the uncertainties of demand for frequency regulation are also considered. The nonlinear characteristics of "power-flow-water head" for the fixed-speed and variable-speed units in the PSHP are modeled and elaborated in the next section.

2.1 Objective function

The objective function includes the revenues of PSHP trading in energy market R^e and the regulation market R^r, as well as the startup/shutdown cost of the pump C^{PH}.

$$\max \quad \sum_{t} \sum_{\omega\in\Omega} \alpha_\omega \left(R^e_{\omega,t} + R^r_{\omega,t}\right) - C^{PH}_t \tag{1}$$

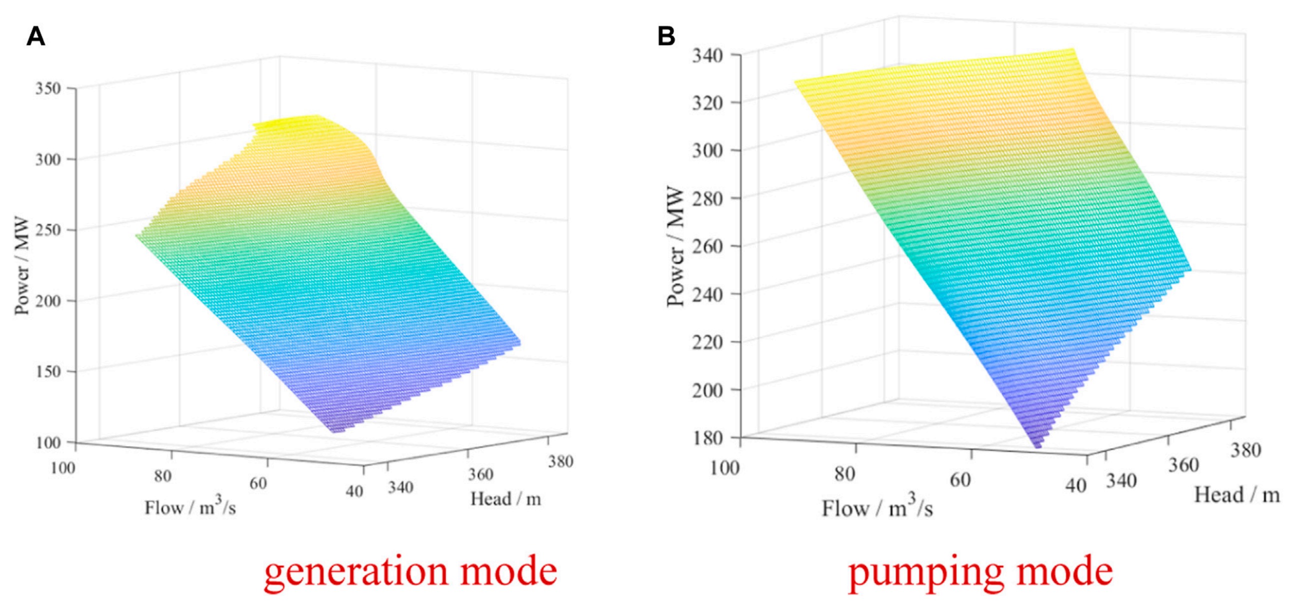

FIGURE 1
Nonlinear relationship among water head, flow and power for a variable-speed unit. **(A)** generation mode, **(B)** pumping mode.

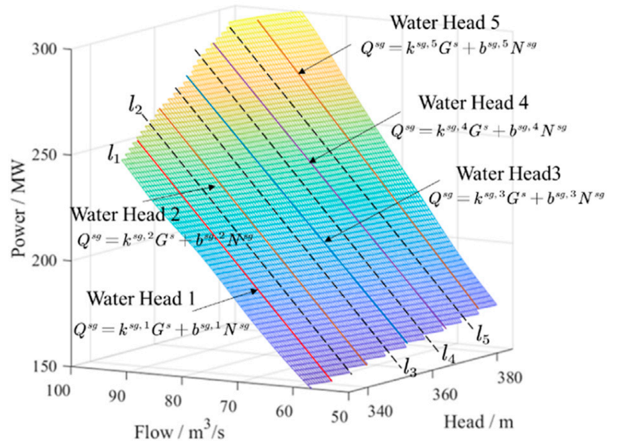

FIGURE 2
Typical water heads and the corresponding function for fixed-speed units in generating mode.

$$R^{e}_{\omega,t} = \left(G^{se}_{\omega,t} + G^{ve}_{\omega,t} - P^{se}_{t} - P^{ve}_{\omega,t}\right)\Delta t \cdot \lambda^{e}_{t} \tag{2}$$

$$R^{r}_{\omega,t} = \left(G^{sr}_{\omega,t} + G^{vr}_{\omega,t} + P^{vr}_{\omega,t}\right) \cdot \left(\lambda^{rp}_{t} + m\lambda^{re}_{t}\right)\Delta t \tag{3}$$

$$C^{PH}_{t} = C^{su}N^{su}_{t} + C^{sd}N^{sd}_{t} + C^{vu}N^{vu}_{t} + C^{vd}N^{vd}_{t} \tag{4}$$

Eq. 2 represents the revenues of PSHP with fixed-speed and variable units in the energy market. Eq. 3 depicts the revenues of the PSHP from the regulation market. The payment of regulation services consists of a capacity payment and a performance-based payment. The former payment indicates the opportunity costs of enabled capacities, while the latter reflects the contribution to the frequency regulation. As uncertainties exist in the Automatic Generation Control (AGC) signal, a scenario-based optimization objective function is established to consider the uncertainty of the AGC signals. Eq. 4 shows the total startup and shutdown cost of the PSHP at time t.

2.2 Constraints

$$N^{sp}_{t+1} = N^{sp}_{t} + N^{su}_{t} - N^{sd}_{t} \tag{5}$$

$$N^{vp}_{t+1} = N^{vp}_{t} + N^{vu}_{t} - N^{vd}_{t} \tag{6}$$

$$N^{sp}_{t}, N^{su}_{t}, N^{sd}_{t}, N^{vp}_{t}, N^{vu}_{t}, N^{vd}_{t} \geq 0 \tag{7}$$

$$\sum_{t} N^{su}_{t}(k) + N^{sd}_{t}(k) \leq A \cdot N_{s} \tag{8}$$

$$\sum_{t} N^{vu}_{t}(k) + N^{vd}_{t}(k) \leq A \cdot N_{v} \tag{9}$$

$$0 \leq P^{state}_{t} + G^{state}_{t} \leq 1 \tag{10}$$

$$0 \leq N^{sg}_{t} \leq N_{s}G^{state}_{t} \tag{11}$$

$$0 \leq N^{sp}_{t} \leq N_{s}P^{state}_{t} \tag{12}$$

$$0 \leq N^{vg}_{t} \leq N_{v}G^{state}_{t} \tag{13}$$

$$0 \leq N^{vp}_{t} \leq N_{v}P^{state}_{t} \tag{14}$$

$$0 \leq N^{sg}_{t} \leq N_{s} - N^{sp}_{t-1} \tag{15}$$

$$0 \leq N^{vg}_{t} \leq N_{v} - N^{vp}_{t-1} \tag{16}$$

Constraints (5–7) show the number of units in the pump state in operation. Constraints (8, 9) limit the number of times that units can switch between the pump state and the generation state. Constraint (10) guarantees that the PSHP cannot pump and generate at the same time. The number of units in operation should be less than the total number of units by constraints (11–14). Besides, units are not

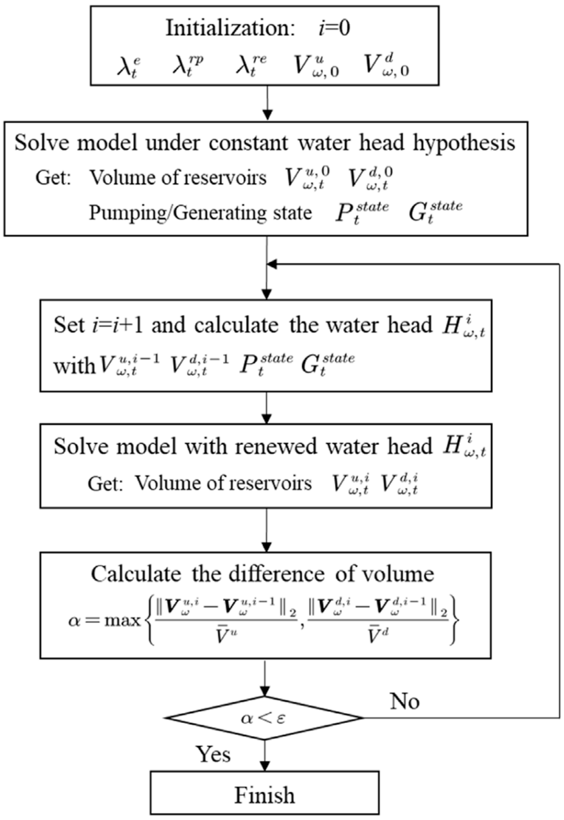

FIGURE 3
Flowchart of the iteration method.

allowed to change from pump state to generation state between adjacent dispatch periods according to constraints (15–16).

$$P_t^{se} = N_t^{sp} \cdot \bar{P}^{sp} \tag{17}$$

$$G_{\omega,t}^{s,\exp} = G_{\omega,t}^{se} + s_{\omega,t}^{up/dw} G_{\omega,t}^{sr} \tag{18}$$

$$0 \le G_{\omega,t}^{sr} \le \bar{G}_{\omega,t}^{sg} - G_t^{se} \tag{19}$$

$$0 \le G_{\omega,t}^{sr} \le G_{\omega,t}^{se} - \underline{G}_{\omega,t}^{sg} \tag{20}$$

$$G_{\omega,t}^{v,\exp} = G_{\omega,t}^{ve} + s_{\omega,t}^{up/dw} G_{\omega,t}^{vr} \tag{21}$$

$$0 \le G_{\omega,t}^{vr} \le \bar{G}_{\omega,t}^{vg} - G_{\omega,t}^{ve} \tag{22}$$

$$0 \le G_{\omega,t}^{vr} \le G_{\omega,t}^{ve} - \underline{G}_{\omega,t}^{vg} \tag{23}$$

$$P_{\omega,t}^{v,\exp} = P_{\omega,t}^{ve} - s_{\omega,t}^{up/dw} P_{\omega,t}^{vr} \tag{24}$$

$$0 \le P_{\omega,t}^{vr} \le \bar{P}_{\omega,t}^{vp} - P_{\omega,t}^{ve} \tag{25}$$

$$0 \le P_{\omega,t}^{vr} \le P_{\omega,t}^{ve} - \underline{P}_{\omega,t}^{vp} \tag{26}$$

Constraints (17–26) show the pumping and generating power constraints for fixed-speed and variable-speed units in the PSHP. The pumping and generating power for each unit should be within the feasible operating region. As the PSHP participates in the regulation market, the expected pump and generation power is limited by constraints (18, 21, 24). The operating power is the sum of the pump and generation power participating in the day-ahead market and the actual power participating in the regulation market.

$$V_{\omega,t}^u = V_{\omega,t-1}^u - Q_{\omega,t}^g \cdot \Delta t + Q_{\omega,t}^p \cdot \Delta t \tag{30}$$

$$V_{\omega,t}^d = V_{\omega,t-1}^d + Q_{\omega,t}^g \cdot \Delta t - Q_{\omega,t}^p \cdot \Delta t \tag{31}$$

$$\bar{V}^u \ge V_t^u \ge \underline{V}^u \tag{32}$$

$$\bar{V}^d \ge V_t^d \ge \underline{V}^d \tag{33}$$

$$\Delta V_{\max} \ge V_{T_{all}}^u - V_0^u \ge -\Delta V_{\max} \tag{34}$$

The volume constraints of the upper and the lower reservoirs are shown in constraints (30–34). Constraints (30, 31) represent the volume balance of the PSHP between dispatch periods. In operation, the amount of water stored in the reservoir should not exceed the reservoir capacity. Besides, the daily volume change should be limited by constraint (34) to guarantee the reservoir's starting volume of the next day.

3 An improved model for the nonlinear "power-flow-water head" characteristics of PSHP

In Section 2, a general day-ahead scheduling model is established for PSHP. However, the model does not include detailed constraints to describe the relationship between the water flow and the power for the fixed-speed and variable-speed units. In this section, an improved mathematical model for the units are established considering the nonlinear relationship among power, flow, and water head.

3.1 Overview of the traditional constant water head (CWH) model

This is the most commonly used model in the previous studies, but it has an obvious error as the hydraulic effects on the power and flow are not considered. Under the constant water head hypothesis, the power is approximately proportional to the flow.

$$Q_{\omega,t}^g = \frac{1}{\eta_g \cdot \rho_0 \cdot g \cdot h_0}\left(G_{\omega,t}^{s,\exp} + G_{\omega,t}^{v,\exp}\right) \tag{35}$$

$$Q_{\omega,t}^p = \frac{\eta_p}{\rho_0 \cdot g \cdot h_0}\left(P_t^{se} + P_{\omega,t}^{v,\exp}\right) \tag{36}$$

Equations (35) and (36) indicate the linear relationship between the water flow and the generating/pumping power of the PSHP. The water head is assumed to maintain as h_0 in operation. However, the water head varies with the volume of the reservoirs, leading to the inaccuracy of the model.

3.2 An improved linearized model based on a cluster of typical water heads (ACTWH)

In operation, the variation of the water head influences the flow and the pumping/generating power. Usually, the detailed parameters can be obtained through experiments before the unit is put into operation.

Figure 1 shows the nonlinear relationship among water head, flow, and power for a variable-speed unit in generating/pumping

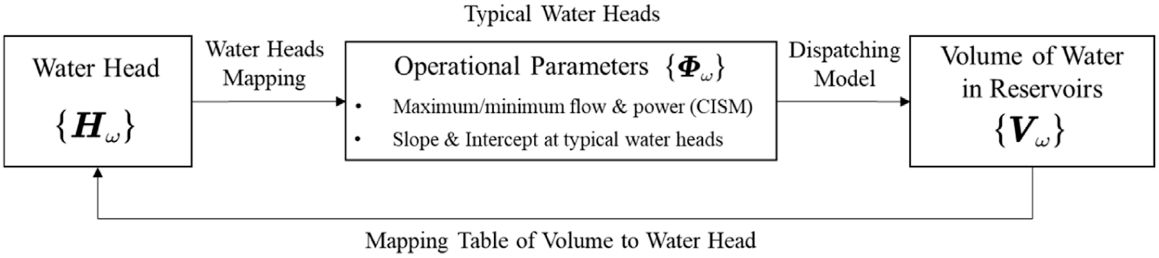

FIGURE 4
Iterative searching process for the optimal dispatch results.

mode, separately. In order to maintain the operating life of the unit up to the design life, there are restrictions on the operating zone of the unit in generating/pumping modes. Without loss of generality, this section constructs a pumping and generating model for the units considering the water head effects according to references (Wood et al., 2013; Mousavi et al., 2019; Alvarez, 2020). The data used can be found at https://pan.baidu.com/s/14QUJkkbEkAr10JZ7XhvUFA?pwd=1111. In real application, the model parameters can be replaced based on experimental data during the optimization.

For the fixed-speed units in generation mode and variable-speed units in generating/pumping mode, the power is approximately proportional to the flow at the same water head. A series of linear functions reflecting the relationship between power and flow can be established by choosing a cluster of water heads at equal distances as shown in Figure 2.

The function of the power varying with flow at any water head can be represented by the linear function at the nearest typical water heads. For example, when the water head is between l_1 and l_2 in Figure 2, the relationship between power and flow can be represented by the line named Water Head 1. Assuming that the water head is close to the jth typical water head at time k under scenario ω, the constraints for the units in generating/pumping mode can be represented by

$$Q_{\omega,t}^{sg} = k_t^{sg,j} G_{\omega,t}^{s,\exp} + b_t^{sg,j} N_{\omega,t}^{sg} \tag{37}$$

$$Q_t^{sp} = b_t^{vg,j} N_t^{sp} \tag{38}$$

$$Q_{\omega,t}^{vg} = k_t^{vg,j} G_{\omega,t}^{v,\exp} + b_t^{vg,j} N_t^{vg} \tag{39}$$

$$Q_{\omega,t}^{vp} = k_t^{vp,j} P_{\omega,t}^{v,\exp} + b_t^{vp,j} N_t^{vp} \tag{40}$$

$$\underline{Q}^{j,sg} \cdot N_t^{sg} \le Q_{\omega,t}^{sg} \le \bar{Q}^{j,sg} \cdot N_t^{sg} \tag{41}$$

$$\underline{G}^{j,sg} \cdot N_t^{sg} \le G_{\omega,t}^{sg} \le \bar{G}_i^{j,sg} \cdot N_t^{sg} \tag{42}$$

$$\underline{Q}^{j,vg} \cdot N_t^{vg} \le Q_{\omega,t}^{vg} \le \bar{Q}^{j,vg} \cdot N_t^{vg} \tag{43}$$

$$\underline{G}^{j,vg} \cdot N_t^{vg} \le G_{\omega,t}^{vg} \le \bar{G}^{j,vg} \cdot N_t^{vg} \tag{44}$$

$$\underline{Q}^{j,vp} \cdot N_t^{vp} \le Q_{\omega,t}^{vp} \le \bar{Q}^{j,vp} \cdot N_t^{vp} \tag{45}$$

$$\underline{P}^{j,vp} \cdot N_t^{vp} \le P_{\omega,t}^{vp} \le \bar{P}^{j,vp} \cdot N_t^{vp} \tag{46}$$

$$\underline{V_j}^{u} \le V_{\omega,t}^{u} \le \bar{V}_j{}^{u} \tag{47}$$

Equations (37)–(40) show the linearized function of flow varying with power at typical water head j in hour t under scenario ω for the fixed-speed and variable-speed units in generating/pumping mode, respectively. Constraints (41–46) demonstrate the upper and lower limits of the power and flow for fixed-speed and variable-speed units in the generation/pumping state. Constraint (47) shows the volume limits of upper reservoir.

4 Customized iterative solution method for the ACTWH based PSHP

The optimal scheduling model for a PSHP with multi-type of units is established based on the ACTWH idea in Sections 2, 3. The objective function of the model is (1), and the constraints of the model are composed of (5)-(34) and (37)-(47). In this model, the indices of the water heads at each hour in (37)-(47) are hyper-parameters, which influences the choice of the solution method.

One solution method is to reformulate the ACTWH-based model as an MILP problem by the Big M technique. In this case, the model can be directly solved by commercial solvers, and details are given in the Appendix. However, there is a dilemma when using the Big M technique based direct solution method (DSM): 1) increasing the typical water heads introduce many auxiliary binary variables and relevant constraints, which significantly increases the complexity and solution time of the model; 2) a small number of typical water heads are unable to precisely reflect the nonlinear relationship between power and flow at different water heads.

4.1 Design of the customized iterative solution method (CISM)

To relieve the bottleneck of DSM, CISM is designed in this section to shorten the solution time and improve the practicability of the ACTWH-based model for PSHPs. The procedures are detailed below and illustrated by Figure 3.

Firstly, the volume of reservoirs (i.e., $V_{\omega,t}^{u,0}$ and $V_{\omega,t}^{d,0}$) and pumping/generating state (i.e., P_t^{state} and G_t^{state}) are calculated by solving the dispatch model of PSHPs under CWH hypothesis, where the objective function is (1) and the constraints include Constraints (5)–(36). This is a simple MILP model and can be rapidly solved by commercial solvers. Set the iteration counter $i = 1$, and the iteration begins.

In the ith iteration, according to $V_{\omega,t}^{u,i-1}$, $V_{\omega,t}^{d,i-1}$, P_t^{state} and G_t^{state} obtained above, the water heads at each hour in daily operation $H_{\omega,t}^{i}$ can be renewed by referring to the design parameters of the PSHP.

TABLE 1 Total revenues of PSHP for different model.

	SMCWHH	CISM	DSM
Revenues in Energy Market (M¥)	1.17	1.15	1.78
Revenues in Regulation Market (M¥)	3.12	2.24	2.30
Startup/Shutdown Cost (M¥)	0.05	0.05	0.05
Total Profits (M¥)	4.24	3.34	4.03
Solving Time(s)	87.6	184.9	15,123

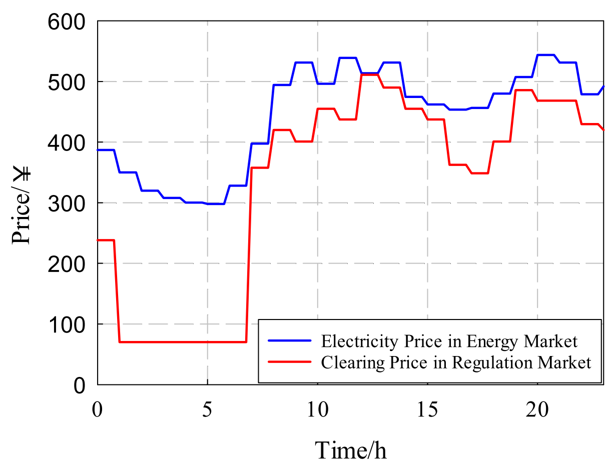

FIGURE 5
Clearing price in energy and regulation market.

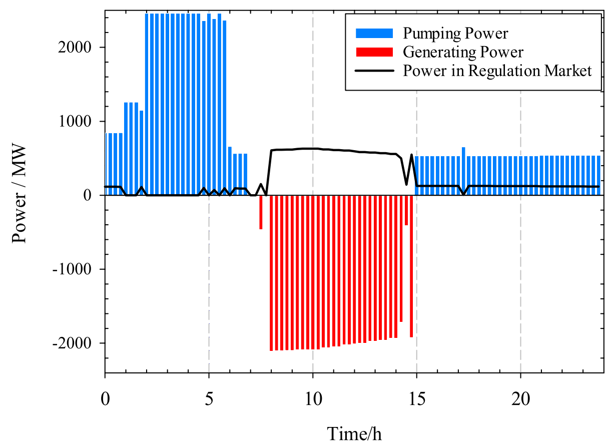

FIGURE 6
Operation results of PSHP with CISM under scenario 5.

Then the renewed water heads $H^i_{\omega,t}$ are introduced into the dispatch model composed of (1), (5)-(34) and (37)-(47) to uniquely determine the hyper-parameter j in constraints (37)–(47). By solving the renewed dispatch model, the updated $V^{u,i}_{\omega,t}$ and $V^{d,i}_{\omega,t}$ are obtained. The iteration continues until the relative errors of $V^{u,i}_{\omega,t}$

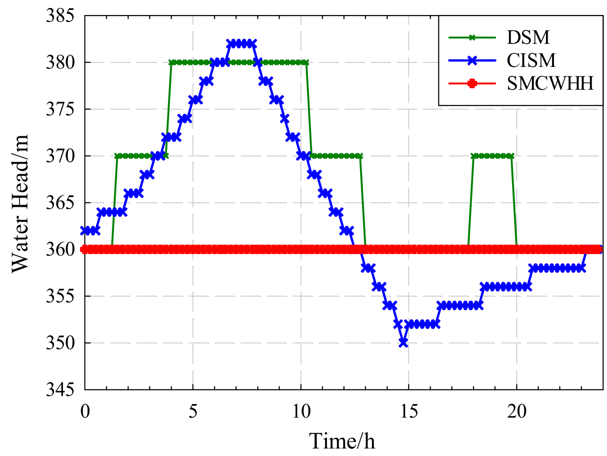

FIGURE 7
Water head profiles under scenario 5 with different solution methods.

and $V^{d,i}_{\omega,t}$ are less than the predefined gap, otherwise run to the (i+1)-th iteration and set $i = i$+1.

Finally, the results of the last iteration are the optimal dispatch strategy considering the effects of the water heads.

With CISM, the renewal of water heads is decoupled with the optimization of the ACTWH-based model, and the auxiliary binary variables and constraints in DSM are all eliminated, which reduce the complexity and solution time.

4.2 Proof of CISM by deduction from the fixed-point view

The iteration of CISM can be described by the interaction between the $\boldsymbol{H}_\omega$, $\boldsymbol{\Phi}_\omega$ and $\boldsymbol{V}_\omega$ presented in Figure 4. Each box in Figure 4 represents some key state variables, and links between boxes stand for the corresponding decision processes formulated above, which constitute one closed directed loop.

For such interacted models, the optimal solutions should make them reach optimality simultaneously, which is not guaranteed. Hence, we adopt the fixed-point theory to analyze whether the solutions exist or not.

First, we define some mappings according to the logic in Figure 4:

- $\{\boldsymbol{\Phi}_\omega\} = M_{\mathrm{WH}}(\{\boldsymbol{H}_\omega\})$: According to the proposed ACTWH idea, with the water head $\boldsymbol{H}_\omega$ given, mapping M_{WH} determines the parameters of the linear function at the typical water heads, which are denoted by $\{\boldsymbol{\Phi}_\omega\}$. It is illustrated by Figure 2 that M_{WH} is constructed by a mapping function instead of an optimization process, so its feasibility is ensured. Besides, the mapping in Figure 2 is obviously continuous because the variation of $\{\boldsymbol{H}_\omega\}$ falling in the domain cannot cause step changes in $\{\boldsymbol{\Phi}_\omega\}$.
- $\{\boldsymbol{V}_\omega\} = M_{\mathrm{DM}}(\{\boldsymbol{\Phi}_\omega\})$: Mapping M_{DM} represents the optimal dispatch model of a PSHP participating in energy and frequency regulation markets, which is an optimal power

TABLE 2 Total revenues of PSHP with different number of variable-speed units.

	Number of variable-speed units					
	0	1	2	3	4	5
Revenue in Energy Market (M¥)	1.20	1.23	1.15	1.00	0.41	0.47
Revenue in Regulation Market (M¥)	1.65	1.87	2.24	2.58	3.37	3.50
Start-up Cost (M¥)	0.05	0.05	0.05	0.05	0.04	0.04
Total Revenue (M¥)	2.80	3.05	3.34	3.53	3.74	3.93

TABLE 3 Total revenues of PSHP in different operation mode.

	Regular mode	HSC mode
Revenue in Energy Market (M¥)	1.15	0.73
Revenue in Regulation Market (M¥)	2.24	2.99
Start-up Cost (M¥)	0.05	0.05
Total Revenue (M¥)	3.34	3.67

flow problem. M_{DM} can also be seen as continuous because: 1) $\boldsymbol{\Phi}_\omega$ is regard as constant parameters instead of optimization variables; 2) $\boldsymbol{V}_\omega$ is not composed of the dual variables of the binding constraints in the optimal dispatch model, so there are no step changes in $\boldsymbol{V}_\omega$ with respect to the variation of $\boldsymbol{\Phi}_\omega$.

- $\{\boldsymbol{H}_\omega\} = M_{\mathrm{VH}}(\{\boldsymbol{V}_\omega\})$: Once the volumes of water in the upper and the lower reservoirs $\boldsymbol{V}_\omega$ are known, the corresponding water levels of the two reservoirs can be easily derived. The difference between the two water levels is the value of the water head $\boldsymbol{H}_\omega$, so the mapping between $\boldsymbol{V}_\omega$ and $\boldsymbol{H}_\omega$ (i.e., M_{VH}) is obviously a continuous function, whose feasibility is also ensured like M_{WH}.

Then the loop in Figure 4 can be described by mapping M_{whole} defined as follows:

$$\{\boldsymbol{H}_\omega\} = M_{\mathrm{whole}}(\{\boldsymbol{H}_\omega\}) = M_{\mathrm{VH}}(M_{\mathrm{DM}}(M_{\mathrm{WH}}(\{\boldsymbol{H}_\omega\}))) \tag{48}$$

With the water head space A defined as

$$A = \left\{\{\boldsymbol{H}_w\} \middle| \boldsymbol{H}_w{}^{\min} \le \boldsymbol{H}_w \le \boldsymbol{H}_w{}^{\max}\right\}, \tag{49}$$

it can be seen that M_{whole} is a self-mapping $A \to A$. Since the mappings M_{WH}, M_{DM}, and M_{VH} are continuous, composite mapping M_{whole} is also continuous.

Due to the continuity, convexity and compactness of A, the fixed point is proved to exist in A according to the Brouwer fixed point theorem (Kellogg et al., 1976), which is the solution to the interacted optimization problem shown in Figure 4. Such fixed-point problem can be commonly solved by the iterative method, which we have given in Figure 4.

5 Case studies

5.1 Simulation settings and parameters

A PSHP with 7 fixed-speed units and 2 variable-speed units is used to perform the case study, with the goal of maximizing its total revenue. The rated power of the fixed-speed units is 300 MW, while the counterpart of variable-speed units is 330 MW. The operation characteristics of the units are shown in Figure 1. Bath County PSHP is chosen to carry out the case studies, and the relationship between the volume and the water head is proportionally scaled by referring to Shisanling PSHP, China. The basic parameters of Bath County PSHP given in (Cao et al., 2021) are used.

The interval between two adjacent typical water heads is 10 m, while the counterpart for a cluster of water heads is 2 m. At each chosen water head, the power approximately shows a linear relationship to the flow. Therefore, the discretized model of the “power-flow-water head” for units can be used to optimize the operation of PSHP. The time interval is 15 min in daily operation, while the bidding power in the regulation market remains the same in an hour.

The forecast prices for energy and regulation are from (Kazempour et al., 2009). The RegA-type regulation signals of PJM market for a whole year are used to establish the energy demand for frequency regulation (Zhang et al., 2018; Cheng et al., 2023; Zhou et al., 2024). The energy demand for frequency regulation are calculated by averaging regulation signals in every 15 min, and the k-means clustering method is used to generate 10 scenarios to represent the potential frequency regulation requirements. The probability of scenarios ranges between 0.8% and 47.1%. Besides, according to historical statistical data, the average regulation mileage is 2.75 (Xia et al., 2016).

The models are solved by using GUROBI 9.1.0 solver and CVX toolbox in MATLAB environment (Grant and Boyd, 2014). Besides, numerical simulations are performed on a laptop containing an Intel Core i7-4700MQ CPU with 2.4 GHz and 16 GB of RAM.

5.2 Simulation results

The objective of the model is to maximize the profits of the PSHP with multi-type of units. According to the simulation results, the solution time of DSM is significantly longer than that of the other methods. This subsection compares the optimization results solved by three solution methods, including Solution Method under CWH Hypothesis (SMCWHH), DSM and CISM.

Table 1 shows the operational results of PSHP under different solution methods. According to the results, the PSHP can get more profits from the regulation market than the energy market. SMCWHH is the fastest solution method, and the total profit is also the largest. In fact, since the water head increases in pump mode, more energy needs to be cost. Besides, the actual generation power declines with the decrease of water head, which further leads to a decrease in revenue. Therefore, more profit is expected to be gotten for SMCWHH, which is an error for the solution method.

Figure 5 demonstrates the clearing price in the energy and regulation market (Kazempour et al., 2009). According to Figure 5, the electricity price is low between 0:00 and 7:00. The total pumping and generating power of PSHP with CISM under scenario 5 is shown in Figure 6. The units pump when the electricity is low, while generating when the electricity is high. For the fixed-speed units, the units can only participate in the regulation market in generation mode. When the PSHP generates power, more power can participate in the regulation market to get more profits.

Figure 7 shows the water head profiles calculated by different solution methods. In daily operation, the water heads with the volume of water stored in the reservoirs. Besides, the change in water heads has an effect on the power and flow. For the constant water head model, the water heads remain the same in daily operations, causing errors to some extent. Since the relationship between power and flow at different heads can be better approximated, the model solved by CISM can give the closest solution to the actual operation.

5.3 Discussions

Sensitivity analyses are performed by setting the different number of variable-speed units in the subsection. Besides, this part also shows the change of revenue of PSHP, when the PSHP is working in the hydraulic short-circuit mode.

5.3.1 Impact of the number of variable-speed units on simulation results

Assuming the total number of units is 9 in the PSHP, Table 2 compares the effect of different variable-speed units on the profits of the PSHP by setting the number of variable-speed units from 0 to 5. According to Table 2, the revenue of the PSHP is mainly from the regulation market. As the generating power of variable-speed units can change in a wider range at the same water head compared to the fixed-speed units and its pumping power can also be changed in pumping mode, a PSHP with more variable-speed units can gain more revenue from the regulation market.

5.3.2 Impact of hydraulic short-circuit (HSC) mode for PSHP

In HSC mode, PSHP can pump and generate simultaneously. Although the efficiency of the HSC mode is less than the regular mode, the power of PSHP can be regulated within a wider range under the HSC mode. Table 3 demonstrates the operation results of PSHP in the regular and HSC mode. When the PSHP works in HSC mode, it loses 0.42 M¥ in the energy market. However, working in HSC mode also makes the PSHP gain 0.75 M¥ more in the regulation market than the PSHP working in the regular mode. According to Table 3, the PSHP can gain more profits from the regulation market in HSC mode.

6 Conclusion

This paper proposes an optimal scheduling method for the PSHP with variable-speed and fixed-speed units considering variable head effects. The objective is to maximize the profits of the PSHP by participating in the energy and regulation markets. The nonlinear relationship of power and flow for the variable-speed and fixed-speed units is established in detail by considering the water head effects. Besides, two iteration solution methods for revising the water head are proposed to optimize the operation of multi-type of units in PSHP. The numerical results show that the scheduling results are closer to the actual operation than the results under the assumption of constant water head. The iteration solution method considering a cluster of water heads can give the most practical solution with less time. Moreover, the PSHP can get more profits by installing more variable-speed units or by operating in short-circuit mode. Besides, PSHP is expected to participate in the intra-day market to further improve the flexibility of the power systems, so its two-stage dispatch model considering the water head effects is among our future work.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

MC: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Visualization, Writing–original draft. ZH: Conceptualization, Investigation, Software, Writing–review and editing. JC: Conceptualization, Formal Analysis, Supervision, Writing–original draft, Writing–review and editing.

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Conflict of interest

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Appendix: Direct Soution Method (DSM) under Typical Water Head

The dispatch model based on ACTWH proposed in Section II and III can be reformulated as an MILP model as follows.

Firstly, to ensure that the water head at any hour is represented by only one typical water head, we add an equality constraint (48).

$$\sum_{j=1}^{H_c} L_{\omega,t}^{j} = 1 \tag{A1}$$

where all $L_{\omega,t}^{j}$ is an 0–1 variable, and only one of them can take 1 in hour t. If $L_{\omega,t}^{j}$ is 1, the water head in scenario ω in hour t is represented by typical water head j.

Then, with the introduction of $L_{\omega,t}^{j}$, constraints (37)–(46) can be reformulated as

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) \le Q_{\omega,t}^{sg} - \left(k^{sg,j}G_{\omega,t}^{sg} + b^{sg,j}N_t^{sg}\right) \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) \tag{A2}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{Q}^{j,sg}\cdot N_t^{sg} \le Q_{\omega,t}^{sg} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{Q}^{j,sg}\cdot N_t^{sg} \tag{A3}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{G}^{j,sg}\cdot N_t^{sg} \le G_{\omega,t}^{sg} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{G}_i^{j,sg}\cdot N_t^{sg} \tag{A4}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) \le Q_{\omega,t}^{sp} - b^{sp,j}N_t^{sp} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) \tag{A5}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) \le Q_{\omega,t}^{vg} - \left(k^{vg,j}G_{\omega,t}^{vg} + b^{vg,j}N_t^{vg}\right) \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) \tag{A6}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{Q}^{j,vg}\cdot N_t^{vg} \le Q_{\omega,t}^{vg} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{Q}^{j,vg}\cdot N_t^{vg} \tag{A7}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{G}^{j,vg}\cdot N_t^{vg} \le G_{\omega,t}^{vg} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{G}^{j,vg}\cdot N_t^{vg} \tag{A8}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) \le Q_{\omega,t}^{vp} - \left(k^{vp,j}P_{\omega,t}^{vp} + b^{vp,j}N_t^{vp}\right) \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) \tag{A9}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{Q}^{j,vp}\cdot N_t^{vp} \le Q_{\omega,t}^{vp} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{Q}^{j,vp}\cdot N_t^{vp} \tag{A10}$$

$$-\mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \underline{P}^{j,vp}\cdot N_t^{vp} \le P_{\omega,t}^{vp} \le \mathrm{M}\left(1-L_{\omega,t}^{j}\right) + \bar{P}^{j,vp}\cdot N_t^{vp} \tag{A11}$$

where a large number M called Big M is introduced to coordinate a series of exclusive constraints at different typical water heads. When $L_{\omega,t}^{j}$ is 1, the power and flow should be limited within the allowable range at water head j, and the flow shows a linear relationship with the power. When $L_{\omega,t}^{j}$ is 0, all of the constraints are relaxed.

$$L_{\omega,t}^{j}\cdot \underline{V_j}^{u} \le V_{\omega,t}^{u,j} \le L_{\omega,t}^{j}\cdot \bar{V}_j^{u} \tag{A12}$$

$$\sum_{j=1}^{H_c} V_{\omega,t}^{u,j} = V_{\omega,t}^{u} \tag{A13}$$

Constraints (A12)-(A13) are supplementary volume constraints for the upper reservoir in operation. As the volume of reservoir is equally divided by the typical water heads, the actual volume at each moment can only be in one of these intervals. When $L_{\omega,t}^{j}$ is 1, it means the volume in scenario ω in hour t is within the range from $\underline{V_j}^{u}$ to $\bar{V}_j^{u}$. Otherwise, the volume is not within the interval.

According to the reformulation above, the model solved by the direct method is constructed as:

1) The objective function is shown as (1);
2) The constraints consist of Constraints (5)–(34) and (A1)–(A13). In theory, such an MILP problem can be directly solved by commercial solvers, which is not elaborated here.

Nomenclature

1 Set and Index

i	Index for times of iteration
j	Index for typical water head
t	Index for hour t
$\boldsymbol{\omega}$	Index for scenario
$\boldsymbol{\Omega}$	Index for set of all scenarios
sp,sg	Index for fixed-speed units in pumping/generating mode
vp,vg	Index for variable-speed units in pumping/generating mode
se,ve	Index for power in energy market for fixed-speed/variable-speed units
sr,vr	Index for power in regulation market for fixed-speed/variable-speed units
CWH	Index for the optimization model under constant water head hypothesis

2 Constant and Parameters

g	Gravitational acceleration, in m/s^2
ρ_0	Water density, in kg/m^3
α_ω	Probability of the scenario ω
A	Total startup/shutdown time of the units in a day
C^{su}, C^{sd}	Startup and shutdown cost for the fixed-speed units, in ¥
C^{vu}, C^{vd}	Startup and shutdown cost for the variable-speed units, in ¥
m	average mileage
N_s, N_v	Number of fixed-speed and variable units
h_0	Average water head, in m
H_c	Total number of typical water heads
λ_t^e	Electricity tariff of energy market in hour t, in ¥/MWh
λ_t^{rp}	Electricity tariff of regulation capacity, in ¥/MW.
λ_t^{re}	Electricity tariff of regulation mileage, in ¥/MWh
$\bar{p}^{(\cdot)}$, $\underline{p}^{(\cdot)}$	Maximum/Minimum rated pumping power for units, in MW.
$\bar{g}^{(\cdot)}$, $\underline{g}^{(\cdot)}$	Maximum/Minimum rated generating power for units, in MW.
$\bar{p}^{(\cdot),j}$, $\underline{p}^{(\cdot),j}$	Maximum/Minimum pumping power for units at water head j, in MW.
$\bar{g}^{(\cdot),j}$, $\underline{g}^{(\cdot),j}$	Maximum/Minimum generating power for units at water head j, in MW.
$k^{(\cdot),j}$	Slope of the linear function mapping power to flow for units at water head j
$b^{(\cdot),j}$	Intercept of the linear function mapping power to flow for units at water head j
$s_t^{up/dw}$	Upward/Downward average regulation signal in hour t
Δt	Time interval
T_{all}	Total number of time intervals in a day
$\bar{V}^u$, $\underline{V}^u$	Maximum/Minimum volume of upper reservoir
$\bar{V}^d$, $\underline{V}^d$	Maximum/Minimum volume of lower reservoir
$\Delta V_{\max}$	Allowable change of the volume of the reservoir

3 Variables

P_t^{state}, G_t^{state}	Binary variables for pumping/generating state in hour t
$L_{\omega,t}^j$	Binary variables for showing water head position in scenario ω in hour t
N_t^{su}, N_t^{sd}	Number of startup/shutdown fixed-speed units
N_t^{vu}, N_t^{vd}	Number of startup/shutdown variable-speed units
$N_t^{(\cdot)}$	Number of units in operation in hour t
$P_t^{(\cdot)}$	Pumping power in hour t, in MW.
$G_t^{(\cdot)}$	Generating power in hour t, in MW.
$G_t^{s,\exp}$	Expectation value of generating power of fixed-speed units participating in electricity market in hour t, in MW
$G_t^{v,\exp}$, $P_t^{v,\exp}$	Expectation value of generating/pumping power of variable-speed units participating in electricity market in hour t, in MW
V_t^u, V_t^d	Volume of upper/lower reservoir in hour t, in m^3
$Q_t^{(\cdot)}$	Pumping/Generating flow for units in hour t, in m^3/s
$R_{\omega,t}^e$, $R_{\omega,t}^r$	Total revenues of PSHP in energy market/regulation market in scenario ω in hour t