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Radiative couple stress Casson hybrid nanofluid flow over an inclined stretching surface due to nonlinear convection and slip boundaries

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The study of fluid dynamics due to the stretching surface is one of the most eminent topics due to its potential industrial applications viz. drawing wire and plastic films, metal and polymer extrusion, fiber and glass production. In the present article, the author is going to study the effects of hybrid nanofluids flow on an inclined plate including CuO (Copper Oxide), and Cu (Copper). The Casson fluid with a couple-stress term has been used in the flow analysis. The surface of the plate is considered slippery. The convection has been taken nonlinear with thermal radiation. The governing equation of the flow of hybrid nanofluids with energy equation has been transformed into highly nonlinear ODEs through similarity transformation. The proposed model has been solved through a numerical RK-4 method. Significant variables of the physical process such as solar radiation, nonlinear convection parameters, heat transfer rates, and their effect on the solar power plant have been noticed.

KEYWORDS

(Cu, CuO,) nanoparticles, thermal radiations, inclined plate, nonlinear convection, numerical RK-4 method

Introduction

Rising energy needs around the world, including irreversible means of energy such as natural fuels, energy storage, heat exchangers, and thermal resources. The production of these real resources, the result in huge detrimental effects on the environment, such as air pollution and global warming. To mitigate these losses, scientists have focused on techniques that improve renewable energy skills, like the production of solar energy. The spotless and cheapest renewable source of energy is solar energy, which may be converted into environmentally sociable thermal energy. These kinds of energies can be found in the shape of (solar collectors) and heat-changing liquids.

The collectors acquire solar rays by an absorbent plate and convey heat further to the absorbent solution such as water, water mixture). Nevertheless, their significant loss is the lower thermal capability of these conventional fluids, as they reveal low thermal efficiency in the alteration development. Converting conventional working liquids into nanofluids is one of the initiatives that has received much attention over the past few years to enhance the thermal efficiency of this technology. Nanofluids refer to a stable dispersion of solid particles of sizes between 1 and 100 nm (Choi, 1995). Nanofluids are widely used in various physical processes such as energy storage (Gul et al., 2019). Mebarek-Oudina (Mebarek-Oudina, 2019) studied the flow of nanofluids using different base liquids Sheikholeslami et al. (2014) presented a complete numerical simulation of nanofluid flow with magnetic and viscous dissipation effects. Because of the excellent use of nano liquid, Li et al. (2019) studied the flow of nano liquid within a porous duct by using external power in the form of the Buongiorno model. The impacts of magnetic hydrodynamics (MHD) on the heat trade-off dynamics of chemically reactive water base nano-liquids containing Cu/Ag in a rotating disk inside the permeable channel have been investigated by Reddy et al. (2017). Many recent studies have been conducted in the area of energy and thermal environments using analytical and numerical methods for handling heat exchange and nano-fluids. For instance, Zaim et al. (2020) and Khan et al. (2019), Hybrid nanofluid is obtained by combining two types of nanomaterials, which are added to conventional liquids, and are used in many heat exchange areas. Khan et al. (2017a) and Khan et al. (2020a) formulated and analyzed the transient flow of$(Cu - Al_2O_3/H_2O)$, and concluded that the Nusselt number performance improved considerably through the addition of 5% nanomaterials. Hayat and Nadeem (Hayat and Nadeem, 2017) investigated the inspiration of energy exchange through the increasing surface of hybrid nanofluids $(Cu - Ag/H_2O)$. Ali Lunda et al. (2020) studied the impacts of viscous dissipation on the flow of hybrid nano liquid $(Cu - Al_2O_3/H_2O)$ by analyzing the stability of the shrinking plate. Aziz et al. (2020) and Khan et al. (2017b) considered the flow of hybrid nano-fluids on an extended plate. Sundar et al. (2020) has been examined the resistive aspect and energy transport phenomenon of$(MWCNT - Fe_3O_4/H_2O)$ hybrid nano liquid. Sohail et al. (2020) examined the three-dimensional movement of nanofluid in a flexible medium along with thermal radiation. Besides the aforementioned research papers, the following current articles can also be cited for getting more knowledge on the hybrid nano liquid process in various geometries. For example, Khan et al. (2021), Tahir et al. (2017), Khan et al. (2020b).

Recently, particles of different shapes on a closed porous surface and liquid moving in the permeable medium have attracted the attention of scientists. Their use can be measured in various fields such as nuclear engineering, and environmental sciences. Various physical processes that require the liquid movement on a porous medium include the use of blood flow in the veins or lungs, chemically catalysts connectors, geothermal energy, porous heat pipes, and porous heat pipes. As a Forchheimer term, it was introduced in Forchheimer (1901). Many researchers have used Darcy-Forchheimer concepts in various geometries for the study of fluid flow in a porous space. We are going to mention a few of them. Saif et al. (2019) discussed the movement of nano-fluids through a porous space. The variation in the motion of a liquid was created an expandable curved surface. Rasool et al. (2019) reported the flow of Darcy-Forchheimer nano-fluids produced by the stretching medium. The Darcy-Forchheimer liquid that flows through a spinning disc was discovered by Sadiq et al. (2020). Sheikholeslami et al. (2020) (Rasool et al., 2022) observed the behavior of non-Darcy liquid within a clear cavity. Hayat et al. (2020b), Sheikholeslami et al. (2020) examined the influence of Darcy-Forchheimer and EMHD on the flow of viscous liquids with joule heating and thermal flux over an extending surface. The numerical outcomes of CNTs nano-liquid flow across the divergent and convergent channels with thermal radiation have been calculated by Kumar et al. (2020) (Hayat et al., 2020a). There are various technical methods available for estimating temperature. The significance of radiation for heat and flow transmission is understood, especially in the fields of glass manufacturing, rocket engineering, furnace construction, nuclear plant, solar farms, physical science, and manufacturing, etc. Of such importance, the imposition of thermal radiation for heat transport is shown in Kumar et al. (2020). The different properties of such fluids were further studied by many researchers. Nanofluid flow reduces (Bilal, 2020; Khan et al., 2020c; Algehyne et al., 2022) the resistance to heat transfer for different flow systems. With the passage of time researchers came to know that the spread of two different types of nanoparticles in a pure fluid further augment the thermal flow characteristics. They termed such fluid as hybrid nanofluid. Rasool et al. (2022) and Wakif et al. (2022) have inspected the dynamics of hybrid nanofluid flow with the influence of thermal radiations and viscous dissipation past a stretching surface and have determined the influence of various parameters. It has been observed by the authors that the upper branch of the solution has been highlighted to be more applicable due to its stable nature (Alghamdi et al., 2021; Xia et al., 2021; Shah et al., 2022). have deliberated an incredible work to discuss the thermal flow improvement for hybrid nanofluid flow by means of different flow geometries and flow conditions. Recently, it has been noticed by researchers that the suspension of three different types of nanoparticles in a pure fluid can enhance the thermal conductivity of such fluid to the best possible limit. Such fluids are termed tri-hybrid nanofluids.

The main aim of the ongoing study is to observe the impact of (Cu, CuO) nano-components on the flow and heat transmission of water as a base fluid. Such formulation of fluid is used further on the inclined plate to improve the efficiency of solar collectors. Thermal radiation, nonlinear convection, and slip conditions are considered while formulating basic equations. The flow-related issues were formulated *via* differential equations which were solved numerically using the BVP-4 technique. Different significant quantities are discussed in terms of temperature and velocity.

Model analysis

The flow of the hybrid nanofluids containing $(Cu - CuO)$ nanoparticles is considered over an inclined plate that makes an angle ψ with the upright axis as shown in the. Figure 1. The surface of the plate is considered slippery to reduce the stability of the external dust particles. The thermal convection is expanded and taking nonlinear. The basic equations in the presence of thermal radiation and Couple stress are displayed as.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho_{hnf}\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = \left(1 + \frac{1}{\beta}\right)\mu_{hnf}\left(\frac{\partial^2 u}{\partial y^2}\right) \pm g(\rho)_{hnf}\left[(\beta_T)_{hnf}(T - T_\infty) + (\beta_T)^2_{hnf}(T - T_\infty)^2\right]\mathrm{Cos}\,\psi - \eta^*\frac{\partial^4 u}{\partial y^4}, \tag{2}$$

$$\left(\rho c_p\right)_{hnf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{hnf}\frac{\partial T^2}{\partial y^2} + \frac{16}{3}\left(\frac{\sigma^* T^3_\infty}{k^*}\frac{\partial^2 T}{\partial y^2}\right), \tag{3}$$

Acceptable boundary conditions are

$$u = u_w + \mu_{hnf}\frac{\partial u}{\partial y},\ v = 0, -k_{hnf}\frac{\partial T}{\partial y} = (T_w - T), \quad at\ y = 0, \qquad u = 0 = v,\ T \to T_\infty, \quad at\ y \to \infty\,. \tag{4}$$

The velocity components u and $v$in the x and y-direction,

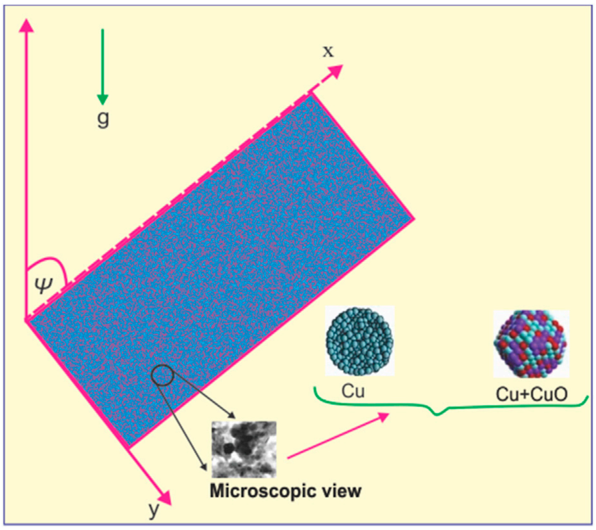

FIGURE 1
The solar collector absorbing model.

Mathematical formulation of thermos-physical properties HNF

$$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \phi_1)^{2.5}\quad(1 - \phi_2)^{2.5}}, \tag{5}$$

$$\frac{\rho_{hnf}}{\rho_f} = \left[(1 - \phi_2)\left\{1 - \left(1 - \frac{\rho_{Cu}}{\rho_f}\right)\phi_1\right\} + \phi_2\frac{\rho_{CuO}}{\rho_f}\right], \tag{6}$$

$$\frac{k_{hnf}}{k_{nf}} = \frac{k_{CuO} + (n-1)k_{nf} - (n-1)\phi_2\left(k_{nf} - k_{CuO}\right)}{k_{CuO} + (n-1)k_{nf} + \phi_2\left(k_{nf} - k_{CuO}\right)}, \qquad \frac{k_{nf}}{k_f} = \frac{k_{Cu} + (n-1)k_f - (n-1)\phi_1\left(k_f - k_{Cu}\right)}{k_{Cu} + (n-1)k_f + \phi_1\left(k_f - k_{Cu}\right)}, \tag{7}$$

$$\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f} = \left[(1 - \phi_2)\left\{1 - \left(1 - \frac{(\rho Cp)_{Cu}}{(\rho Cp)_f}\right)\phi_1\right\} + \phi_2\frac{(\rho Cp)_{CuO}}{(\rho Cp)_f}\right], \tag{8}$$

In the above mathematical expression, ϕ_1 denotes the nanomaterials (Cu) volume fraction whereas ϕ_2 denotes the volume fraction of (CuO) nanocomponents.

Introduction to useful similarity transformation variables as follows:

$$u = f'(\eta)\,bx,\ v = -\sqrt{b v_f}\,f(\eta),\ (T_w - T_\infty)\,\theta(\eta) = T - T_\infty,\ \eta = y\sqrt{\frac{b}{v_f}}, \tag{9}$$

In the light of Eq.11, the Eqs 1–5 become

$$\left(1 + \frac{1}{\beta}\right)f''' + \frac{\rho_{hnf}}{\rho_f}\frac{\mu_f}{\mu_{hnf}}\left[f f'' - \left(f'\right)^2\right] \pm \frac{\mu_f}{\mu_{hnf}}\left[Gr\theta + Gr^*(\theta)^2\right]\mathrm{Cos}\,\psi - k\,f^{v} = 0, \tag{10}$$

$$\left(\frac{k_{hnf}}{k_f} + Rd\right)\theta'' + \mathrm{Pr}\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f}f\theta' = 0, \tag{11}$$

$$f(0) = 0, f'(0) = 1 + \frac{\Lambda}{(1 - \phi_1)(1 - \phi_2)}f''(0), \theta'(0) = -B_i(1 - \theta(0)), f(\infty) = \theta(\infty) \to 0. \tag{12}$$

In terms of mutual boundary conditions:

$$Gr = \frac{g\beta_T(T_w - T_\infty)}{bu_w}, Gr^* = \frac{g\beta_T^2(T_w - T_\infty)^2}{bu_w}, Rd = \frac{16}{3}\frac{\sigma^* T^3_\infty}{k^* k}, k = \frac{\eta^* b}{v_f^2 \rho_f}, \mathrm{Pr} = \frac{v_f}{\alpha_f}, \Lambda = \mu_f\sqrt{\frac{b}{v_f}}. \tag{13}$$

The above-mentioned physical numbers are Grashof numbers, Couple stress term, Radiation factor, Slip velocity, parameter, and Prandtl number, Velocity slip, and Biot number.

In addition, some of the most useful physical numbers are, as, (Sherwood number), Skin friction coefficient (C_{fx}), and Nusselt number (Nu_x).

$$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho_{hnf}\left(u_w\right)^2}, Nu_x = \frac{xq_w}{k_{hnf}\left(T_w - T_\infty\right)}. \tag{14}$$

Where q_w represents heat flux close to the surface, and τ_w denotes shear stress, employing Eqs. 9 and Eq. 14 gives,

$$\begin{aligned} C_{fx}Re_x^{0.5} &= \frac{2}{\left(1-\phi_1\right)^{2.5}\left(1-\phi_2\right)^{2.5}} f''(0), \\ Nu_x Re_x^{-0.5} &= -\left(\frac{k_{hnf}}{k_f} + Rd\right)\Theta'(0). \end{aligned} \tag{15}$$

Numerical method

The RK-4 numerical method has been used. The modeled Eqs 10, 11 are transformed to the first order by considering.

$$\begin{aligned} &x_1 = \eta, x_2 = f, x_3 = f', x_4 = f'', x_5 = f''', x_6 = f^{iv}, \\ &x_7 = \theta, x_8 = \theta'. \end{aligned} \tag{16}$$

The first order ODEs system (Zaydan et al., 2022) has been solved with the efforts of the projected variables as shown in Eq. 16. The first order system using the RK4 method has been obtained as:

$$\begin{aligned} &D_\eta x_1 = 1, D_\eta x_2 = x_3, D_\eta x_3 = x_4, D_\eta x_5 = x_6, D_\eta x_7 = x_8 \\ &D_\eta x_6 = \frac{1}{k}\left[\left(1+\frac{1}{\beta}\right)x_5 + \frac{\rho_{hnf}}{\rho_f}\frac{\mu_f}{\mu_{hnf}}\left(x_2x_4 - \left(x_3\right)^2\right) \pm \frac{\mu_f}{\mu_{hnf}}\left(Grx_7 + Gr^* x_7x_7\right)\mathrm{Cos}\,\psi\right], \\ &D_\eta x_8 = -\left(\frac{k_{hnf}}{k_f} + Rd\right)^{-1}\Pr\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f}x_2x_8. \end{aligned} \tag{17}$$

The first order system solution obtained and the results are analyzed for various embedded parameters.

Results and discussion

The hybrid nanofluid flow over a slippery surface of an inclined plane is considered for the applications of heat transfer. The impact of $f'(\eta)$ (velocity profile) and $\Theta(\eta)$ (temperature profile) quantitatively *via* differents tables and graphs for various active variables such as Λ (slip parameter), Gr, Gr^* (Gravitational parameters), B_i (Biot number), ϕ_1, ϕ_2 (Volume fractions), and Rd (Radiation factor) while considering (Cu + Water), (CuO + Water) nano liquid and hybrid nanofluid. The schematic diagram of the flow field is shown in Figure 1. Figure 2 The flow of fluid is increases over the slippery surface and this increase is more effective due to the larger values of the slip parameter Λ. As the fluid moves over the surface, it traps more heat from the sheet, causing significant thermal dispersion. Intriguingly, even under favorable thermal transmission conditions, the entropy rate decreases to improve the quality of the slow-moving barrier. This can improve the slip to influence the suspended components and consequently the

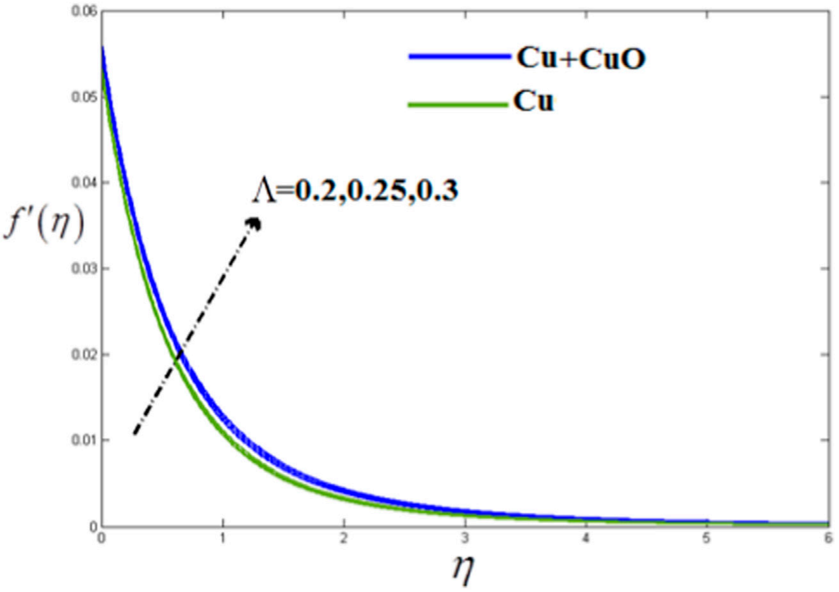

FIGURE 2
Λ versus $f'(\eta)$. When, $\phi_1, \phi_2 = 0.02, k = 0.2, Gr = Gr^* = Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

fluid motion is increases. Figure 3 The influence of the couple stress parameter versus velocity profile is shown in Figure 3. It is perceived that the development in couple stress terms results, decreases in the fluid motion at all locations. The rise in the couple stress parameter values improves the resistance force, to decline the velocity of the fluid. Figure 4 Incremented fractional volume through particle strength in porosity, hybrid nano-fluids also affects the flow rate. As a result, it increases the production of thermal transport and entropy in both types of flowing fluids and reduces the velocity profile by incrementing the volume fraction. Figure 5 Incremented Casson fluid parameter, increasing the resistive force and declining the fluid motion. The decline effect is compared for both Cu and CuO hybrid nano-fluids (Figures 6, 7).The increment in the positive value of the gravity parameters Gr, Gr^{*}will raise the velocity profile and the velocity of the liquid is improved by the positive value of the gravity parameter. The opposite impact declines the fluid motion. The comparative analysis for both Cu and Cu-CuO shown has been displayed in Figures 6, 7 which signifies that the velocity profile decrement due to the negative value of the gravity parameter.

Figure 8 Biot number B_i denotes an incremental convectively thermal rates that can affect the area of interest related to temperature. According to the limitation to heat production, the Biot number B_i tends to increase the current thermal rates in the flow region but makes the lower portion of the sheet. According to previous entropy and thermal plots, the Biot number B_i is also a factor in the elevation of the thermal diffusion, which simultaneously increases the rate of entropy. Figure 9 As the volume friction value rises which reduces the temperature profile. will decrease. This is because when nanoparticles are put in the conventional fluid, the nanoparticles increase in temperature, which will increase the temperature profile.

Figure 10 illustrates the state of thermal radiation sequentially, for the growing values of the thermal radiation barrier. Radiation heat raises the thermal state of the environment of interest, which places a greater thermal transfer load on the passing fluid, which increases the thermal conductivity to the radiation barriers. The thermal properties of the solid materials and base fluid are displayed in Table. 1. The skin friction improved with the increasing values of the parameters$\phi_1, \phi_2, k, Gr, Gr^{*}$as displayed in Table. 2. Physically, the greater values of these factors enrich the resistance force and subsequently skin friction enhancing. The heat transfer rate growths with the cumulative values of the nanoparticle volume fraction and Radiation factors as displayed in Table 3. The heat transfer rate enhancing for the nanofluids Cu up to 8.1501% using $\phi_1 = 0.01$ and 8.473% increases for $Cu + CuO$ hybrid nanofluids using $\phi_1, \phi_2 = 0.01$. Similarly, the rate of heat transfer escalats gradually with the growing amount of nanoparticle volume fraction. From Table 4 it has been detected that hybrid nanofluids are more proficient for heat transfer enhancement. The comparison of the current results is compared with the existing literature considering base fluid only and shown in Table 4.

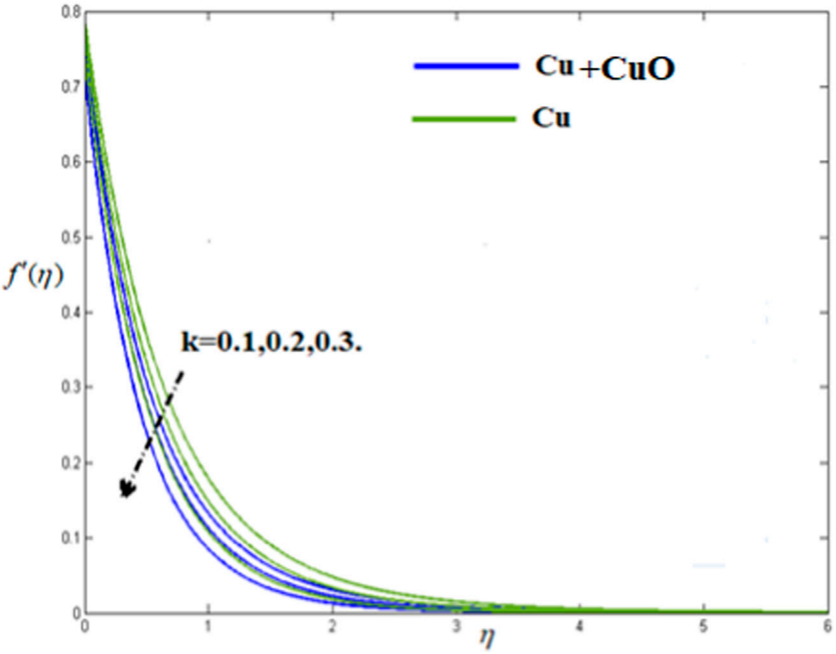

FIGURE 3
k versus $f'(\eta)$. When, $\phi_1, \phi_2 = 0.02, \Lambda = 0.4, Gr = Gr^{*} = Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

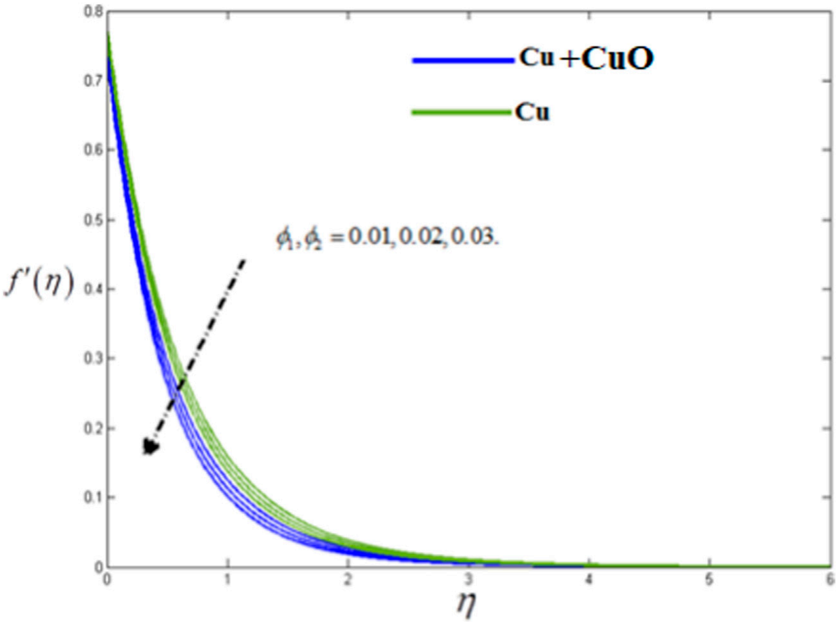

FIGURE 4
ϕ_1, ϕ_2 versus $(f'(\eta))$. When, $k = 0.2, \Lambda = 0.4, Gr = Gr^* = Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

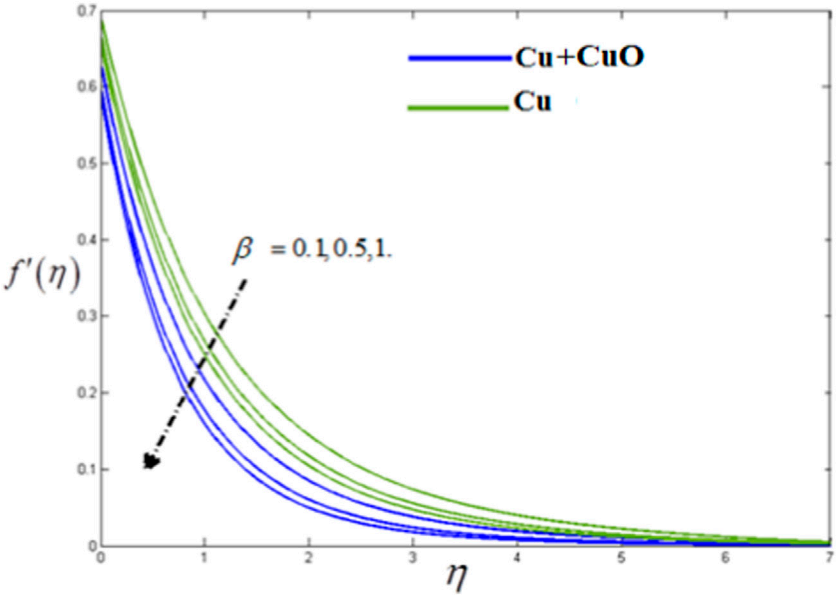

FIGURE 5
β versus $(f'(\eta))$. When, $k = 0.2, \Lambda = 0.4, \phi_1, \phi_2 = 0.02, Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

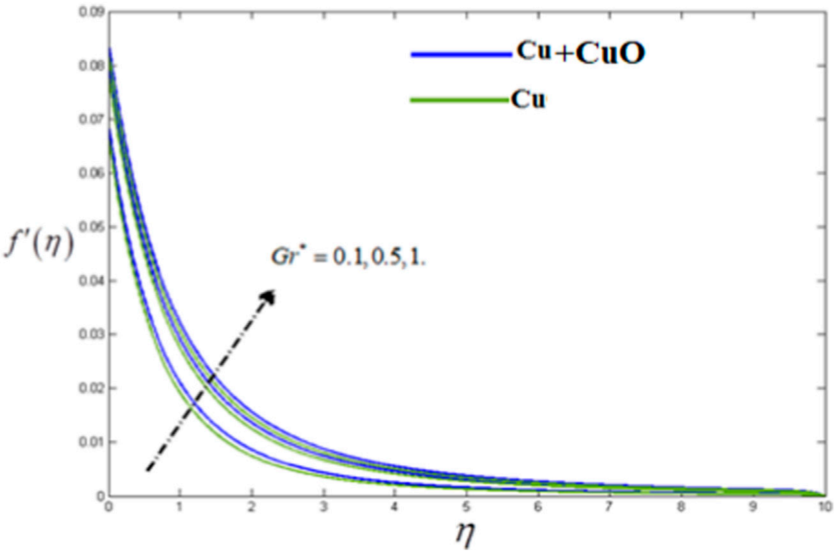

FIGURE 6
Gr^* versus $(f'(\eta))$. When, $k = 0.2, \Lambda = 0.4, \phi_1, \phi_2 = 0.02, Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

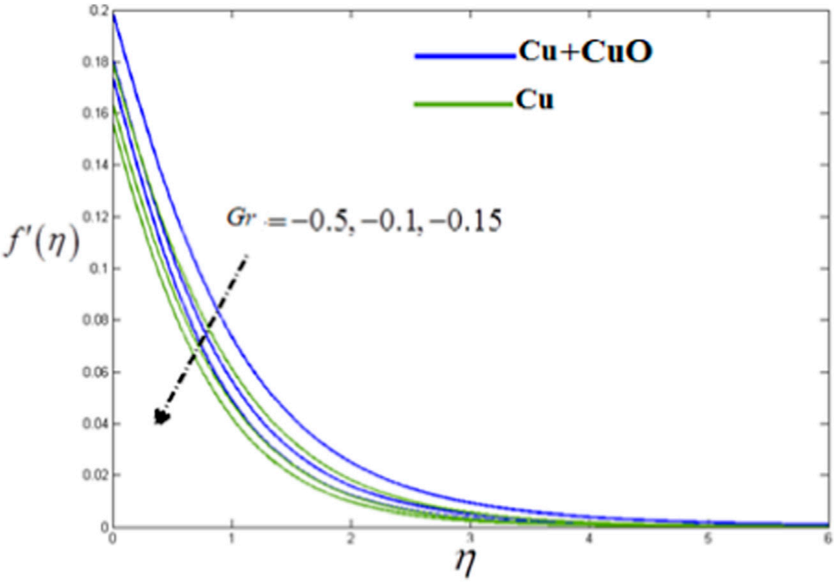

FIGURE 7
Gr versus $f'(\eta)$. When, $k = 0.2, \Lambda = 0.4, \phi_1, \phi_2 = 0.02, Rd = 0.3, B_i = 0.1, \Pr = 6.2$.

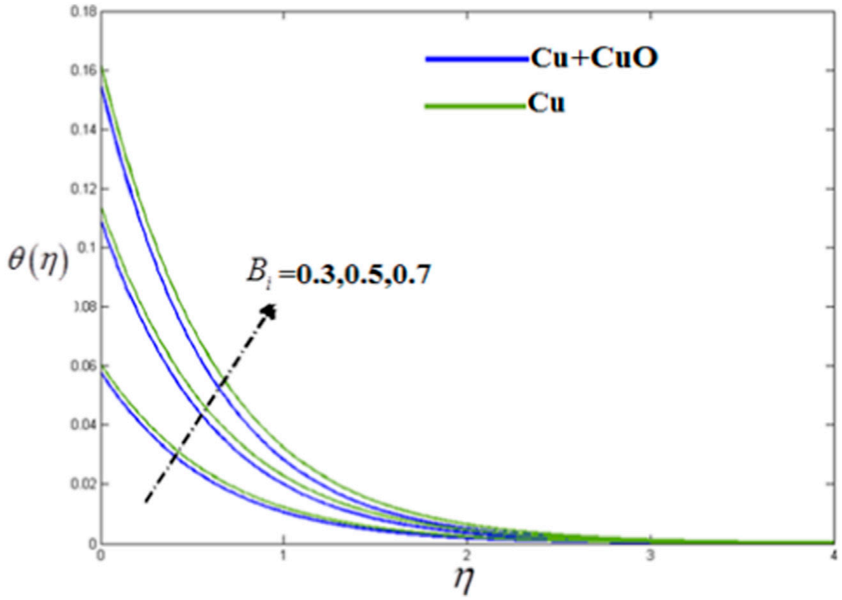

FIGURE 8
B_i versus $\theta(\eta)$. When, $k = 0.2, \Lambda = 0.4, \phi_1, \phi_2 = 0.02, Rd = 0.3, Gr = 0.1, \Pr = 6.2$.

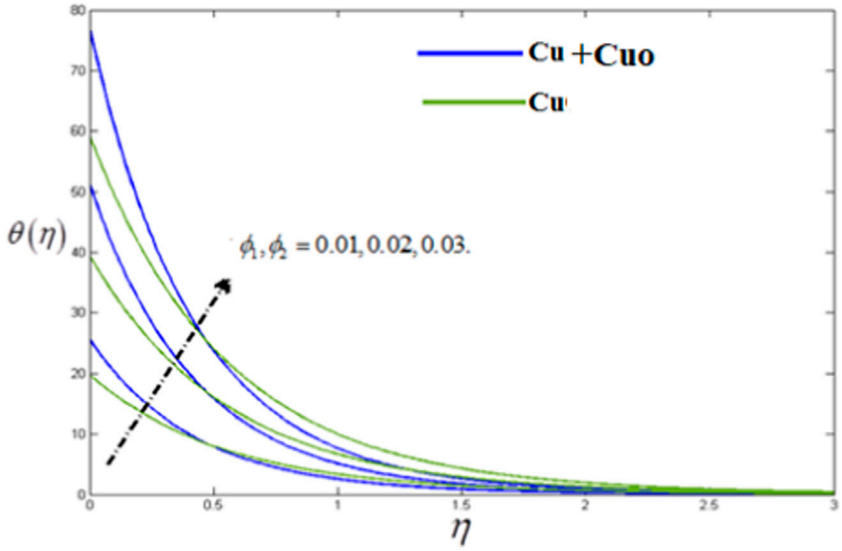

FIGURE 9
ϕ_1, ϕ_2 versus $\theta(\eta)$. When, $k = 0.2, \Lambda = 0.4, B_i = 0.2, Rd = 0.3, Gr = 0.1, \Pr = 6.2$.

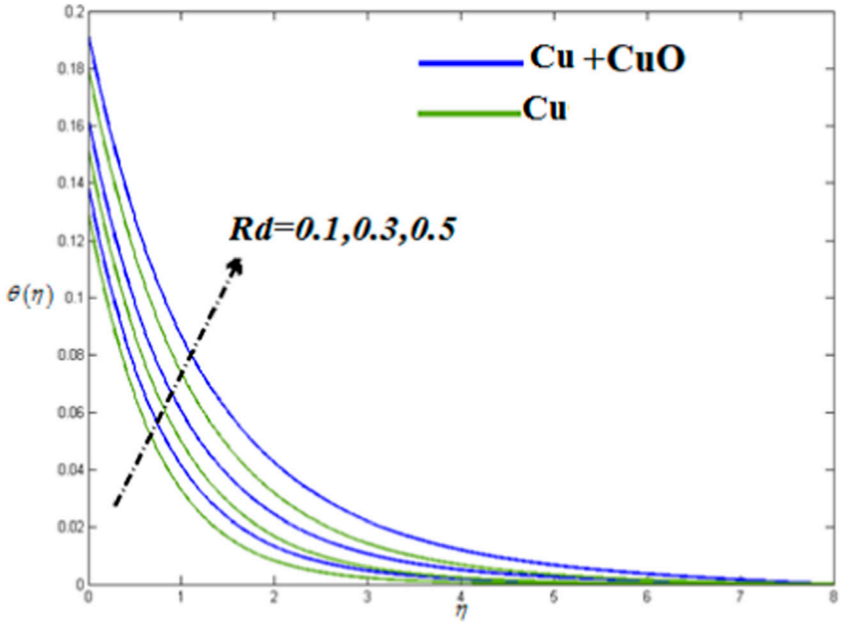

FIGURE 10
Rd versus $\theta(\eta)$. When, $k = 0.2, \Lambda = 0.4, B_i = 0.2, \phi_1, \phi_2 = 0.02, Gr = 0.1, \Pr = 6.2$.

TABLE 1 *Cu &CuO*thermo-physical properties.

Property	Cu	CuO
$\rho(Kgm^{-3})$	6320	3580
$c_p(Kg^{-1}K^{-1})$	531.8	960
$k(Wm^{-1}K^{-1})$	76.5	48.4

TABLE 2 $C_{fx}Re_x^{0.5}$ numerical outcomes versus different values of emerging parameters.

Gr	ϕ_1, ϕ_2	k	Gr^*	$Cf_x - Cu$	$Cf_x - Cu\&CuO$
0.2	0.0	0.2	0.2	0.472836	0.473952
0.3				1.473962	1.4745742
0.4				1.4749321	1.47575322
	0.01			1.4735318	1.474421331
	0.02			1.4744321	1.47538012
		0.4		1.483120	1.4846352
		0.6		1.49431275	1.49532125
			0.4	1.47343546	1.47446874
			0.6	1.47412023	1.4753214091

TABLE 3 $Nu_xRe_x^{-0.5}$ numerical outcomes versus different values of emerging parameters and % enhancement in heat transfer rate.

ϕ_1, ϕ_2	Rd	$Nu_x - Cu$	$Nu_x - Cu\&CuO$
0.0	0.1	0.132421	0.132421
0.01		0.143214	0.1436422
		8.1501%	8.473%
0.02		0.14491	0.146474
		9.431%	10.612
0.03		0.1458321	0.147623
		10.127%	11.48%
	0.2	0.523416	0.5243215
	0.3	0.621571	0.6223532
	0.5	0.62314	0.6243212

TABLE 4 Quantitative analogy with [39], [40] using $Nu_xRe_x^{-0.5}$.

Pr	**Wang** Wang (1989)	**Golra and sidawi** Gorla and Sidawi (1994)	**Recent**
6.3	0.24532	0.24545	0.24548
6.5	0.194522	0.194642	0.194653
6.7	0.135281	0.135393	0.135412

Conclusion

The impact of the Cu and CuO hybrid nanofluid flow for the enhancement of heat transfer rate has been examined through a slippery surface of the inclined plate. The convection is taken quadratic and due to the stretching of the plate, the gravity force is assumed negative and positive. The significant physical characteristics of Nusselt number versus radiation and other physical constraints have been noticed. In this article, some significant points present our conclusion in the following remarks. For Gr, Gr^* impact have been analyzed and the comparative results for the Cu& CuO hybrid nanofluid are observed. The higher value of the nanoparticle volume fractions ϕ_1, ϕ_2 incremented the temperature distribution $\Theta(\eta)$. The heat transfer rate enhancing for Cu up to 10.127% using $\phi_1 = 0.01, 0.02, 0.03$ and 11.48% increases for $Cu + CuO$ hybrid nanofluids using $\phi_1, \phi_2 = 0.01, 0.02, 0.03$. The Biot number increases the temperature distribution for its larger amount. The larger value of the parameter Rd enhances the rate of heat transfer and, as a result, the Nusselt number rises. Nanoliquids are more viscous than ordinary liquids, which reflects that the boiling point of nanoliquids is more than that of conventional base liquids. This would improve the heat transfer power of the solar collectors.

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Author contributions

TG, modeling and solution; SM, writing draft WA, editing. ZR, Validated, SEA and ETE, participated in revision and provide funding source.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

References

Algehyne, E. A., Wakif, A., Rasool, G., Saeed, A., and Ghouli, Z. (2022). Significance of Darcy-forchheimer and lorentz forces on radiative alumina-water nanofluid flows over a slippery curved geometry under multiple convective constraints: A renovated buongiorno's model with validated thermophysical correlations. *Waves Random Complex Media* 2022, 1–30. doi:10.1080/17455030.2022.2074570

Alghamdi, M., Wakif, A., Thumma, T., Khan, U., Baleanu, D., and Rasool, G. (2021). Significance of variability in magnetic field strength and heat source on the radiative-convective motion of sodium alginate-based nanofluid within a Darcy-Brinkman porous structure bounded vertically by an irregular slender surface. *Case Stud. Therm. Eng.* 28, 101428. doi:10.1016/j.csite.2021.101428

Ali Lunda, L., Omara, Z., Khan, I., Seikh, A., El-Sayed, H., Sherif, M., et al. (2020). Stability analysis and multiple solution of Cu–Al_2O_3/H_2O nanofluid contains hybrid nanomaterials over a shrinking surface in the presence of viscous dissipation. *J. Mat. Res. Technol.* 9 (1), 421–432. doi:10.1016/j.jmrt.2019.10.071

Aziz, A., Jamshed, W., Ali, Y., and Shams, M. (2020). Heat transfer and entropy analysis of Maxwell hybrid nanofluid including effects of inclined magnetic field, Joule heating and thermal radiation. *Discrete Contin. Dyn. Syst. Ser. S* 13 (10), 101428. doi:10.3934/dcdss.20201.42

Bilal, M. (2020). Micropolar flow of EMHD nanofluid with nonlinear thermal radiation and slip effects. *Alexandria Eng. J.* 59 (2), 965–976. doi:10.1016/j.aej.2020.03.023

Choi, U. S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *AS-MEFED* 231, 99–103.

Forchheimer, P. (1901). Wasserbewegung durch boden. *Z. Des. Vereins Dtsch. Ingenieure* 45, 1782–1788.

Gorla, R. S. R., and Sidawi, I. (1994). Free convection on a vertical stretching surface with suction and blowing. *Appl. Sci. Res.* 52 (3), 247–257. doi:10.1007/bf00853952

Gul, T., Noman, W., Sohail, M., and Khan, M. A. (2019). Impact of the Marangoni and thermal radiation convection on the graphene-oxide-water-based and ethylene-glycol-based nanofluids. *Adv. Mech. Eng.* 11 (6), 1687814019856773. doi:10.1177/1687814019856

Hayat, T., Haider, F., and Alsaedi, A. (2020a). Darcy-Forchheimer flow with nonlinear mixed convection. *Appl. Math. Mech.* 41 (11), 1685–1696. doi:10.1007/s10483-020-2680-8

Hayat, T., Khan, S. A., Alsaedi, A., and Fardoun, H. M. (2020b). Heat transportation in electro-magnetohydrodynamic flow of Darcy-Forchheimer viscous fluid with irreversibility analysis. *Phys. Scr.* 95 (10), 105214. doi:10.1088/1402-4896/abb7aa

Hayat, T., and Nadeem, S. (2017). Heat transfer enhancement with Ag–CuO/water hybrid nanofluid. *Results Phys.* 7, 2317–2324. doi:10.1016/j.rinp.2017.06.034

Khan, A., Saeed, A., Tassaddiq, A., Gul, T., Mukhtar, S., Kumam, P., et al. (2021). Bio-convective micropolar nanofluid flow over thin moving needle subject to Arrhenius activation energy, viscous dissipation and binary chemical reaction. *Case Stud. Therm. Eng.* 25, 100989. doi:10.1016/j.csite.2021.100989

Khan, S. U., Tlili, I., Waqas, H., and Imran, M. (2020). Effects of nonlinear thermal radiation and activation energy on modified second-grade nanofluid with Cattaneo–Christov expressions. *J. Therm. Anal. Calorim.* 143 (2), 1175–1186. doi:10.1007/s10973-020-09392-6

Khan, U., Zaib, A., and Mebarek-Oudina, F. (2020). Mixed convective magneto flow of SiO_2–MoS_2/$C_2H_6O_2$ hybrid nanoliquids through a vertical stretching/shrinking wedge: Stability analysis. *Arab. J. Sci. Eng.* 45 (11), 9061–9073. doi:10.1007/s13369-020-04680-7

Khan, U., Ahmed, N., and Mohyud-Din, S. T. (2017). Influence of the shape factor on the flow and heat transfer of a water-based nanofluid in a rotating system. *Eur. Phys. J. Plus* 132 (4), 166–211. doi:10.1140/epjp/i2017-11410-4

Khan, U., Ahmed, N., and Mohyud-Din, S. T. (2020). Surface thermal investigation in water functionalized Al2O3 and γAl2O3 nanomaterials-based nanofluid over a sensor surface. *Appl. Nanosci.*, 1–11. doi:10.1007/s13204-020-01527-3

Khan, U., Alkanhal, T. A., Ahmed, N., Khan, I., and Mohyud-Din, S. T. (2019). Stimulations of thermophysical characteristics of nano-diamond and silver nanoparticles for nonlinear radiative curved surface flow. *IEEE Access* 7, 55509–55517. doi:10.1109/access.2019.2907304

Khan, U., Asadullah, M., Ahmed, N., and Mohyud-Din, S. T. (2017). Influence of Joule heating and viscous dissipation on MHD flow and heat transfer of viscous fluid in converging/diverging stretchable channels. *J. nanofluids* 6 (2), 254–263. doi:10.1166/jon.2017.1327

Kumar, K. G., Rahimi-Gorji, M., Reddy, M. G., Chamkha, A. J., and Alarifi, I. M. (2020). Enhancement of heat transfer in a convergent/divergent channel by using carbon nanotubes in the presence of a Darcy–Forchheimer medium. *Microsyst. Technol.* 26 (2), 323–332. doi:10.1007/s00542-019-04489-x

Li, Z., Saleem, S., Shafee, A., Chamkha, A. J., and Du, S. (2019). Analytical investigation of nanoparticle migration in a duct considering thermal radiation. *J. Therm. Anal. Calorim.* 135 (3), 1629–1641. doi:10.1007/s10973-018-7517-z

Mebarek-Oudina, F. (2019). Convective heat transfer of Titania nanofluids of different base fluids in cylindrical annulus with discrete heat source. *Heat. Trans. Res.* 48 (1), 135–147. doi:10.1002/htj.21375

Rasool, G., Shafiq, A., Khalique, C. M., and Zhang, T. (2019). Magnetohydrodynamic Darcy-Forchheimer nanofluid flow over a nonlinear stretching sheet. *Phys. Scr.* 94, 105221. doi:10.1088/1402-4896/ab18c8

Rasool, G., Shah, N. A., El-Zahar, E. R., and Wakif, A. (2022). Numerical investigation of EMHD nanofluid flows over a convectively heated riga pattern positioned horizontally in a Darcy-forchheimer porous medium: Application of passive control strategy and generalized transfer laws. *Waves Random Complex Media* 2022, 1–20. doi:10.1080/17455030.2022.2074571

Reddy, P. S., Sreedevi, P., and Chamkha, A. J. (2017). MHD boundary layer flow, heat and mass transfer analysis over a rotating disk through porous medium saturated by Cu-water and Ag-water nanofluid with chemical reaction. *Powder Technol.* 307, 46–55. doi:10.1016/j.powtec.2016.11.017

Sadiq, M. A., Haider, F., Hayat, T., and Alsaedi, A. (2020). Partial slip in Darcy-Forchheimer carbon nanotubes flow by rotating disk. *Int. Commun. Heat Mass Transf.* 116, 104641. doi:10.1016/j.icheatmasstransfer.2020.104641

Saif, R. S., Hayat, T., Ellahi, R., Muhammad, T., and Alsaedi, A. (2019). Darcy-Forchheimer flow of nanofluid due to a curved stretching surface. *Int. J. Numer. Methods Heat. Fluid Flow.* 29, 2–20. doi:10.1108/hff-08-2017-0301

Shah, N. A., Wakif, A., El-Zahar, E. R., Ahmad, S., and Yook, S. J. (2022). Numerical simulation of a thermally enhanced EMHD flow of a heterogeneous micropolar mixture comprising (60%)-ethylene glycol (EG),(40%)-water (W), and copper oxide nanomaterials (CuO). *Case Stud. Therm. Eng.* 35, 102046. doi:10.1016/j.csite.2022.102046

Sheikholeslami, M., Abelman, S., and Ganji, D. D. (2014). Numerical simulation of MHD nanofluid flow and heat transfer considering viscous dissipation. *Int. J. Heat. Mass Transf.* 79, 212–222. doi:10.1016/j.ijheatmasstransfer.2014.08.004

Sheikholeslami, M., Arabkoohsar, A., and Ismail, K. A. R. (2020). Entropy analysis for a nanofluid within a porous media with magnetic force impact using non-Darcy model. *Int. Commun. Heat Mass Transf.* 112, 104488. doi:10.1016/j.icheatmasstransfer.2020.104488

Sohail, M., Naz, R., and Abdelsalam, S. I. (2020). On the onset of entropy generation for a nanofluid with thermal radiation and gyrotactic microorganisms through 3D flows. *Phys. Scr.* 95, 045206. doi:10.1088/1402-4896/ab3c3f

Sundar, L. S., Singh, M. K., and Sousa, A. C. M. (2020). Enhanced heat transfer and friction factor of MWCNT–Fe_3O_4/water hybrid nanofluids. *Int. Commun. Heat Mass Transf.* 52, 73–83. doi:10.1016/j.icheatmasstransfer.2014.01.012

Tahir, F., Gul, T., Islam, S., Shah, Z., Khan, A., Khan, W., et al. (2017). Flow of a nano-liquid film of Maxwell fluid with thermal radiation and magneto hydrodynamic properties on an unstable stretching sheet. *J. nanofluids* 6 (6), 1021–1030. doi:10.1166/jon.2017.1400

Wakif, A., Zaydan, M., Alshomrani, A. S., Muhammad, T., and Sehaqui, R. (2022). New insights into the dynamics of alumina-(60% ethylene glycol+ 40% water) over an isothermal stretching sheet using a renovated buongiorno's approach: A numerical gdqllm analysis. *Int. Commun. Heat Mass Transf.* 133, 105937. doi:10.1016/j.icheatmasstransfer.2022.105937

Wang, C. Y. (1989). Free convection on a vertical stretching surface. *Z. Angew. Math. Mech.* 69 (11), 418–420. doi:10.1002/zamm.19890691115

Xia, W. F., Animasaun, I. L., Wakif, A., Shah, N. A., and Yook, S. J. (2021). Gear-generalized differential quadrature analysis of oscillatory convective Taylor-Couette flows of second-grade fluids subject to Lorentz and Darcy-Forchheimer quadratic drag forces. *Int. Commun. Heat Mass Transf.* 126, 105395. doi:10.1016/j.icheatmasstransfer.2021.105395

Zaim, A., Aissa, A., Mebarek-Oudina, F., Mahanthesh, B., Lorenzini, G., Sahnoun, M., et al. (2020). Galerkin finite element analysis of magneto-hydrodynamic natural convection of Cu-water nanoliquid in a baffled U-shaped enclosure. *Propuls. Power Res.* 9 (4), 383–393. doi:10.1016/j.jppr.2020.10.002

Zaydan, M., Hamad, N. H., Wakif, A., Dawar, A., and Sehaqui, R. (2022). Generalized differential quadrature analysis of electro-magneto-hydrodynamic dissipative flows over a heated riga plate in the presence of a space-dependent heat source: The case for strong suction effect. *Heat. Trans.* 51 (2), 2063–2078. doi:10.1002/htj.22388

Nomenclature

Greek symbols

u, v Velocities components (ms^{-1})

μ_{nf} Dynamic viscosity of nanofluid (mPa)

f Dimensional velocity profiles

μ_f Dynamic viscosity of base fluid (mPa)

T Fluid temperature (K)

ρ_{nf} Nanofluid density (Kgm^{-3})

T_w Wall surface temperature (K)

ρ_f Base fluid density (Kgm^{-3})

T_∞ Free surface temperature (K)

ξ Similarity variable

f Dimensional velocity profilesDimensional

ϕ volume fraction of CuO, $TiO_2$and Al_2O_3 nanoparticles

B_i Biot number

θ Dimensional heat profiles

P Pressure

σ_{nf} electrical conductivity of nanofluid Sm^{-1}

Pr Prandtl number

η Couple stress parameter

Re Local Reynolds number

σ^* Stefan Boltzmann constant

ψ Stream function

a_k Coefficient of mean absorption

Nu Nusselt number

C_f Skin friction coefficient

$(C_p)_f$ Specific heat of base fluid (J/kgK)

k_{nf} Thermal conductivity $(Wm^{-1}K^{-1})$

Subscripts

Thnf Tri-hybrid Nanofluid

nf Nanofluid

f Base fluid