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Intelligent Command Filter Design for Strict Feedback Unmodeled Dynamic MIMO Systems With Applications to Energy Systems

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This study presents a command filtered control scheme for multi-input multi-output (MIMO) strict feedback nonlinear unmodeled dynamical systems with its applications to power systems. To deal with dynamic uncertainties, a dynamic signal is introduced, together with radial basis function neural networks (RBFNNs) to overcome the influences of the dynamic uncertainties. Command filters (CFs) are used to prevent the explosion of complexity, where the compensating signals can eliminate the effect of filter errors. Compared with single-input single-output strict feedback nonlinear systems, the method proposed in this study has more suitability. In the end, the simulation experiments are carried out by applying the developed algorithm to power systems, where the simulation results verify the efficacy of the approach proposed.

Keywords: power system, dynamic uncertainty, command filter, MIMO system, strict feedback nonlinear system

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1 INTRODUCTION

In recent years, adaptive control has become a hotspot because of its strong disturbance-rejection property. Related theories, such as model reference control, robust adaptive control, and adaptive dynamic programming (Mukherjee et al., 2017; Yang et al., 2021b; Han and Liu, 2020; Yang et al., 2021d; L'Afflitto, 2018; Yang et al., 2021e), have been applied to many fields, including power systems, wind energy systems, and multi-agent systems (Li et al., 2020; Xu et al., 2018; Wu et al., 2017; Ghaffarzdeh and Mehrizi-Sani, 2020; Zou et al., 2020b; Ghosh and Kamalasadan, 2017; Namazi et al., 2018; Zou et al., 2020a). Moreover, applications of adaptive control on energy systems are also widely reported (Deese and Vermillion, 2021; Quan et al., 2020; Liu et al., 2022; Nascimento Moutinho et al., 2008; Liu et al., 2021). Among them, backstepping is a powerful tool since many energy systems can essentially be modeled as strict feedback systems, which can be analyzed through the backstepping technique.

The main idea of backstepping is to divide the whole system into a series of subsystems so that they can be analyzed individually. In this way, the control design and stability analysis can both be simplified, especially for large-scale systems (Yang et al., 2021a). Meanwhile, for unmodeled dynamical systems, if the unmodeled dynamics are ignored, the disturbance from dynamic uncertainties may result in unbounded evolution. Therefore, the dynamic uncertainties need to be paid enough attention, which is not considered in the aforementioned literatures. Zhao J. et al. (2021) presented a fuzzy adaptive control approach with an observer design for unmodeled dynamical systems. Xia et al. developed an output feedback control design with quantized performance for dynamic uncertainties in Xia and Zhang (2018). Wang et al. (2017)investigated nonstrict feedback systems with unmodeled dynamics and dead zones through output feedback-based control methods. Although the aforementioned results can successfully tackle dynamic uncertainties, they are not able to deal with the explosion of complexity and avoid the influences of filter errors.

In the backstepping process, the explosion of complexity often occurs because the virtual control is repeatedly differentiated. Meanwhile, the computational complexity increases significantly, which results in the presented design not being suitable for applications (Yang et al., 2020). To deal with this issue, the dynamic surface control method is proposed (Wang and Huang, 2005). The dynamic surface control method uses first-order filters, where the virtual control is replaced by the filter states in each subsystem (Yang et al., 2021c). In this way, the repeated differentiation issue can be evaded. However, filter errors are introduced simultaneously, which degrades the control precision. Thus, command filters (CFs) are developed (Farrell et al., 2009). Based on the dynamic surface control approach, CFs additionally introduce compensating signals to compensate for the loss caused by filter errors, which further improves the control accuracy compared with the dynamic surface control method. Owing to this advantage, CFs are widely applied to many systems. For example, Zhu et al. (2018)investigated a command filtered robust adaptive neural network (NN) control for strict feedback nonlinear systems with input saturation. Zhao L. et al. (2021)presented an adaptive finite-time tracking control design with CFs. The adaptive fuzzy backstepping control approach of uncertain strict feedback nonlinear systems is developed by Wang et al. (2016). However, the applications of the backstepping technique in energy systems are not taken into consideration in these works. In addition, the systems of interest in these works are single-input single-output systems, which may give conservative results. Therefore, in this study, for multi-input multi-output (MIMO) strict feedback nonlinear unmodeled dynamical systems, a command filtered control method is developed and applied to energy systems.

The contributions of this study are two-fold. First, this study designs an adaptive backstepping control scheme for MIMO strict feedback nonlinear unmodeled dynamical systems with CFs, the compensating signal design and controller design are improved such that they can get higher tracking precision. Second, this study investigates the applications of the presented CF-based adaptive backstepping control approach on power systems, and a MIMO circuit system is used in the simulation experiments to verify the effectiveness of the method developed.

The rest of this article is organized as follows. **Section 2** provides the problem formulation and necessary assumptions. In **Section 3**, the control design is proposed. The stability analysis of the system with the presented design is carried out in **Section 4**. In **Section 5**, a voltage source converter-high voltage direct current transmission system is used to verify the efficacy of the proposed method. The conclusion is made in **Section 6**.

2 PROBLEM FORMULATION

In this study, the circuit system under consideration is modeled as

$$\begin{aligned} &\dot{\varsigma} = q(\varsigma,\ X),\\ &\dot{X}_i = F_i\left(\underline{X}_i\right) + G_i X_{i+1} + D_i + \Delta_i(\varsigma,\ X),\\ &\dot{X}_i = F_n(X) + G_n U + D_n + \Delta_n(\varsigma,\ X),\\ &y = X_1, \end{aligned} \tag{1}$$

where $X = [X_1 \dots X_n]^{\mathrm{T}} \in \mathbb{R}^{nm}$, $y \in \mathbb{R}^m$, and $U \in \mathbb{R}^m$ are the system state, output, and the control input, respectively. $F_i(\cdot) : \mathbb{R}^{im} \to \mathbb{R}^m$ is a known continuous function, $q(\cdot,\cdot) : \mathbb{R} \times \mathbb{R}^{nm} \to \mathbb{R}$ is an unknown continuous function, $G_i \neq 0$ is a known constant, $D_i \in \mathbb{R}^m$ is an unknown constant vector, $\underline{X}_i = [X_1,\ \cdots,\ X_i]^{\mathrm{T}} \in \mathbb{R}^{im}$, $\varsigma \in \mathbb{R}$ is the unmeasured portion of the state, and $\Delta_i \in \mathbb{R}^m$ is the unmodeled dynamics.

In this study, the following assumptions are needed.

Assumption 1: Jiang and Praly (1998): The dynamic uncertainty $\Delta_{\rm i}$ in **Eq. 1** is assumed to satisfy

$$\|\Delta_i(\varsigma,\ X)\| \leq \phi_{i1}\left(\left\|\underline{X}_i\right\|\right) + \phi_{i2}(\|\varsigma\|), \quad i = 1,\ \dots, n \tag{2}$$

with unknown smooth functions $\phi_{i1}(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ and $\phi_{i2}(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$. In addition, $\phi_{i2}(\cdot)$ is assumed to be strictly increasing.

Assumption 2: Jiang and Praly (1998): There exists an input-to-state practically stable Lyapunov function $V_\varsigma(\varsigma)$ for $\dot{\varsigma} = q(\varsigma, X)$ in **Eq. 1** such that

$$\begin{aligned} &\omega_1(\|\varsigma\|) \leq V_\varsigma(\varsigma) \leq \omega_2(\|\varsigma\|),\\ &\frac{\partial V_\varsigma}{\partial \varsigma} q(\varsigma, X) \leq -c_0 V_\varsigma(\varsigma) + \vartheta(\|X_1\|) + d_0, \end{aligned} \tag{3}$$

with ω_1 and ω_2 belonging to class $\mathcal{K}_\infty$ *functions*, $\vartheta(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$, and c_0 and d_0 being positive constants.
To deal with the dynamic uncertainty, a dynamic signal is designed with the following dynamics,

$$\dot{r} = -\bar{c}r + \bar{\vartheta}(\|X_1\|) + d_0, \qquad r(0) = r_0, \tag{4}$$

where $\bar{\vartheta}(X_1) \geq \vartheta(\|X_1\|)$, $\bar{c} \in (0, c_0)$, and $c_0 > 0$ and r_0 are constants.

Lemma 1: Hardy et al. (1952): For any $\xi_0 > 0$, one has

$$0 \leq \|\xi_0\| - \xi_0 \tanh\left(\frac{\xi_0}{\chi}\right) \leq 0.2785\chi,$$

where $\chi > 0$ is a constant.

Lemma 2: Jiang and Praly (1998):

For the unmeasured partial state $\varsigma(t)$ with initial state ς_0, $V_\varsigma(\varsigma)$ given in Assumption 2, the dynamic signal $r(t)$ in **Eq. 4**, and all $t \geq 0$, there is a non-negative function $B(t)$ *such that*

$$V_\varsigma(\varsigma) \leq r(t) + \Phi(t). \tag{5}$$

In addition, there is a limited time $T_0 = T_0(\bar{c}_0, r_0, \varsigma_0)$ such that $\Phi(t) = 0$ for all $t \geq T_0$.

With no loss of generality, choose $\overline{\Theta}(X_1)$ as $\overline{\Theta}(X_1) = X_1^2\Theta(X_1^2)$. Accordingly, the dynamic signal $r(t)$ is designed as

$$\dot{r} = -\bar{c}r + X_1^2\Theta(X_1^2) + d_0, \qquad r(0) = r_0. \tag{6}$$

The control objective of this study can be formulated as follows.

Control Objective: Consider the reference output X_d satisfying $\{X_d, \dot{X}_d, \ddot{X}_d\}$ are bounded. Under Assumptions 1–2, design a neuro-adaptive controller for the system (1), such that,

1. the system output X_1 can track the reference X_d asymptotically, and
2. all signals in the closed-loop system keep bounded.

3 NEURO-ADAPTIVE CONTROLLER DESIGN

First, the tracking errors E_i, filter errors Z_i, and the compensated tracking errors Λ_i are defined for each subsystem as

$$\begin{aligned} E_i &= X_i - A_{i-1},\ i = 1,2,3, \\ Z_i &= A_i - S_i,\ i = 1,2, \\ \Lambda_i &= E_i - B_i,\ i = 1,2,3, \end{aligned} \tag{7}$$

where A_i is the filter state, $A_0 = X_d$, S_i is the virtual control, and B_i is the compensating signal.

For the subsequent design and analysis, denote $\Theta_i = \|W_i^*\|^i,\ i = 1,\ldots,n$ with W_i^* being the ideal weight vector of the RBFNNs. In addition, denote $\hat{\Theta}_i(t)$ as the estimation of Θ_i with an estimation error $\tilde{\Theta}_i(t) = \Theta_i - \hat{\Theta}_i(t)$.

3.1 Adaptive Backstepping Design

3.1.1 Step 1

Based on **Eqs 1**, **7**, taking a derivative of E_1 yields

$$\begin{aligned} \dot{E}_1 &= F_1(X_1) + G_1X_2 + D_1 + \Delta_1 - \dot{X}_d \\ &= F_1(X_1) + G_1E_2 + G_1S_1 + G_1Z_1 + D_1 + \Delta_1 - \dot{X}_d. \end{aligned} \tag{8}$$

For the first subsystem, the virtual control S_1 is designed as

$$S_1 = \frac{1}{G_1}\left(-F_1 - K_1E_1 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + \dot{X}_d\right), \tag{9}$$

with $K_1 = \mathrm{diag}\{K_{11}, \cdots, K_{1m}\}$ is a positive definite matrix, and $\eta_1 > 0$. To avoid repeated differentiation of the virtual control, a CF is designed as

$$\dot{A}_1 = \frac{S_1 - A_1}{\tau_1}, A_1(0) = S_1(0), \tag{10}$$

with a positive constant τ_1. To eliminate the effect of filter errors, the compensating signal is developed as

$$\dot{B}_1 = -K_1B_1 + G_1B_2 + G_1Z_1, B_1(0) = 0. \tag{11}$$

To compensate for the unknown dynamics, the adaptive law for Θ_1 is presented as

$$\dot{\hat{\Theta}}_1 = \frac{1}{2\eta_1}\Lambda_1^{\mathrm{T}}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 - \gamma_1\hat{\Theta}_1, \hat{\Theta}_1(0) = 0, \tag{12}$$

where $\gamma_1 > 0$ is a constant.

3.1.2 Step *i* $(2 \le i \le n-1)$

From **Eqs 1**, **7**, differentiating E_i leads to

$$\begin{aligned} \dot{E}_i &= F_i + G_iX_{i+1} + D_i + \Delta_i - \dot{A}_{i-1} \\ &= F_i + G_iE_{i+1} + G_iS_i + G_iZ_i + D_i + \Delta_i - \dot{A}_{i-1}. \end{aligned} \tag{13}$$

The virtual control design S_i is developed as

$$S_i = \frac{1}{G_i}\left(-F_i - G_{i-1}E_{i-1} - K_iE_i - \frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i + \dot{A}_{i-1}\right), \tag{14}$$

where $K_i = \mathrm{diag}\{K_{i1}, \cdots, K_{im}\}$ is a positive definite matrix, and $\eta_i > 0$. To obviate repeated differentiation of the virtual control S_i, a CF is given as

$$\dot{A}_i = \frac{S_i - A_i}{\tau_i}, A_i(0) = S_i(0), \tag{15}$$

with a positive design parameter τ_i. To diminish the influences of filter errors, the compensating signal is proposed as

$$\dot{B}_i = -G_{i-1}B_{i-1} - K_iB_i + G_iB_{i+1} + G_iZ_i, B_i(0) = 0. \tag{16}$$

To deal with the parameter estimation, the adaptive law to estimate Θ_i is designed as

$$\dot{\hat{\Theta}}_i = \frac{1}{2\eta_i}\Lambda_i^{\mathrm{T}}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i - \gamma_i\hat{\Theta}_i, \hat{\Theta}_i(0) = 0, \tag{17}$$

with a constant $\gamma_i > 0$.

3.1.3 Step *n*

According to **Eqs 1**, **7**, the differentiation of E_n can be transformed as

$$\dot{E}_n = F_n + G_nU + D_n + \Delta_n - \dot{A}_{n-1}. \tag{18}$$

The controller design is given as

$$U = \frac{1}{G_n}\left(-F_n - G_{n-1}E_{n-1} - K_nE_n - \frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + \dot{A}_{n-1}\right), \tag{19}$$

with design parameters $K_n = \mathrm{diag}\{K_{n1}, \cdots, K_{nm}\}$ is a positive definite matrix, and $\eta_n > 0$. The compensating signal for this step is presented as

$$\dot{B}_n = -G_{n-1}B_{n-1} - K_nB_n, B_n(0) = 0. \tag{20}$$

The adaptive law is developed as

$$\dot{\hat{\Theta}}_n = \frac{1}{2\eta_n}\Lambda_n^{\mathrm{T}}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n - \gamma_n\hat{\Theta}_n, \hat{\Theta}_n(0) = 0, \tag{21}$$

where $\gamma_n > 0$ is a constant.

4 STABILITY ANALYSIS

In this section, we analyze the stability of the closed-loop system (**Eq. 1**) with the presented design of the virtual control (**Eqs 9**, **14**), controller (**Eq. 19**), adaptive laws (**Eqs 12**, **17**, **21**), CFs (**Eq. 10**) and (**15**), and compensating signals (**Eqs 11**, **16**, **20**).

4.1 Step 1

Inserting **Eq. 9** into **Eq. 8**, we obtain

$$\dot{E}_1 = -K_1 E_1 + G_1 E_2 + G_1 Z_1 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1 \varphi_1^{\mathrm{T}}\varphi_1 + D_1 + \Delta_1. \quad (22)$$

From the aforementioned equation and **Eq. 11**, one has

$$\dot{\Lambda}_1 = -K_1 \Lambda_1 + G_1 \Lambda_2 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1 \varphi_1^{\mathrm{T}}\varphi_1 + D_1 + \Delta_1. \quad (23)$$

The Lyapunov function is defined as $V_1(\Lambda_1, \tilde{\Theta}_1) = \frac{1}{2}\Lambda_1^{\mathrm{T}}\Lambda_1 + \frac{1}{2}\tilde{\Theta}_1^{\mathrm{T}}\tilde{\Theta}_1$. From Assumption 1, the term $\Lambda_1^{\mathrm{T}}\Delta_1$ satisfies

$$\Lambda_1^{\mathrm{T}}\Delta_1 \le \|\Lambda_1\|\phi_{11}(\|X_1\|) + \|\Lambda_1\|\phi_{12}(\|\varsigma\|). \quad (24)$$

For the term $\|\Lambda_1\|\phi_{11}(\|X_1\|)$ in the aforementioned equation, based on Lemma 1, one has

$$\|\Lambda_1\|\phi_{11}(X_1,\ \Lambda_1) \le \Lambda_1^{\mathrm{T}}\hat{\phi}_{11}(\|X_1\|) + \varepsilon'_{11},\ \varepsilon'_{11} = 0.2785\varepsilon_{11}, \quad (25)$$

with ε'_{11} and ε_{11} being positive constants and

$$\hat{\phi}_{11}(X_1,\ \Lambda_1) = \phi_{11}(\|X_1\|)\tanh\left(\frac{\Lambda_1^{\mathrm{T}}\phi_{11}(\|X_1\|)}{\varepsilon_{11}}\right).$$

Consider the term $\|\Lambda_1\|\phi_{12}(\|\varsigma\|)$ in **Eq. 24**, according to Lemma 2, we have

$$\|\Lambda_1\|\phi_{12}(\|\varsigma\|) \le \|\Lambda_1\|\phi_{12}(\omega_1^{-1}(r+\Phi)). \quad (26)$$

It is to be noted that $\phi_{12}(\cdot)$ is strictly increasing and non-negative from Assumption 1, together with the fact that $r + \Phi \le \max\{2r, 2\Phi\}$, one has

$$\begin{aligned}\|\Lambda_1\|\phi_{12}(\omega_1^{-1}(r+\Phi)) \le\ & \|\Lambda_1\|\phi_{12}(\omega_1^{-1}(2r)) \\ & + \|\Lambda_1\|\phi_{12}(\omega_1^{-1}(2\Phi)). \quad (27)\end{aligned}$$

From Lemma 1, we can obtain

$$\|\Lambda_1\|\phi_{12}(\omega_1^{-1}(2r)) \le \Lambda_1^{\mathrm{T}}\hat{\phi}_{12}(\Lambda_1, r) + \varepsilon'_{12},\ \varepsilon'_{12} = 0.2785\varepsilon_{12}, \quad (28)$$

where ε'_{12} and ε_{12} are positive constants, and

$$\hat{\phi}_{12}(\Lambda_1, r)\,\phi_{12}(\omega_1^{-1}(2r))\tanh\left(\frac{\Lambda_1\phi_{12}(\omega_1^{-1}(2r))}{\varepsilon_{12}}\right),$$

$$\|\Lambda_1\|\phi_{12}\|(\omega_1^{-1}(2\Phi))\| \le \frac{1}{4}\Lambda_1^{\mathrm{T}}\Lambda_1 + d_1(t), \quad (29)$$

where $d_1(t) = \phi_{12}^2(\omega_1^{-1}(2\Phi(t)))$. From **Eqs 23–29**, the derivative of V_1 can be expressed as

$$\begin{aligned}\dot{V}_1 =\ & \Lambda_1^{\mathrm{T}}\left(-K_1\Lambda_1 + G_1\Lambda_2 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + D_1 + \Delta_1\right) - \tilde{\Theta}_1^{\mathrm{T}}\dot{\hat{\Theta}}_1 \\ \le\ & -\Lambda_1^{\mathrm{T}}K_1\Lambda_1 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1^{\mathrm{T}}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + G_1\Lambda_1^{\mathrm{T}}\Lambda_2 + \frac{1}{2}\Lambda_1^{\mathrm{T}}\Lambda_1 \\ & + \frac{1}{2}D_1^{\mathrm{T}}D_1 - \tilde{\Theta}_1^{\mathrm{T}}\dot{\hat{\Theta}}_1 + \Lambda_1^{\mathrm{T}}\hat{\phi}_{11}(x_1,\ \Lambda_1) + \varepsilon'_{11} + \Lambda_1^{\mathrm{T}}\hat{\phi}_{12}(\Lambda_1, r) \\ & + \varepsilon'_{12} + \frac{1}{4}\Lambda_1^{\mathrm{T}}\Lambda_1 + d_1(t). \quad (30)\end{aligned}$$

Using RBFNNs satisfies

$$\begin{aligned}\dot{V}_1 \le\ & -\Lambda_1^{\mathrm{T}}K_1\Lambda_1 - \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1^{\mathrm{T}}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + G_1\Lambda_1^{\mathrm{T}}\Lambda_2 \\ & + \Lambda_1 H_1(Y_1) + \frac{1}{2}D_1^{\mathrm{T}}D_1 + \varepsilon'_{11} + \varepsilon'_{12} + d_1(t) - \tilde{\Theta}_1^{\mathrm{T}}\dot{\hat{\Theta}}_1, \quad (31)\end{aligned}$$

where $H_1(Y_1) = \hat{\phi}_{11}(x_1, \Lambda_1) + \hat{\phi}_{12}(\Lambda_1, r) + \frac{3}{4}\Lambda_1$, $Y_1 = [X_1, \Lambda_1, r]^{\mathrm{T}}$. It is to be noted that $H_1(Y_1)$ is an unknown function. Then, according to the universal approximation theory, the unknown function $H_1(Y_1)$ can be approximated by the RBFNNs in the following form,

$$\hat{H}_1(Y_1|\,W_1^*) = W_1^{*T}\varphi_1(Y_1), \quad (32)$$

with W_1^* being the ideal weight vector defined as

$$W_1^* = \arg\min_{W_1\in\Omega_{W_1}}\left[\sup_{Y_1\in\Omega_{Y_1}}\|\hat{H}_1(Y_1|\,W_1) - H_1(Y_1)\|\right],$$

where Ω_{W_1} and Ω_{Y_1} are compact regions for W_1 and Y_1, respectively. The corresponding approximation error ε_1^* is defined as

$$\varepsilon_1^* = H_1(Y_1) - \hat{H}_1(Y_1|\,W_1^*),$$

with $\|\varepsilon_1^*\| \le \varepsilon_1$ and a positive constant ε_1.

Based on the definition of Θ_1, combining with Young's inequality, we have

$$\Lambda_1^{\mathrm{T}}H_1(Y_1) \le \frac{\Theta_1}{2\eta_1}\Lambda_1^{\mathrm{T}}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + \frac{\eta_1}{2} + \frac{1}{2}(\Lambda_1^{\mathrm{T}}\Lambda_1 + \varepsilon_1^2). \quad (33)$$

Inserting **Eq. 33** into **Eq. 31** yields

$$\begin{aligned}\dot{V}_1 \le & -\Lambda_1^{\mathrm{T}}K_1\Lambda_1 - \frac{\Lambda_1\Lambda_i}{L_i} + \frac{\tilde{\Theta}_1}{2\eta_1}\Lambda_1^{\mathrm{T}}\Lambda_1\varphi_1^{\mathrm{T}}\varphi_1 + \frac{\eta_1}{2} \\ & + \frac{1}{2}(\Lambda_1^{\mathrm{T}}\Lambda_1 + \varepsilon_1^2) + \varepsilon'_{11} + \varepsilon'_{12} + d_1(t) - \tilde{\Theta}_1^{\mathrm{T}}\dot{\hat{\Theta}}_1. \quad (34)\end{aligned}$$

4.2 Step $i\,(2 \le i \le n-1)$

Inserting the virtual control design **Eq. 14** into **Eq. 13**, we have

$$\dot{E}_i = -G_{i-1}E_{i-1} - K_iE_i + G_iE_{i+1} + G_iZ_i - \frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i + D_i + \Delta_i. \quad (35)$$

On the basis of **Eq. 16** and the aforementioned equation, one can obtain

$$\dot{\Lambda}_i = -G_{i-1}\Lambda_{i-1} - K_i\Lambda_i + G_i\Lambda_{i+1} - \frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i + D_i + \Delta_i. \quad (36)$$

To analyze the stability of the i-th subsystem through the Lyapunov theory, define the Lyapunov function for Λ_i and $\tilde{\Theta}_i$ as $V_i(\Lambda_i, \tilde{\Theta}_i) = \frac{1}{2}\Lambda_i^{\mathrm{T}}\Lambda_i + \frac{1}{2}\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i$. Based on Assumption 1, the term $\Lambda_i^{\mathrm{T}}\Delta_i$ satisfies

$$\Lambda_i^{\mathrm{T}}\Delta_i \le \|\Lambda_i\|\phi_{i1}(\|\underline{X}_i\|) + \|\Lambda_i\|\phi_{i2}(\|\varsigma\|). \quad (37)$$

Consider the term $\|\Lambda_i\|\phi_{i1}(\|\underline{X}_i\|)$ in **Eq. 37**, on account of Lemma 1, one has

$$\|\Lambda_i\|\phi_{i1}(\|\underline{X}_i\|) \le \Lambda_i^{\mathrm{T}}\hat{\phi}_{i1}(\underline{X}_i,\ \Lambda_i) + \varepsilon'_{i1},\ \varepsilon'_{i1} = 0.2785\varepsilon_{i1}, \tag{38}$$

with $\varepsilon'_{i1} > 0$, $\varepsilon_{i1} > 0$, and

$$\hat{\phi}_{i1}(\underline{X}_i,\ \Lambda_i) = \phi_{i1}(\|\underline{X}_i\|)\tanh\left(\frac{\Lambda_i\phi_{i1}(\|\underline{X}_i\|)}{\varepsilon_{i1}}\right).$$

For the term $\|\Lambda_i\|\phi_{i2}(\|\varsigma\|)$ in **(37)**, according to Lemma 2, we can obtain

$$\|\Lambda_i\|\phi_{i2}(\|\varsigma\|) \le \|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(r+\Phi)). \tag{39}$$

Since ϕ_{i2} is strictly increasing and non-negative from Assumption 1, based on the fact $r+\Phi \le \max\{2r, 2\Phi\}$, one has

$$\begin{aligned}\|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(r+\Phi)) \le \|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(2r)) \\ + \|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(2\Phi)).\end{aligned} \tag{40}$$

On the basis of Lemma 1, we can obtain

$$\|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(2r)) \le \Lambda_i^{\mathrm{T}}\hat{\phi}_{i2}(\Lambda_i, r) + \varepsilon'_{i2},\ \varepsilon'_{i2} = 0.2785\varepsilon_{i2}, \tag{41}$$

with $\varepsilon'_{i2} > 0$, $\varepsilon_{i2} > 0$, and

$$\hat{\phi}_{i2}(\Lambda_i, r) = \phi_{i2}(\omega_1^{-1}(2r))\tanh\left(\frac{\Lambda_i\phi_{i2}(\omega_1^{-1}(2r))}{\varepsilon_{i2}}\right).$$

Using Young's inequality, we have

$$\|\Lambda_i\|\phi_{i2}(\omega_1^{-1}(2\Phi)) \le \frac{1}{4}\Lambda_i^{\mathrm{T}}\Lambda_i + d_i(t), \tag{42}$$

where $d_i(t) = \phi_{i2}^2(\omega_1^{-1}(2\Phi(t)))$.

From **Eqs 36–42**, the derivative of V_i becomes

$$\begin{aligned}\dot{V}_i &= \Lambda_i\Bigg(-G_{i-1}\Lambda_{i-1} - K_i\Lambda_i + G_i\Lambda_{i+1} \\ &\quad -\frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i + D_i + \Delta_i\Bigg) - \tilde{\Theta}_i^{\mathrm{T}}\dot{\hat{\Theta}}_i \\ &\le -G_{i-1}\Lambda_{i-1}^{\mathrm{T}}\Lambda_i - \Lambda_i^{\mathrm{T}}K_i\Lambda_i + G_i\Lambda_i^{\mathrm{T}}\Lambda_{i+1} - \frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i^{\mathrm{T}}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i \\ &\quad + \frac{1}{2}\Lambda_i^{\mathrm{T}}\Lambda_i + \frac{1}{2}D_i^{\mathrm{T}}D_i + \Lambda_i^{\mathrm{T}}\hat{\phi}_{i1}(\underline{X}_i,\ \Lambda_i) + \varepsilon'_{i1} + \Lambda_i^{\mathrm{T}}\hat{\phi}_{i2}(\Lambda_i, r) \\ &\quad + \varepsilon'_{i2} + \frac{1}{4}\Lambda_i^{\mathrm{T}}\Lambda_i + d_i(t) - \tilde{\Theta}_i^{\mathrm{T}}\dot{\hat{\Theta}}_i.\end{aligned} \tag{43}$$

Applying RBFNNs yields

$$\begin{aligned}\dot{V}_i &\le -G_{i-1}\Lambda_{i-1}^{\mathrm{T}}\Lambda_i - \Lambda_i^{\mathrm{T}}K_i\Lambda_i + G_i\Lambda_i^{\mathrm{T}}\Lambda_{i+1} - \frac{\hat{\Theta}_i}{2\eta_i}\Lambda_i^{\mathrm{T}}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i \\ &\quad + \Lambda_i^{\mathrm{T}}H_i(Y_i) + \frac{1}{2}D_i^{\mathrm{T}}D_i + \varepsilon'_{i1} + \varepsilon'_{i2} + d_i(t) - \tilde{\Theta}_i^{\mathrm{T}}\dot{\hat{\Theta}}_i,\end{aligned} \tag{44}$$

where $H_i(Y_i) = \hat{\phi}_{i1}(\underline{X}_i,\ \Lambda_i) + \hat{\phi}_{i2}(\Lambda_i, r) + \frac{3}{4}\Lambda_i, Y_i = [\underline{X}_i^{\mathrm{T}},\ \Lambda_i,\ r]^{\mathrm{T}}$. The unknown function $H_i(Y_i)$ can be approximated in the following form:

$$\hat{H}_i(Y_i|W_i^*) = W_i^{*T}\varphi_i(Y_i), \tag{45}$$

where W_i^* is the ideal weight vector defined as

$$W_i^* = \underset{W_i\in\Omega_{W_i}}{\operatorname{argmin}}\left[\sup_{Y_i\in\Omega_{Y_i}}\|\hat{H}_i(Y_i|W_i) - H_i(Y_i)\|\right],$$

with Ω_{W_i} and Ω_{Y_i} being compact regions for W_i and Y_i, respectively. The approximation error ε_i^* is defined as

$$\varepsilon_i^* = H_i(Y_i) - \hat{H}_i(Y_i|W_i^*),$$

where $\|\varepsilon_i^*\| \le \varepsilon_i$ and $\varepsilon_i > 0$.

Based on the definition of Θ_i, using Young's inequality, one has

$$\Lambda_i^{\mathrm{T}}H_i(Y_i) \le \frac{\Theta_i}{2\eta_i}\Lambda_i^{\mathrm{T}}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i + \frac{\eta_i}{2} + \frac{1}{2}(\Lambda_i^{\mathrm{T}}\Lambda_i + \varepsilon_i^2). \tag{46}$$

Inserting **Eq. 46** into **Eq. 44**, one can obtain

$$\begin{aligned}\dot{V}_i &\le -G_{i-1}\Lambda_{i-1}^{\mathrm{T}}\Lambda_i - \Lambda_i^{\mathrm{T}}K_i\Lambda_i + G_i\Lambda_i^{\mathrm{T}}\Lambda_{i+1} + \frac{\tilde{\Theta}_i}{2\eta_i}\Lambda_i^{\mathrm{T}}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i \\ &\quad + \frac{\eta_i}{2} + \frac{1}{2}(\Lambda_i^{\mathrm{T}}\Lambda_i + \varepsilon_i^2) + \frac{1}{2}D_i^{\mathrm{T}}D_i + \varepsilon'_{i1} + \varepsilon'_{i2} + d_i(t) \\ &\quad - \tilde{\Theta}_i^{\mathrm{T}}\dot{\hat{\Theta}}_i.\end{aligned} \tag{47}$$

4.3 Step n

Inserting **Eq. 19** into **Eq. 18** results in

$$\dot{E}_n = -G_{n-1}E_{n-1} - K_nE_n - \frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + D_n + \Delta_n. \tag{48}$$

Based on the aforementioned equation and **Eq. 20**, we have

$$\dot{\Lambda}_n = -G_{n-1}\Lambda_{n-1} - K_n\Lambda_n - \frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + D_n + \Delta_n. \tag{49}$$

To investigate system stability through the Lyapunov theory, the Lyapunov function is defined for Λ_n and $\tilde{\Theta}_n$ as $V_n(\Lambda_n, \tilde{\Theta}_n) = \frac{1}{2}\Lambda_n^{\mathrm{T}}\Lambda_n + \frac{1}{2}\tilde{\Theta}_n^2$. According to Assumption 1, the term $\Lambda_n^{\mathrm{T}}\Delta_n$ satisfies

$$\Lambda_n^{\mathrm{T}}\Delta_n \le \|\Lambda_n\|\phi_{n1}(\|X\|) + \|\Lambda_n\|\phi_{n2}(\|\varsigma\|). \tag{50}$$

For the term $\|\Lambda_n\|\phi_{n1}(\|x\|)$ in **Eq. 50**, one can obtain

$$\|\Lambda_n\|\phi_{n1}(\|X\|) \le \Lambda_n^{\mathrm{T}}\hat{\phi}_{n1}(X,\ \Lambda_n) + \varepsilon'_{n1},\ \varepsilon'_{n1} = 0.2785\varepsilon_{n1}, \tag{51}$$

with ε'_{n1} and ε_{n1} being positive constants and

$$\hat{\phi}_{n1}(X,\ \Lambda_n) = \phi_{n1}(\|X\|)\tanh\left(\frac{\Lambda_n\phi_{n1}(\|X\|)}{\varepsilon_{n1}}\right).$$

For the term $\|\Lambda_n\|\phi_{n2}(\|\varsigma\|)$, from Lemma 2, we have

$$\|\Lambda_n\|\phi_{n2}(\|\varsigma\|) \le \|\Lambda_n\|\phi_{n2}(\omega_1^{-1}(r+\Phi)). \tag{52}$$

Based on the facts that $\phi_{n2}(\cdot)$ is strictly increasing and non-negative from Assumption 1 and $r+\Phi \le \max\{2r, 2\Phi\}$, one has

$$\begin{aligned}\|\Lambda_n\|\phi_{n2}(\omega_1^{-1}(r+\Phi)) \le \|\Lambda_n\|\phi_{n2}(\omega_1^{-1}(2r)) \\ + \|\Lambda_n\|\phi_{n2}(\omega_1^{-1}(2\Phi)).\end{aligned} \tag{53}$$

From Lemma 1, we can obtain

$$\|\Lambda_n\|\phi_{n2}\left(\omega_1^{-1}(2r)\right) \le \Lambda_n^{\mathrm{T}}\hat{\phi}_{n2}\left(\Lambda_n, r\right) + \varepsilon'_{n2}, \ \varepsilon'_{n2} = 0.2785\varepsilon_{n2}, \quad (54)$$

where $\varepsilon'_{n2} > 0$ and $\varepsilon_{n2} > 0$ are constants and

$$\hat{\phi}_{n2}\left(\Lambda_n, r\right) = \phi_{n2}\left(\omega_1^{-1}(2r)\right)\tanh\left(\frac{\Lambda_n\phi_{n2}\left(\omega_1^{-1}(2r)\right)}{\varepsilon_{n2}}\right).$$

Applying Young's inequality, we have

$$\|\Lambda_n\|\phi_{n2}\left(\omega_1^{-1}(2\Phi)\right) \le \frac{1}{4}\Lambda_n^{\mathrm{T}}\Lambda_n + d_n(t), \quad (55)$$

with $d_n(t) = \phi_{n2}^2\left(\omega_1^{-1}(2\Phi(t))\right)$. From **Eqs 48–55**, the derivative of V_n becomes

$$\begin{aligned}\dot{V}_n &= \Lambda_n\Bigg(-G_{n-1}\Lambda_{n-1} - K_n\Lambda_n \\ &\quad -\frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + D_n + \Delta_n\Bigg) - \tilde{\Theta}_n^{\mathrm{T}}\dot{\hat{\Theta}}_n \\ &\le -G_{n-1}\Lambda_{n-1}^{\mathrm{T}}\Lambda_n - \Lambda_n^{\mathrm{T}}K_n\Lambda_n - \frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n^{\mathrm{T}}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n \\ &\quad + \frac{1}{2}D_n^{\mathrm{T}}D_n + \frac{1}{2}\Lambda_n^{\mathrm{T}}\Lambda_n + \Lambda_n^{\mathrm{T}}\hat{\phi}_{n1}(X, \Lambda_n) + \varepsilon'_{n1} \\ &\quad + \Lambda_n^{\mathrm{T}}\hat{\phi}_{n2}(\Lambda_n, r) + \varepsilon'_{n2} + \frac{1}{4}\Lambda_n^{\mathrm{T}}\Lambda_n + d_n(t) - \tilde{\Theta}_n^{\mathrm{T}}\dot{\hat{\Theta}}_n. \quad (56)\end{aligned}$$

Inserting **Eqs 19**, **51**, **52** into **Eq. 56** results in

$$\begin{aligned}\dot{V}_n &\le -G_{n-1}\Lambda_{n-1}^{\mathrm{T}}\Lambda_n - \Lambda_n^{\mathrm{T}}K_n\Lambda_n - \frac{\hat{\Theta}_n}{2\eta_n}\Lambda_n^{\mathrm{T}}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n \\ &\quad + \Lambda_n^{\mathrm{T}}H_n(Y_n) + \frac{1}{2}D_n^{\mathrm{T}}D_n + \varepsilon'_{n1} + \varepsilon'_{n2} + d_n(t) - \tilde{\Theta}_n^{\mathrm{T}}\dot{\hat{\Theta}}_n, \quad (57)\end{aligned}$$

where $H_n(Y_n) = \hat{\phi}_{n1}(X, \Lambda_n) + \hat{\phi}_{n2}(\Lambda_n, r) + \frac{3}{4}\Lambda_n$, $Y_n = [X, \Lambda_n, r]^{\mathrm{T}}$. The unknown function $H_n(Y_n)$ can be estimated as

$$\hat{H}_n\left(Y_n \middle| W_n^*\right) = W_n^{*T}\varphi_n(Y_n), \quad (58)$$

with W_n^* being the ideal weight vector defined as

$$W_n^* = \underset{W_n\in\Omega_{W_n}}{\operatorname{argmin}}\left[\sup_{Y_n\in\Omega_{Y_n}}\left\|\hat{H}_n\left(Y_n\middle|W_n\right) - H_n(Y_n)\right\|\right],$$

where Ω_{W_n} and Ω_{Y_n} are compact regions for W_n and Y_n, respectively, with the approximation error ε_n^* defined as

$$\varepsilon_n^* = H_n(Y_n) - \hat{H}_1\left(Y_n\middle|W_n^*\right),$$

with ε_n^* satisfying $\|\varepsilon_n^*\| \le \varepsilon_n$ and a positive constant ε_n.

From the definition of Θ_n, combining with Young's inequality, we can obtain

$$\Lambda_n H_n(Y_n) \le \frac{\Theta_n}{2\eta_n}\Lambda_n^{\mathrm{T}}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + \frac{\eta_n}{2} + \frac{1}{2}\left(\Lambda_n^{\mathrm{T}}\Lambda_n + \varepsilon_n^2\right). \quad (59)$$

Applying Young's inequality, substituting **Eqs 21**, **59** into **Eq. 57** yields

$$\begin{aligned}\dot{V}_n &\le -G_{n-1}\Lambda_{n-1}^{\mathrm{T}}\Lambda_n - \Lambda_n^{\mathrm{T}}K_n\Lambda_n + \frac{\tilde{\Theta}_n}{2\eta_n}\Lambda_n^{\mathrm{T}}\Lambda_n\varphi_n^{\mathrm{T}}\varphi_n + \frac{\eta_n}{2} \\ &\quad + \frac{1}{2}\left(\Lambda_n^{\mathrm{T}}\Lambda_n + \varepsilon_n^2\right) + \frac{1}{2}D_n^{\mathrm{T}}D_n + \varepsilon'_{n1} + \varepsilon'_{n2} + d_n(t) \\ &\quad - \tilde{\Theta}_n\dot{\hat{\Theta}}_n. \quad (60)\end{aligned}$$

Theorem 1: Under Assumptions 1–2, with the virtual control (**Eqs 9**, **14**), the CF design (**Eqs 10**, **15**), the adaptive laws (**Eqs 12**, **17**, **21**), the compensating signals (**Eqs 11**, **16**, **20**), and the controller (**Eq. 19**), the following facts hold.

1. The tracking errors will converge to the neighborhood of the origin asymptotically.
2. The boundedness of all signals in the closed-loop system (**Eq. 1**) can be guaranteed.

Proof: Define $V = \sum_{i=1}^{n} V_i$, applying Young's inequality yields

$$\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i \le \frac{1}{2}\Theta_i^{\mathrm{T}}\Theta_i - \frac{1}{2}\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i.$$

Based on **Eqs 34**, **47**, **60**, the overall Lyapunov function satisfies

$$\begin{aligned}\dot{V} &\le -\sum_{i=1}^{n}\Lambda_i^{\mathrm{T}}\left(K_i - \frac{1}{2}I_m\right)\Lambda_i - \sum_{i=1}^{n}\frac{\gamma_i}{2}\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\left[\eta_i + \varepsilon_i^2 + D_i^{\mathrm{T}}D_i + \gamma_i + \Theta_i^{\mathrm{T}}\Theta_i + 2\varepsilon'_{i1}\right. \\ &\quad \left. + 2\varepsilon'_{i2} + 2d_i(t)\right] \\ &\le -aV + b,\end{aligned}$$

where I_m is the m-dimension identity matrix,

$$a = \min_{i=1,\ldots,n}\left\{\lambda_{\min}(2K_i - I_m),\ \gamma_i\right\},$$
$$b = \frac{1}{2}\sum_{i=1}^{n}\left[\eta_i + \varepsilon_i^2 + D_i^{\mathrm{T}}D_i + \gamma_i + \Theta_i^{\mathrm{T}}\Theta_i + 2\varepsilon'_{i1} + 2\varepsilon'_{i2} + 2d_i(t)\right].$$

Therefore, Λ_i, $\tilde{\Theta}_i$, and $\hat{\Theta}_i$ are bounded. Next, we investigate the boundedness of Z_i, and the dynamics of the filter error Z_i can be expressed as

$$\dot{Z}_i = \dot{A}_i - \dot{S}_i = -\frac{Z_i}{\tau_i} - \dot{S}_i, \quad (61)$$

where

$$\begin{aligned}\dot{s}_i = \frac{1}{G_i}\Bigg(&-\dot{F}_i - G_{i-1}\dot{E}_{i-1} - K_i\dot{E}_i - \frac{\dot{\hat{\Theta}}_i}{2\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\varphi_i - \frac{\hat{\Theta}_i}{2\eta_i}\dot{\Lambda}_i\varphi_i^{\mathrm{T}}\varphi_i \\ &-\frac{\hat{\Theta}_i}{\eta_i}\Lambda_i\varphi_i^{\mathrm{T}}\dot{\varphi}_i + \ddot{A}_{i-1}\Bigg)\end{aligned}$$

is continuous on the compact set $\Omega_i \times \Omega_{X_d}$ with

$$\Omega_{X_d} = \left\{\left(X_d, \dot{X}_d, \ddot{X}_d\right)\middle| X_d^2 + \dot{X}_d^2 + \ddot{X}_d^2 \le R_0\right\},$$
$$\Omega_i = \left\{\left(E_i, Z_i, \tilde{\Theta}_i\right)\middle| E_i^2 + Z_i^2 + \tilde{\Theta}_i^2 \le R_i\right\},$$

and $R_0 > 0, R_i > 0$. Thus, $\dot{S}_i$ is bounded, which derives that Z_i is also bounded from **Eq. 61**. According to **Eqs 11**, **16**, **20**, B_i is bounded. Thus, E_i, A_i, S_i, U, and X_i are all bounded, which invokes ς, Δ, and r to be bounded based on Lemma 2 and **Eq. 6**. In the end, we can conclude that the boundedness of all the signals in the closed-loop system can be guaranteed. This completes the proof.

5 SIMULATION STUDY

The system considered in this section is a voltage source converter-high voltage direct current transmission system with the following dynamics (Hu et al. (2020)).

$$
\begin{aligned}
\dot{\varsigma} &= q(\varsigma,\ x),\\
\dot{x}_1 &= -b_2 x_1 - \frac{x_n}{L_2} + \omega x_2 + T_1 + \delta_1(\varsigma, x),\\
\dot{x}_2 &= -b_2 x_2 - \frac{x_4}{L_2} - \omega x_1 + \delta_2(\varsigma, x),\\
\dot{x}_3 &= \frac{x_1 - x_5}{C_2} + \omega x_4 + \delta_3(\varsigma, x),\\
\dot{x}_4 &= \frac{x_2 - x_6}{C_2} + \omega x_3 + \delta_4(\varsigma, x),\\
\dot{x}_5 &= -b_1 x_5 + \frac{x_3}{L_1} + \omega x_6 - \frac{u_d}{L_1} + \delta_5(\varsigma, x),\\
\dot{x}_6 &= -b_1 x_6 + \frac{x_4}{L_1} + \omega x_5 - \frac{u_q}{L_1} + \delta_6(\varsigma, x),
\end{aligned}
$$

where L_1 and L_2 are the electrical inductances, and C_1 and C_2 are the capacitances. Applying variable transformation $X_i = [x_{2i-1},\ x_{2i}]^{\mathrm{T}}$, $\overline{X}_i = [x_{2i},\ x_{2i-1}]^{\mathrm{T}}$, $X = [X_1,\ X_2,\ X_3]^{\mathrm{T}}$, $\overline{T} = [T_1,\ 0]^{\mathrm{T}}$, $c_1 = \operatorname{diag}\{1,\ -1\}$, and $U = \left[u_d,\ u_q\right]^{\mathrm{T}}$, the aforementioned equation becomes

$$
\begin{aligned}
\dot{\varsigma} &= q(\varsigma, X),\\
\dot{X}_1 &= -b_2 X_1 - \frac{X_2}{L_2} + \omega c_1 \overline{X}_1 + \overline{T} + \Delta_1(\varsigma, X),\\
\dot{X}_2 &= \frac{X_1 - X_3}{C_2} + \omega \overline{X}_2 + \Delta_2(\varsigma, X),\\
\dot{X}_3 &= -b_1 X_3 + \frac{X_2}{L_1} + \omega \overline{X}_3 - \frac{U}{L_1} + \Delta_3(\varsigma, X).
\end{aligned}
$$

By applying the presented control scheme, the control design is developed as

$$
\begin{aligned}
S_1 &= L_2\left(-b_2 X_1 + K_1 E_1 + \frac{\hat{\Theta}_1}{2\eta_1}\Lambda_1 \varphi_1^{\mathrm{T}}\varphi_1 + \omega c_1 \overline{X}_1 - \dot{X}_d\right),\\
S_2 &= C_2\left(\frac{X_1}{C_2} - \frac{E_1}{L_2} + K_2 E_2 + \omega \overline{X}_2 + \frac{\hat{\Theta}_2}{2\eta_2}\Lambda_2 \varphi_2^{\mathrm{T}}\varphi_2 - \dot{A}_1\right),\\
U &= L_1\left(-b_1 X_3 + \frac{X_2}{L_1} + \omega \overline{X}_3 - \frac{E_2}{C_2} + K_3 E_3 \right.\\
&\quad \left. + \frac{\hat{\Theta}_3}{2\eta_3}\Lambda_3 \varphi_3^{\mathrm{T}}\varphi_3 - \dot{A}_2\right),
\end{aligned}
$$

with the compensating signal design

$$
\begin{aligned}
\dot{B}_1 &= -K_1 B_1 - \frac{B_2}{L_2} - \frac{Z_1}{L_2}, B_1(0) = 0.\\
\dot{B}_2 &= \frac{B_1}{L_2} - K_2 B_2 - \frac{B_3}{C_2} - \frac{Z_2}{C_2}, B_2(0) = 0.\\
\dot{B}_3 &= \frac{B_2}{C_2} - K_3 B_3, B_3(0) = 0.
\end{aligned}
$$

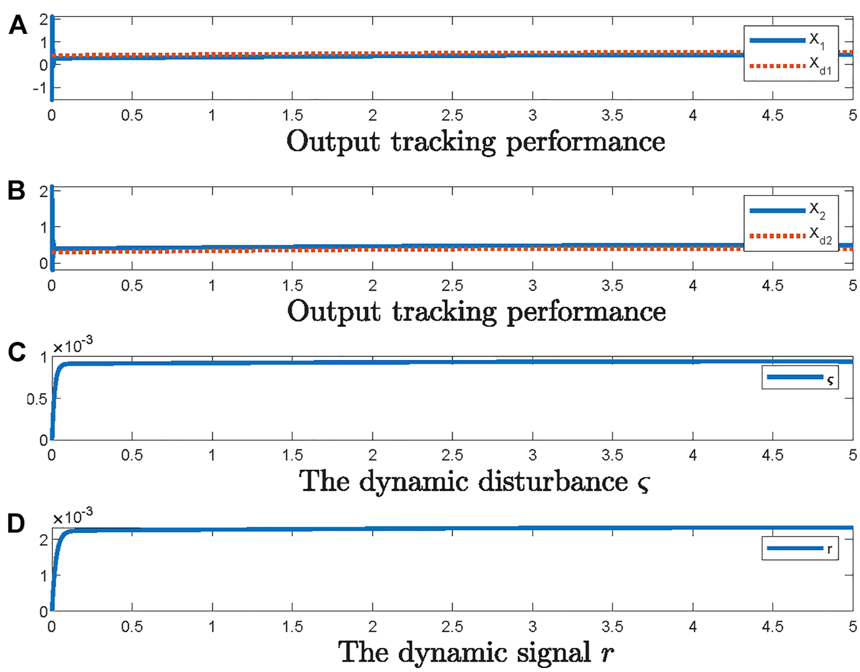

FIGURE 1 | Output tracking performance and evolution of dynamic uncertainties.

In addition, the CF design and adaptive law design are the same as **Eqs 10**, **11**, **15**, **16**, **20**.

The design parameters are given as $L_1 = 4$ mH, $L_2 = 8$ mH, $C_2 = 0.1\mu$F, $\overline{T} = [0.01, 0.02]^{\mathrm{T}}$, $\omega = 100\pi$ rad/s, $K_1 = \mathrm{diag}\{1258,\ 1646\}$, $K_2 = \mathrm{diag}\{124630,\ 161622\}$, $K_3 = \mathrm{diag}\{188539,\ 138474\}$, $\gamma_1 = 0.00085$, $\gamma_2 = 0.00066$, $\gamma_3 = 0.00059$, $\eta_1 = 0.00005$, $\eta_2 = 0.000003$, $\eta_3 = 0.000004$.

The RBFNNs are chosen in typical Gaussian form. To be specific, the RBFNN $\varphi_1(X_1, \Lambda_1, r)$ contains 32 nodes with the center and width being $[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]$ and 2, respectively. RBFNN $\varphi_2\left(\underline{X}_2, \Lambda_2, r\right)$ contains 128 nodes and the center and width are distributed in $[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]$ and 2. RBFNN $\varphi_3(X, \Lambda_3, r)$ contains 512 nodes with the center and width selected as $[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]$ and 2, respectively.

The simulation results are shown in **Figure 1**. From **Figure 1**, it can be observed that the output tracking objective can be achieved and the system output can track the reference output asymptotically. The dynamic uncertainties can also converge with the convergence of system states.

6 CONCLUSION

In this study, a control approach for MIMO strict feedback nonlinear unmodeled dynamical systems with CFs is developed. The dynamic signal design introduced together with RBFNNs can efficiently prevent the effect of the dynamic uncertainties. The CFs employed in the controller design can not only prevent the explosion of complexity, but can also eliminate the effect of filter errors through the compensating signal design. Compared with single-input single-output strict feedback nonlinear systems, the approach proposed in this study is suitable for more general cases. Finally, in the simulation experiments, the presented method is applied to power systems, where the simulation results validate the effect of the scheme proposed.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

XF, LS, and YZ contributed to conception and design of this study. XF investigated the theoretical analysis for the command filter design. LS performed the simulation study with application to an energy system. YZ organized the writing of the manuscript. XF, LS, and YZ collaborated to write all the sections of the manuscript. All authors contributed to manuscript revision, and read and approved the submitted version.

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