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A New Test for Ridge Wind Directional Data Under Neutrosophic Statistics

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The statistical tests under classical statistics can be only applied when the data is linear and has certain observations. The existing statistical tests cannot be applied for circular/angles data. In this paper, the Watson-Williams test under neutrosophic is introduced to analyze having uncertain, imprecise, and indeterminate circular/angles data. The neutrosophic test statistic is introduced and applied to wind direction data. From the real example and simulation study, it can be concluded the proposed neutrosophic Watson-Williams test performs better than the Watson-Williams test under classical statistics.

Keywords: watson-williams test, circular data, directional data, neutrosophic statistics, classical statistics

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INTRODUCTION

In practice, the population parameters are unknown and estimated on the basis of sample information. The testing of a hypothesis is a procedure that is applied for testing the unknown parameters using sample information. The Z-test and *t*-test are very popular for testing the mean of unknown population parameters when the sample size is larger than 30 and less than 30, respectively. These traditional tests are used for linear data and cannot be applied for the angles/circular data. In many scientific areas such as wind directions, animal movement, ocean directions, radar data, and bone-fracture plane data are recorded in radians or degrees, see (Fisher, 1995). For the data recorded in radians or degrees, the traditional statistical tests can be applied for testing the mean of circular observations. Watson-Williams test is a popular test that is applied to test whether the mean angles of circular observations differ significantly or not. The test is applied under the assumption that the data follow the von Mises distribution with the same value of concentration parameter, see (Kanji, 2006). (Fitak and Johnsen, 2017), (Ruxton, 2017) and (Landler et al., 2018) used the statistical tests for circular biological data. (Landler et al., 2019) proposed the circular test for non-continue data. More information on tests for circular data can be seen in (Mulder and Klugkist, 2021).

According to (Farrugia and Micallef, 2006), "Wind is a vector quantity having both a magnitude and a three-dimensional direction. This would make wind a spherical variable. However, usually, only the horizontal component is considered. Thus, the wind is mainly treated as a circular variable with an associated magnitude". The wind directional data is also analyzed using circular statistics. The decision-makers may be interested to test whether the mean wind direction on two edges is the same or different (Bowers et al., 2000) presented the statistical analysis for wind and waves data. (Farrugia and Micallef, 2006) presented the comparative analysis using the wind direction data. More applications of statistical tests for wind data can be seen in (Hassan et al., 2009), (Qin et al., 2010), (Heckenbergerova et al., 2015), (Arias-Rosales and Osorio-Gómez, 2018), (Katinas et al., 2018), (Min et al., 2019) and (Ul Haq et al., 2020).

The aforementioned statistical tests cannot be applied when the decision-makers are uncertain in sample size selection or imprecise circular data is recorded from the complex system. To deal with such data, the statistical tests using fuzzy logic can be helpful in making a decision about the unknown parameters. (Yang and Pan, 1997), (Chen et al., 2013), (Pewsey et al., 2013), (Kesemen et al., 2016), (Lubiano et al., 2016), (Benjamin et al., 2019) and (Pewsey and García-Portugués, 2020) presented various tests to analyze fuzzy data.

(Smarandache, 2014) introduced neutrosophic statistics (NS) to deal with the data having neutrosophic numbers. The neutrosophic statistics were found to be more efficient than classical statistics in terms of informative and flexibility, (Aslam, 2019a). The NS is found to be more efficient than classical statistics, see (Chen et al., 2017a) and (Chen et al., 2017b), (Aslam, 2019a), (Aslam, 2019b) and (Aslam, 2020). (Aslam, 2021) proposed the neutrosophic statistical test to analyze radar data (Khan et al., 2020). proposed variance chart under neutrosophic. More applications to deal with the neutrosophic numbers can be seen in (Ye, 2018), (Ye et al., 2018), (Pramanik and Banerjee, 2018), (Pramanik and Dey, 2018), (Mondal et al., 2018), (Pramanik and Dey, 2019), (Maiti et al., 2020) and (Mondal et al., 2021).

The Watson-Williams test cannot be applied when uncertainty is recorded in circular/angles data. By exploring the literature and to the best of our knowledge, no work on the Watson-Williams test under NS is found in the literature. In this paper, the neutrosophic Watson-Williams test will be introduced for the first time. The test statistic of the Watson-Williams test is introduced under NS. The testing of the hypothesis procedure will be given and applied using the wind direction data. It is expected that the proposed Watson-Williams test will perform better than in the existing test in uncertainty.

DESIGN OF THE PROPOSED TEST

Watson-Williams (W-W) test under classical statistics is applied for testing the average angles of two independent circular observations which are drawn from von Mises distribution having the same value of concentration parameter k. The null hypothesis H_0: the mean angles are the same *vs.* H_1: the mean angles differ significantly. The existing (W-W) test is applied when all the circular observations are determined and précised. In this section, the neutrosophic Watson-Williams (N-W-W) test will be introduced on testing H_0 when circular observations are indeterminate or recorded from the complex systems. The methodology of the proposed N-W-W test is stated as: Suppose that $\Phi_{n_N} = \Phi_{n_L} + \Phi_{n_U} I_{\Phi N}; I_{\Phi N} \varepsilon [I_{\Phi L}, I_{\Phi U}]$ be the first neutrosophic random sample of size $n_N = n_L + n_U I_{nN}; I_{nN} \varepsilon [I_{nL}, I_{nU}]$ and $\Psi_{m_N} = \Psi_{m_L} + \Psi_{m_U} I_{\Psi N}; I_{\Psi N} \varepsilon [I_{\Psi L}, I_{\Psi U}]$ be the second neutrosophic random sample of size $m_N = m_L + m_U I_{mN}; I_{mN} \varepsilon [I_{mL}, I_{mU}]$. Note that Φ_{n_L}, n_L, Ψ_{m_L} and m_L presents the determinate parts and Φ_{n_U}, $n_U I_{nN}$, $\Psi_{m_U} I_{\Psi N}$ and $m_U I_{mN}$ present indeterminate parts and $I_{\Phi N} \varepsilon [I_{\Phi L}, I_{\Phi U}]$, $I_{nN} \varepsilon [I_{nL}, I_{nU}]$, $I_{\Psi N} \varepsilon [I_{\Psi L}, I_{\Psi U}]$ and $I_{mN} \varepsilon [I_{mL}, I_{mU}]$ is the associated measures of indeterminacy/uncertainty. The components of neutrosophic results vectors are calculated as follows

$$C_{1N} = \sum_{i=1}^{n_L} \cos \Phi_{iL} + \sum_{i=1}^{n_U} \cos \Phi_{iU} I_{C1N}; I_{C1N} \varepsilon [I_{C1L}, I_{C1U}] \quad (1)$$

$$S_{1N} = \sum_{i=1}^{n_L} \sin \Phi_{iL} + \sum_{i=1}^{n_U} \sin \Phi_{iU} I_{S1N}; I_{S1N} \varepsilon [I_{S1L}, I_{S1U}] \quad (2)$$

$$C_{2N} = \sum_{i=1}^{n_L} \cos \Psi_{iL} + \sum_{i=1}^{n_U} \cos \Psi_{iU} I_{C2N}; I_{C2N} \varepsilon [I_{C2L}, I_{C2U}] \quad (3)$$

$$S_{2N} = \sum_{i=1}^{n_L} \sin \Psi_{iL} + \sum_{i=1}^{n_U} \sin \Psi_{iU} I_{C2N}; I_{S2N} \varepsilon [I_{S2L}, I_{S2U}] \quad (4)$$

The neutrosophic resultant vectors are defined as

$$R_{1N} = \left(C_{1L}^2 + S_{1L}^2\right)^{\frac{1}{2}} + \left(C_{1U}^2 + S_{1U}^2\right)^{\frac{1}{2}} I_{R1N}; I_{R1N} \varepsilon [I_{R1L}, I_{R1U}] \quad (5)$$

$$R_{2N} = \left(C_{2L}^2 + S_{2L}^2\right)^{\frac{1}{2}} + \left(C_{2U}^2 + S_{2U}^2\right)^{\frac{1}{2}} I_{R2N}; I_{R2N} \varepsilon [I_{R2L}, I_{R2U}] \quad (6)$$

For the combined neutrosophic sample, the components of the neutrosophic resultant vectors are given by

$$C_N = (C_{1L} + C_{2L}) + (C_{1U} + C_{2U}) I_{CN}; I_{CN} \varepsilon [I_{CL}, I_{CU}] \quad (7)$$

$$S_N = (S_{1L} + S_{2L}) + (S_{1U} + S_{2U}) I_{SN}; I_{SN} \varepsilon [I_{SL}, I_{SU}] \quad (8)$$

The length of the neutrosophic resultant vector is calculated as follows

$$R_N = \left(C_L^2 + S_U^2\right)^{\frac{1}{2}} + \left(C_L^2 + S_U^2\right)^{\frac{1}{2}} I_{RN}; I_{RN} \varepsilon [I_{RL}, I_{RU}] \quad (9)$$

The neutrosophic statistic to test the unknown neutrosophic mean angles are given by

$$F_N = g_L (N_L - 2) \frac{R_{1L} + R_{2L} - R_L}{N_L - (R_{1L} + R_{2L})} + g_U (N_U - 2) \frac{R_{1U} + R_{2U} - R_U}{N_U - (R_{1U} + R_{2U})} I_{FN}; I_{FN} \varepsilon [I_{FL}, I_{FU}] \quad (10)$$

where the first part denotes determinate parts and the second part denotes indeterminate parts and $N_N = n_N + m_N$ and $g_N = 1 - 3/8\hat{k}_N$ and $\hat{k}_N$ is determined by

$$\bar{R}_N = \left(\frac{R_{1N} + R_{2N}}{N_N}\right) \quad (11)$$

The proposed test will be operated using the following steps

Step-1: state H_0: the mean angles are the same *vs.* H_1: the mean angles differ significantly

Step-2: Set the level of significance α and select the critical value from F-table at F_{1,N_N-2}.

Step-3: Do not reject H_0 if $F_N \varepsilon [F_L, F_U] < F_{1,N_N-2}$, otherwise, accept H_1.

The operational process of the N-W-W test is also shown in **Figure 1**.

APPLICATION USING WIND DIRECTIONAL DATA

In this section, the application of the proposed N-W-W test will be given using the wind directional (angles) data recorded near Corls Ridge, Michaux State Forest. The data is reported from opposite sides of two ridges. The decision-makers are interested to test either angles of the two groups are the same *vs.* the alternative hypothesis that the angles of the two groups are different. Suppose that the decision-makers are uncertain about the first and the second sample size with the measure of indeterminacy $I_{nU} = 0.28$ and $I_{mU} = 0.3$. Under uncertainty, to

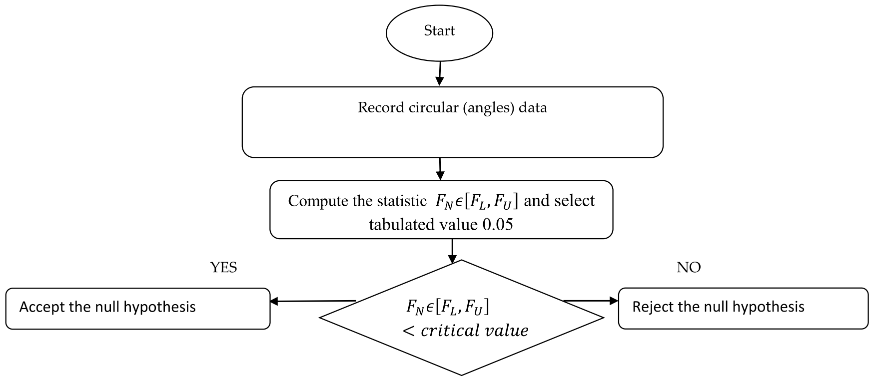

FIGURE 1 | The procedure of the proposed N-W-W test.

test H_0: angles of two ridges are the same *vs.* H_1: the angles of two ridges are different; the decision-makers want to select the first sample from 5 to 7 and the second sample from 4 to 6. To apply the proposed test, the decision-makers decided to select the first sample size is 5 and the second sample size is 7. Note that for application of the proposed test, the data of the first sample (55, 57, 60, 63, 64) is extracted from http://webspace.ship.edu/pgmarr/Geo441/Lectures/Lec%2016%20-%20Directional%20Statistics.pdf and the values from the second sample (55, 57, 60, 63, 64, 66, 67) are selected from the same source. The neutrosophic forms of two samples are: $n_N = 5 + 7I_{nN}; I_{nN}\varepsilon[0, 0.28]$ and $m_N = 4 + 6I_{mN}; I_{mN}\varepsilon[0, 0.28]$. The components of neutrosophic results vectors for the wind speed directional data are calculated as follows

$$C_{1N} = 2.5105 + 3.3079 I_{C1N}; I_{C1N}\varepsilon[0, 0.2411]$$
$$S_{1N} = 4.3137 + 6.1477 I_{SN}; I_{SN}\varepsilon[0, 0.2983]$$
$$C_{2N} = 1.5299 + 2.1312 I_{C2N}; I_{C2N}\varepsilon[0, 0.2821]$$
$$S_{2N} = 3.6937 + 5.6011 I_{C2N}; I_{S2N}\varepsilon[0, 0.3405]$$

The neutrosophic resultant vectors for the real data are given as

$$R_{1N} = 4.9910 + 6.9811 I_{R1N}; I_{R1N}\varepsilon[0, 0.2851]$$
$$R_{2N} = 3.9980 + 5.9928 I_{R2N}; I_{R2N}\varepsilon[0, 0.3328]$$

For the combined neutrosophic sample, the components of the neutrosophic resultant vectors for the real data are given by

$$C_N = 4.0404 + 5.4391 I_{CN}; I_{CN}\varepsilon[0, 0.2572]$$
$$S_N = 8.0074 + 11.7488 I_{SN}; I_{SN}\varepsilon[0, 0.3184]$$

The length of the neutrosophic resultant vector for the real data is given by

$$R_N = 8.9690 + 12.9467 I_{RN}; I_{RN}\varepsilon[0, 0.3072]$$

The neutrosophic statistic is calculated as

$$F_N = 12.8147 + 13.4394 I_{FN}; I_{FN}\varepsilon[0, 0.0464]$$

where $N_N\varepsilon[9, 13]$ and $g_N = 1 - 3/8\hat{k}_N$ and $\hat{k}_N$ is determined by $\bar{R}_N\varepsilon[0.9987, 0.998]$. From (Kanji, 2006), $\hat{k}_N\varepsilon[50.24, 50.24]$ and $g_N\varepsilon[1, 1]$.

The Proposed Test Will Be Operated Using the Following Steps

Step-1: state H_0: The mean angles are the same *vs.* H_1: the mean angles differ significantly

Step-2: Set the level of significance $\alpha = 0.05$ and select the critical values from F-table that are 236.8 and 242.9.

Step-3: Do not reject H_0 if $F_N\varepsilon[12.8147, 13.4394] <$ [236.8, 242.9].

From the study, it is concluded that both groups of wind directional data have the same mean angles.

Advantages of the Proposed N-W-W Test

The proposed N-W-W test under neutrosophic statistics is a generalization of the existing W-W test under classical statistics. The proposed N-W-W reduces to the existing W-W test when no uncertainty is recorded in the data. In this section, the efficiency of the proposed N-W-W test will be discussed in terms of the measure of indeterminacy, adequacy, information, and flexibility. To discuss the advantages, the neutrosophic statistic of the N-W-W test of real example is considered. The neutrosophic form of the test statistic is $F_N = 12.8147 + 13.4394 I_{FN}; I_{FN}\varepsilon[0, 0.0464]$. This neutrosophic form consists of test statistic of classical statistics and indeterminate parts. The neutrosophic form of the proposed test reduces to the W-W test when $I_{FL} = 0$. It means that the value 12.8147 presents the value of test statistic of the existing W-W test under classical statistics. The second part $13.4394 I_{FN}$ presents the indeterminate part with the measure of indeterminacy is 0.0464. From the information, it is clear that, under uncertainty, the proposed N-W-W test can take values from 12.8147 to 13.4394. On the other hand, the existing W-W test considers only a single value that is adequate in uncertainty. For testing H_0: the angles of two ridges are the same *vs.* H_1: the angles of two ridges are different, the proposed test gives the chance of accepting H_0 is 0.95, the chance of committing a type-I error is 0.05, and the

TABLE 1 | The effect of indeterminacy on $F_N \varepsilon [F_L, F_U]$.

I_{FU}	$F_N = 12.8147 + 13.4394 I_{FN}$	Decision About H_0
0	12.81	Do not reject H_0
0.10	14.16	Do not reject H_0
0.20	15.50	Do not reject H_0
0.30	16.85	Do not reject H_0
0.40	18.19	Do not reject H_0
0.50	19.53	Do not reject H_0
0.60	20.88	Do not reject H_0
0.70	22.22	Do not reject H_0
0.80	23.57	Do not reject H_0
0.90	24.91	Do not reject H_0
1.00	26.25	Do not reject H_0

measure of indeterminacy associated with the test is 0.0464. The existing test is unable to give information about the measure of indeterminacy. From the study, it is concluded that the proposed N-W-W test is more flexible and more informative than the existing W-W test.

SIMULATION STUDY

In this section, the effect of the measure of indeterminacy will be studied on the proposed N-W-W test. The various values of the indeterminacy parameter I_{FU} will be selected to see its effects on the test statistic $F_N \varepsilon [F_L, F_U]$. The values of $F_N \varepsilon [F_L, F_U]$ for different values of I_{FU} are shown in **Table 1**.

From **Table 1**, it is clear that the values of statistic $F_N \varepsilon [F_L, F_U]$ increase as the values of I_{FU} increases. For example, when $I_{FU} = 0.10$, the values of $F_N \varepsilon [12.81, 14.16]$ and when $I_{FU} = 0.90$, the values of $F_N \varepsilon [12.81, 26.25]$. The decision about H_0 for different values of I_{FU} is also shown in **Table 1**. From the simulation study, it can be concluded that the measure of indeterminacy I_{FU} affects the values of test $F_N \varepsilon [F_L, F_U]$.

CONCLUSION

In this paper, the Watson-Williams test under neutrosophic was introduced to analyze having uncertain, imprecise, and indeterminate circular/angles data. The proposed test was the extension of the existing Watson-Williams test under classical statistics. The neutrosophic statistic for the Watson-Williams test was introduced. The application using wind direction data, simulation, and comparative studies of the proposed Watson-Williams test was given. From these studies, it is concluded that the proposed test is more efficient than the existing Watson-Williams test in terms of flexibility, applicability, and information. The meteorologists can apply the proposed test for testing whether the angles of two wind groups have the same average or not. The proposed Watson-Williams test for big circular data can be studied as future research.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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