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Global Exponential Stability of a Class of Memristor-Based RNN and Its Application to Design Stable Voltage Circuits

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In this article, we study the global exponential stability of the equilibrium point for a class of memristor-based recurrent neural networks (MRNNs). The MRNNs are based on a realistic memristor model and can be implemented by a very large scale of integration circuits. By introducing a proper Lyapunov functional, it is proved that the equilibrium point of the MRNN is globally exponentially stable under two less conservative assumptions. Furthermore, an algorithm is proposed for the design of MRNN-based circuits with stable voltages. Finally, an illustration example is performed to show the validation of the proposed theoretical results; an MRNN-based circuit with stable voltages is designed according to the proposed algorithm.

Keywords: memristor, voltage, circuit, recurrent neural network, stability

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1 INTRODUCTION

Recurrent networks have been one of the necessary tools to character system states since their wide applications in optimization (Li et al., 2021; Ma and Bian, 2021), games (Wu et al., 2019, 2021; Cheng et al., 2021), control (Yang et al., 2015; Jianmin et al., 2021; Toyoda and Wu, 2021), and so on (Wang et al., 2007; Shen et al., 2020; Shen and Raksincharoensak, 2021). In recent years, a new type of recurrent network was proposed based on a new two-terminal circuit element called the memristor (Chua, 1971; Strukov et al., 2008). Note that a memristor works like a biological synapse (Anthes, 2011; Qin et al., 2015) and has the ability of automatic information storage. Thus, memristors replaced resistors as synapses in recurrent neural networks, that is, memristor-based recurrent neural networks (MRNNs) (Anthes, 2011; Wen et al., 2013; Zhang et al., 2013). In recent years, the stability and stabilization of Boolean networks have been extensively investigated (Chen et al., 2018; Guo et al., 2019, 2021).

MRNNs have been a promising architecture in neuromorphic systems by virtue of their non-volatility, high-density, and physical storable feature. According to the realistic structure of MRNNs, several different mathematical models for MRNNs were proposed (Hu and Wang, 2010; Wu et al., 2011; Li et al., 2014; Chen et al., 2015; Jianmin et al., 2019). Meanwhile, notice that the MRNN, a special recurrent network, depends on the stability of its equilibrium points in application scenarios. Therefore, many interesting works were addressed to analyze the stability for the MRNNs (Hu and Wang, 2010; Wu et al., 2011; Li et al., 2014; Chen et al., 2015; Jianmin et al., 2019). A mathematical model of MRNN was proposed, and its global uniform asymptotic stability was investigated in a Lyapunov sense (Hu and Wang, 2010). A simple model of MRNN was introduced by Wu et al. (2011) by means of the typical current–voltage characteristics of memristors. A stochastic MRNN was proposed by Li et al. (2014) based on the work by Wang et al. (2007), in which there was some unavoidable noise in real networks. Furthermore, the global exponential stability for the stochastic MRNN was studied under the framework of Filppov's solution; three sufficient conditions with the form of linear inequalities were provided to determine the global exponential stability of the stochastic MRNN. The global asymptotic stability and synchronization of a class of fractional-order memristor-based delayed neural networks were investigated by Chen et al. (2015). The existence and global exponential stability were discussed by Jianmin et al. (2019) for an uncertain MRNN with mixed time delay under two assumptions.

Motivated by the aforementioned works, the global exponential stability of the equilibrium point is investigated for a class of MRNNs with time-varying delay, and its application to stabilize the voltage in a circuit network is carried out in this study. A sufficient condition is obtained for the global exponential stability of MRNNs. Based on this condition, an algorithm is proposed to stabilize the voltage of the MRNN-based circuit. The time-varying delay was considered in the activation functions of MRNN in this study. In addition, the activation functions in the MRNN are not necessarily non-decreasing, while the activation functions are non-decreasing in the works by Hu and Wang (2010); Wu et al. (2011); Li et al. (2014); Chen et al. (2015); Jianmin et al. (2019). Thus, the MRNN considered in this study is the extension from the view of activation functions compared with those in the works by Hu and Wang (2010); Wu et al. (2011); Li et al. (2014); Chen et al. (2015); Jianmin et al. (2019). Meanwhile, the stable voltage is a necessary prerequisite for obtaining high-quality electric energy in power systems, such as wind power converters (Kobravi et al., 2007). Consequently, the obtained theretical results are successfully applied to design the MRNN-based circuit system with global exponential stability, which makes it possible to apply the MRNN to power converters.

The structure of this article is given as follows: an MRNN with time-varying delay and some notations is introduced in **Section 2**. In **Section 3**, the global exponential stability of the equilibrium point for the MRNN is obtained, and an example is given to show the effectiveness of the obtained results. Then, an algorithm to design the MRNN-based circuit with stable voltage is proposed, and a simple application is carried out in **Section 4**. Finally, the main conclusions are given in **Section 5**.

2 MEMRISTOR-BASED RECURRENT NEURAL NETWORK

In this section, some notations are introduced, and an MRNN is described under two assumptions based on the mathematical models by Wen et al. (2013); Jianmin et al. (2019).

Notation: $\mathbb{R}$ denotes the set of real numbers. $x = (x_1, x_2, \ldots, x_m)^T$ is an $m-$ dimensional column, and the superscript T stands for the transpose operator. $\| x \| := (\sum_{i=1}^{m} x_i^2)^{1/2}$. $A = (a_{ij}) \in \mathbb{R}^{m\times m}$ is a matrix. $\| A \| = \sqrt{\lambda_M(A^T A)}$, where $\lambda_M(A)$ represents the maximum eigenvalue of A. $I \in \mathbb{R}^{m\times m}$ stands for an identity matrix. For a real symmetric matrix A, $A > 0 (A < 0)$ means that A is positive (negative) definite.

Consider the following MRNN, which was originated from Wen et al. (2013),

$$C_i \dot{x}_i(t) = -\left[\sum_{j=1}^{n}\left(\frac{1}{R_{fij}} + \frac{1}{R_{gij}}\right) + W_i(x_i(t))\right] x_i(t) + \sum_{j=1}^{n} \frac{\text{sign}_{ij}}{R_{fij}} f_j(x_j(t)) + \sum_{j=1}^{n} \frac{\text{sign}_{ij}}{R_{gij}} g_j(x_j(t-\tau_j(t))) + I_i. \tag{1}$$

Here, $f_j(\cdot)$ is the activation function, $\tau_j(\cdot)$ is the time-varying delay, C_i is the capacitance of the capacitor, and $x_i(t)$ is the voltage of the capacitor. $R_{f_{ij}}$ is the resistor between the feedback function $f_j(x_j(t))$ and the state $x_i(t)$, and $R_{g_{ij}}$ is the resistor between the feedback function $g_j(x_j(t-\tau_j(t)))$ and the state $x_i(t)$. sign_{ij} is defined as

$$\text{sign}_{ij} = \begin{cases} 1, & \text{if } i \neq j; \\ 0, & \text{if } i = j. \end{cases} \tag{2}$$

$W_i[x_i(t)]$ is the memductance of the $i-$ th memristor satisfying

$$W_i(x_i(t)) = \begin{cases} W_i', & \text{if } x_i(t) \leq 0; \\ W_i'', & \text{if } x_i(t) > 0. \end{cases} \tag{3}$$

I_i is an external input or bias and $i, j = 1, 2, \ldots, n$. Let

$$\widetilde{W}_i = \frac{W_i'' - W_i'}{2C_i}. \tag{4}$$

From Jianmin et al. (2019), the MRNN (**Eq. 1**) is transformed into:

$$\dot{x}_i(t) = -d_i x_i(t) - \widetilde{W}_i |x_i(t)| + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_j(t))) + U_i. \tag{5}$$

Here,

$$d_i = \sum_{j=1}^{n}\left[\frac{1}{C_i R_{f_{ij}}} + \frac{1}{C_i R_{g_{ij}}} + \frac{W_i' + W_i''}{2C_i}\right], \quad a_{ij} = \frac{\text{sign}_{ij}}{C_i R_{f_{ij}}},\ b_{ij} = \frac{\text{sign}_{ij}}{C_i R_{g_{ij}}},\ U_i = \frac{I_i}{C_i}. \tag{6}$$

Next, let $D = \text{diag}\{d_1, d_2, \ldots, d_n\}$, $\widetilde{W} = \text{diag}\{\widetilde{W_1}, \widetilde{W_2}, \ldots, \widetilde{W_n}\}$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $|x(t)| = (|x_1(t)|, |x_2(t)|, \ldots, |x_n(t)|)^T$, $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t))^T$, and $U = (U_1, U_2, \ldots, U_n)^T$. Then, **Eq. 5** is rewritten as:

$$\dot{x}(t) = -Dx(t) - \widetilde{W}|x(t)| + Af(x(t)) + Bg(x(t-\tau(t))) + U. \tag{7}$$

In addition, there are two assumptions and one lemma, which will be needed in the sequel, for the MRNN (**Eq. 7**). The first assumption about the activation function f_i is from Wen et al. (2013). The second assumption about the time-varying delay τ_j is from Wen et al. (2013).

$\mathbf{S_1}$. For $i \in \{1,2,\ldots,n\}$, the activation function f_i is bounded continuous, and $\forall r_1, r_2 \in \mathbb{R}$, there exists real number $l_i > 0$ such that

$$0 \le \frac{f_i(r_1) - f_i(r_2)}{r_1 - r_2} \le l_i. \quad (8)$$

Here, we set $L_f = \text{diag}\{l_1, l_2, \ldots, l_n\}$.

For $i \in \{1,2,\ldots,n\}$, the activation function g_i is bounded continuous, $g_i(0) = 0$, and $\forall r_1, r_2 \in \mathbb{R}$, there exists real number $l_i' > 0$ such that

$$-l_i' \le \frac{g_i(r_1) - g_i(r_2)}{r_1 - r_2} \le l_i'. \quad (9)$$

Here, we set $L_g = \text{diag}\{l_1', l_2', \ldots, l_n'\}$.

$\mathbf{S_2}$. For $i \in \{1,2,\ldots,n\}$, $\tau_i(t)$ satisfies

$$0 \le \tau_i(t) \le \overline{\tau}_i,\ \dot{\tau}_i(t) \le \mu_i < 1. \quad (10)$$

Here, we let $\overline{\tau} = \max\{\overline{\tau}_1, \ldots, \overline{\tau}_n\}$, and $\mu = \max\{\mu_1, \ldots, \mu_n\}$.

Remark 1. From **Eq. 9**, the activation functions $g_i[x_i(t)]$ are non-monotonic in this study. On the other hand, we notice that the activation functions of MRNNs in the works by Hu and Wang (2010; Wu et al. (2011); Li et al. (2014); Chen et al. (2015); Jianmin et al. (2019) are non-decreasing. Thus, **Eq. 1** is the extension from the view of activation functions compared with those references.

3 GLOBALLY EXPONENTIAL STABILITY

In this section, we will prove that the MRNN (**Eq. 1**) is globally exponentially stable under the assumptions $\mathbf{S_1}$ and $\mathbf{S_2}$. A sufficient condition with the form of linear matrix inequalities can be obtained for globally exponential stability of MRNN by constructing a suitable Lyapunov functional.

Theorem 1. Assume that $\mathbf{S_1}$ and $\mathbf{S_2}$ hold. If there exist a matrix $P = diag\{p_1, p_2, \ldots, p_n\} > 0$, a constant $k > 0$, and small enough constants $\xi > 0$ and $\vartheta > 0$ such that

$$\begin{aligned}\Phi &:= \xi P - 2PD + 2P|\widetilde{W}| + \left(1 + \frac{1}{k}\right)I + 2\vartheta\left(\xi L_f P + L_f P|\widetilde{W}|\right)\\ &\quad + \frac{(1+\vartheta)}{1-\mu}\,\|PB\|^2 e^{\xi\overline{\tau}} L_g L_g < 0,\\ \Psi &:= -2\vartheta L_f^{-1}PD + \vartheta\left(PA + A^T P\right) + \vartheta I + k\,\|PA\|^2 I < 0. \end{aligned} \quad (11)$$

Then, the equilibrium point of the MRNN (**Eq. 1**) is globally exponentially stable.

Proof: To simplify the proof, we make the following transformation:

$$z = x - x^*, \quad (12)$$

where x^* is the equilibrium point of the MRNN (**Eq. 1**). Then, the MRNN (**Eq. 1**) can be rewritten equivalently as

$$\begin{aligned}\dot{z}(t) &= -Dz(t) - \widetilde{W}(|z(t) + x^*| - |x^*|)\\ &\quad + A\mathscr{f}(z(t)) + B\mathscr{g}(z(t - \tau(t))), \end{aligned} \quad (13)$$

where $\mathscr{f}(z(t)) = f(z(t) + x^*) - f(x^*)$ and $\mathscr{g}(z(t - \tau(t))) = g(z(t - \tau(t)) + x^*) - g(x^*)$. It is obvious that $\mathscr{f}_i(0) = 0$ and $\mathscr{g}_i(0) = 0$. By the assumption $\mathbf{S_1}$, we get

$$\begin{aligned}&\mathscr{f}^T(z(t))z(t) \ge \mathscr{f}^T(z(t))L_f^{-1}\mathscr{f}(z(t)),\\ &\mathscr{f}^T(z(t))z(t) \le z^T(t)L_f z(t),\\ &\mathscr{g}(z(t)) \le L_g|z(t)|. \end{aligned} \quad (14)$$

We define a Lyapunov functional as follows:

$$\mathscr{V}(t,z) = \mathscr{V}_0(t,z) + \mathscr{V}_1(t,z) + \mathscr{V}_2(t,z), \quad (15)$$

where

$$\begin{aligned}&\mathscr{V}_0(t,z) = e^{\xi t} z^T P z,\\ &\mathscr{V}_1(t,z) = 2\vartheta e^{\xi t}\sum_{i=1}^{n} p_i \int_0^{z_i} \mathscr{f}_i(s)\,ds,\\ &\mathscr{V}_2(t,z) = \eta \sum_{i=1}^{n} \int_{t-\tau_i(t)}^{t} \mathscr{g}_i^2(z_i(s))\, e^{\xi(s+\overline{\tau}_i)} ds. \end{aligned} \quad (16)$$

Here, ξ, ϑ are small positive constants, and η is a positive constant to be determined.

First, calculating the time derivative of $\mathscr{V}_0(t,z)$ along the trajectories of the MRNN (**Eq. 13**), we have

$$\begin{aligned}\frac{d}{dt}\mathscr{V}_0(t,z(t)) &= \xi e^{\xi t} z^T(t)Pz(t) + 2e^{\xi t} z^T(t)P\dot{z}(t)\\ &= \xi e^{\xi t} z^T(t)Pz(t) - 2e^{\xi t} z^T(t)PDz(t)\\ &\quad - 2e^{\xi t} z^T(t)P\widetilde{W}(|z(t) + x^*| - |x^*|)\\ &\quad + 2e^{\xi t} z^T(t)PA\mathscr{f}(z(t))\\ &\quad + 2e^{\xi t} z^T(t)PB\mathscr{g}(z(t - \tau(t))). \end{aligned} \quad (17)$$

In addition,

$$\begin{aligned}-2e^{\xi t} z^T(t)P\widetilde{W}(|z(t) + x^*| - |x^*|) &\le 2e^{\xi t}|z(t)|^T P|\widetilde{W}|\\ &\quad \times |(|z(t) + x^*| - |x^*|)|\\ &\le 2e^{\xi t}|z(t)|^T P|\widetilde{W}| \cdot |z(t)|\\ &= 2e^{\xi t} z^T(t)P|\widetilde{W}|z(t), \end{aligned} \quad (18)$$

$$\begin{aligned}&2e^{\xi t} z^T(t)PA\mathscr{f}(z(t))\\ &\quad \le e^{\xi t}\left[\frac{1}{k}z^T(t)z(t) + \mathscr{f}^T(z(t))k\,\|PA\|^2\,\mathscr{f}(z(t))\right], \end{aligned} \quad (19)$$

$$\begin{aligned}2e^{\xi t} z^T(t)PB\mathscr{g}(z(t - \tau(t))) &\le e^{\xi t}\left[z^T(t)z(t) + \mathscr{g}^T(z(t - \tau(t)))\right.\\ &\quad \left.\times \|PB\|^2\,\mathscr{g}(z(t - \tau(t)))\right]. \end{aligned} \quad (20)$$

Here, the parameter k is a positive constant. Substituting **Eqs 18–20** into **Eq. 17**, we obtain

$$\begin{aligned}\frac{d}{dt}\mathscr{V}_0(t,z(t)) &\le e^{\xi t} z^T(t)\left[\xi P - 2PD + 2P|\widetilde{W}| + \left(1 + \frac{1}{k}\right)I\right]z(t)\\ &\quad + e^{\xi t}\mathscr{f}^T(z(t))k\,\|PA\|^2\,\mathscr{f}(z(t)) + e^{\xi t}\mathscr{g}^T(z(t - \tau(t)))\\ &\quad \times \|PB\|^2\,\mathscr{g}(z(t - \tau(t))). \end{aligned} \quad (21)$$

Second, by calculating the time derivative of $\mathscr{V}_1(t,z)$ along the trajectories of the MRNN (**Eq. 13**), it follows

$$\begin{aligned}\frac{d}{dt}\mathscr{V}_1(t,z(t)) &= 2\xi\vartheta e^{\xi t}\sum_{i=1}^{n}p_i\int_0^{z_i}f_i(s)\,ds + 2\vartheta e^{\xi t}f^T(z(t))P\dot{z}(t)\\ &\le 2\xi\vartheta e^{\xi t}f^T(z(t))Pz(t) - 2\vartheta e^{\xi t}f^T(z(t))PDz(t)\\ &\quad - 2\vartheta e^{\xi t}f^T(z(t))P\widetilde{W}(|z(t)+x^*|-|x^*|)\\ &\quad + 2\vartheta e^{\xi t}f^T(z(t))PAf(z(t)) + 2\vartheta e^{\xi t}f^T(z(t))PB\\ &\quad \times g(z(t-\tau(t))). \end{aligned} \tag{22}$$

By **Eq. 14**, we have

$$\begin{aligned}&2\xi\vartheta e^{\xi t}f^T(z(t))Pz(t) \le 2\xi\vartheta e^{\xi t}z^T(t)L_fPz(t),\\ &\quad -2\vartheta e^{\xi t}f^T(z(t))PDz(t) \le -2\vartheta e^{\xi t}f^T(z(t))L_f^{-1}PDf(z(t)),\end{aligned} \tag{23}$$

$$\begin{aligned}&-2\vartheta e^{\xi t}f^T(z(t))P\widetilde{W}(|z(t)+x^*|-|x^*|)\\ &\le 2\vartheta e^{\xi t}|z(t)|^TL_fP|\widetilde{W}\parallel z(t)|\\ &= 2\vartheta e^{\xi t}z^T(t)L_fP|\widetilde{W}|z(t).\end{aligned} \tag{24}$$

Notice that

$$\begin{aligned}&2\vartheta e^{\xi t}f^T(z(t))PBg(z(t-\tau(t)))\\ &\le \vartheta e^{\xi t}f^T(z(t))f(z(t)) + e^{\xi t}g^T(z(t-\tau(t)))\vartheta\parallel PB\parallel^2\\ &\quad\times g(z(t-\tau(t))).\end{aligned} \tag{25}$$

Substituting **Eqs 23–25** into **Eq. 22**, we have

$$\begin{aligned}&\frac{d}{dt}\mathscr{V}_1(t,z(t))\\ &\le 2\xi\vartheta e^{\xi t}z^T(t)L_fPz(t) - 2e^{\xi t}f^T(z(t))\vartheta L_f^{-1}PDf(z(t))\\ &\quad + 2\vartheta e^{\xi t}z^T(t)L_fP|\widetilde{W}|z(t) + 2\vartheta e^{\xi t}f^T(z(t))PAf(z(t))\\ &\quad + \vartheta e^{\xi t}f^T(z(t))f(z(t)) + e^{\xi t}g^T(z(t-\tau(t)))\vartheta\parallel PB\parallel^2\\ &\quad\times g(z(t-\tau(t)))\\ &= 2\vartheta e^{\xi t}z^T(t)\left[\xi L_fP + L_fP|\widetilde{W}|\right]z(t) + 2e^{\xi t}f^T(z(t))\\ &\quad\times\left[-\vartheta L_f^{-1}PD + \vartheta PA + \frac{1}{2}\vartheta I\right]f(z(t))\\ &\quad + e^{\xi t}g^T(z(t-\tau(t)))\vartheta\parallel PB\parallel^2 g(z(t-\tau(t))).\end{aligned} \tag{26}$$

Third, calculating the time derivative of $\mathscr{V}_2(t,z)$ along the trajectories of the MRNN (**Eq. 13**), we have

$$\begin{aligned}&\frac{d}{dt}\mathscr{V}_2(t,z(t))\\ &= \eta\sum_{i=1}^{n}e^{\xi(t+\overline{\tau}_i)}g_i^2(z_i(t)) - \eta\sum_{i=1}^{n}\left(1-\dot{\tau}_i(t)\right)e^{\xi(t-\tau_i(t)+\overline{\tau}_i)}\\ &\quad\times g_i^2\left(z_i\left(t-\tau_i(t)\right)\right)\\ &\le \eta e^{\xi(t+\overline{\tau})}g^T(z(t))g(z(t)) - \eta(1-\mu)e^{\xi t}g^T(z(t-\tau(t)))\\ &\quad\times g(z(t-\tau(t)))\\ &\le e^{\xi t}z(t)^T\left[\eta e^{\xi\overline{\tau}}L_gL_g\right]z(t) - \eta(1-\mu)e^{\xi t}g^T(z(t-\tau(t)))\\ &\quad\times g(z(t-\tau(t))).\end{aligned} \tag{27}$$

Hence, by **Eqs 21**, **26**, **27**, we have

$$\begin{aligned}&\frac{d}{dt}\mathscr{V}(t,z(t))\\ &= \frac{d}{dt}\mathscr{V}_0(t,z(t)) + \frac{d}{dt}\mathscr{V}_1(t,z(t)) + \frac{d}{dt}\mathscr{V}_2(t,z(t))\\ &\le e^{\xi t}z^T(t)\Big[\xi P - 2PD + 2P|\widetilde{W}| + \left(1+\frac{1}{k}\right)I\\ &\quad + 2\vartheta\left(\xi L_fP + L_fP|\widetilde{W}|\right) + \eta e^{\xi\overline{\tau}}L_gL_g\Big]z(t)\\ &\quad + e^{\xi t}f^T(z(t))\left[-2\vartheta L_f^{-1}PD + \vartheta\left(PA + A^TP\right) + \vartheta I + k\parallel PA\parallel^2 I\right]\\ &\quad\times f(z(t)) + e^{\xi t}g^T(z(t-\tau(t)))\left[(1+\vartheta)\parallel PB\parallel^2 - \eta(1-\mu)\right]\\ &\quad\times g(z(t-\tau(t))).\end{aligned} \tag{28}$$

Let $\eta = \frac{(1+\vartheta)}{1-\mu}\parallel PB\parallel^2$ in **Eq. 28**. It means that

$$\begin{aligned}&\frac{d}{dt}\mathscr{V}(t,z(t))\\ &\le e^{\xi t}z^T(t)\Big[\xi P - 2PD + 2P|\widetilde{W}| + \left(1+\frac{1}{k}\right)I\\ &\quad + 2\vartheta\left(\xi L_fP + L_fP|\widetilde{W}|\right) + \frac{(1+\vartheta)}{1-\mu}\parallel PB\parallel^2 e^{\xi\overline{\tau}}L_gL_g\Big]z(t)\\ &\quad + e^{\xi t}f^T(z(t))\left[-2\vartheta L_f^{-1}PD + \vartheta\left(PA + A^TP\right) + \vartheta I\right.\\ &\quad\left. + k\parallel PA\parallel^2 I\right]f(z(t))\\ &= e^{\xi t}z^T(t)\Phi z(t) + e^{\xi t}f^T(z(t))\Psi f(z(t)).\end{aligned} \tag{29}$$

Since $\Phi < 0$, $\Psi < 0$, and by **Eq. 29**, we have

$$\frac{d}{dt}\mathscr{V}(t,z(t)) \le 0, \tag{30}$$

which means that $e^{\xi t}z^T(t)Pz(t) = \mathscr{V}_0(t,z(t)) \le \mathscr{V}(t,z(t)) \le \mathscr{V}(0,z(0))$. More precisely,

$$\parallel x(t) - x^*\parallel = \parallel z(t)\parallel \le Me^{-\frac{\xi}{2}t}, \tag{31}$$

where $M = [p\mathscr{V}(0,z(0))]^{\frac{1}{2}}$ and $p = \max\{p_i : i = 1,\ldots,n\}$, that is, the unique equilibrium point x^* of the MRNN (**Eq. 1**) is globally exponentially stable.

Remark 2. Motivated by the representation of the Lyapunov functional in the work by Jianmin et al. (2019), we construct a new Lyapunov functional $\mathscr{V}(t,x(t))$, in order to overcome the difficulty brought by the nonmonotone activation functions in MRNN (**Eq. 1**) in the proof of Theorem 1.

Now, we give an example to illustrate that the equilibrium point of the MRNN is globally exponentially stable when the conditions in Theorem 1 are satisfied.

Example 1. Consider an MRNN (**Eq. 1**) with four state voltages, for which the parameter values of MRNN (**Eq. 1**) are originated from the work by Jianmin et al. (2019), especially the capacitors $C_1 = 2$, $C_2 = 3$, $C_3 = 2$, and $C_4 = 7$; the external inputs $I_1 = 9$, $I_2 = 3$, $I_3 = 9.5$, and $I_4 = 6$; the memductances $W_1' = 1$, $W_2' = 3$, $W_3' = 9.5$, and $W_4' = 1$ for $x_i(t) \le 0$; the memductances $W_1'' = 4$, $W_2'' = 1.5$, $W_3'' = 2$, and $W_4'' = 3.5$ for $x_i(t) \ge 0$; and the resistors $R_f := (R_{f_{ij}})$ and $R_g := (R_{g_{ij}})$ are given as follows:

$$R_f = \begin{bmatrix}1 & 3 & 1.5 & 2\\ 2 & 4 & 6 & 12\\ 1.8 & 2.6 & 3.5 & 4\\ 7 & 3 & 4 & 3\end{bmatrix},$$

$$R_g = \begin{bmatrix} 1 & 3 & 1.5 & 2 \\ 2 & 2 & 2 & 2 \\ 1.8 & 2.6 & 3.5 & 4 \\ 7 & 3 & 4 & 3 \end{bmatrix}.$$

Next, by the aforementioned parameters and **Eqs 2–6**, it follows that D, $\widetilde{W}$, U, A, and B. Let the activation functions

$$f_i(x_i(t)) = \frac{1}{2}(|x_i(t)+1| - |x_i(t)-1|),$$

$$g_i(x_i(t)) = \sin(x_i(t)),$$

and the time-varying delays

$$\tau_i(t) = 1.8 + 0.5\sin t,$$

for $i = 1,2,3,4$. It is obvious that the assumptions $\mathbf{S}_1$ and $\mathbf{S}_2$ are satisfied. Then, by assumptions $\mathbf{S}_1$ and $\mathbf{S}_2$, we have $L = \text{diag}\{1,1,1,1\}$, $\overline{\tau} = 2.3$, and $\mu = 0.5$.

Now, by fixing the parameters $k = 1000$, $\xi = 0.001$, and $\vartheta = 0.001$ in Theorem 1 and substituting the matrices $A, B, D, \widetilde{W}$ into the linear matrix inequalities (**Eq. 11**), we get a positive definite diagonal matrix

$$P = diag\{0.0499, 0.0499, 0.0499, 0.0499\},$$

namely, by Theorem 1, the equilibrium point of the MRNN (**Eq. 1**) is globally exponentially stable.

The initial values of the neural network (**Eq. 1**) are set at $(0.1,0.1,0.1,0.1)^{\mathrm{T}}$, $(0.5,0.5,0.5,0.5)^{\mathrm{T}}$, and $(0.9,0.9,0.9,0.9)^{\mathrm{T}}$. The solution trajectories of **Eq. 1** are illustrated in **Figure 1**. From **Figure 1**, we see that the equilibrium point of the MRNN is globally exponentially stable, which shows the validation of the obtained result from Theorem 1.

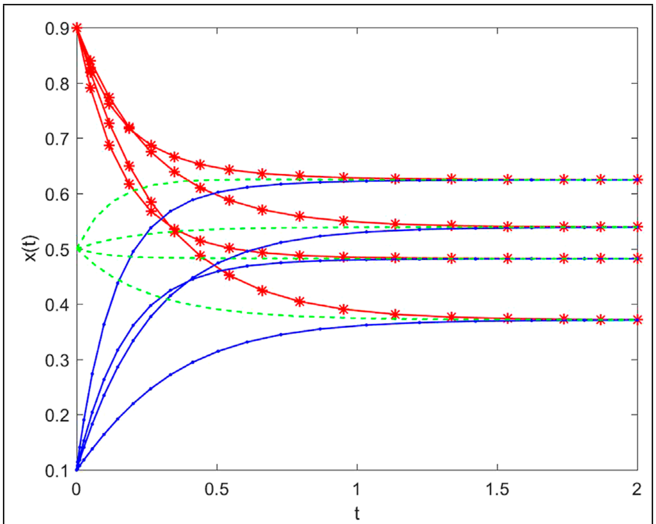

FIGURE 1 | Solution trajectories of the MRNN (1).

4 AN ALGORITHM TO DESIGN THE MRNN-BASED CIRCUIT WITH STABLE VOLTAGES

Note that the stable voltage is a necessary prerequisite for obtaining high-quality electric energy in power systems. In this section, the two linear inequalities in Theorem 1 are used to design the MRNN-based circuit with globally exponentially stable voltages, which make it possible to apply the MRNN to power converters. The design process is described by the following four steps:

Step 1: Fix the values of capacitor C_i, external input I_i, and the resistors $R_{f_{ij}}$ and $R_{g_{ij}}$ in **Eq. 1** for $i,j = 1,2,\ldots,n$.

Step 2: For the given time-varying delay $\tau_i(t)$ and the activation functions f_i, g_i, calculate the matrices L_f, L_g in the assumption $\mathbf{S}_1$ and the parameters $\overline{\tau}$ and μ in the assumption $\mathbf{S}_2$.

Step 3: Determine the parameters W_i' and W_i'' in the memductance $W_i(x_i(t))$ of the $i-$ th memristor in **Eq. 1** for $i = 1,2,\ldots,n$.

- Fix a matrix $P > 0$ and the parameters k, ξ, and ϑ in Theorem 1.
- Substitute C_i, I_i, $R_{f_{ij}}$, and $R_{g_{ij}}$ into a_{ij}, b_{ij}, and U_i in **Eq. 6** to obtain matrices A, B, D, and U.
- Substitute the matrices P, D, A, B, and U into the linear matrix inequalities (11).
- Solve **Eq. 11** to obtain the matrix $\widetilde{W}$.
- Calculate W_i' and W_i'' by the d_i and $\widetilde{W}_i$ in **Eq. 5**.

Step 4: By substituting $C_i, I_i, R_{f_{ij}}, R_{g_{ij}}, W_i'$, and W_i'' into **Eq. 1**, the MRNN-based circuit with stable voltages is obtained.

Remark 3. From Step 3, the parameters W_i' and W_i'' in the MRNN (**Eq. 1**) can be determined at the same time by the parameter d_i and $\widetilde{W}_i$ for $i = 1,2,\ldots,n$ in (6). Consequently, we can select or make the memristor guarantee the MRNN-based circuit with stable voltage when the other elements are given beforehand by means of the proposed algorithm.

Next, we will design an MRNN-based circuit with four stable voltages by the proposed algorithm, where the activation functions and some of the parameters in the MRNN-based circuit in this example are the same as those in the first example.

Example 2. It is declared that the activation functions $f_i(x_i(t))$, $g_i(x_i(t))$, the time-varying delay $\tau_i(t)$, and the values of parameters C_i, I_i, $R_{f_{ij}}$, and $R_{g_{ij}}$ for the MRNN-based circuit are the same as those in the first example. Next, by Step 3, we determine the values of W_i' and W_i'' in the memductance $W_i[x_i(t)]$ *of the* $i-$ th memristor in **Eq. 1** for $i,j = 1,2,3,4$.

- Fix the values of parameters $k = 1000$, $\xi = 0.001$, and $\vartheta = 0.001$ in Theorem 1 and a matrix $P = diag\{5,5,5,5\} > 0$ and $D = diag\{30,20,25,10\}$. Substitute the matrices P, D, B, and U into the linear matrix inequalities (11). Then, solve **Eq. 11** to obtain the matrix $\widetilde{W}$:

$$\widetilde{W} = \{25.5148, 15.9082, 20.7115, 6.3017\}.$$

- Calculate W_i' and W_i'' by d_i and $\widetilde{W}_i$ in **Eq. 6**, especially $W_1'' = 60.9574$, $W_2'' = 56.3541$, $W_3'' = 47.3558$,

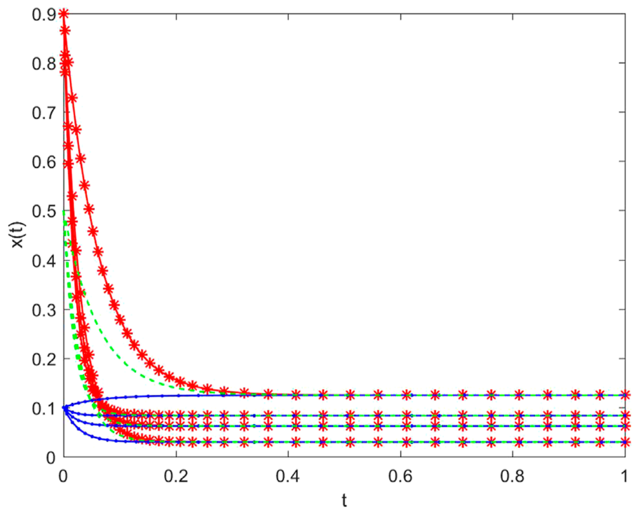

FIGURE 2 | Solution trajectories of the designed MRNN-based circuit.

$W_4'' = 17.1508$; $W_1' = 35.4426$, $W_2' = 40.4459$, $W_3' = 26.6443$, and $W_4' = 10.8492$.

By Step 4, substituting C_i, I_i, $R_{f_{ij}}$, $R_{g_{ij}}$, W_i', and W_i'' into **Eq. 1**, we obtain the MRNN-based circuit with stable voltages. The initial values of **Eq. 1** are given as same as those in Example 1. The solution trajectories of the designed MRNN-based circuit are depicted in **Figure 2**, which means that we obtain a MRNN-based circuit with stable voltages through selecting the suitable parameter values in the memductance of the memristor.

5 CONCLUSION

In this study, the global exponential stability of the equilibrium point of the MRNN is investigated for a class of general activation functions. A sufficient condition with the form of linear matrix inequalities is obtained for the global exponential stability. Furthermore, the proposed results are applied to design the MRNN-based circuits with stable voltages. From the view of the MRNN-based circuit, some elements of the MRNN-based circuit with stable voltages can be determined by the proposed algorithm. Note that the earth's environmental pollution and the lack of energy restrict the survival and development of the human society. Wind energy, an environment-friendly renewable resource, has become one of the effective ways to solve these two difficulties. The conversion of wind energy into electric energy can rely on wind power converters. The mathematical model of the power system of new wind turbines was described by a recurrent network. Thus, further research will focus on transforming the output voltage of the wind power converter to ensure the stable amplitude of its output voltage based on MRNN with stability.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

ZY contributed to the globally exponential stability of MRNN by considering the proper assumptions and constructing a suitable Lyapunov functional. YL drafted the manuscript and contributed to the algorithm of design of the MRNN-based circuit with stable voltages, experiments, and conclusions. All authors agree to be accountable for the content of the work.

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