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Electrical Circuits Described by General Fractional Conformable Derivative

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The general fractional conformable derivative (GCD) and its attributes have been described by researchers in the recent times. Compared with other fractional derivative definitions, this derivative presents a generalization of the conformable derivative and follows the same derivation formulae. For electrical circuits, such as RLC, RC, and LC, we obtain a new class of fractional-order differential equations using this novel derivative, The use of GCD to depict electrical circuits has been shown to be more adaptable and lucrative than the usual conformable derivative.

Keywords: general fractional conformable derivative, conformable derivative, electrical RC circuit, electrical LC circuit, electrical RLC circuits

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INTRODUCTION

Fractional calculus (FC) is a natural evolution of regular calculus that includes noninteger-order derivatives and integrals. FC has received considerable attention in the last 3 decades because it is an effective and commonly used approach for better modeling and control of processes in many sectors in science and engineering (Baleanu et al., 2010; Caponetto et al., 2010; Monje et al., 2010; Golmankhaneh Alireza and Lambert, 2012; Valsa and Vlach, 2013; Bao et al., 2015; Hartley et al., 2015; Kaczorek and Rogowski, 2015; Soltan et al., 2016). Because fractional derivatives (FDs) are defined using integrals, they are non-local operators. As a result, FDs in time incorporate information about the function at previous positions, resulting in a memory effect and nonlocal spatial effects. In reality, they consider the background of the system as well as nonlocal scattered effects, all of which are crucial for a more accurate and precise description and analysis of complex and dynamic control systems. FDs and integrals are now defined in various ways (Capelas de Oliveira and Tenreiro Machado, 2014). The references cited therein are examples of these concepts (Oldham and Spanier, 1974; Miller and Ross, 1993; Samko et al., 1993; Podlubny, 1999; Uchaikin, 2013; Caputo and Fabrizio, 2015; Atangana and Baleanu, 2016). One issue in this discipline is determining which FD is used to replace the ordinary derivative in a particular scenario. The Riemann–Liouville and Caputo FDs are the most frequently used definitions (Li et al., 2011). In addition, a two-scale FD occurs see (Ji-Huan et al., 2021). Classical applications of FC, such as the autochrone issue, have demonstrated its potential (Abel, 1839a). Other types of applications include the fractional diffusion equation (Wyss, 1986), models based on memory mechanisms (Caputo and Mainardi, 1971), and new linear capacitor theory (Westerlund, 1994). Teka et al. (2017) studied several elements of the fractional-order faulty integrate and burn model offered by sophisticated multiple timescale brain dynamics. The linearity attribute is satisfied by all FD definitions. However, almost all FDs lack mathematical features such as product rules and chain rules. These among other irregularities have caused several issues in real-world applications, limiting opportunities to investigate fractional computations.

By contrast, the fractal concept has been extensively studied in literature (Wang, 2022a; Wang, 2022b). For example, in Wang (2022a), fractal calculus was used to illustrate a shallow water wave with irregular borders, and He's variational method was used to successfully find its exact fractal solitary wave solution. Numerical examples demonstrate the simplicity, efficiency, and convenience of this method. Finally, we illustrate the physical features of fractal solitary solutions using certain graphs. In addition, the goal of this study () is to define the coupled nonlinear fractal Schrödinger system using fractal derivatives and to establish its variational concept using the fractal semi-inverse approach.

Regarding fractal electrical circuits, one can cite recent works (Banchuin, 2022). Banchuin (2022) derived fractal integrodifferential equations for RL, RC, LC, and RLC circuits subjected to zero-mean additive white Gaussian noise specified on a fractal set.

To overcome these problems, Khalil et al. (2014) developed an innovative approach that extends the standard limit definitions of a function's derivatives, termed as conformable FD. This definition allows for expansions of some classical calculus theorems required in fractional differential models, but not allowed by existing definitions. Researchers are interested in this conformable derivative because it appears to meet all of the standard derivative criteria (Katugampola, 2014; Abdeljawad, 2015). Furthermore, computation with this new derivative is considerably easier than that with existing FD formulations. Consequently, this new definition is being used in a large number of projects. (Hammad and Khalil, 2014; Atangana et al., 2015; Al Horani et al., 2016; Zhao and Li, 2016; Cenesiz et al., 2017). By contrast (Zhao and Luo, 2015; Li et al., 2020), investigated an extension of the classical conformable FD. The authors of (Zhao and Luo, 2015) defined a new type of FD known as general fractional conformable derivative (GCD). Furthermore, the authors demonstrated several unique results for the diffusion equation solution (Li et al., 2020). Some further additional efforts to the conformable derivative are recently done by researchers, for example, Exact solutions of conformable time fractional Zoomeron equation *via* IBSEFM (Demirbilek et al., 2021), Fuzzy systems (Younus et al., 2021a; Younus et al., 2021b), the Solutions of Fractional Cauchy Problem Featuring Conformable Derivative (Yavuz and Özdemir, 2018) and Fundamental Results of Conformable Sturm–Liouville Eigenvalue Problems (Al-Refai and Abdeljawad, 2017). The conformable derivative's broad application is exemplified by the large number of recent research publications, which demonstrate the derivative's importance in solving diverse problems in science and engineering. Certain notions remain unaddressed by the conformable derivative, and it represents an unexplored subject of study.

Electrical circuits are modeled using mathematical representations, whose requirement stems from the following question: what is the best mathematical model that approximates the real one? According to our findings, it has been proven in the literature that conformable derivatives are preferable to integer-order derivatives and other types of FDs. Thus, in our study, we provide a general conformable derivative as a solution to describe electrical circuits and prove that this choice is more flexible, providing a large set of equations that simplify the modelization problem. The authors of (Martínez et al., 2018) compared conformable derivatives with other types of FDs. Thus, the fundamental contribution of our work is the introduction of a novel GCD and comparison of our results with those of (Martínez et al., 2018). Indeed, after interpretation and analysis, we determined that our choice is not only more appropriate but also offers a wider range of model alternatives.

The structure of this article is organized as follows. *Preliminaries* presents the preliminaries. *General Fractional Conformable RC Circuit* discusses the general fractional-conformable RC circuits. *General Fractional Conformable LC Circuit* introduces a general fractional-conformable LC circuit. *General Fractional Conformable RLC Circuit* describes a general fractional-conformable RLC circuit. Finally, the conclusions are presented in *Conclusion*.

PRELIMINARIES

This section begins with a review of some theorems, definitions, and lemmas (Samko et al., 1993; Podlubny, 1999; Hermann, 2011; Hartley et al., 2015).

Definition 1. Let $\theta \in (n, n+1]$. Assume a function ϕ, which is defined in $[0\, b)$; then, the general conformable derivative of ϕ is defined by:

$$T^{\theta,\psi}\phi(t) = \lim_{\varepsilon\to 0} \frac{\phi^{([\theta]-1)}\left(t + \varepsilon\psi(t,\theta)\right) - \phi^{([\theta]-1)}(t)}{\varepsilon} \tag{1}$$

For all $t > 0$, where $[\theta]$ is the smallest integer greater than or equal to θ, and $\psi(t,\theta)$ is a continuous non-negative function that depends on t and satisfies

$$\psi(t, n+1) = 1,$$
$$\psi(.,\theta_1) \neq \psi(.,\theta_2), \ \ where\ \theta_1 \neq \theta_2\ and\ \theta_1, \theta_2 \in (n, n+1) \tag{2}$$

If $T^{\theta,\psi}$ exists, for every $t \in (0, c)$ and for some $c > 0$, and $\lim_{t\to 0^+} T^{\theta,\psi}\phi(t)$ exists, then:

$$T^{\theta,\psi}\phi(0): = \lim_{t\to 0^+} T^{\theta,\psi}\phi(t) \tag{3}$$

Remark 1. The general conformable derivative generalizes the classical derivative ($\theta = 1$) and the conformable derivative $\psi(t) = t^{1-\theta}$ (see (Lu et al., 2021)).

Remark 2. To further study the properties of the general conformable derivative, we assumed that $\psi(t,\theta) > 0$ for all $t > 0$, and $\frac{1}{\psi}(.,\theta)$ are locally integrable.

GENERAL FRACTIONAL CONFORMABLE RC CIRCUIT

An RC circuit's behavior is governed by the equation:

$$\frac{dV(t)}{dt} + \frac{1}{T}V(t) = \frac{e(t)}{T} \tag{4}$$

where $T = RC$, C is the capacitance, R is the resistance, $e(t)$ is the source, and the capacitor's voltage is $V(t)$.

Rosales et al. followed a comprehensive method for constructing fractional differential equations in Rosales et al. (2011), achieved in earlier investigations (Rosales et al., 2012; Gómez et al., 2013) using the Caputo fractional derivative. It comprises the following presentation of the elements ∂_t and ∂_x with appropriate dimensions (Rosales et al., 2011):

$$\frac{d}{dt} = \frac{1}{\partial_t^{1-\alpha}}\frac{d^\alpha}{dt}, \frac{d}{dx} = \frac{1}{\partial_x^{1-\alpha}}\frac{d^\alpha}{dx} \tag{5}$$

where $\alpha \in (0, 1]$ is the derivative order.

Transformation (5) describes the electrical circuits RC, LC, and RLC. Thus, they obtained the following transformation for an RC circuit:

$$\frac{d}{dt} \rightarrow \frac{1}{T^{1-\alpha}}\frac{d^\alpha}{dt} = \frac{t^{1-\alpha}}{T^{1-\alpha}}\frac{d}{dt} \tag{6}$$

Inspired by the method used in Martínez et al. (2018), we introduce the following transformation for GCD:

$$\frac{d}{dt} \rightarrow \frac{1}{\psi_1(T,\alpha)}\frac{d^\alpha}{dt} = \frac{\psi_1(t,\alpha)}{\psi_1(T,\alpha)}\frac{d}{dt} \tag{7}$$

where $\psi_1(t,\alpha)$ is a continuous nonnegative function that depends on t and satisfies (2).

Replacing the GCD in **Eq. 4**, we obtain:

$$\frac{1}{\psi_1(T,\alpha)}\frac{d^\alpha V(t)}{dt^\alpha} + \frac{1}{T}V(t) = \frac{e(t)}{T} \tag{8}$$

Then,

$$\frac{dV(t)}{dt} + \frac{a}{\psi_1(t,\alpha)}V(t) = \frac{a}{\psi_1(t,\alpha)}e(t) \tag{9}$$

where $a = \frac{\psi_1(T,\alpha)}{T}$.

Consider $e(t) = e_0$, where e_0 is a real constant, and the initial condition $V(0) = 0$. We get:

$$V(t) = e_0\left[-e^{-a\int_0^t \frac{1}{\psi_1(s,\alpha)}ds} + 1\right] \tag{10}$$

Remark 1. If we consider $\psi_1(t,\alpha) = t^{1-\alpha}$, we obtain the same result as in Martínez et al. (2018) (the classical conformable fractional derivative) with the following solution:

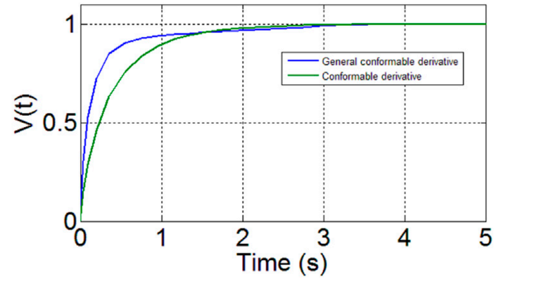

FIGURE 1 | Evolution of $V(t)$ with $\alpha = 0.8$.

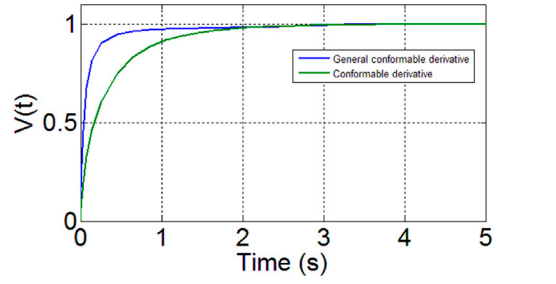

FIGURE 2 | Evolution of $V(t)$ with $\alpha = 0.7$.

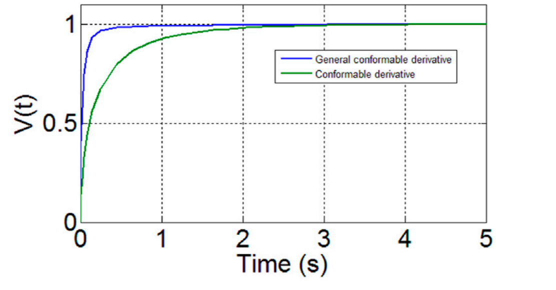

FIGURE 3 | Evolution of $V(t)$ with $\alpha = 0.6$.

$$V(t) = e_0\left[-e^{-\frac{1}{\alpha}\left(\frac{t}{T}\right)^\alpha} + 1\right] \tag{11}$$

When $e(t) = e_0\cos(\omega t)$ the oscillatory source with ω is the angular frequency. We obtain:

$$\frac{dV(t)}{dt} + \frac{a}{\psi_1(t,\alpha)}V(t) = \frac{a}{\psi_1(t,\alpha)}e_0\cos(\omega t) \tag{12}$$

For the simulation, we chose the same parameter values as the ones in Martínez et al. (2018): $R = 10\,M\Omega$, $C = 46\,nF$ and $f = 60\,Hz$. When the source $e(t) = e_0 = 1V$, one can select $\psi_1(t,\alpha) = t^{1-\alpha}((1-\alpha)g^2(t) + 1)$, where $g(t) = \sqrt{30}\sin(t)$. The simulation results are as follows:

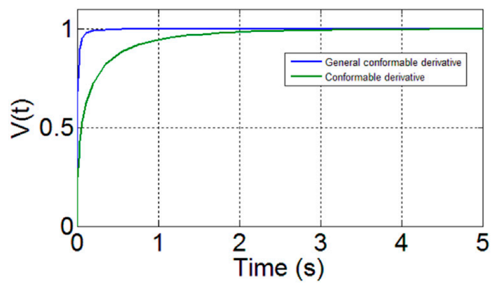

FIGURE 4 | Evolution of $V(t)$ with $\alpha = 0.5$.

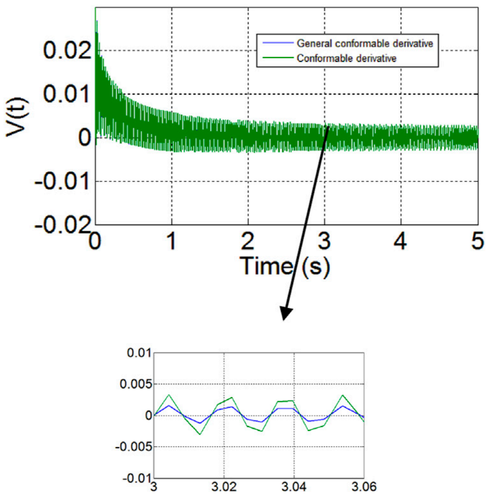

FIGURE 5 | Evolution of $V(t)$ with $\alpha = 0.7$.

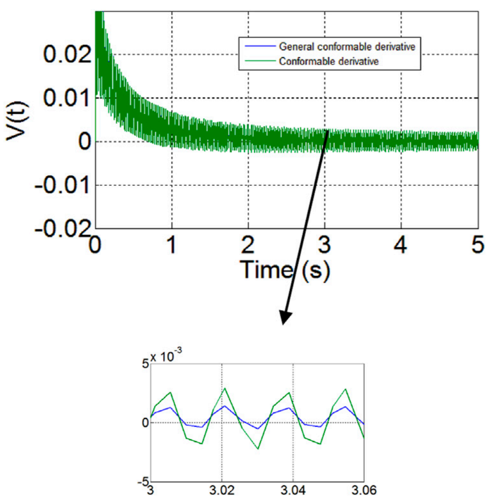

FIGURE 6 | Evolution of $V(t)$ with $\alpha = 0.6$.

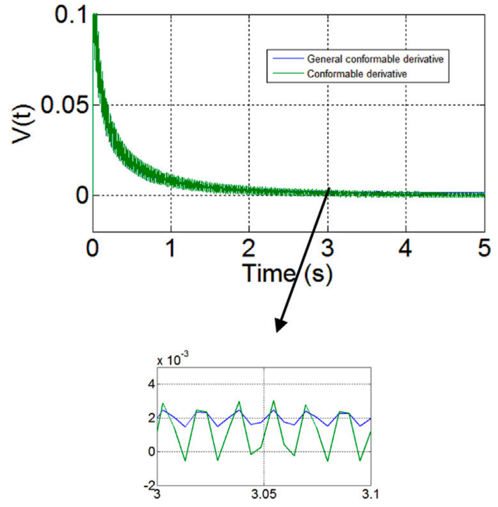

FIGURE 7 | Evolution of $V(t)$ with $\alpha = 0.4$.

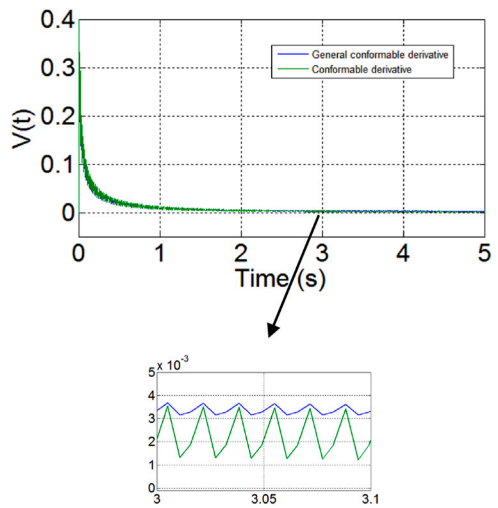

FIGURE 8 | Evolution of $V(t)$ with $\alpha = 0.2$.

As shown in **Figures 1–4**, with the chosen function $\psi_1(t,\alpha) = t^{1-\alpha}((1-\alpha)g^2(t)+1)$, it has been discovered that the general conformable derivative approaches the steady state faster than the conformable derivative.

Now, in the case when $e_0 = e_0\cos(\omega t)$, one can select $\psi_1(t,\alpha) = t^{1-\alpha}((1-\alpha)g^2(t)+1)$ where $g(t) = \sqrt{0.01}\ \frac{1}{e^{-t}}$. We obtained the following simulation results for different values of α.

Based on **Figures 5–8**, with the chosen function $\psi_1(t,\alpha) = \sqrt{0.01}\ \frac{1}{e^{-t}}$, it can be observed that when the value of α is reduced, the oscillations with GCD drop more quickly than with the classical conformable one.

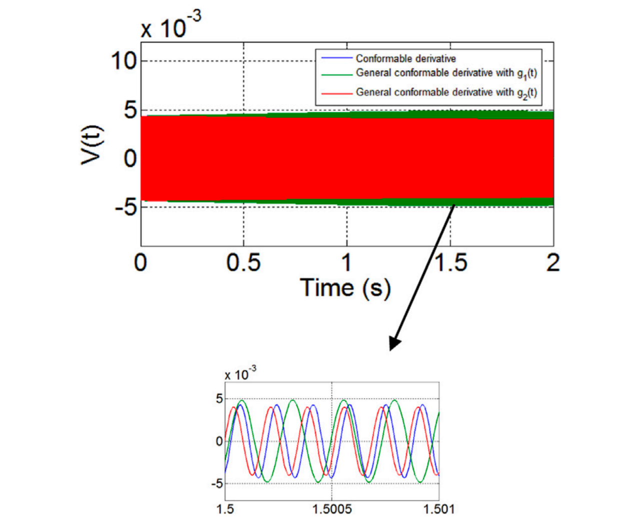

FIGURE 9 | Evolution of $V(t)$ with $\alpha = 0.98$.

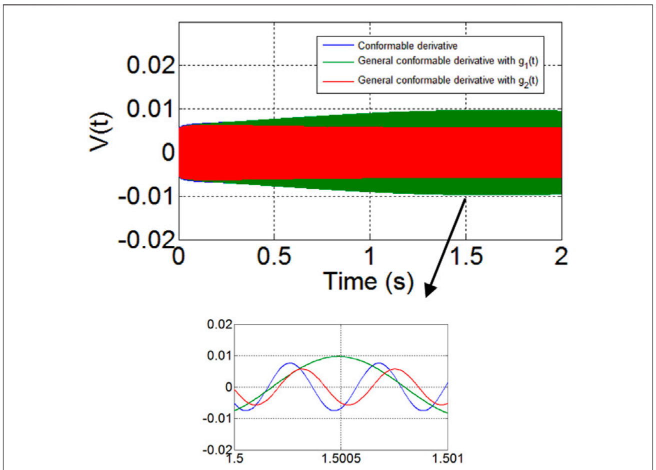

FIGURE 10 | Evolution of $V(t)$ with $\alpha = 0.9$.

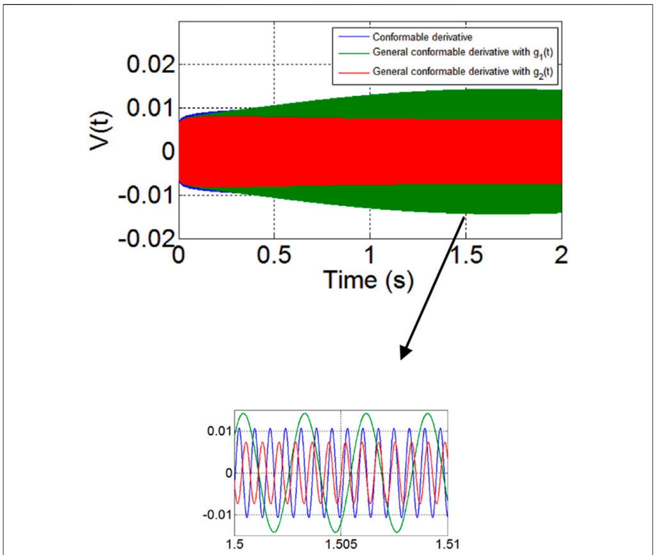

FIGURE 11 | Evolution of $V(t)$ with $\alpha = 0.85$.

GENERAL FRACTIONAL CONFORMABLE LC CIRCUIT

We assume an LC circuit that does not have a driving force. When the capacitor is originally charged and subsequently closed, the current in the circuit and the charge on the capacitor fluctuate between the positive and negative values. The voltage variation in the charge of the capacitor with respect to time is specified by a smooth second-order linear differential equation as follows:

$$\frac{d^2V(t)}{dt^2} + \frac{1}{LC}V(t) = 0 \tag{13}$$

The typical solution will then be:

$$V(t) = V_0 cos\left(\omega\frac{t}{0}\right) \tag{14}$$

where $\omega_0^2 = \frac{1}{LC}$ is the circuit angular frequency, and V_0 is the initial voltage at $t = 0$.

We can write the time general fractional conformable transform using **Eq. 7**.

$$\frac{d^2}{dt^2} \rightarrow \psi_2(\omega_0, 2\alpha)\frac{d^{2\alpha}}{dt^{2\alpha}} = \psi_2(\omega_0, 2\alpha)\psi_2(t, 2\alpha)\frac{d^2}{dt^2} \tag{15}$$

Then, considering this relationship, we derive its corresponding general conformable differential equation for (14):

$$\psi_2(\omega_0, 2\alpha)\psi_2(t, 2\alpha)\frac{d^2V(t)}{dt^2} + \omega_0^2V(t) = 0 \tag{16}$$

$$\frac{d^2V(t)}{dt^2} + \frac{\omega_0^2}{\psi_2(\omega_0, 2\alpha)\psi_2(t, 2\alpha)}V(t) = 0 \tag{17}$$

The relevant general conformable differential equation for a harmonic source with angular frequency is given by:

$$\psi_2(\omega_0, 2\alpha)\psi_2(t, 2\alpha)\frac{d^2V(t)}{dt^2} + \omega_0^2V(t) = \frac{1}{\psi_2(\omega_0, 2\alpha)\psi_2(t, 2\alpha)}V_0\cos(\omega t) \tag{18}$$

Remark 2. If consider $\psi_2(t, 2\alpha) = t^{2-2\alpha}$, we obtain the same result as in Martínez et al. (2018) (the classical conformable fractional derivative).

The values considered for the frequency, capacitance, voltage, and inductance are $f = 60\,Hz$, $C = 47\,\mu F$ $V_0 = 1\,V$, $L = 10\,\mu H$ respectively.

The numerical results of the general conformable differential equation are plotted for different values of ψ_2. We can select $\psi_2(t, 2\alpha) = t^{2-2\alpha}((2-2\alpha)g^2(t) + 1)$. Two expressions of $g(t)$, $g_1(t) = \sqrt{30}\sin(t)$ and $g_2(t) = \sqrt{10}e^{-t}$, can be written.

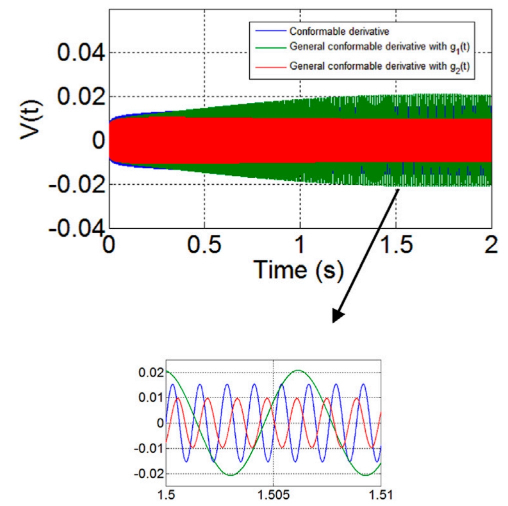

FIGURE 12 | Evolution of $V(t)$ with $\alpha = 0.8$.

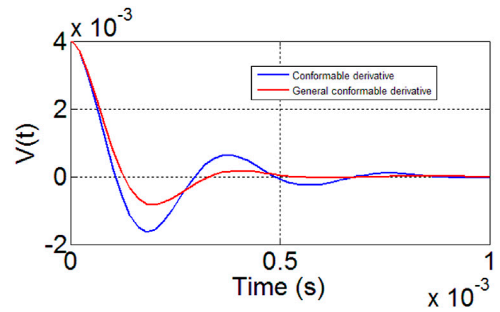

FIGURE 13 | Evolution of $V(t)$ with $\alpha = 0.98$.

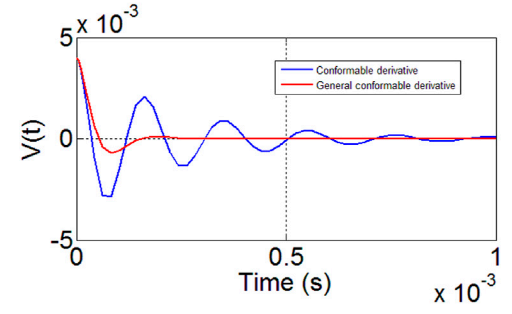

FIGURE 14 | Evolution of $V(t)$ with.$\alpha = 0.9$.

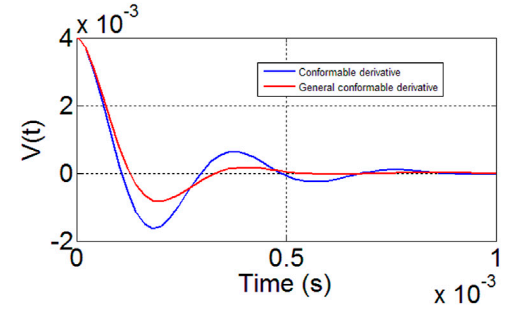

FIGURE 15 | Evolution of $V(t)$ with $\alpha = 0.85$.

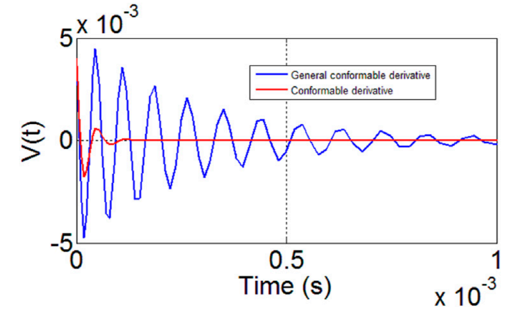

FIGURE 16 | Evolution of $V(t)$ with $\alpha = 0.8$.

The following simulation results were obtained by considering the existence of a harmonic source with angular frequency.

According to **Figures 9–12**, if we select $g_1(t) = \sqrt{30}\sin(t)$, the amplitudes expand, and the waves shift in comparison to the classical conformable situation. If $g_2(t) = \sqrt{10}e^{-t}$, the opposite is noted.

GENERAL FRACTIONAL CONFORMABLE RLC CIRCUIT

In the case of the fractional conformable derivative, the equation of the series RLC circuit with the driving force is presented as follows (Martínez et al., 2018):

$$\frac{d^2V(t)}{dt^2} + 2\xi\omega_0^{\alpha}t^{\alpha-1}\frac{dV(t)}{dt} + \omega_0^{2\alpha}t^{2(\alpha-1)}V(t) = V_0\omega_0^{2(\alpha-1)}t^{2(\alpha-1)}\cos(\omega t) \tag{19}$$

where $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$ is the damping factor.

Based on the preceding considerations, the general conformable fractional differential equation is as follows:

$$\frac{d^2V(t)}{dt^2} + 2\xi\frac{\omega_0\psi_1(t,\alpha)}{\psi_1(\omega_0,\alpha)\psi_2(t,2\alpha)}\frac{dV(t)}{dt} + \frac{\omega_0^2}{\psi_2(t,2\alpha)}V(t) = \frac{V_0\cos(\omega t)}{\psi_2(t,2\alpha)} \tag{20}$$

Remark 3. By considering $\psi_1(t,\alpha) = t^{1-\alpha}$ and $\psi_2(t,2\alpha) = t^{2-2\alpha}$, we obtain **Eq. 19**.

Assume $\psi_1(t,\alpha) = t^{1-\alpha}((1-\alpha)f^2(t)+1)$, where $f(t) = \frac{\sqrt{30}}{t-1}$ and $\psi_2(t,2\alpha) = t^{2-2\alpha}$. Considering the same parameters as in the previous case, we obtain the following simulation results of **Eq. 20** for different values of α.

According to **Figures 13–16**, if we consider $\psi_1(t,\alpha) = t^{1-\alpha}((1-\alpha)f^2(t)+1)$, where $f(t) = \frac{\sqrt{30}}{t-1}$ and $\psi_2(t,2\alpha) = t^{2-2\alpha}$, by decreasing α, the oscillations increase with the conformable derivative and the response time also increases. However, for the general conformable derivative, we obtain the opposite.

CONCLUSION

The fractional derivative of the electrical circuits is a mathematical modelization. The necessity for this presentation arises from the following question: What is the best mathematical model that approximates the real one? It has been established in the literature that employing conformable derivatives is more appropriate than using integer-order derivatives as well as other types of fractional derivatives. Thus, our study presents a general conformable derivative as a solution to represent electrical circuits. We demonstrated that this choice is more flexible and provides a broad set of equations that make the modelization problem easy to deal with. Authors in Martínez et al. (2018) employed conformable derivatives and compared them to other types of FDs. Thus, the primary contribution of our work is to introduce a unique GCD and compare our findings with those of (Martínez et al., 2018). With interpretation and analysis, we discovered that our selection is appropriate and provides a broad range of model possibilities.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

OK and ON contributed to conception and design of the study. ME wrote the first draft of the manuscript. HA and BK wrote sections of the manuscript. All authors contributed to manuscript revision, read, and approved the submitted version.

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