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# Dynamics of MHD second-grade nanofluid flow with activation energy across a curved stretching surface

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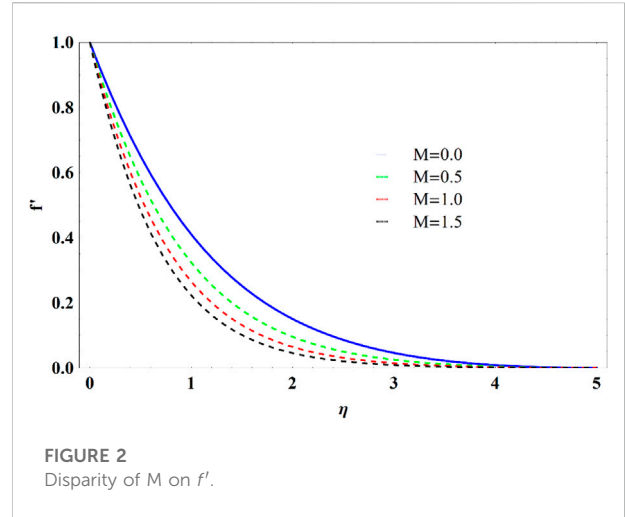
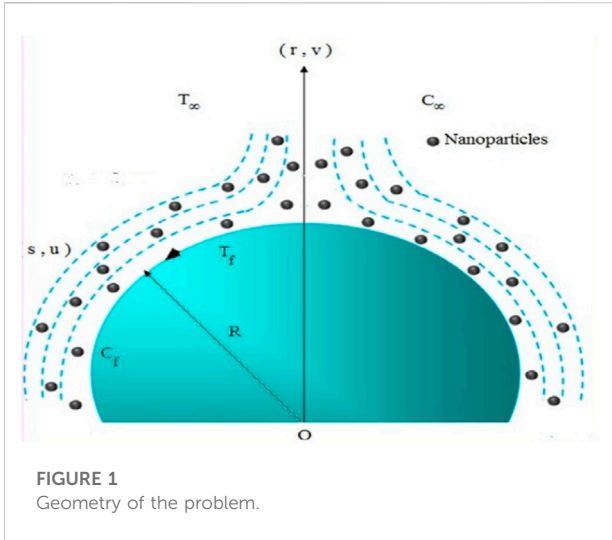
This analysis addresses the influence of activation energy on the MHD flow of second-grade nanofluid over a convectively heated curved stretched surface. The impact of heat generation/absorption, thermophoresis, and Brownian motion are also incorporated. This current study in addendum reveals the solution narrating the nanofluid flow behaviour of the stretched curve to better the performance of the system. Hence, the mathematical construction of governing partial differential equations (PDEs) is transmitted into nonlinear ODEs by employing appropriate transformations. The attained ODEs are conducted numerically via ND-Solve. It is consequential to report that fluid velocity and temperature fields significantly rise with concurrent enhancing values of the fluid parameter and curvature parameter. Moreover, the concentration field enhances considering the energy activation variable and suppresses with the reaction rate constant while thermophoresis escalates the temperature distribution as the Nusselt number lowers with a stronger internal heat source parameter  $Q$ .

## KEYWORDS

activation energy, heat generation, nanofluid, second-grade, MHD

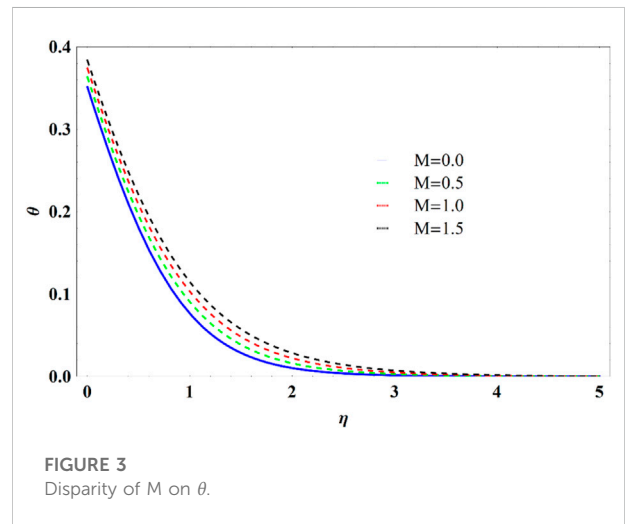
## Introduction

In recent advancements, nanofluids have obtained immense attention due to their notable thermal transfer and fascinating applications in numerous fields such as computer processes, hybrid power, fuel cell, and other high-energy devices. The fluids are prepared by suspending nanoparticles in base fluids. The size of nanoparticles  $\leq 100$  nm. The prime idea of nanofluid was coined by [Choi and Eastman \(1995\)](#). Later, [Buongiorno \(2006\)](#) suggested the two main characteristics, namely, Brownian movement and thermophoretic

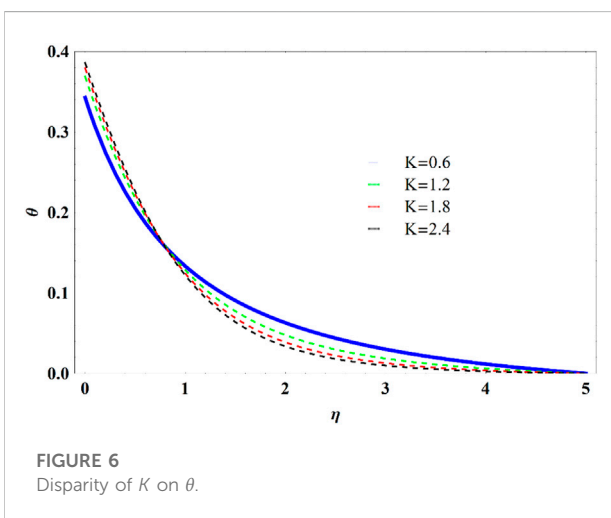
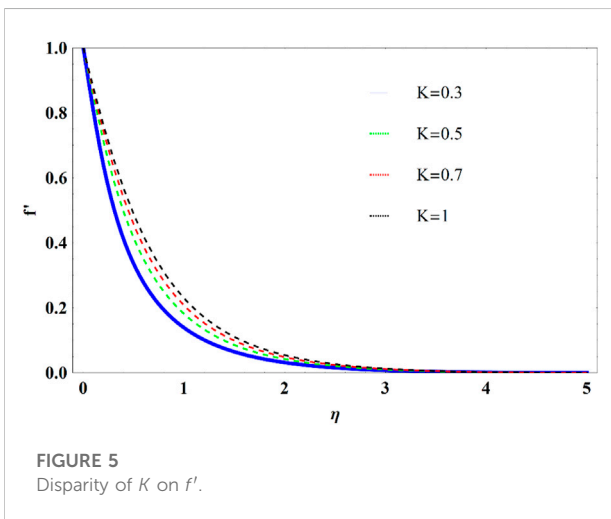
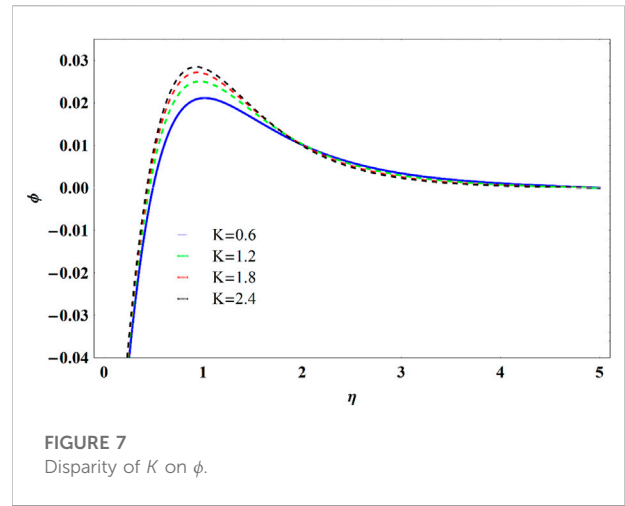
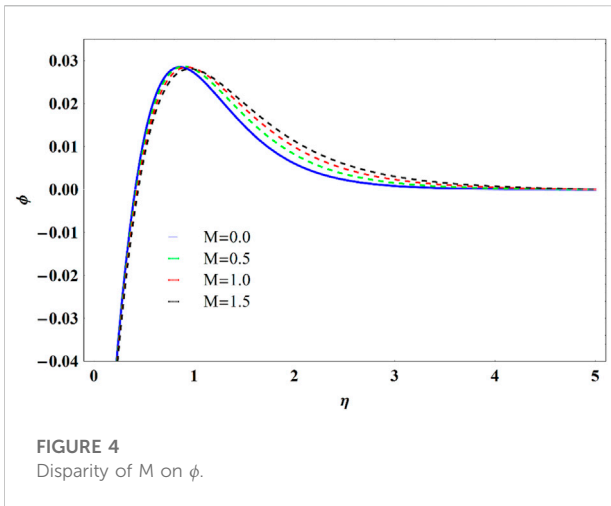


force, to enhance the ability of the ordinary fluid. He found that enhancing the Nusselt number leads to a rise in the nanoparticle volume fraction. [Sheikholeslami et al. \(2014\)](#) discussed the impact of MHD copper nanofluid flow. The theoretical investigation of  $Al_2O_3$  water nano liquid was examined by [Malvandi et al. \(2015\)](#). They discovered that the enforced heat irregularity alters the path of nanoparticle movement and modifies the patterns of the fields. [Mahanthesh et al. \(2016\)](#) reported the squeezing effect of nanofluids which escalates the thermal layer and leads to a depreciation of the rate of heat transport. [Ibáñez et al. \(2016\)](#) explored the MHD nanofluid flow in a porous channel with a radiation effect. [Eid et al. \(2017\)](#) considered gold nanoparticles in the flow of Sisko nanofluid and revealed that radiation production boosts thermal transport. The Burgers flow of nanofluid with the effect of Cattaneo–Christov was examined by [Hayat et al. \(2017a\)](#). Also, [Hayat et al. \(2017b\)](#) swotted numerically the flow of nanofluid over a revolving disk in the presence of the slip effect. [Zuhra et al. \(2018\)](#) considered MHD second-grade nanofluid comprising gyrotactic microorganisms. They concluded that microorganism density leads to increases with momentum slip. The MHD Carreau nanofluid over a permeable stretching sheet was considered by [Khan and Shehzad \(2020\)](#). [Reddy et al. \(2019\)](#) analyzed the MHD nanofluid *via* a stretching sheet. [Shah et al. \(2021\)](#) considered numerical computation of entropy production of nanofluid due to a porous surface. [Khan et al. \(2020\)](#) thrashed out the consequences of entropy minimization of Casson nanofluid over a rotating cylinder.

The exploration of non-Newtonian fluid has inspired scholars due to its several applications, such as the production of plastic, food processing, and exclusion of tumors. In this current investigation is a subcategory of non-Newtonian fluid termed second-grade fluid. This model takes into consideration the consequences of normal stress in flow conditions, as well as

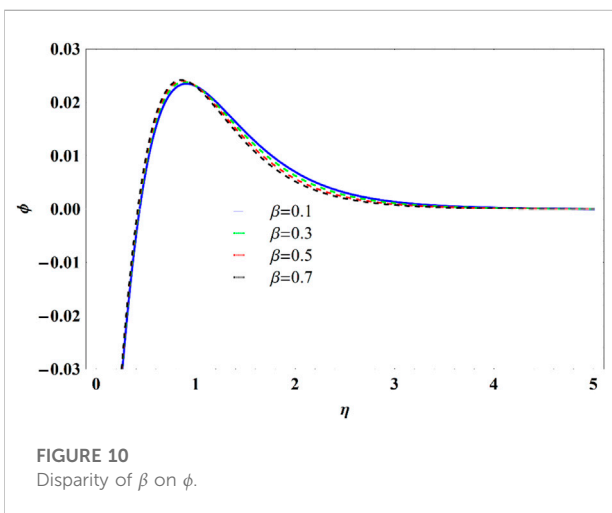
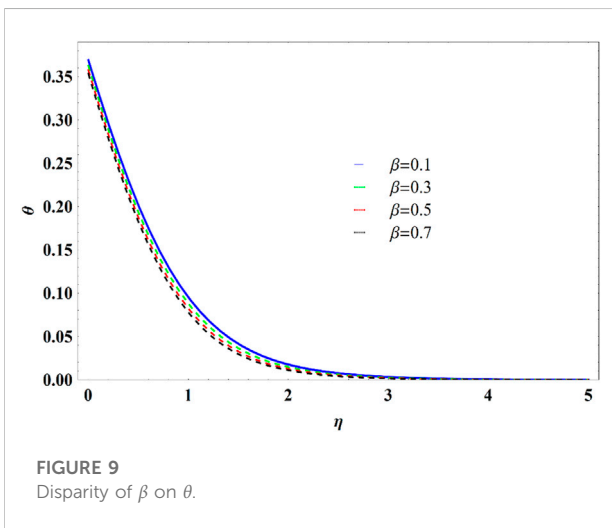
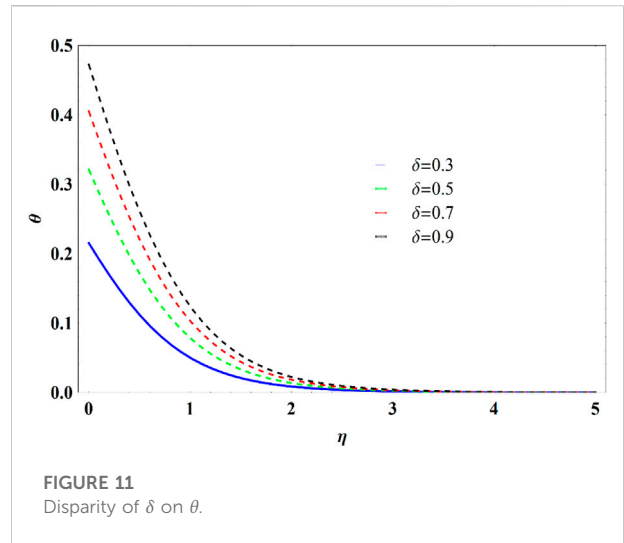
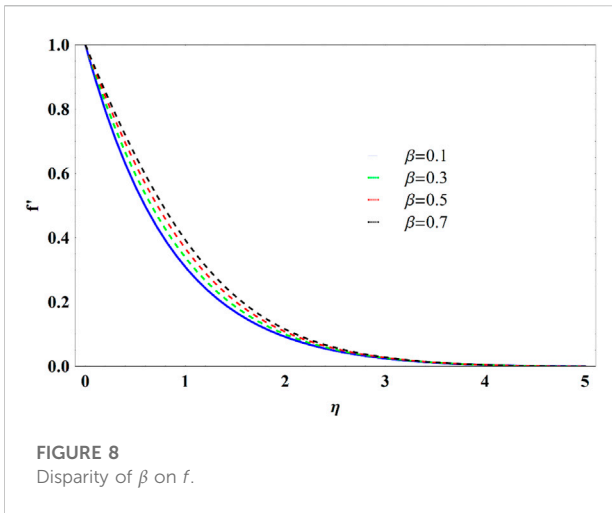


shear thinning and thickening. [Tan and Masuoka \(2005\)](#) described the flow of nanofluid with variable thermal conductivity for the second grade. [Rashidi and Majid \(2010\)](#) analyzed the time-dependent squeezing flow for second-grade fluid. The thermal and species transport analysis of second-grade fluid over a surface with heat flux was deliberated by [Das et al. \(2016\)](#). [Jamil et al. \(2011\)](#) reported the helical flow of second-grade fluid over coaxial cylinders. [Turkylmazoglu \(2012\)](#) analytically examined the flow of second-grade non-Newtonian fluid with mass transfer over a shrinking sheet. [Khan and Pop \(2010\)](#), [Makinde and Aziz \(2011\)](#), and [Hsiao \(2010\)](#) evaluated the magnetohydrodynamic flow of liquid of second grade with electromagnetic dispersion and non-uniform heat source/sink. [Akinbobola and Okoya \(2015\)](#) swotted second-grade fluid with heat generation. The exact solution of a second-grade fluid *via* coaxial cylinders was reported by [Erdogan and Imrak \(2008\)](#). [Nadeem et al. \(2012\)](#) explored second-grade fluid over a horizontal cylinder.



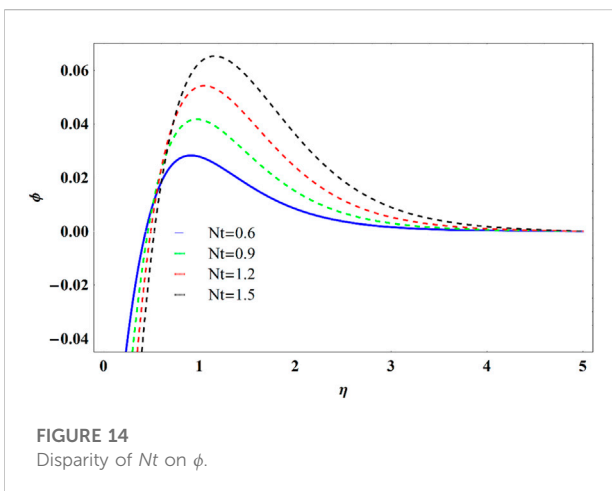
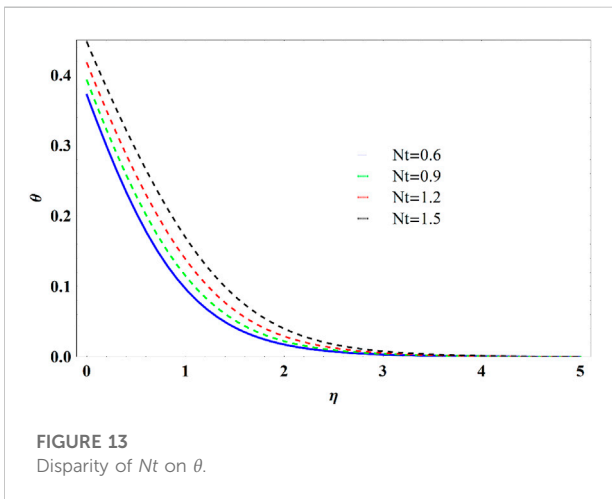
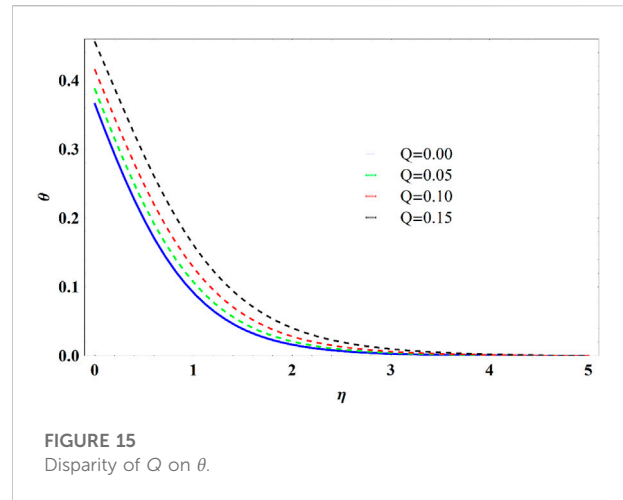
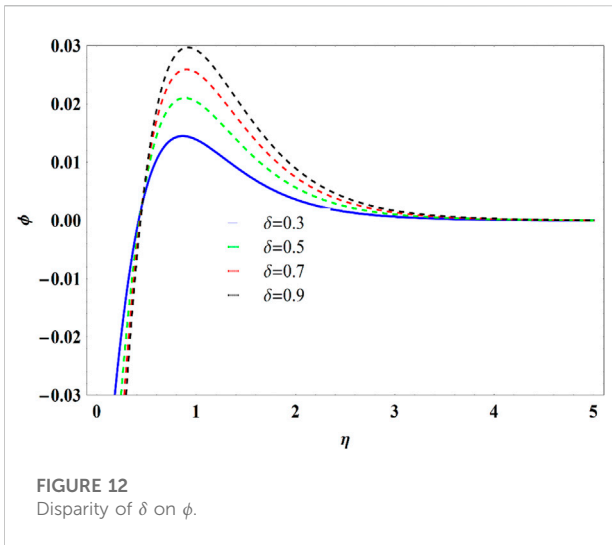
Magnetohydrodynamics (MHD) flow has gained the interest of scholars due to its remarkable applications in the industry and engineering. As contained in the structure of the MHD generator, the cooling system is filled with fluids metal, the deposit of energy, pumps, and flow meters. Theoretically, the magnetic fields can persuade a drag identified as Lorentz force in a moving liquid, which depreciates the fluid velocity. Thus, boosting the fluid temperature and concentration. Numerous researchers have analyzed the impact of magnetic parameters, specifically in the boundary layer problem. The impact of magnetic field flow on permeable surfaces with slip conditions was elaborated by [Hayat et al. \(2011\)](#). [Makinde et al. \(2013\)](#) delineated the influence of MHD on nanoliquid with buoyancy effect. The flow of nanoliquid through two-phase models was visualized by [Sheikholeslami et al. \(2015\)](#). [Hayat et al. \(2016\)](#) addressed the influence of second-grade nanofluid. MHD flows over radially shrinking/stretching disks were reported by [Soid et al. \(2018\)](#). [Hayat et al. \(2015\)](#) considered the 3D flow of MHD nanoliquid. [Sharif et al. \(2019\)](#) elucidated MHD nanoliquid *via* an exponential sheet. [Shah et al. \(2020\)](#) conducted water base nanoparticles consisting of SWCNT and MWCNT over a vertical cone. [Shoaib et al. \(2020\)](#) evaluated numerically MHD hybrid nanoliquid with thermal radiation. They found that thermal transport rate enhances with growing values of magnetic effect and biot number. Recently, [Alamri et al. \(2019\)](#) discussed the second-grade fluid in the presence of Fourier's heat flux theory.

Activation energy (AE) is the minimal amount of energy needed for a reaction to occur. The activation energy required to transfer energized particles or macromolecules to a location where they would undergo physical transit can be overestimated. The notion of activation energy is commonly useful in thermal engineering. The Bestman ([Bestman, 1990](#)) was initially coined with the activation



energy of a binary amalgam phenomenon in a porous space. The unsteady natural convective flow was reported by Makinde et al. (2011) with AE. The influences of activation energy on a magnetic nanofluid were investigated by Hamid et al. (2018). Mustafa et al. (2017) and Dhlamini et al. (2022) elaborated on the behaviour of magneto nanoliquid with activation energy. Dawar et al. (2020) conducted a magnetic field flow of nanoliquid with activation energy. Hayat et al. (2022) addressed the effect of AE on the MHD flow of third-grade nanofluid over convective condition. The 3D flow of Casson nanoliquid for the thermal radiative flow with AE was examined by Khan et al. (2019). Hayat et al. (2018) inspected the 3D flow of Darcy–Forchheimer rotating AE. The 3D time-dependent flow of Carreau nanofluid on chemical reaction and AE was explored by Irfan et al. (2019). Other materials that have added value to this work are in the studies by Asogwa et al. (2013); Khan et al. (2017); Ali et al. (2020); Bilal et al. (2021); Jayaprakash et al. (2021); Ali et al. (2022a); Adnan et al. (2022); AdnanAshraf, (2022); AdnanAshraf et al. (2022); AdnanMurtaza et al. (2022); Asogwa et al. (2022); Ali et al. (2022b); Goud et al. (2022); and Weera et al. (2022).

Considering the overview of the abovementioned work, the prime focus of the current analysis is to scrutinize the impact of Arrhenius activation energy on the MHD flow of second-grade nanofluid toward a curved stretched surface. By employing the transformation procedure, PDEs have been transmuted into ODEs, which are then established numerically by ND-Solve using Mathematica. Our obtained physical parameters are prescribed through tables and graphs.



### Modeled equation

The second-grade fluid Cauchy stress tensor is given by [Mabood and Das \(2016\)](#).

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2$$

Where  $\mu, I, A_1, A_2, \alpha_1,$  and  $\alpha_2$  are the identity tensor, dynamic viscosity, first and second Rivlin Ericksen tensor, and material constant.

$$A_1 = \nabla V + (\nabla V)^T$$

$$A_n = (\nabla V)^T A_1 + A_1 (\nabla V) + \frac{dA_{n-1}}{dt}$$

### Mathematical formulation

We consider the 2D flow of MHD second-grade nanoliquid flow due to the curved stretched surface. Featuring AE, chemical reaction (binary), and heat generation. We consider  $(r,s)$  to be the curvilinear coordinates (see [Figure 1](#)). The stretched sheet in  $s$  path with  $U_w = as$  and  $r$  is considered orthogonal to  $s$ . Then,  $B_0$  is applied along the transverse path of flow from the magnetic field. Assuming that the moderate magnetic field in the form of  $Re_s$  generated is ignored, a nanostructured materials framework is utilized for erratic motion and thermophoresis. Under the aforementioned assumptions, equations ([Mabood and Das, 2016](#); [Imtiaz et al., 2019](#)) are written for the boundary layer as follows:

$$(r + R) \frac{\partial v}{\partial r} + v + R \frac{\partial v}{\partial s} = 0 \tag{1}$$

$$\frac{u^2}{r + R} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \tag{2}$$

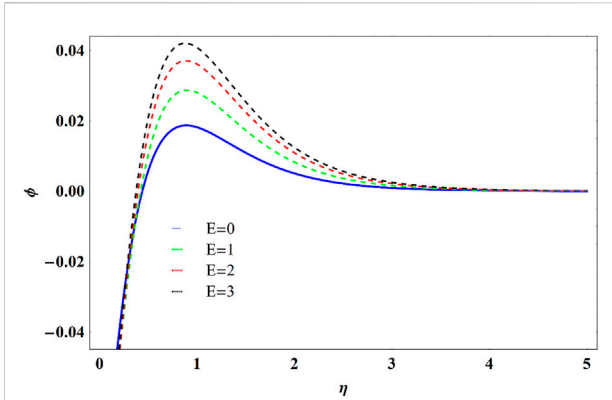


FIGURE 16  
Disparity of  $E$  on  $\phi$ .

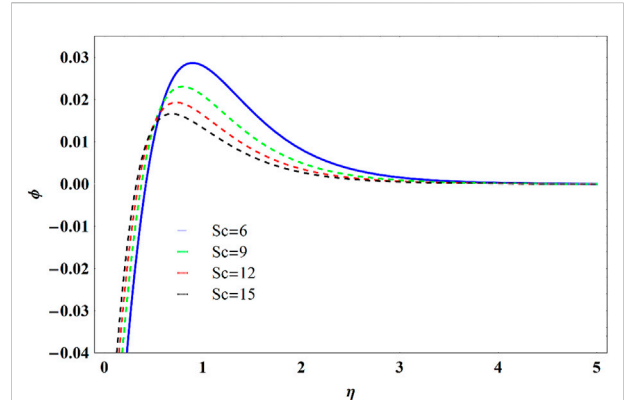


FIGURE 19  
Disparity of  $Sc$  on  $\phi$ .

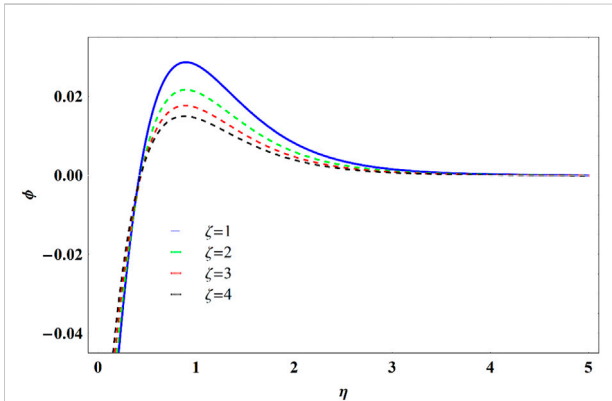


FIGURE 17  
Disparity of  $\zeta$  on  $\phi$ .

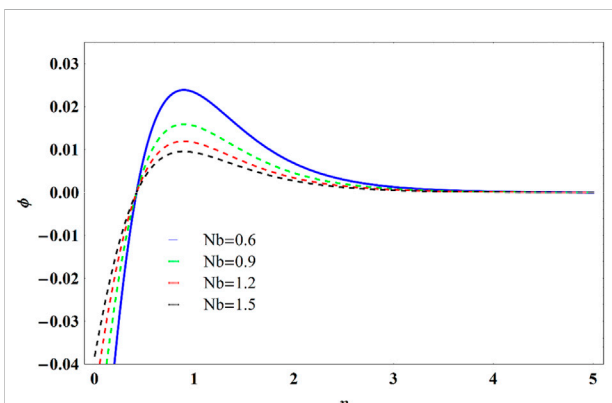


FIGURE 18  
Disparity of  $Nb$  on  $\phi$ .

$$\begin{aligned}
 v \frac{\partial u}{\partial r} + \frac{Ru}{r+R} \frac{\partial u}{\partial s} + \frac{uv}{r+R} = & \\
 -\frac{1}{\rho} \frac{R}{r+R} \frac{\partial p}{\partial s} + v \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R} \frac{\partial u}{\partial r} - \frac{u}{(r+R)^2} \right) & \\
 + \frac{\alpha_1}{\rho} \left\{ \frac{2R}{r+R} \frac{\partial^2 u}{\partial r^2} \frac{\partial u}{\partial s} - \frac{2R}{(r+R)^2} \frac{\partial u}{\partial r} \frac{\partial u}{\partial s} + \frac{2}{r+R} \frac{\partial v}{\partial r} \frac{\partial u}{\partial r} \right. & \\
 + \frac{2}{r+R} v \frac{\partial^2 u}{\partial r^2} - \frac{2}{(r+R)^2} v \frac{\partial u}{\partial r} - u \frac{4R}{(r+R)^2} \frac{\partial^2 u}{\partial s \partial r} & \\
 \left. - u \frac{4}{(r+R)^2} \frac{\partial v}{\partial r} + u \frac{2R}{(r+R)^3} \frac{\partial u}{\partial s} \right\} - \frac{\sigma B_0^2 u}{\rho} & \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 v \frac{\partial T}{\partial r} + \frac{Ru}{r+R} \frac{\partial T}{\partial s} = \alpha \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R} \frac{\partial T}{\partial r} \right) + \tau \left[ D_B \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} \right. & \\
 \left. + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial r} \right)^2 \right] + \frac{Q_0}{\rho c_p} (T - T_w) & \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 v \frac{\partial C}{\partial r} + \frac{Ru}{r+R} \frac{\partial C}{\partial s} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r+R} \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} \left[ \frac{\partial^2 T}{\partial r^2} \right. & \\
 \left. + \frac{1}{r+R} \frac{\partial T}{\partial r} \right] - T_r^2 \left( \frac{T}{T_\infty} \right)^n e^{-\frac{Ea}{RT}} (C - C_\infty) & \quad (5)
 \end{aligned}$$

With associated BCs (Mabood and Das, 2016) – (Imtiaz et al., 2019)

$$\left. \begin{aligned}
 u = U_w = cs, \quad v = 0, \quad \frac{\partial T}{\partial r} = -hf(T_f - T_\infty), \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \text{ at } y = 0 \\
 u \rightarrow 0, \quad \frac{\partial u}{\partial r} \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } y \rightarrow \infty
 \end{aligned} \right\} \quad (6)$$

Transformation consideration

TABLE 1 Contrasting outputs of  $-f''(0)$  for selected values of  $M$  when  $K = \infty$  and  $\beta = 0$ .

M	Reference (Mabood and Das, 2016)	Reference (Imtiaz et al., 2019)	Present results
1	1.4142135	1.4142266	1.414328374
5	2.4494897	2.4495271	2.449788955
10	3.31666	3.3166679	3.316366745
50	7.1414284	7.1414769	7.143182469
100	10.049875	10.049924	10.048234644

TABLE 2 Friction co-efficient for various values  $M, K,$  and  $\beta$ .

K	M	$\beta$	$\frac{1}{2}C_f Re_s^{1/2}$
1.0	1	0.2	1.81567
1.5			1.76525
2.0			1.75389
3	0.0		1.28664
	0.4		1.49317
	0.8		1.67268
		0	2.10848
		0.1	1.91158
		0.2	1.75485

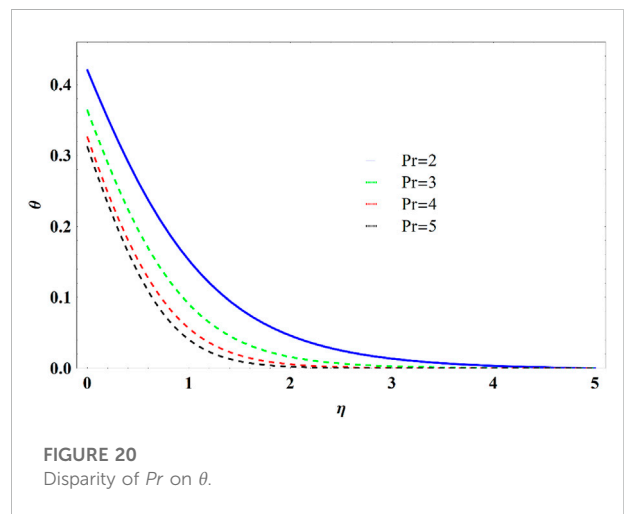


FIGURE 20 Disparity of Pr on  $\theta$ .

$$\eta = \sqrt{\frac{c}{v}}r, \quad u = csf'(\eta), \quad v = \frac{R}{r+R}\sqrt{cv}f(\eta), \quad \tau = \frac{(\rho_c)_p}{(\rho_c)_f} \quad (7)$$

$$p = \rho c^2 s^2 P(\eta), \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Eq. 3 is satisfied, and with the help of the abovementioned transformation Eqs 1–5 are reduced

$$\frac{\partial P}{\partial \eta} = \frac{f'^2}{n+K} \quad (8)$$

$$\begin{aligned} \frac{2K}{\eta+K}P = f''' + \frac{1}{\eta+K}f - \frac{1}{(\eta+K)^2}f^2 + \frac{K}{n+K}ff'' + \frac{K}{(\eta+K)^2}ff' \\ - \frac{K}{(\eta+K)}f'^2 + \beta \left\{ \frac{2K}{\eta+K}ff'' - \frac{2K}{(\eta+K)^2}ff''' - \frac{8K}{(\eta+K)^2}ff'' \right. \\ \left. + \frac{4K}{(\eta+K)^3}ff'' + \frac{6K}{(\eta+K)^3}f'^2 - \frac{4K}{(\eta+K)^4}ff' \right\} - Mf' \\ - \frac{K}{(\eta+K)}f'^2 + \beta \left\{ \frac{2K}{\eta+K}ff'' - \frac{2K}{(\eta+K)^2}ff''' - \frac{8K}{(\eta+K)^2}ff'' \right\} \quad (9) \end{aligned}$$

$$\theta'' + \frac{1}{\eta+K}\theta' + Pr \left( \frac{K}{\eta+K}f\theta' + Nb\theta'\phi' + Nt\theta'^2 \right) + Q\theta = 0 \quad (10)$$

$$\begin{aligned} \phi'' + \frac{1}{\eta+K}\phi' + \frac{K}{\eta+K}Scf\theta' + \frac{Nt}{Nb} \left( \theta'' + \frac{1}{\eta+K}\theta' \right) \\ - Sc\zeta(1 + \delta_1\theta)^m \exp\left(\frac{-E}{1 + \delta_1\theta}\right)\phi \\ = 0 \quad (11) \end{aligned}$$

with transformed BCs

$$f'(0) = 1, \quad \theta'(0) = -\delta(1 - \theta(0)), \quad Nb\phi'(0) + Nt\theta'(0) = 0 \quad \text{at } \eta = 0$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (12)$$

We have,  $\beta = \frac{\alpha_1 c}{\eta}$ ,  $\delta = \frac{h_f}{k} \sqrt{\frac{v}{a}}$ ,  $Nb = \frac{\tau D_B (C_w - C_\infty)}{v}$ ,  
 $K = R \sqrt{\frac{c}{v}}$ ,  $Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty v}$ ,  
 $Sc = \frac{v}{D_B}$ ,  $E = \frac{E_a}{kT_\infty}$ ,  $Q = \frac{Q_0}{\rho C_p}$ ,  $\zeta = \frac{k_c^2}{c}$ ,  $Pr = \frac{v}{\alpha}$ ,  $\delta_1 = \frac{T_w - T_\infty}{T_\infty}$ .

Now, removing the P between Eqs 8 and 9, we get

$$\begin{aligned} f''' + \frac{2}{\eta+K}f'' - \frac{1}{(\eta+K)^2}f'' + \frac{1}{(\eta+K)^3}f' + \frac{K}{\eta+K}(ff''' + f'f'') + \frac{K}{(\eta+K)^2}(ff'' + f'^2) \\ - \frac{K}{(\eta+K)^3}ff' + \beta \left\{ \frac{2K}{\eta+K}ff'' - \frac{2K}{(\eta+K)^2}ff''' - \frac{8K}{(\eta+K)^2}ff'' \right. \\ \left. + \frac{4K}{(\eta+K)^3}ff'' + \frac{6K}{(\eta+K)^3}f'^2 - \frac{4K}{(\eta+K)^4}ff' \right\} - Mf'' - \frac{M}{(\eta+K)}f' \quad (13) \end{aligned}$$



TABLE 3 Local Nusselt number for various values  $K, \delta, Nt, Nb, Sc, E,$  and  $Q$ .

$K$	$\delta$	$Nt$	$Nb$	$Sc$	$Q$	$E$	$NuRe_s^{-1/2}$	
1.0	0.5	0.2	0.2	5	0.1	1	0.30889	
1.5							0.31461	
2							0.31716	
3	0.6						0.35622	
							0.38809	
							0.41584	
	0.7						0.31559	
							0.31162	
							0.30729	
	0.8	0.3					0.31926	
							0.31926	
							0.31926	
		0.4		0.2				0.32053
					0.4			0.31881
					0.5			0.31780
	0.5			0.2	3			0.34414
					6			0.31926
					9			0.31926
				6	0		0.31926	
					0.1		0.25176	
					0.2		0.31873	
					0.5	0.31963		
					1.5	0.31996		
					2.5	0.31996		

Expressions for the local  $C_{fs}$  and  $Nu_s$  gives

$$C_{fs} = \frac{\mu}{\frac{1}{2}\rho U_w^2} \tau_{rs}, \quad Nu_s = \frac{sq_w}{k(T_w - T_\infty)} \tag{14}$$

$$\tau_{rs} = \mu \left[ \frac{\partial u}{\partial r} - \frac{u}{r+R} + \frac{2\alpha_1}{\mu} \left( \frac{R}{r+R} \frac{\partial u}{\partial r} \frac{\partial u}{\partial s} + \frac{\nu}{r+R} \frac{\partial u}{\partial r} - \frac{2Ru}{(r+R)^2} \frac{\partial u}{\partial s} - \frac{2uv}{(r+R)^2} \right) \right]_{r=0} \tag{15}$$

$$q_w = -k \left( \frac{\partial T}{\partial y} \right)_{r=0} \tag{16}$$

Non-dimensional Nusselt number and friction coefficient

$$Re_s^{-1/2} C_{fs} = \left[ f''(0) - \frac{f'(0)}{K} + \beta \left( f'(0) f''(0) - \frac{2}{K} f'(0)^2 \right) \right]$$

$$Re_s^{-1/2} Nu_s = -\theta'(0)$$

where  $Re_s = \frac{c_s \delta^2}{\nu}$

## Results and discussion

The key emphasis of this article is highlighted *via* the numerical approach integrated by utilizing the NDSolve technique by using Mathematica. The main emphasis of pertained physical variables on  $f'(\eta), \theta(\eta),$  and  $\phi(\eta)$  fields, as well as drag fraction and Nusselt number, are elaborated and delineated through Figures 2–20 and Tables 1–3. Table 1 is adorned to check the compatibility of the current analysis by constructing a contrasting  $-f''(0)$ . An excellent achievement has been found with a previously published result. 8. The reference values in the current study have been taken as  $K = 5, \delta = 0.8, Nt = Nb = 0.4, Sc = 5, Q = 0.1, \beta = 0.3, M = 1, E = 1,$  and  $\zeta = 1$  kept constant throughout the computation, and the variations have been mentioned in the graphs and tables accordingly.

Table 2 is equipped to check the variance in  $\frac{1}{2} C_f Re_s^{1/2}$ . For empowered fluid parameter  $\beta$  and curvature parameter  $K$ , friction factor decreases, whereas the reverse trend is observed with  $M$ . Table 3 is elaborated in variance in Nusselt number for selected values of  $K, \delta, Nt, Nb, Sc, Q,$  and  $E$ . It is noticed that  $Nu Re_s^{-1/2}$  rises with uplifting values of  $K, \delta$  and  $E$ .  $Nu Re_s^{-1/2}$  declines with rising values of  $Nt, Sc$  and  $Q$  and there are no significant changes with  $Nb$ .

The impact of  $M$  on  $f'(\eta), \theta(\eta),$  and  $\phi(\eta)$  fields is illustrated in Figures 2–4. It is revealed that velocity diminishes with an escalation in  $M$ . This notifies that increase in magnetic field obtains the resistive force (Lorentz force) leading to a reduction in the fluid velocity. This also means a reduction in the thickness of the thermal boundary layer. The heat generated causes resistance of the fluid for a greater value of  $M$ , which escalates the fluid temperature, and similar behaviour is also seen for the concentration field. Figures 5–7 presents the disparity of curvature parameter  $K$  on  $f'(\eta), \theta(\eta),$  and  $\phi(\eta)$  fields. Figure 5 displays the enhancing values of curvature parameter  $K$  on velocity field. Here, velocity escalates with a larger value of  $K$ . This is because enhancing values of  $K$  lessens the kinematic viscosity of the fluid, which causes a reduction in viscosity and, as a result, the velocity of the fluid gains momentum. Temperature and concentration profiles for various values of  $K$  are revealed in Figures 6 and 7. It is seen that temperature and concentration decline for augmenting the value of  $K$ .

Figures 8–10 analyze the fluid parameter  $\beta$  on  $f'(\eta), \theta(\eta),$  and  $\phi(\eta)$  fields. It can be seen from Figure 8 that velocity profile enhances for a larger value of  $\beta$ . Physically, fluid parameter  $\beta$  has a reverse relation with viscosity. In contrast, temperature and concentration profiles (Figures 9 and 10) offer a reducing behaviour with a rising value of  $\beta$ . Figures 11 and 12 presented the effect of Biot number  $\delta$  on  $\theta(\eta)$  and  $\phi(\eta)$  fields. Physical involvement of a larger heat transfer coefficient corresponds to enhancement on  $\theta(\eta)$  and  $\phi(\eta)$  fields. Therefore,  $\theta(\eta)$  and  $\phi(\eta)$  layer thickness is escalated by increasing Biot number  $\delta$ .

Figure 13 affirms the increasing activity of temperature profile and thermal boundary layer with  $Nt$ . The  $Nt$  increase produces a much strong thermophoretic force, causing nanoparticles to move



away from the plate. This results in the growth of the  $\theta(\eta)$  Figure 14 symbolizes an upsurge in concentration profile for different values of the thermophoresis variable  $Nt$ . Because microscopic particles move from the hot region towards the cold region during the process of thermophoresis. Figure 15 plotted the numerous values of heat source input  $Q$  against the temperature profile. Physically, the existence of  $Q$  in the boundary layer improves energy and causes a boost in temperature. Figure 16 explicates the enhancing trend of activation energy  $E$  on nanoparticle concentration. The activation energy reduces the modified Arrhenius function and causes boosting generative chemical reaction.

Figure 17 presents the dimensionless concentration on the chemical reaction variable  $\zeta$ . The concentration of nanoparticle is depreciated on the rise of  $\zeta$ . This behaviour characterizes the decreasing effect of buoyancy force due to concentration gradient, which causes the reduction of concentration profile.

Figure 18 exhibits the variation of the concentration field to diverse values of the  $Nb$ . The thickness of the boundary layer and concentration profile decline on increasing  $Nb$  input. The reason behind this is that this enriches the pace at which tiny particles drive with various velocities in separate unexpected directions. Figure 19 affirms the declining trend in the concentration field on the escalating values of  $Sc$ . This happens because of decaying mass diffusion. Fluctuation for differing values of Prandtl number on temperature is evident in Figure 20.  $\theta(\eta)$  minifies with upshot values of  $Pr$  because larger  $Pr$  results in lesser thermal diffusivity, thereby bringing about a decrease in temperature.

## Conclusion

In this analysis of MHD second grade nanoliquid over a curved stretched sheet with activation energy is explored. The observations are concluded as follows:

- Increment exists in the momentum and thermal boundary layer thickness when the fluid parameter enhances.
- Velocity distributions show decreasing phenomenon with enhancement in  $M$ .
- Both velocity and temperature have increasing behaviour for higher  $K$ .

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- Convective heating condition enhances the thermal field significantly.
- Brownian movement  $Nb$  depreciate for concentration profile.
- Concentration is diminished for higher value of  $Sc$ .
- Increment in fluid parameter  $\beta$  decreases the friction factor.
- Thermophoresis reduces the volume fraction field and enhances the temperature field.
- The temperature profile increases due to temperature heat source  $Q$ . An opposite trend is noted in the heat transfer rate.

## Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

## Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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## Glossary

$\alpha_1$  fluid parameter

$\alpha$  thermal diffusivity

$s$  &  $r$

**curvilinear co-ordinates**

$p$  pressure

$Sc$  Schmidt number

$\sigma$  electrical conductivity

$\nu$  kinematic viscosity

$T$  fluid temperature

$D_B$  Brownian diffusion coefficient

$R$  radius

$Q$  heat generation/absorption

$T_\infty$  ambient temperature

$C$  concentration of the fluid

$B_0$  magnetic field strength

$D_T$  thermophoretic diffusion coefficient

$u$  and  $v$  velocity components

$T_w$  surface temperature

$C_\infty$  ambient fluid concentration

$k_r^2$  reaction rate

$\rho$  fluid density

$k$  Boltzmann constant

$E_a$  coefficient of activation energy

$\tau$  nanoparticle heat capacity against base fluid heat capacity

$Pr$  Prandtl number

$\beta$  fluid parameter

**BCs** boundary conditions

$K$  curvature parameter

$\delta$  Biot number

$Nb$  Brownian motion

$C_w$  surface concentration

$T$  fluid temperature/fluid temperature

$Re_s$  local Reynolds number

$Nt$  thermophoresis

$E$  energy parameter

$\zeta$  reaction rate parameter

$\delta_1$  temperature difference parameter

$\tau_{rs}$  shear stress

$q_w$  heat flux