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Robust Vehicle Dynamics Control for a Sharp Curve With Uncertain Road Condition

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Recently, more and more research has been conducted to develop Connected Autonomous Vehicles (CAVs) applications that ensures the safety driving of CAVs under some extreme situations. This brief presents a robust control strategy for CAVs to preserve a precise tracking performance and maintain the stability of lateral dynamics when passing a sharp curve with uncertain road friction coefficient changes. In the proposed robust lateral dynamics control, robust optimization-based lateral dynamics controller is designed to achieve the stability of the lateral dynamics with the consideration of the road friction coefficient uncertainty. Simulation validations are carried out to evaluate the proposed control strategy. The results show that the robust optimization-based lateral dynamics can improve the robustness even with the uncertainty of the road friction coefficient.

Keywords: model predictive conrol, robust optimisation, vehicle dynamic, uncertainty, stability

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1 INTRODUCTION

Autonomous vehicles will meet more emergency scenarios when leaving the research laboratory and entering public roads (Kritayakirana and Gerdes, 2012; Shen and Raksincharoensak, 2021). Vehicle stabilization under uncertain scenarios is one of the most important issues in the control of autonomous vehicles (Yue et al., 2019; Shen et al., 2020a; Guo et al., 2020). Recently, Model Predictive Control (MPC) has been used to improve the vehicle dynamics stability (Yuan et al., 2019). In (Taghavifar, 2019), neural network autoregressive with exogenous input system has been applied to obtain an accurate and explicit model in order to contribute to the control of the system over the prediction horizon. (Weiskircher et al., 2017). proposed a MPC-based predictive trajectory guidance and tracking control framework for autonomous and semiautonomous vehicles in dynamic public traffic. Moreover, a data-driven predictive control is proposed in (Li and Schutter, 2021) which is model-free predictive control method.

However, the normal MPC without considering the uncertainty is not able to address the problem caused by environment uncertainty. The state space model-based prediction has large variance and even mean bias if there are any uncertainties in disturbance or the system parameters (Shen et al., 2020b). If there is uncertain road friction changes when passing a sharp curve and the model used in MPC cannot reflect the uncertainty, MPC will lose some precise on the lateral dynamics control. To improve the robustness against uncertainty, it is necessary to design a robust controller. In (Heshmati-Alamdari et al., 2020), a robust predictive controller is designed for underwater robotic vehicles which forms a high robust closed-loop system against parameter uncertainties.

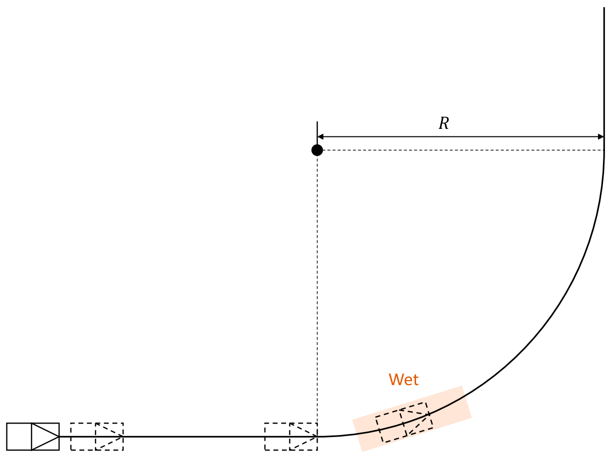

FIGURE 1 | Passing a sharp curve with water-covered surface.

Besides, (Gao et al., 2021), proposed a robust lateral trajectory following control for autonomous vehicles. Robust model predictive control is a potential solution to the issue caused by uncertain road friction in this research. In the problem formulation of robust model predictive control, the road friction is regarded as a uncertain variable. For all possible realizations of uncertain variable, a fixed control law has a cost. We focused on finding a control law that minimize the upper bound of the cost for all possible realizations of uncertain variable. In this way, the robustness of the control strategy is able to be attained. To achieve robust model predictive controller, it is essentially to solve a robust optimization problem or a chance constrained optimization problem in every time step (Nemirovski and Shapiro, 2006; Shen et al., 2019). Although it is NP-hard to solve a robust optimization problem or a chance constrained optimization problem (Hong et al., 2011; Geletu et al., 2017; Pena-Ordieres et al., 2020), the approximate solution can be obtained by formulating a solvable approximate problem of the original one (Luedtke and Ahmed, 2008; Shen et al., 2021; Campi and Garatti, 2019, 2011). Robust model predictive control was widely applied in water qulity management (Takyi and Lence, 1999) and other process control applications (Henrion and Moller, 2003). Recently, robust model predictive control has been applied to the automotive powertrain control to optimize the fuel efficiency with stochastic constraint on the knock (Shen et al., 2017; Shen and Shen, 2017) and the energy management system in hybrid electric vehicle (Shen et al., 2016). Robust model predictive control can also be applied to ensure the robustness for an autonomous vehicle when it passes a sharp curve with uncertain road condition.

This paper presents a novel robust model predictive control strategy for automated vehicles to preserve a precise tracking performance and maintain the stability of lateral dynamics. The optimal feedback control input is obtained in every step by solving a robust optimization problem. The robust optimization problem is solved by scenario approach introduced in (Calariore and Campi, 2006). Simulation validations are carried out to evaluate the proposed control strategy.

2 PROPOSED METHOD

2.1 Background and Problem Description

In **Figure 1**, the vehicle passed a sharp curve with water-covered surface. The water-covered surface is the area with orange color. The single track model of vehicle dynamics can be described by the following equations:

$$\dot{y}_c = \dot{y}_c, \tag{1}$$

$$\ddot{y}_c = -\frac{2\left(C_f + C_r\right)}{mV}\dot{y}_c + \frac{2\left(C_f + C_r\right)}{m}\phi + \frac{2\left(l_r C_r - l_f C_f\right)}{mV}\dot{\phi} + \frac{2C_f}{m}\delta_f, \tag{2}$$

$$\dot{\phi} = \dot{\phi}, \tag{3}$$

$$\ddot{\phi} = \frac{2\left(l_r C_r - l_f C_f\right)}{IzV}\dot{y}_c + \frac{2\left(l_f C_f - l_r C_r\right)}{IzV}\phi - \frac{2\left(l_f^2 C_f + l_r^2 C_r\right)}{IzV}\dot{\phi}. \tag{4}$$

Here, y_c is the lateral distance. $\dot{\phi}$ is the yaw rate. m is the mass of the vehicle. δ_f is the steer angle. V is the vehicle speed.

In order to apply MPC, the vehicle lateral dynamics model is transformed to the lateral deviation from the reference model. The used linear model is as

$$\begin{bmatrix} \dot{y}_{cr} \\ \ddot{y}_{cr} \\ \dot{\phi}_{cr} \\ \ddot{\phi}_{cr} \end{bmatrix} = A \begin{bmatrix} y_{cr} \\ \dot{y}_{cr} \\ \phi_{cr} \\ \dot{\phi}_{cr} \end{bmatrix} + B\delta_f + C\frac{1}{R} \tag{5}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2\left(C_f + C_r\right)}{mV} & \frac{2\left(C_f + C_r\right)}{m} & \frac{2\left(l_r C_r - l_f C_f\right)}{mV} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2\left(l_r C_r - l_f C_f\right)}{I_z V} & \frac{2\left(l_f C_f - l_r C_r\right)}{I_z V} & -\frac{2\left(l_f^2 C_f + l_r^2 C_r\right)}{I_z V} \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 \\ \frac{2C_f}{m} \\ 0 \\ \frac{2l_f C_f}{m} \end{bmatrix} \tag{7}$$

$$C = \begin{bmatrix} 0 \\ -V^2 + \frac{2l_r C_r - 2l_f C_f}{m} \\ 0 \\ -\frac{2l_f^2 C_f + 2l_r^2 C_r}{Iz} \end{bmatrix}. \tag{8}$$

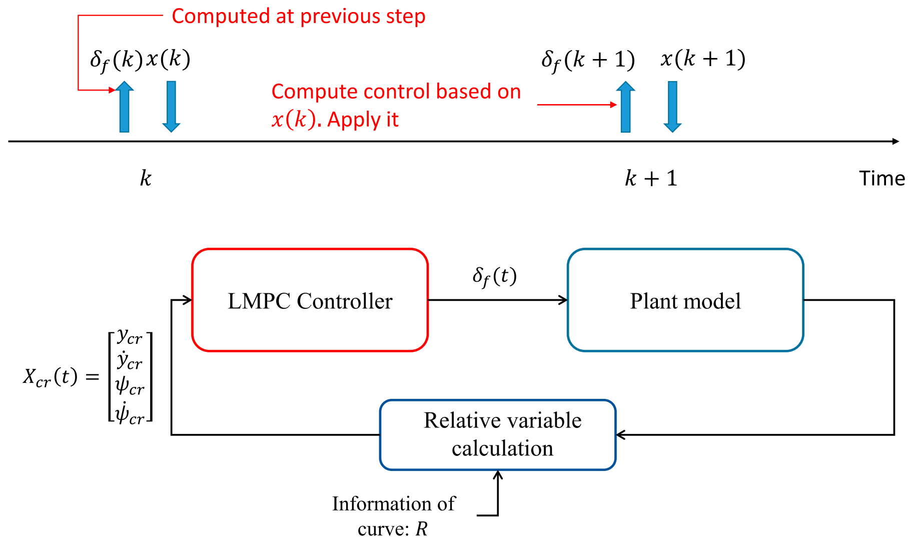

FIGURE 2 | Implementation of robust MPC.

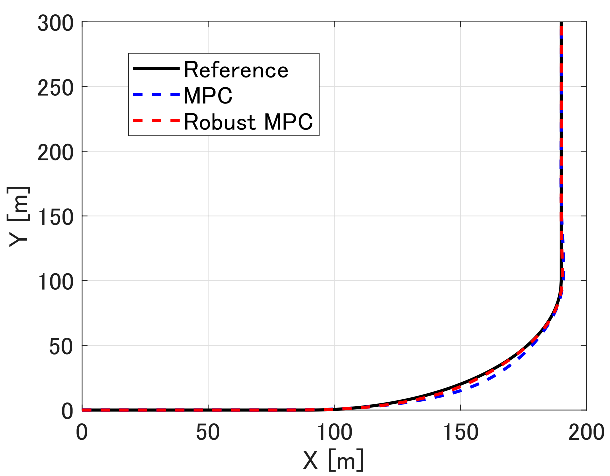

FIGURE 3 | Validation result.

Here, y_{cr} is the lateral deviation from the reference trajectory. $\dot{\phi}_{cr}$ is the yaw rate. R is the radius of the curve. m is the mass of the vehicle. δ_f is the steer angle. V is the vehicle speed.

Notice that C_f and C_r are both decided by the road friction coefficient. Since the road friction coefficient is uncertain, C_f and C_r are both uncertain variable as well.

Equation 5 is a continuous differential equation and can be transformed to a discrete state-space model by Euler method. Since at every time step, the state variable is decided by the input δ_f and the state variable in the previous step. The state variable at $k + 1$ can be expressed by the previous input sequence $\delta_f(0), \ldots, \delta_f(k)$ and the state variable at the initial step. Since the objective is to minimize the difference between the actual trajectory and the reference one, the cost function is a function of the input sequence and known state variable at initial step. To obtain the optimal input, a robust optimization problem should be solved. The problem can be formulated generally by

$$\begin{aligned} &\min_{u\in\mathcal{U}\subset\mathbf{R}^{n_u}} \ J(u) \\ &s.t. \quad h(u,\delta)\le 0,\ \ \delta\in\Delta\subset\mathbf{R}^{n_\delta}. \end{aligned} \tag{9}$$

Here, $u = [\delta_f(0), \ldots, \delta_f(K-1), E]^T$ if we consider K steps forward. δ is the uncertain variable. In our problem, it includes C_f and C_r. $J(u) = E$ and $h\ (u, \delta)$ is defined as

$$\sum_{k=1}^{K} y_{cr}(k) - E. \tag{10}$$

2.2 Scenario Approach

In scenario approach, independent samples $\delta^{(i)}$, $i = 1, \ldots, N$ is identically extracted from Δ randomly, a deterministic convex optimization problem can be formed as (Calariore, 2017; Campi et al., 2018; Campi and Garatti, 2018)

$$\begin{aligned} &\min_{u\in\mathcal{U}\subset\mathbf{R}^{n_u}} \ J(u) \\ &s.t. \quad h\left(u,\delta^{(i)}\right)\le 0,\ \ i=1,\ldots,N \end{aligned} \tag{11}$$

which is a standard finitely constrained optimization problem. The optimal solution $\hat{u}_N$ of the program **Eq. 11** is called as the scenario solution for program **Eq. 9** generally. Moreover, since the extractions $\delta^{(i)}$, $i = 1, \ldots, N$ is randomly chosen, the optimal solution $\hat{u}_N$ is random variable. If $\hat{u}_N$ is expected to satisfy

$$\Pr{}^N\left\{\left(\delta^{(1)},\ldots\delta^{(N)}\in\Delta^N : V(\hat{u}_N)\le\alpha\right\}\ge 1-\beta, \beta\in(0,1), \tag{12}$$

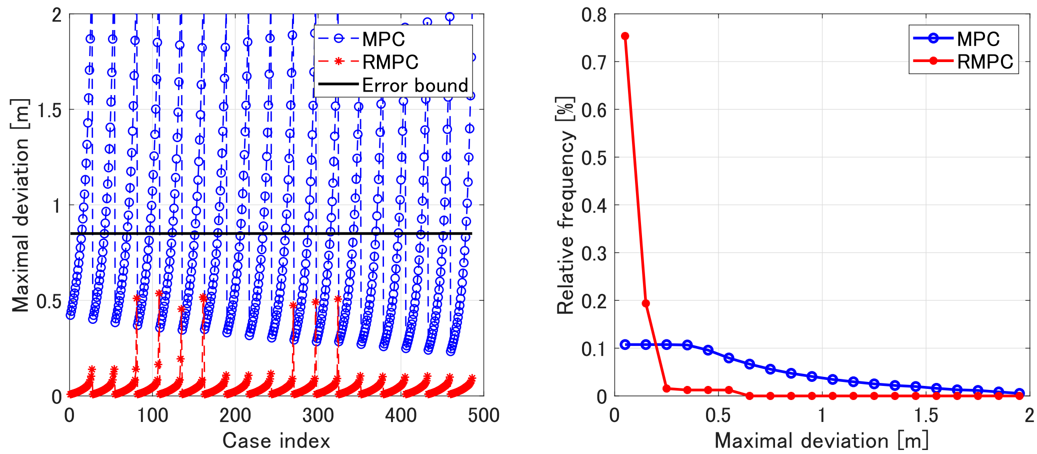

FIGURE 4 | Validation result.

then, N should have a lower limitation N_l

$$N \geq \frac{2}{\alpha}\ln\frac{1}{\beta} + 2n_u + \frac{2n_u}{\alpha}\ln\frac{2}{\alpha}. \tag{13}$$

Note that β is an important factor and choosing $\beta = 0$ makes $N_l = \infty$. Namely, if the number of chosen samples gets larger, the probability of satisfying the original probabilistic constraints approaches 1. Actually, when number of chosen samples becomes infinity, the samples cover the whole sample space. The feasible area determined by probabilistic constraints is only a subset of whole sample space. Then, it becomes a problem which requires total robustness. Therefore, the scenario approach conducts to a solution with total robustness which is more conservative than the probabilistic constraints require.

2.3 Implementation of Robust Model Predictive Control

The implementation of robust MPC is shown in **Figure 2**. At time step $k + 1$, it uses the first element of u calculated in time step k as the input. Namely, $\delta_f(k) = u\ (1)$. $x(k)$ denotes the state variable vector at time step k. Moreover, since the LMPC controller takes relative variable calculation as feedback, there will be a relative variable calculation. In the relative variable calculation, the relative variable is calculated based on the feedback state variable from plant model or real vehicle and the information of curve, for example, radius value R.

3 VALIDATION RESULTS AND CONCLUSTION

The validation is implemented by simulation. Since the real vehicle is not available, a plant model is established and used instead of the real vehicle. The plant model adopts the single track nonlinear model described by

$$\dot{v}_x - v_y\dot{\phi} = \frac{1}{m}\left(F_f^{x_T}\cos\delta_f + F_r^{x_T} - F_f^{y_T}\sin\delta_f\right), \tag{14}$$

$$\dot{v}_y + v_x\dot{\phi} = \frac{1}{m}\left(F_f^{y_T}\cos\delta_f + F_r^{y_T} + F_f^{x_T}\sin\delta_f\right), \tag{15}$$

$$I_{zz}\ddot{\phi} = l_f F_f^{y_T}\cos\delta_f - l_r F_r^{y_T} + l_f F_f^{x_T}\sin\delta_f, \tag{16}$$

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \mathcal{R}(\phi)\begin{bmatrix} v_x \\ v_y \end{bmatrix}. \tag{17}$$

The magic formula is used to model the friction forces which refers to (Yuan et al., 2019).

For the simulation conditions, the radius has six options: 100, 110, 120, 130, 140, and 150 m. For each R, three coefficients of friction for the wet road is randomly generated from (0.4,0.6). For each pair of a value of R and a value of coefficients of friction, the following longitudinal velocity values have be tested:

$$[0.4, 0.42, \ldots, 0.92] \times \sqrt{R\mu_{wet}g}. \tag{18}$$

Figure 3 shows one example of the validation results. The friction coefficient of dry road is $\mu_{dry} = 0.8$ which the one of wet road is $\mu_{wet} = 0.5$. The radius of the curve is 100 m. The middle part of the road is wet. The longitudinal velocity for passing the curve is $V = 65\ km/h$. If MPC is used by setting C_r and C_f according to $\mu_{dry} = 0.8$, the tracking error increases during the wet road. However, by considering $\mu \in [0.4, 0.9]$, the robust MPC keeps the tracking performance stable during the wet road.

Figure 4 shows a comprehensive statistical validation results of all cases. Obviously, the robust MPC succeeded to decrease the maximal deviation into the error bound. However, the normal MPC failed in most cases since the model has a very large bias compared to the real dynamics due to the uncertain friction coefficient.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.